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Joint determination of process mean, price differentiation, and production decisions with demand leakage: A multi-objective approach

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ABSTRACT

The selection of an optimal process mean is an important problem in production planning and quality control research. Most of the literature in this area has focused on the single objective problem of maximizing the profit for a fixed exogenous price. However, it is known that considering multiple objectives (such as gross income from sales, profit, and expected product uniformity) while allowing process mean, production and pricing to vary can significantly improve the profitability and performance of a firm. This article addresses this multi-objective problem while allowing the firm to sell two classes of products at differentiated prices based on their quality characteristics. These products are sold at differentiated prices depending upon their quality characteristics into primary and secondary markets at full and discounted prices respectively. Any nonconforming items are reworked at an additional cost. Due to customers heterogeneity, the firm experiences demand leakage between the two market segments. The proposed joint decision control for the firm includes the joint determination of full and discounted prices, the process mean selection, and the production quantities for each of the two product classes along with expected reworked items. A mathematical formulation of the objectives is first provided and then the multi-objective problem is transformed into a goal-programming problem. A solution procedure is developed using simulation-based optimization to identify Pareto-optimal solutions. Some important characteristics of the solution procedure are discussed and the performance of the approach is corroborated through detailed numerical experiments.

1. Introduction

The process mean determination has received a significant amount of attention from academic researchers and industry practitioners alike in revenue management. It is also commonly referred to as a process targeting problem and mainly addresses the optimal selection of the mean of a process. This problem has a rich history which dates back to early 1950s. Among early studies on process targeting is research reported in [1] which determined the optimal process mean for a canning process. Most studies in process mean selection have found several industrial applications. Alkhedher and Darwish

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[2] found that process targeting is often a difficult decision in production processes such as canning/filling, metal plating, and grinding; though they were able to find many applications in the glass, fiber, and steel industries. Often the specification limits are determined by a manufacturer on quality performance measures, these measures may include a quantitative assessment on weight, volume, concentration, thickness, length, etc. Depending upon the conformance to these specification limits, a production is classified (graded). The products that do not comply with these specification limits are sold at a discounted price, or reprocessed/scrapped. In order to avoid nonconforming items, manufacturers tend to set the process mean to a higher level in order to meet the specification limits and to mitigate nonconformity among the manufactured products [3].

There is a rich literature in the area of process mean selection, but we only discuss the main studies related to this paper. Bisgaard et al. [4] presented an economic analysis for the problem of selecting the most favorable quality distribution for an industrial process. The analysis takes into account the stochastic nature of the process, discusses the issue of overfill, and considers lognormal and Poisson distributions. Golhar [5] revisited the problem of optimum process mean in a canning process where the cans filled above a specification limit are sold at a fixed price while the under-filled cans are emptied and refilled at a reprocessing cost. Later, in [6], sampling inspection plans were introduced into the problem in place of 100% inspections. Optimal process mean with a rectifying inspection was addressed in [7] for the targeting problem in a filling process with a time dependent process mean. Shao et al. [8] proposed strategies for determining the optimal process mean of industrial processes when rejected goods can be held and sold to other customers in the same market at a later time. Bowling et al. [9] explored the targeting problem in the context of a multi-stage serial production process. Darwish [10] developed an integrated and hierarchical model of optimal process targeting in a single-vendor single-buyer supply chain. Based on a quality loss function, Chen and Kao [11] computed both the optimal process mean and screening limits. A reverse programming routine that identifies the relationship between the process mean and the settings within an experimental factor space was developed by [12] and [13]. Aforementioned literature clearly reveals that researchers have developed many process targeting models for optimizing firms' profitability in terms of product uniformity and cost reduction. This has been done primarily by approaching products' characteristics in various production processes with multiple (or single) stages or inputs of production. Recently, several research studies have identified that there is a practice in many industries to set the process mean higher to meet specification limits. Nonetheless, Roan et al. [3] have pointed out that this practice results in an additional "giveaway" cost by over-meeting the specification limits. Hariga and Al-Fawzan[14] developed a model for a multi-echlon production system with process targeting. They integrated the decisions of process targeting and production run for deterministic demand, then proposed simple solutions to find optimal control parameters. Shao et al. [8] optimized the target for a process with multiple markets with variable holding costs. Park et al. [15] addressed the problem of optimum common process mean and screening limits for a production process with multiple products. In an extension of the work of [14], [16] considered stochastic demand in an inventory model along with process mean selection.

A multi-objective optimization approach to the problem of process targeting is a very recent development in the field. Duffuaa and El-Gaaly [17] used the Taguchi loss function [18] to develop a multi-objective model with 100% inspection in which they considered three objectives: total (gross) income, profit, and product uniformity. Later they extended their proposed model in [19] by incorporating sampling plans. Duffuaa and El-Gaaly [20] also studied the effect of inspection error on the model they developed in [19]. Tamiz et al. [21] reported a survey regarding the use of goal programming for decision making. Aouni and Kettani [22] discussed the history of goal programming models and their promising future. A complete survey of the recent developments in goal programming research is out of the scope of this paper, but readers can find many recent applications of goal programming models to various real-life situations (see [23], [24], [25], [26], [27], [28]). As far as other multi-objective optimization techniques are concerned, there are numerous methods reported in [29]. Meta-heuristics are also commonly to achieve a multi-objective optimization of various problems. Deb [30] has provided a detailed study on evolutionary algorithms for multi-objective optimization. Jones et al. [31] reported a survey of literature on various existing meta-heuristics that are utilized for multi-objective optimization.

There is a growing interest in simultaneous decision making in process mean selection along with other production related controls. To this end, Jeang [32] proposed a model for an optimal joint determination of process means, process tolerances, and resetting cycle for production process under process shifting behavior. In a related study, Jeang [33] studied simultaneous determination of production lot size and process parameters under process deterioration and process breakdown. Recently, Jeang [34] also developed a robust optimization framework for simultaneous process mean, process tolerance, and product specification determination. Besides integration of process mean selection with other production system control parameters, an interesting avenue would be to re-visit jointly a firm's operations decisions and its marketing decisions. In the present context, the operation decisions would consist of process mean and production decisions. Marketing decisions would include market segmentation and pricing related decisions. This interfacing of the two business cores is expected to augment the profitability of a firm with more emphasis on pricing, as price is a paramount factor that significantly impacts the purchase behavior of customers [35]. Recently, several studies have integrated pricing decisions with the production (ordering) decisions at a firm's level. Yield Management, also referred to as Revenue Management (RM), is commonly described as the science of profitability. Price differentiation is among the most successful tools of RM to improve a firm's profitability [36,37]. The price differentiation tool mainly tries to sell the same (or slightly different) product at distinct prices. A firm does not always use this price differentiation on products which are distinct in their attributes, but it often offers almost the same or slightly different product at differentiated prices by using some distinct distribution channels, such as online versus in-store sale. This tool mainly takes the benefit of the customers' heterogeneity in their willingness-to-pay (WTP). But there is no established mechanism to estimate the true WTP for a customer, and it is not uncommon to observe each customer has a distinct WTP. Zhang and Bell [38] discussed the taxonomy of fencing in RM context, but indeed most fencing schemes cannot make a market segmentation perfect due to customers' heterogeneity. Thus, the market segmentation that is incurred due to price differentiation also observes demand leakage, which is also known in marketing literature as *cannibalization*. Although the evidence of increased profitability is well established under perfect market segmentation, in most practical situations demand leakages are inevitable. Zhang et al. [39] presented an integrated optimization framework for a firm's decision for fencing investment to mitigate demand leakage and at the same time to determine the prices and the order quantities for two market segments. In addition, Zhang et al. [39] also assumed predetermined market share (maximum perceived demand) in each market segment. However, with demand leakage from the full price to the discounted price the market segment is dependent on the price differential between the two market segments. Zhang and Bell [38] have presented numerous examples in which a firm may experience a similar situation. For instance, a very commonly observed example is an online versus in-store sales, where the products might be offered by a firm at discounted prices for online sales without the option of touch and feel. On the other hand, retail stores sales are set to a higher price and the customers can interact with the products or the sales staff. Airlines have been practicing fare pricing differentiation for over four decades using different strategies such as fare classes, sale conditions, advanced-purchase rules, cancellation penalties, No-show fees, etc. Regardless of the market segment strategy, once the market is segmented, a firm can adopt various strategies to mitigate cannibalization and to maintain the fences that differentiate the market segments. Zhang et al. [39] recently addressed the necessity of fencing improvement to mitigate the customers' spill-over from one market segment to another. Zhang and Bell [38] presented an overview of price fencing in RM's practice and its taxonomy. However, a realistic optimal control of price differentiation and capacity allocation for each market segment would still need to consider the demand leakage effects. In contrast to studies published in [38], [39], and [40] the research reported in [37] divides a single deterministic market demand into two market segments using a notion (also a variable) differentiation price, as a price widget. Philips [37] also considered the demand leakage effects which were referred to as *cannibalization* for price-dependent deterministic demand. Raza [41] extended the work in [37] by studying the impact of differentiation price on a firm's profitability when the firm experiences price-dependent stochastic demand. The study also re-visited the impact when the distribution of the price-dependent stochastic demand is unknown [42]. Later, [43] proposed an integrated framework for price differentiation and inventory decisions with demand leakage effect for price-dependent stochastic demand whose distribution may also be unknown. The fundamental difference between research reported in [37], [41,43] and that reported in [40], and [39] is the use of notion of a *differentiation price*. By using a differentiation price, the cumulative perceived maximum demand (the market share) is divided into two or more market segments, and indeed the differentiation price is a decision variable. Thus a firm is able to have an additional control in terms of optimizing its market share since the market segments are created from a single marketshare. In contrast, Zhang et al. [38], [39], and [40] mainly study the effect of demand leakage between the two pre-existing market segments with fixed market shares for each market segment.

This paper integrates a firm's decisions by interfacing both the operational decisions, which we refer to as production and process mean (target) decisions; and the firm's marketing decisions which we regard here as pricing and market segmentation with the consideration of the customers' cannibalization. We address this problem using a multi-objective model for a process targeting manufacturing firm. We consider three well-known objectives from literature [17,19,20]: sales-gross income to improve the market share of the firm; profit to augment the (net) profitability; and product uniformity in order to improve the quality, and thus the customers' satisfaction. The multi-objective mathematical model provides a joint decision framework for market segmentation using a differentiation price, process mean, along with production, pricing decisions for each market segment. We assumed the demand is price-dependent and deterministic, and the market segmentation incurred is imperfect; thus there is a demand leakage between the market segments. We transform the proposed multi-objective optimization model into an equivalent goal program model. We propose a simulation-based optimization procedure for repeatedly solving the goal program formulated in order to obtain a non-dominated (Pareto-optimal) solution. Finally, we utilize a methodology to calibrate the performance of the proposed simulation-based optimization procedure integration to goal program formulation in terms of identifying the true Pareto-optimal front.

The rest of the paper is organized as follows: In Section 2, we develop a multi-objective model. Section 3 reformulates the multi-objective model into a goal program model. Section 4 outlines the proposed solution procedure that uses a simulation-based optimization approach by utilizing Monte-Carlo simulations in order to compute the diverse viable solutions for the multi-objective nonlinear optimization problem. The suggested simulation-based optimization approach enables us to mitigate its inability to find diverse solutions for multi-objective nonlinear problems using the goal programming formulation. Later, we use standard approaches from literature (see [30] for details) to identify the non-dominated (Pareto-optimal) solutions. Section 5 reports the computational experiment results on an illustrative example from literature and characterizes the non-dominated (Pareto-optimal) solutions to the multi-objective problem. Finally, in Section 6, we summarize our results and outline suggestions for future research.

2. Model development

In this section, we propose a mathematical model of Revenue Management (RM) problem for a monopolist manufacturing firm selling two products depending upon a quality characteristic, x, in a single selling period. The product quality characteristic, x, is acquired using a manufacturing process which is normally distributed with a probability distribution function, $\phi(x)$, and a cumulative probability distribution function, $\Phi(x)$. The assumption of a normally distributed process has been widely used in process target literature (see [14], [44], [17,19,20] for details). The normally distributed process has a mean μ and a standard deviation, σ. The products manufactured are distinguished by the quality characteristic, x, based on which the products are classified into three classes. If the quality characteristic, x, is such that $x \geq l_1$, then the product is classified as a class (grade) 1 product and it is sold in a primary market at price p_1 per unit. Whereas, if the product quality characteristic follows $l_2 \leq x < l_1$, then the product is considered class (grade) 2, and this product is sold in a secondary market at a discounted price p_2 per unit. Lastly, the products with quality characteristic, $x < l_2$ are reworked at a rework cost, r per unit. As mentioned earlier in this paper, most recent research has studied the problem of process targeting for predetermined fixed prices, and with a single objective of profit maximization. This paper extends the existing literature in process (mean) targeting by incorporating pricing and market segmentation decisions with demand leakage effects. In addition to previously described modeling framework, we postulate some additional *assumptions* in the following which are taken into an account to formulate the mathematical model proposed here:

1. The firm operates in a monopoly and sells its products in a single selling period
2. The market segmentation is solely achieved using a price differentiation (price widget), and it costs a negligible investment
3. Market segmentation is considered imperfect and therefore the demand leakage is experienced only from full price market segment to the discounted price segment. This demand leakage is only dependent on the price difference among the two market segments
4. The price-dependent demand is uncorrelated in each market segment, and it is observed simultaneously
5. Products are classified based on a single quality characteristic
6. The inspection process assesses 100% of the products, and it is assumed to be error-free.
7. There are no machine breakdowns
8. There is no drift in the normally distributed process mean and standard deviation
9. Inspection incurs a firm a negligible cost.
10. The price-dependent demand is deterministic

In this paper we look at the interaction between process mean selection in production research and decisions made about pricing and inventory (production) quantity within a NewsVendor Problem (NVP) framework form of pricing. Assumptions 1-4 are standard in the NVP framework (see [45,46], and [43] for details). Assumptions 5-9 are frequently observed in process mean selection research [14,33]. Assumption 10 mainly facilitates the model tractability, and in this study we have also assumed an additive linear demand model, the additive linear modeling is very widely used in many related studies on joint pricing and production problems [39]. Earlier studies have shown deterministic approximation for stochastic demand works very well [42,43].

This study extends work reported in [37] in which a firm's single market demand is divided into two segments using a differentiation price. We also assume that the price-dependent demand a firm experiences is deterministic. Table 1 outlines a comprehensive list of symbols that are used in the development of our model. It is assumed that the total perceived maximum deterministic demand (market share) for the firm is, α, which has a demand price sensitivity, β, and follows a negative linear relationship. Linear demand curves are most commonly used in the literature because of their simplicity and their ability to catch important *managerial implications* (see [39,47,48], etc. for details). Thus, the total price-dependent deterministic demand to the firm would be, $[\alpha - \beta\, p]^+$, where $[x]^+ = \max\{x, 0\}$, $x \in \mathbb{R}$, $\mathbb{R}$ is a real set . Next, we introduce a differentiation price, $\upsilon \geq 0$, that is utilized to divide the firm's market share, α, into two segments. Given that the firm exercises a differentiation price (the price widget), υ, such that $p_1 \geq \upsilon$, and $\upsilon \geq p_2$, the maximum deterministic demand that the firm can perceive in the full price market segment 1, would be $\alpha - \beta\,\upsilon$. It is obvious that the remaining market share which is $\beta\,\upsilon$ would be allocated for the discounted price market segment 2. Recall that the price-dependent deterministic demand is modeled using a linear curve, and thus the price-dependent deterministic demand, $u_1(p_1) = [\alpha - \beta\, p_1]^+$ would be observed for the full price market segment. Likewise, the price-dependent deterministic demand for the discounted price market segment 2 would be $u_2(p_2, \upsilon) = [\beta\,\upsilon - \beta\, p_2]^+$. Since both $u_1(p_1)$ and $u_2(p_2, \upsilon)$ are linear demand curves they have the property of *increasing price elasticity* [35]. Clearly $0 \leq \upsilon \leq \frac{\alpha}{\beta}$, and therefore $p_1 \geq p_2$. The maximum price in the full price market segment would be, $\overline{p}_1 = \frac{\alpha}{\beta}$, and due to differentiation pricing constraint, the maximum price in the discounted price market segment would be, $\overline{p}_2 = \upsilon$. Next, it is assumed that the fences (segments) observed due to the proposed differentiation price, υ, are imperfect and thus there is a θ proportion of customers who have WTP of p_1 or more and though they are perceived to be associated with the product class 1, such customers would cannibalize (move) to the discounted price market segment that offers product class 2 at the discounted price, p_2. With no exception to this study, many pricing models in literature consider that the customers are homogenous. But actually much of the recent research has revealed that the customers are far more heterogenous in terms of their WTP (see [37,49] for details). Although in this paper we assume homogeneous demand, we balance this by capturing the customers' heterogeneity in their WTP, and at the same time we consider the effects of demand leakage. An obvious benefit is that an assumption of homogenous demand keeps the modeling tractable and also allows us to capture customers' heterogeneity to an acceptable extent. It is supposed that there is a θ proportion of customers' demand which belongs to the full price market segment and cannibalizes (moves) to

Table 1
Notations.

Parameters:	
σ	Process variability, i.e., standard deviation
l_1	Upper specification limit for class (grade) 1 products
l_2	Lower specification limit for class (grade) 2 products
a	Fixed production cost per unit item
b	Variable production cost per unit item
$c_i = c_i(\mu)$	Expected cost per unit for product class (grade) i, $\forall\ i = \{1, 2, 3\}$
$\phi(x)$	Probability distribution of quality characteristic x
$\Phi(x)$	Cumulative probability distribution of quality characteristic x
$\delta_1 = (l_1 - \mu)/\sigma$	Standard normal transformation of l_1
$\delta_2 = (l_2 - \mu)/\sigma$	Standard normal transformation of l_2
$\emptyset(\delta_i)$	Standard normal probability distribution of $\delta_i, \forall\ i = \{1, 2\}$
$\varphi(\delta_i)$	Standard normal cumulative probability distribution of $\delta_i, \forall\ i = \{1, 2\}$
θ	Proportion of demand leakage, $0 \le \theta \le 1$
α	Maximum perceived cumulative deterministic (riskless) demand
β	Price sensitivity of deterministic demand
ρ	Quality loss coefficient (constant) [18]
γ	Coefficient of demand leakage sensitivity per unit price difference, $\gamma \ge 0$
$u_1 = u_1(p_1)$	Price dependent deterministic demand in market segment 1
$u_2 = u_2(p_2, \upsilon)$	Price dependent deterministic demand in market segment 2
$y_1 = y_1(p_1, \theta)$	Adjusted price-dependent deterministic demand in market segment 1 with demand leakage
$y_2 = y_2(p_1, p_2, \upsilon, \theta)$	Adjusted price-dependent deterministic demand for market segment 2 with leakage
C	Total yield capacity
r	Estimated rework cost per nonconforming item
$\pi_1 = \pi_1(\mu, p_1, p_2, \upsilon)$	Total sales (gross income) to the firm
$\pi_2 = \pi_2(\mu, p_1, p_2, \upsilon)$	Total profit to the firm
$\pi_3 = \pi_3(\mu, p_1, p_2, \upsilon)$	Total uniformity to the firm
$E(\cdot)$	Expected value of a parameter
Decision variables	
p_i	Price for product class i, $\forall i = \{1, 2\}$
υ	Differentiation price, $\upsilon \ge 0$
μ	Target process mean
z	Number of nonconforming products reworked

the discounted price market segment. As a result, the adjusted demand for each product class would be:

$$y_1 = (1 - \theta)u_1, \tag{1}$$

$$y_2 = \theta\, u_1 + u_2. \tag{2}$$

Unlike in [37] and [43], our model assumes that the proportion of the demand leakage, θ, is not a constant. In this research, we have extended this conceptualization by assuming that θ is dependent upon the price difference between the prices of the two products. We have modeled θ by customizing a logit function in order to best represent this demand leakage [50]. In this paper, we have suggested the following expression for the proportion of demand leakage:

$$\theta = 1 - e^{-\gamma\,(p_1 - p_2)}. \tag{3}$$

The proposed model for θ in Eq. (3) precisely fits the required modeling needs which is also demonstrated in Fig. 1. For instance, when the coefficient of demand leakage sensitivity per unit price difference is zero, that is, $\gamma = 0$, the proportion of demand leakage will be zero, $\theta = 0$, which means a perfect market segmentation with no demand leakage. Similarly, when the price difference is zero, $p_1 - p_2 = 0$, then again we can find that, $\theta = 0$, which means the two market segments are indifferent and are not differentiated with the distinct prices, but rather the products are sold at a single price.

In the following, we develop the mathematical expressions for aforementioned three objectives:

2.1. Total sales income

We formulate the total sales, π_1, which is the gross income and it is dependent upon prices, p_i, $\forall i = \{1, 2\}$. We recall that the firm experiences the price-dependent deterministic demand, y_i, $\forall i = \{1, 2\}$ for each market segment. Each product unit sold in the market would yield in gross income to the firm, thus the total sales income, π_1, would simply be formulated in Eq. (4) as follows:

$$\pi_1 = \sum_{i=1}^{2} p_i y_i. \tag{4}$$

For simplicity in Eq. (4), we have ignored the expected sales incomes that may be yielded from reworked products. Potentially there could be two reasons to support this. First, the reworked items may not become available to be sold in that

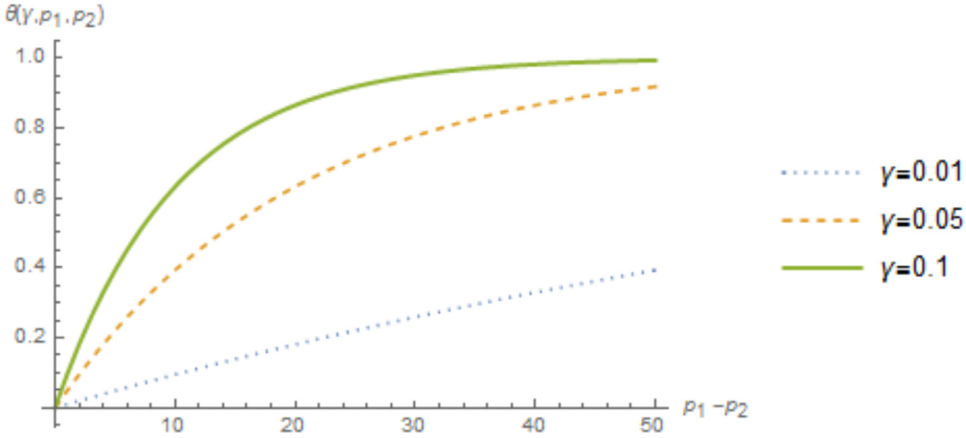

Fig. 1. θ relationship with price difference, $p_1 - p_2$.

selling period. Second, the sales of these items may actually need more cost investment than their price which mainly violates the fundamental cost based pricing principle, but this aspect will be addressed in developing the mathematical model for total expected profit.

2.2. Total expected profit

in this section, we develop the mathematical model for total (net) profit, π_2, which is a random variable and depends on decision variables, μ, $p_i, \forall i = \{1, 2\}$, and υ. The objective is to develop the total expected profit, $E(\pi_2)$, but we first formulate π_2 in Eq. (5) in the following:

$$\pi_2 = \begin{cases} (p_1 - c(x))y_1 - by_1(x - l_1) & x \geq l_1; \\ (p_2 - c(x))y_2, & l_2 \leq x < l_1; \\ E(\pi_2) - rz - zc(x), & x < l_2. \end{cases} \tag{5}$$

The total expected profit, $E(\pi_2)$ would be:

$$E(\pi_2) = \int_{l_1}^{\infty} ((p_1 - c(x))y_1 - by_1(x - l_1))\phi(x)\,dx + \int_{l_2}^{l_1} (p_2 - c(x))y_2\phi(x)\,dx$$
$$+ \int_{-\infty}^{l_2} (E(\pi_2) - rz - zc(x))\phi(x)\,dx. \tag{6}$$

Where in Eq. (6), the first term represents the expected profit obtained from selling class 1 products, the second term is the expected profit from selling products class 2 and the last term is the expected profit (or expected expense incurred) from the rework products.

Next, we simplify the expected profit function $E(\pi_2)$ established in Eq. (6). In Eq. (7), we use the following notations: $1 - \varphi(\delta_1) = \int_{l_1}^{\infty}\phi(x)dx$, $\varphi(\delta_1) - \varphi(\delta_2) = \int_{l_2}^{l_1}\phi(x)dx$, $\varphi(\delta_2) = \int_{-\infty}^{l_2}\phi(x)dx$, $C_1 = \int_{l_1}^{\infty}c(x)dx$, $C_2 = \int_{l_2}^{l_1}c(x)dx$, $C_3 = \int_{-\infty}^{l_2}c(x)dx$, and $g = \int_{l_1}^{\infty}(x-l_1)\phi(x)dx$, and we simplify the profit function in the following:

$$E(\pi_2) = p_1y_1\int_{l_1}^{\infty}\phi(x)dx - y_1\int_{l_1}^{\infty}c(x)\phi(x)dx - by_1\int_{l_1}^{\infty}(x - l_1)\phi(x)dx$$
$$+ p_2y_2\int_{l_2}^{l_1}\phi(x)dx - y_2\int_{l_2}^{l_1}c(x)\phi(x)dx$$
$$+ (E(\pi_2) - rz)\int_{-\infty}^{l_2}\phi(x)dx - z\int_{-\infty}^{l_2}c(x)\phi(x)dx. \tag{7}$$

We consider that the direct unit production cost for the finished product is a linear function of the quality characteristic x. Many previous studies have followed this framework (e.g., [14], [8], and [4]). Thus, the production cost per unit at quality characteristic x is $c(x) = a + bx$, where a is the fixed production cost and b is the cost of obtaining a specific quality characteristic for one unit of finished product. Therefore, the expected unit production cost of class 1 products is given by c_1 as follows:

$$\begin{aligned} c_1 &= \int_{l_1}^{\infty} c(x)\phi(x)dx \\ &= \int_{l_1}^{\infty} (a+bx)\phi(x)\,dx \\ &= (a+b\mu)(1-\varphi(\delta_1)) + \sigma\, b\emptyset(\delta_1), \end{aligned} \tag{8}$$

where $\emptyset(\delta_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\delta_1^2}$.

Next, in order to determine the giveaway cost, we first determine the expected additional quality characteristic, g, that is given away when a process mean is set to a mean μ. Thus, following recent related studies on process mean (target) determination in [17,19], we can write:

$$\begin{aligned} g &= \int_{l_1}^{\infty} (x-l_1)\,\phi(x)dx \\ &= E(x|x \geq l_1)(1-\varphi(\delta_1)) \\ &= \left(\mu + \sigma \frac{\emptyset(\delta_1)}{1-\varphi(\delta_1)}\right)(1-\varphi(\delta_1)) \\ &= \mu(1-\varphi(\delta_1)) + \sigma\emptyset(\delta_1). \end{aligned} \tag{9}$$

Notice here that Eq. (9) gives the expected quality characteristic of giveaway. Since b is the cost per unit [14], the expected giveaway cost per unit would be b/g.

Also the expected production cost for a grade 2 item, $c_2 = \int_{l_2}^{l_1} c(x)\,\phi(x)dx$ is determined similarly as follows:

$$\begin{aligned} c_2 &= \int_{l_2}^{l_1} c(x)\phi(x)\,dx \\ &= \int_{l_2}^{l_1} (a+bx)\phi(x)\,dx \\ &= (a+b\mu)(\varphi(\delta_1)-\varphi(\delta_2)) + \sigma b(\emptyset(\delta_2)-\emptyset(\delta_1)), \end{aligned} \tag{10}$$

where in Eq. (10), $\varphi(\delta_1)-\varphi(\delta_2) = \int_{l_2}^{l_1}\phi(x)dx$, $\emptyset(\delta_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\delta_1^2}$, $\emptyset(\delta_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\delta_2^2}$, $\delta_1 = \frac{l_1-\mu}{\sigma}$ and $\delta_2 = \frac{l_2-\mu}{\sigma}$.

Lastly, the expected production of a non-conforming item, $c_3 = \int_{-\infty}^{l_2} c(x)dx$ is determined in the following:

$$\begin{aligned} c_3 &= \int_{-\infty}^{l_2} (a+bx)\phi(x)\,dx \\ &= (a+b\mu)\varphi(\delta_2) - \sigma\, b\emptyset(\delta_2). \end{aligned} \tag{11}$$

We must notice here that the firm has capacity, C. Thus if the process mean is set to μ, the total expected number of nonconforming items would be, $z = C\int_{-\infty}^{l_2}\phi(x)dx = C\varphi(\delta_2)$.

Using the costs' expressions derived earlier, the simplified expected profit function, $E(\pi_2)$ would be:

$$\begin{aligned} E(\pi_2) = {} & p_1 y_1(1-\varphi(\delta_1)) - c_1 y_1 - bgy_1 \\ & + p_2 y_2(\varphi(\delta_1)-\varphi(\delta_2)) - c_2 y_2 \\ & + (E(\pi_2) - r\varphi(\delta_2)C)\varphi(\delta_2) - c_3\varphi(\delta_2)C. \end{aligned} \tag{12}$$

Finally, after rearranging the terms in the expected profit function, we get:

$$E(\pi_2) = \frac{1}{1-\varphi(\delta_2)}\{p_1 y_1(1-\varphi(\delta_1)) - c_1 y_1 - bgy_1 + p_2 y_2(\varphi(\delta_1)-\varphi(\delta_2)) - c_2 y_2 - (r\varphi(\delta_2)+c_3)\varphi(\delta_2)C\}. \tag{13}$$

2.3. Total expected product uniformity

Earlier [17,19] and [20] evaluated product uniformity using the loss function approach with a larger-the-better criterion [18]. In this paper, we have also followed the same approach. The notion of total expected product uniformity measures the losses a firm experiences due to variation between the product quality and the specification limits for the product. Total expected product uniformity is a measure which can also function as a proxy measure for customer dissatisfaction and is therefore a potential measure for a customer's satisfaction index [17,19,20]. Ideally with a larger-the-better criterion, the total expected product uniformity measure would be zero. Naturally the process (target) mean being a decision variable could be shifted or adjusted near or at the specification limits. Although the process variance is known and constant, it does not guarantee product uniformity but rather requires optimization [34]. The product uniformity maximization objective is equivalent to the minimization of the expected total deviation of the products from their specification limits which is measured in this study using the Taguchi quadratic loss function [18]. There can be a number of assignable causes for this variability, and these variations may result from the natural variability of a production process, raw material variation, environmental factors, tool/equipment conditions, etc. Use of the larger-the-better criterion enables us to formulate the product uniformity function, π_3, which is a random variable and depends on μ, $p_i, \forall i = \{1, 2\}$, and υ as follows:

$$\pi_3 = \begin{cases} \left(\frac{\rho}{x^2} + b\,(x - l_1)\right) y_1, & x \geq l_1; \\ \left(\frac{\rho}{x^2} + (p_1 - p_2)\right) y_2, & l_2 \leq x < l_1; \\ \left(\frac{\rho}{x^2} + p_1 + r\right) z, & x < l_2. \end{cases} \tag{14}$$

Likewise the previous objective function, the total expected uniformity , $E(\pi_3)$, would be:

$$\begin{aligned} E(\pi_3) = & \int_{l_1}^{\infty} \left(\frac{\rho}{x^2} + b\,(x - l_1)\right) y_1\,\phi(x) d\,x \\ & + \int_{l_2}^{l_1} \left(\frac{\rho}{x^2} + (p_1 - p_2)\right) y_2\,\phi(x) d\,x \\ & + \int_{-\infty}^{l_2} \left(\frac{\rho}{x^2} + (p_1 + r)\right) z\,\phi(x) d\,x. \end{aligned} \tag{15}$$

After simplification, we obtain:

$$\begin{aligned} E(\pi_3) = & \rho \left(y_1 \int_{l_1}^{\infty} \frac{\phi(x)}{x^2} + y_2 \int_{l_2}^{l_1} \frac{\phi(x)}{x^2} + C\,\varphi(\delta_2) \int_{-\infty}^{l_2} \frac{\phi(x)}{x^2} \right) \\ & + b g y_1 + (p_1 - p_2)\, y_2\, (\varphi(\delta_1) - \varphi(\delta_2)) + (p_1 + r)\, \varphi^2(\delta_2)\, C. \end{aligned} \tag{16}$$

Next, we construct a Multi-Objective Nonlinear Optimization Problem (MONLP) using the aforementioned three objectives which are subject to nonlinear constraints. For convenience in this presentation, we suppose, $f_1 = \pi_1$ from Eq. (4) , $f_2 = E(\pi_2)$ from Eq. (13), and $f_3 = E(\pi_3)$ from Eq. (16).

Thus, the firm's optimization problem for the price-dependent deterministic demand in both market segments is formulated as follows:

$$\text{MONLP}: \quad \max\{f_1, f_2, -f_3\}, \tag{17}$$

subject to:

$$p_1 - \upsilon \geq 0, \tag{18}$$

$$\upsilon - p_2 \geq 0, \tag{19}$$

$$(1 - \varphi(\delta_1))\, C - q_1 \geq 0, \tag{20}$$

$$(\varphi(\delta_1) - \varphi(\delta_2))\, C - q_2 \geq 0, \tag{21}$$

$$(1 - \varphi(\delta_2))\, C - \sum_{i=1}^{2} q_i \geq 0, \tag{22}$$

$$\mu - l_2 \geq 0. \tag{23}$$

In Eq. (17) three objectives are listed, where f_1 represents the total sales (gross income) which mainly intends to improve the market share of the firm. In addition to this, f_2 outlines the total expected profit (net income) to the firm. Lastly, f_3 is the total expected uniformity. All of these objective functions have a multi-variate decision control. These variables include μ, p_1, p_2, and υ. In problem, MNOP, Eqs. (18) and (19) are the price differentiation constraints. These constraints imply that the price, p_1, for the product class 1, must not fall below the price differentiation, υ. We remark here that υ is utilized as a price widget that is used to divide the single market into two segments. Also the price, p_2, for the product class 2, must not exceed the price differentiation, υ. The constraints in Eqs. (20)–(22) state the capacity limitations. Eq. (20) implies that the production quantity, q_1, sold cannot exceed the expected number of class 1 items, that is $(1 - \varphi(\delta_1))\,C$. Likewise, the constraint in Eq. (21) states a similar limitation for class 2 products which is that the production quantity q_2 cannot exceed the allocated capacity, $(\varphi(\delta_1) - \varphi(\delta_2))\,C$. Similarly, the constraint in Eq. (22) limits the total production of the manufacturing firm to its capacity C. As discussed before, the products with quality characteristic $x < l_2$ are regarded as nonconforming products, therefore it is obvious that the firm must maintain a process mean, $\mu \geq l_2$, in order to ensure the conformity of the products and the market demands. Indeed, in most practical situations the process mean must lie such that $l_2 \leq \mu \leq \overline{\mu}$, where $\overline{\mu}$ is the maximum permissible process mean [17,19].

3. A goal programming based approach

Goal programming (GP) has a rich history in managing multi-objective optimization problems [29]. When we use a GP approach for multi-objective optimization; we first attain predefined target values (also referred to as ideal goals) for each of the single objectives by optimizing it irrespective of the other objectives. Using these targets, we can normalize each of the objectives and later preference-based weights are assigned to each of these normalized goals. Thus the multi-objective problem is transformed into a single objective by using a linear combination of these weighted normalized goals. Solving GP for a combination of weights and goals results a single solution to the multi-objective problem. However, this solution is not guaranteed to be a Pareto-optimal solution. But naturally, since multi-objective optimization problems have several conflicting objectives, it is very important for a solution methodology to be able to find several equally viable Pareto-optimal solutions for a multi-objective problem. To address this shortcoming of the GP method, simulation-based optimization procedures are integrated with the GP. Gosavi [51] provided a discussion on simulation-based optimization approaches, and [52] have used a simulation-based optimization approach which utilizes Monte-Carlo simulations for solving the GP repeatedly with randomly generated weights for ideal goals, as well as for randomly generated goals. Each run gives a solution, once a set of solutions are obtained in a simulation experiment, one can identify non-dominated (Pareto-optimal) solutions among these solutions using the techniques presented in [30]. Once the Pareto-optimal solutions are computed decision makers can select among these Pareto-optimal solutions based on some other higher-level information and preferences. The motivation of incorporating simulation-based optimization with the GP is to overcome the limitation of standard GP, and develop an efficient multi-objective optimization solution procedure able to perform the following three main tasks [30]:

1. The computed non-dominated (Pareto-optimal) solutions should be close enough to the true Pareto front. Ideally, the non-dominated solutions should be a subset of the Pareto-optimal set.
2. The non-dominated solutions obtained should have a uniform spread over of the Pareto front in order to provide the decision-maker a true insight of trade-offs among available non-dominant solutions.
3. The non-dominated solutions should be able to capture the whole spectrum of the Pareto front. This requires investigating non-dominated solutions at the extreme ends of the objective functions space.

Besides the GP approach, evolutionary algorithms (see [30] for details) are widely used to obtain efficient solutions to multi-objective optimization problems. Although in the literature we find many successful implementations of evolutionary algorithms and other meta-heuristics such as Tabu Search, Simulated Annealing, Harmony Search (see [53], [54], [31], [55] for details), none of these methods guarantee a true Pareto-optimal frontier. These methods are highly dependent on their respective algorithmic parameters (for instance in the case of Genetic algorithm, population size, cross-over, stopping criterion, etc.) to find a near true Pareto-optimal frontier. Fine tuning these algorithmic parameters is therefore essential. In contrast to these evolutionary algorithms and meta-heuristics, using a goal programming based approach for multi-objective optimization does not need any parametric fine tuning. But, like evolutionary algorithms, goal programming based approaches do not guarantee finding a true Pareto-optimal frontier for most of the multi-objective optimization problems.

To solve the multi-objective model, MONLP, we utilized a GP approach. Similar to [52], we use Monte-Carlo simulations to repeatedly solve the GP as discussed previously. This simulation-based optimization approach allows us to handle a major drawback found in standard goal programming methods; namely that the method only offers a single solution which is highly dependent on the decision makers choice of the goals and the weights of the deviation from the predefined goals. In the following, we develop a GP model using the three objectives listed in Eq. (17). The proposed GP formulation uses a commonly employed preference-based approach in which pre-defined weights are utilized to describe a decision maker's preferences for normalized goals. This reduces the multi-objective problem into a single objective problem which is a weighted average of normalized goals. All of these solution methodologies find a single Pareto-optimal (possibly a non-dominant) solution to the problem. However, in most real life applications, there is a need to obtain more than just a single Pareto-optimal solution to the problem. Based on their preferences, the decision makers would be able to select a Pareto-optimal solution to the problem. In the following, we transform MONLP into a GP model. Both the goals and the objective functions are normalized in order to avoid the scalability and dimensionality problem. We achieve normalization for each of the three objectives as follows:

$$f_k^{\text{normalized}} = \frac{f_k - f_k^{\min}}{f_k^{\max} - f_k^{\min}}, \ \forall k = \{1, 2, 3\}, \tag{24}$$

$$goal_k^{\text{normalized}} = \frac{goal_k - f_k^{\max}}{f_k^{\max} - f_k^{\min}}, \ \forall k = \{1, 2, 3\}. \tag{25}$$

We recall here that the GP model has an additional constraint expressed in Eq. (27). This constraint mainly minimizes the deviation of each of the three objectives, $f_k, \forall k = \{1, 2, 3\}$. Thus,

$$\text{GP}: \quad \min w_1 d_1^- + w_2 d_2^- + w_3 d_3^+, \tag{26}$$

subject to:

$$f_k^{\text{normalized}} + d_k^- - d_k^+ = goal_k^{\text{normalized}}, \ \forall k = \{1, 2, 3\}, \tag{27}$$

$$p_1 - \upsilon \geq 0, \tag{28}$$

$$\upsilon - p_2 \geq 0, \tag{29}$$

$$(1 - \varphi(\delta_1))C - y_1 \geq 0, \tag{30}$$

$$(\varphi(\delta_1) - \varphi(\delta_2))C - y_2 \geq 0, \tag{31}$$

$$(1 - \varphi(\delta_2))C - \sum_{i=1}^{2} y_i \geq 0, \tag{32}$$

$$\mu - l_2 \geq 0. \tag{33}$$

4. Solution procedure

In this section, we propose a simulation-based optimization procedure to solve the GP [51]. The procedure resembles an hybrid algorithm suggested in [52] to solve a multi-objective mixed integer nonlinear program for a maintenance scheduling problem. The following procedure is utilized to solve the GP model and to obtain the non-dominated (Pareto-optimal) solutions to MONLP.

- **Step 1**: Compute the single objective optimal solution. The minimum, $f_k^{\min}, \forall k = \{1, 2, 3\}$, and the maximum, $f_k^{\max}, \forall k = \{1, 2, 3\}$. $f_k^{\min}, \forall k = \{1, 2, 3\}$ and $f_k^{\max}, \forall k = \{1, 2, 3\}$ are determine by constrained optimization of $f_k, \forall k = \{1, 2, 3\}$ subject to the constraints in Eqs. (28)–(33).
- **Step 2** Using $f_k^{\min}, \forall k = \{1, 2, 3\}$, and $f_k^{\max}, \forall k = \{1, 2, 3\}$, determine three types of goal, these goals are ideal, uniformly, and normally distributed goals
  1. For ideal goal, $goal_k = f_k^{\max} \, \forall k = \{1, 2\}$, and $goal_3 = f_3^{\min}$
  2. For random uniform goal, $goal_k^{\text{uniform}} \sim U[f_k^{\min}, f_k^{\max}] \, \forall k = \{1, 2, 3\}$
  3. For random normal goal, $goal_k^{\text{normal}} \sim N[f_k^{\min}, f_k^{\max}] \, \forall k = \{1, 2, 3\}$.
- **Step 3** Normalize the goals; for ideal goals, $goal_k^{\text{normalized}} = 1 \, \forall k = \{1, 2\}$, and $goal_3^{\text{normalized}} = 0 \, \forall k = \{1, 2\}$. For random uniform and normal goals use Eq. (25).
- **Step 4** Generate the random weights, $w_k' = rand(0, 1), \ \forall k = \{1, 2, 3\}$. The random weights are normalized, $w_k = \frac{w_k'}{\sum_{k=1}^{3} w_k'}$.
- **Step 5** Solve GP model for each of the three (ideal, random uniform, random normal) goals.
- **Step 6** Repeat **Step 2** to **Step 5** for maximum number of simulation runs (i.e., 300 simulations for each goal in this study).
- **Step 7** Determine the Pareto-optimal solutions for three types of goals using procedure suggested in [30].

5. Computational results

We report the results of the computational experiments with the proposed solution procedure described in Section 4. The objective of this computational study is to identify the true Pareto-optimal frontier for this problem. The procedure enables us to find diverse non-dominated (Pareto-optimal) solutions and characterize a firm's decision pattern for the proposed integrated control framework on market segmentation using a differentiation price, pricing, production quantities, and process mean decisions in a multi-objective optimization context. As this study is merging the production systems from operations research and the pricing from the economics and marketing, we have selected problem related parameters for the following example from recently published research articles in the fields of pricing and process targeting. In Table 2, we present the data selected for the parameters for the GP model as an illustrative example from literature in process targeting and the price research from RM. The parameters that are related to process targeting literature are taken from [17,19,20]. Whereas the pricing related parameters are taken from relevant recently published studies in RM from [37], and [43]. Regarding the dimensions (units) of the parameters mentioned in Table 2, we only describe units for a parameter in that table, if it is also reported in the literature. Often studies in the economics of pricing within a NVP framework do not imply any

Table 2
Parameters selection for numerical experimentation.

Parameters	Value(s)	Source
C	300	Best guess of the authors
l_1	20 μm	Best guess of the authors perceived from process targeting literature [17,19]
l_2	15 μm	
a	$ 2	Best guess of the authors perceived from process targeting literature [14]
b	$ 0.5	
α	500	Best guess of the authors perceived from pricing literature [37,39]
β	7	
r	$ $\frac{1}{2} \times c_1$	Best guess of the authors
σ	5 μm	
$\overline{\mu}$	35 μm	Best guess of the authors
γ	0.01	
ρ	$r \times l_1^2$	See [17,19,20]

Table 3
Pareto-optimal solutions for ideal goals.

w_1	w_2	w_3	Sales goal	Profit goal	Uniformity goal	Sales	Profit	Uniformity	μ^*	p_1^*	p_2^*	υ^*	y_1^*	y_2^*
0.102	0.508	0.390	10727	3856.6	0	10533.902	3183.425	4.54E+13	21.868	43.804	34.664	43.804	176.480	80.872
0.479	0.295	0.226	10727	3856.6	0	9979.953	3533.906	3.79E+12	23.951	44.033	38.006	44.033	180.556	53.402
0.228	0.599	0.172	10727	3856.6	0	10415.638	3284.254	2.55E+13	22.384	43.806	35.441	43.806	177.845	74.067
0.257	0.301	0.442	10727	3856.6	0	10200.804	3424.768	9.70E+12	23.203	43.875	36.750	43.875	179.608	63.141
0.050	0.425	0.525	10727	3856.6	0	10601.384	3114.181	6.45E+13	21.541	43.815	34.195	43.815	175.563	85.071
0.446	0.388	0.167	10727	3856.6	0	10103.100	3476.517	6.38E+12	23.542	43.935	37.315	43.935	180.130	58.668
0.298	0.248	0.454	10727	3856.6	0	10087.120	3484.423	5.96E+12	23.596	43.946	37.405	43.946	180.201	57.962
0.217	0.400	0.384	10727	3856.6	0	10330.552	3345.186	1.72E+13	22.723	43.823	35.973	43.823	178.647	69.542
0.003	0.229	0.768	10727	3856.6	0	10708.515	2974.701	1.28E+14	20.878	44.287	33.709	44.287	170.916	93.125
0.122	0.308	0.570	10727	3856.6	0	10407.434	3290.494	2.45E+13	22.418	43.807	35.493	43.807	177.929	73.619

dimensions for profit functions as well as the corresponding optimal pricing decisions (see [36,37] for details). Because of this, units for the parameters related to pricing are not reported. Similarly in the literature, the three objectives: sales, profit, and uniformity are often measured without any specific units. However, if we can quantify sales and profit objectives using, for example, a monetary unit it can be measured in $. The uniformity objective is calibrated in quality loss units (see [18]) which will be zero when ideal uniformity is achieved.

Next, the proposed solution procedure discussed earlier in Section 4 is implemented in MATLAB Release 2013a. A built-in goal programming based function, "fgoalattain" was customized and simulation runs were conducted on an Intel Core i7-3520M processor 2.9 GHz and 8 GB RAM, with Windows 7, 64-bit operating system. In order to compute the single objective goals, $f_k^{min}, \forall k = \{1, 2, 3\}$, and also $f_k^{max}, \forall k = \{1, 2, 3\}$, we have also utilized a built-in MATLAB Release 2013a function "Global Search" with its default settings to solve the nonlinear constrained single objective optimization problem for each of the three objectives. While carrying out the single objective optimization for the problem in MATLAB, we obtained, $f_1^{max} = 10727$, $f_2^{max} = 3856.6$, and $f_3^{max} = 8.0743 \times 10^{15}$. Also, $f_1^{min} \approx 0$, $f_2^{min} = -3654.8$, and $f_3^{min} \approx 0$. Next, the simulation-based optimization procedure is repeated for over 300 runs for each of the three goals. As mentioned in the procedure in Section 4 , for each replication we have also randomly generated weights, w_k, $\forall k = \{1, 2, 3\}$ [52]. The purpose of random generation of goals and weights for the GP is to diversify the (non-dominated) solutions, and therefore help to improve the ability of GP formulation to obtain a true Pareto-optimal frontier. We also note here that in [52], a total of 150 (50 for each goal type) replications were performed. In this study, we have conducted over a total of 900 simulation runs with 300 replications for each goal type which could potentially improve the performance of the proposed solution procedure.

5.1. Distribution and trade-off analysis of pareto-optimal solutions

Among 300 simulation runs, we have found over 292 Pareto-optimal solutions, but in Table 3, we only present 10 non-dominant solutions arbitrarily as a sample from these Pareto-optimal solutions along with details on the corresponding optimal control parameters. But, Fig. 2 presents the 3D-plot of 292 non-dominated (Pareto-optimal) solutions for the three objectives. We can find here that perhaps the true Pareto-optimal front is closely obtained and therefore the solutions are quite diverse. We may observe here that a firm may choose to increase its market share by offering primarily one product only. This will improve its performance in terms of quality uniformity and could yield a reasonable profit using differentiated prices as a tool. Alternatively the firm could opt for diversity and improve its profitability while slightly compromising its market share from a particular product and some loss in the product uniformity objective. In Fig. 3 the trade-off curves of the objective functions and distributions of the obtained solutions are also graphed using a plot-matrix chart. It can

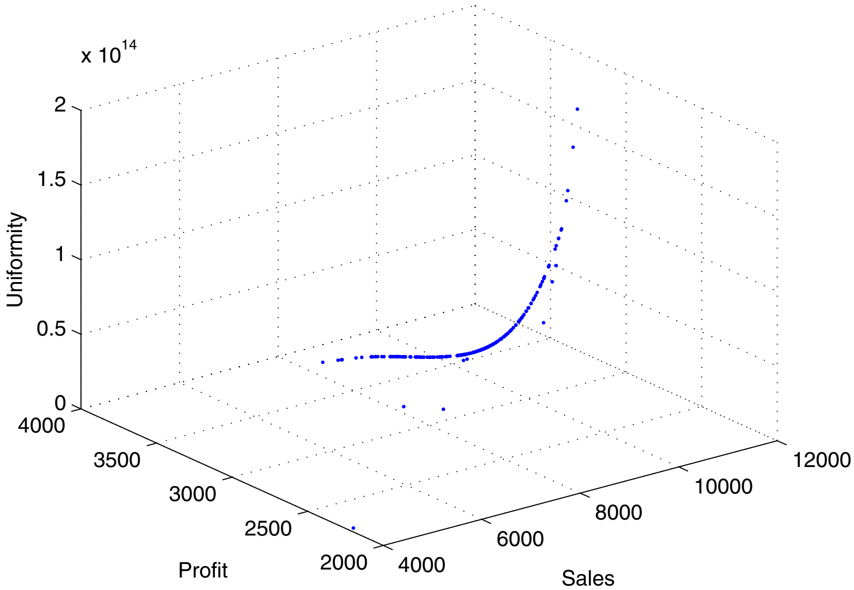

Fig. 2. Pareto-optimal plot ideal goal.

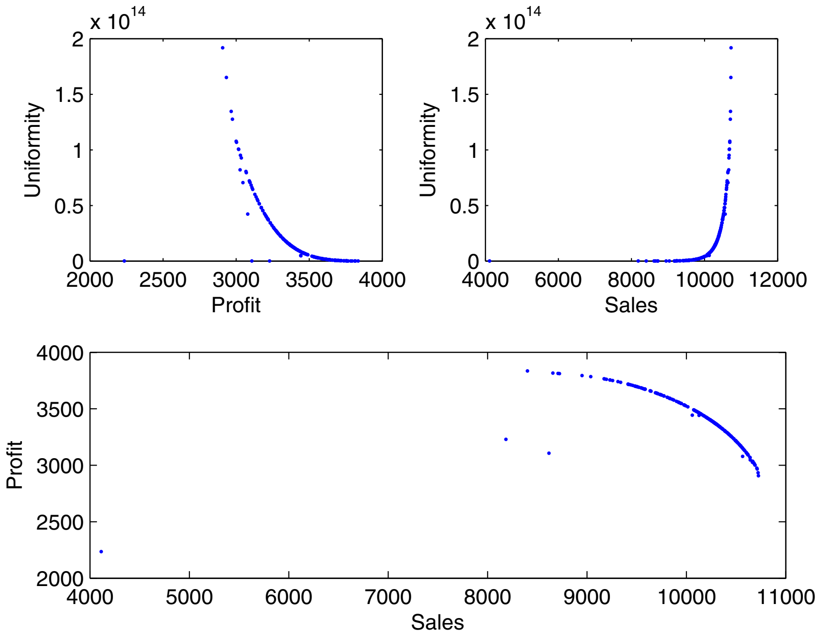

Fig. 3. Trade-off plot ideal goal.

be observed that the non-dominated solutions are uniformly distributed over wide ranges of the objective functions, and the space is able to capture the existing trade-off among the three objectives: sale, profit, and uniformity. The first task in solving a multi-objective optimization problem is to identify non-dominated solutions as close as possible to the true Pareto-optimal front [30,52]. Other necessary features to be realized are that the obtained non-dominated solutions must be uniformly and widely distributed in the Pareto-optimal region reflecting the existing trade-off among different objective functions. This task is achieved by generating uniformly and normally distributed goals. The weights for the deviations of these goals are also generated randomly using uniform distribution.

Uniformly distributed goals are randomly generated such that, $goal_k \in \mathrm{U}[f_k^{\min}, f_k^{\max}],\ \forall k = \{1, 2, 3\}$. In the numerical simulation, we found 118 Pareto-optimal solutions out of 300 simulation runs for the uniformly generated goals. Similar to ideal goals, we have only reported 10 arbitrary non-dominated solutions along with their details in Table 4. Like with ideal goals, the 3-D plot of all the Pareto-optimal solutions obtained for uniformly generated goals are presented in Fig. 4. Due

Table 4
Pareto-optimal solutions for uniform goals.

w_1	w_2	w_3	Sales goal	Profit goal	Uniformity goal	Sales	Profit	Uniformity	μ^*	p_1^*	p_2^*	υ^*	y_1^*	y_2^*
0.683	0.191	0.127	2256.247	1255.785	4.12E+15	10725.945	2910.935	1.88E+14	20.479	45.629	34.425	45.629	161.453	97.575
0.396	0.430	0.174	4628.889	−599.206	6.79E+15	10030.671	3511.223	4.69E+12	23.785	43.989	37.724	43.989	180.412	55.523
0.135	0.572	0.293	9998.413	2790.007	3.28E+15	10209.894	3419.630	1.01E+13	23.170	43.871	36.697	43.871	179.551	63.571
0.359	0.369	0.272	3181.175	−1448.897	6.32E+15	10049.758	3502.351	5.09E+12	23.721	43.974	37.616	43.974	180.346	56.338
0.547	0.397	0.056	4238.688	−171.303	6.12E+15	10631.338	3079.762	7.60E+13	21.386	43.823	33.979	43.823	175.125	87.020
0.215	0.391	0.394	8416.604	1195.010	7.01E+15	10178.113	3437.342	8.79E+12	23.283	43.888	36.883	43.888	179.744	62.077
0.560	0.204	0.235	4942.748	1810.601	7.07E+15	10487.214	3225.898	3.59E+13	22.079	43.801	34.977	43.801	177.055	78.107
0.112	0.340	0.548	9353.066	1105.521	7.39E+15	10387.679	3305.181	2.24E+13	22.498	43.810	35.618	43.810	178.125	72.549
0.313	0.366	0.321	6129.451	−1365.989	3.17E+15	10221.529	3261.866	6.33E+12	23.548	39.662	33.261	39.662	208.578	58.593
0.041	0.771	0.188	10466.184	310.874	7.23E+15	10670.971	3029.865	9.51E+13	21.168	43.835	33.686	43.835	174.510	89.689

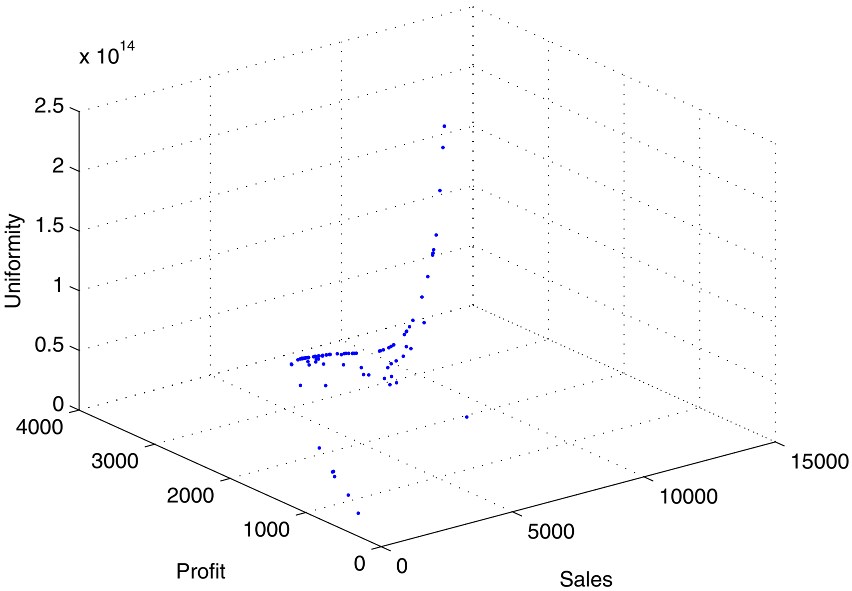

Fig. 4. Pareto-optimal plot uniform goal.

Table 5
Pareto-optimal solutions normal goals.

w_1	w_2	w_3	Sales goal	Profit goal	Uniformity goal	Sales	Profit	Uniformity	μ^*	p_1^*	p_2^*	υ^*	y_1^*	y_2^*
0.070	0.631	0.300	10042.339	3764.293	2.43E+15	10004.356	3523.150	4.20E+12	23.871	44.011	37.870	44.011	180.492	54.415
0.270	0.446	0.284	7466.292	−160.966	4.61E+15	10429.071	3273.851	2.72E+13	22.328	43.804	35.355	43.804	177.705	74.808
0.458	0.356	0.186	3525.070	−649.612	5.16E+15	10548.141	3169.654	4.88E+13	21.801	43.806	34.567	43.806	176.295	81.737
0.210	0.560	0.230	7729.800	−1127.221	5.33E+15	10176.361	3438.299	8.72E+12	23.289	43.889	36.893	43.889	179.754	61.996
0.445	0.448	0.107	7532.289	739.369	4.34E+15	10706.778	2978.035	1.25E+14	20.902	44.209	33.668	44.209	171.475	92.849
0.561	0.073	0.367	6526.214	1772.597	2.18E+15	10632.721	3047.761	6.43E+13	21.544	42.150	32.655	42.150	186.390	85.025
0.673	0.232	0.095	6919.529	2170.252	8.57E+14	10640.718	3068.433	8.01E+13	21.335	43.826	33.911	43.826	174.981	87.642
0.279	0.416	0.305	6462.356	−640.474	5.50E+15	10295.996	3367.787	1.47E+13	22.854	43.834	36.183	43.834	178.935	67.781
0.590	0.056	0.354	2498.350	282.721	3.79E+15	10684.912	3008.908	9.76E+13	21.143	43.420	33.269	43.420	177.130	89.990
0.339	0.632	0.030	9626.053	1653.359	3.20E+15	10675.553	3023.730	9.77E+13	21.142	43.839	33.653	43.839	174.421	90.009

to the diversity in the uniformly generated random goals, we have also observed a variation in the non-dominated solutions obtained for this goal type. Trade-off plot for this goal type is presented in Fig. 5. Although the solutions are diverse, the trade-off trends in the non-dominated solutions for this goal type are comparable with the ideal goal type which was discussed earlier in this section.

Here we continue the numerical simulations and also generate normally distributed goals such that $goal_k \in \mathrm{N}[f_k^{\min}, f_k^{\max}], \forall k = \{1, 2, 3\}$ with a mean, $\frac{f_k^{\min}+f_k^{\max}}{2}$, and a standard deviation, $\frac{f_k^{\max}-f_k^{\min}}{\sqrt{12}}$. This selection makes uniformly and normally distributed goals stochastically generated with the same parameters of mean and standard deviation. It is important to note that the normal distribution utilized here is truncated since $goal_k \in \mathrm{N}[f_k^{\min}, f_k^{\max}], \forall k = \{1, 2, 3\}$ which makes it consistent with the uniform distribution (see [42], [56], and [57] for more details). The 3-D plot of the Pareto-optimal solutions and the trade-off matrix plot are presented in Figs. 6 and 7 respectively. We found 123 Pareto-optimal solutions out of 300 simulation runs for the normally generated goals. Similar to the earlier two simulation studies, in Table 5, we

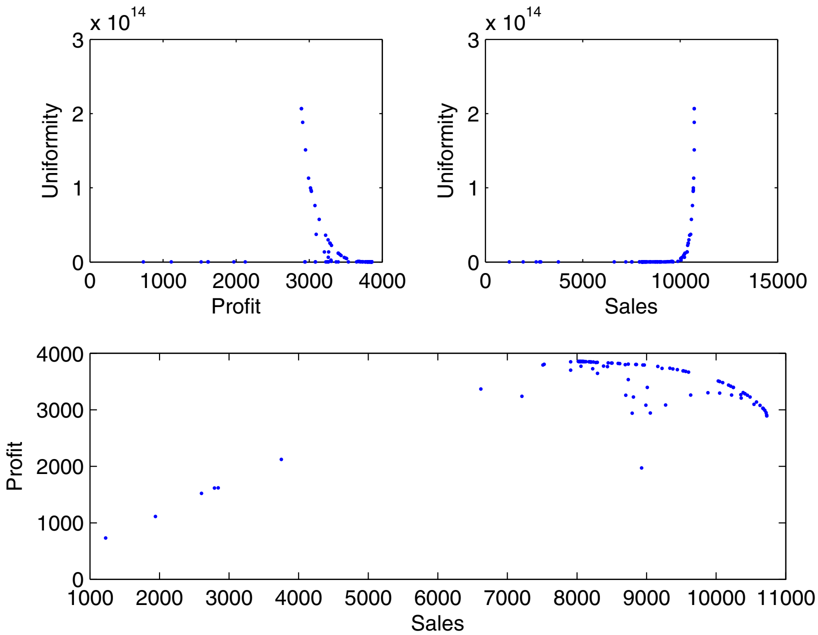

Fig. 5. Trade-off plot uniform goal.

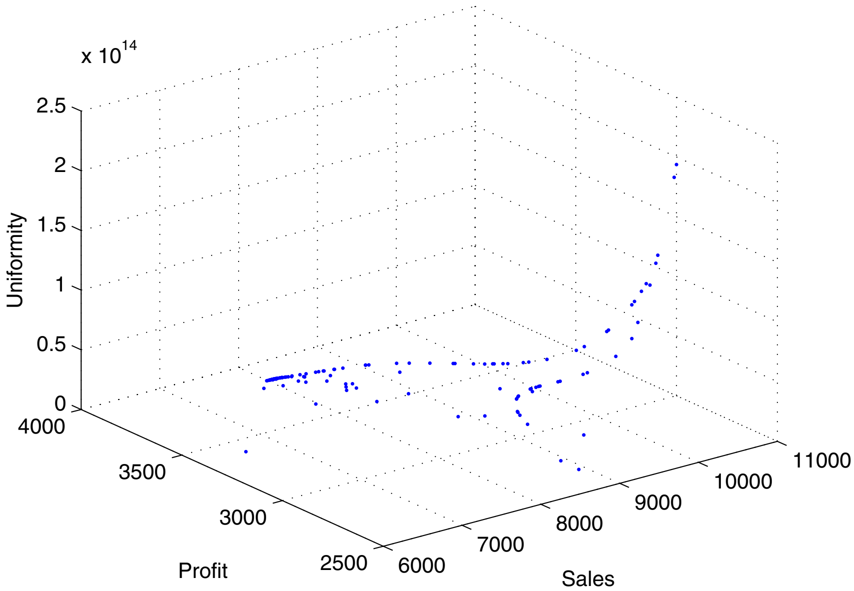

Fig. 6. Pareto−optimal plot normal goal.

only report 10 arbitrarily selected non-dominated solutions out of 123 solutions obtained. We conclude from the plots in Figs. 6 and 7 that normally generated goals also enable us to find more diversified non-dominated solutions to the multi-objective problem.

In summary, we can conclude from these plots that the Pareto-optimal frontier is identifiable. The firm seeks to improve product uniformity by offering more grade 1 products, but alternatively a firm may also decide to offer both product grades and obtain additional profitability. Indeed due to this product diversity, the firm would also incur some improvements in the product uniformity which would improve customers' satisfaction. This finding is more easily observed in the trade-off matrix plots. Higher sales (gross income) are related to higher profits, and the firm potentially obtains higher profits from the class 1 products, and therefore we can observe that optimal uniformity and the profit are inversely related. In this situation, the firm seeks to set the process mean to produce as many class 1 items as possible, as a results this decision also minimizes the optimal cost of uniformity simultaneously.

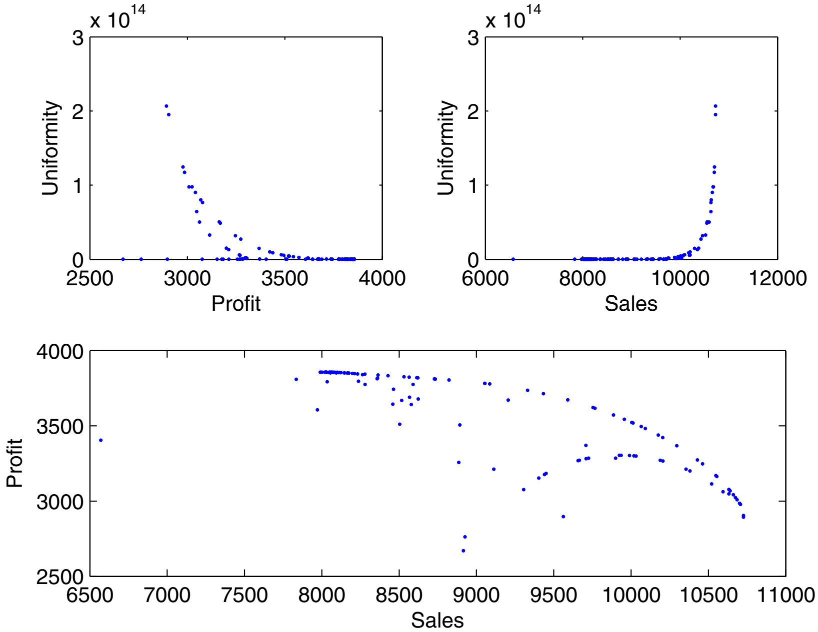

Fig. 7. Trade-off plot normal goal.

5.2. Analysis and characterization of Pareto-optimal solutions

In order to characterize the Pareto-optimal solutions and the *managerial insights* for a firm's decision making, we re-visit Tables 3–5. These tables show the details of the solution of some of the sample of the Pareto-optimal points obtained using the proposed solution procedure. It is quite evident that a firm sets an optimal process target mean more than, l_1, $\mu^* > l_1$, in order to ensure products of grade 1 produced at larger amount than the products of grade 2. This setting also enables a firm to mitigate the expected number of nonconforming items manufactured in the production process in a single production period. In most Pareto-optimal solutions, we found that $p_1^* = \upsilon^*$, which means that a firm segments the market using an optimal differentiation price, υ^* such that it is equivalent to p_1^*. This finding was also reported in an earlier study in [37] and [43]. This particular pattern of the Pareto-optimal solution can be utilized in a future study to reduce the dimensionality of the problem. It may also enable the development of some heuristic based solution methodologies for large scale optimization with complex problems that involve several process means and product classes to be optimized. In the following, we have conducted a sensitivity analysis to calibrate the performance of the proposed solution method. In many multi-objective optimization studies a very well-known sensitivity analysis is utilized to achieve multi-objective optimization. Much of the earlier published research reported a sensitivity analysis for the performance calibration of the multi-objective solution procedure and often did not emphasize using the sensitivity analysis on the problem related parameters (see [30], [58], and [52] for details). In this paper, we have also reported the performance analysis with the proposed solution procedure.

5.3. Performance analysis of the proposed solution algorithm

In this section, we quantitatively evaluate and compare the performance of three goal types generated using two distinct randomization schemes. Van-Veldhuizen [59] proposed a hyper volume metric that calculates an approximation of the hypercube volume formed by the non-dominated solutions in the objective function space. Thus, this metric can measure some necessary tasks in solving multi-objective optimization problems such as closeness to the true Pareto-optimal front and the spread of non-dominated solutions as $\mathbf{f}_{\text{nadir}} = (f_{1,\text{nadir}}, f_{2,\text{nadir}}, f_{3,\text{nadir}})$ in order to form the diagonal corners of the hypercube. A volume, v_i can be computed for each non-dominated solution and a selected reference point, for instance here, $\mathbf{f}_{\text{nadir}}$ to construct the diagonal corners of the hypercube. Then, the approximate volume of the hypercube can be found as the union of all these non-dominated solution points, Q, which is presented in Eq. (34) (see [52], and [59], [60] for more details).

$$HV = \text{volume}\left(\bigcup_{i=1}^{|Q|} v_i\right). \tag{34}$$

Table 6
Evaluation of the closeness and diversity of the non-dominated solutions.

Distribution pattern	$HV_{normalized}$
Ideal goal	0.2248
Uniformly distributed random goals	0.4357
Normally distributed random goals	0.2816

In order to compute the hyper volume (HV), a reference point such as the nadir solution is needed. Here among the non-dominated solutions, we have taken the f_{nadir} among these Pareto-optimal solution as the least favorable solution for each objective in 900 simulation runs. We found that $\mathbf{f}_{\text{nadir}} = (f_{1,\text{nadir}}, f_{2,\text{nadir}}, f_{3,\text{nadir}}) = (1225.8, 730.51, 2.0672 \times 10^{14})$ as the reference point. As we mentioned earlier each of the three objectives have different magnitude and therefore we normalize $\mathbf{f}_{\text{nadir}}$. This normalization yields, $\mathbf{f}^{\text{normalized}}_{\text{nadir}} = (f^{\text{normalized}}_{1,\text{nadir}}, f^{\text{normalized}}_{2,\text{nadir}}, f^{\text{normalized}}_{3,\text{nadir}}) = (0, 0, 1)$. Next, we proceed to compute the normalized hyper volume, $HV_{\text{normalized}}$ whose maximum value is less than or equal to 1. $HV_{\text{normalized}}$ is dimensionless. The larger the normalized hyper volume, the greater the ability of the multi-objective optimization algorithm to find more diverse non-dominated solutions. In order to avoid overlapping, we first sort the non-dominated solutions such that, $f_{1,\,1}$ is the first (lowest observed) value of the first objective function (the sales). Similarly, $f_{2,\,1}$ is the first value (again the lowest observed) of the second objective (the profit), and likewise, $f_{3,\,1}$ is first (maximum observed) value of the third objective (the total expected uniformity). Now, we compute the normalized hyper volume, $HV_{\text{normalized}}$ following the relationship outlined in Eq. (35).

$$HV_{\text{normalized}} = \left(f^{\text{normalized}}_{1,1} - f^{\text{normalized}}_{1,\text{nadir}}\right) \times \left(f^{\text{normalized}}_{2,1} - f^{\text{normalized}}_{2,\text{nadir}}\right) \times \left(f^{\text{normalized}}_{3,1} - f^{\text{normalized}}_{3,\text{nadir}}\right) + \sum_{i=2}^{|Q|} \left(f^{\text{normalized}}_{1,i} - f^{\text{normalized}}_{1,\text{nadir}}\right) \times \left(f^{\text{normalized}}_{2,i} - f^{\text{normalized}}_{2,\text{nadir}}\right) \times \left(f^{\text{normalized}}_{3,i} - f^{\text{normalized}}_{3,\text{nadir}}\right). \quad (35)$$

Table 6 reports the normalized hyper volume formed by the normalized non-dominated (Pareto-optimal) solutions using randomly generated goals following uniform and normal distributions. The normalized hyper volume is also obtained for ideal goals. By observing the hyper volume values we can infer that such values for randomized goals are better than for ideal goals. We recall that in a simulation run of 300 with each of the normally and uniformly distributed goals, experimentation with normal goals yielded a greater number of non-dominant (Pareto-optimal) solutions compared to the corresponding uniformly distributed goals. We can therefore conclude that the randomly generated goals significantly improve the capability of the proposed algorithm to generate non-dominant (Pareto-optimal) solutions very close to the true Pareto-optimal front. But, this particular observation is specific to this experimental setup and problem structure.

6. Conclusions, limitations and future research suggestions

This research proposes a multi-objective model for a process targeting firm. We consider three objectives: the total expected sales, total expected profit, and the total expected uniformity. The model suggests a joint optimal decision approach for a firm to determine market segmentation using a differentiation price, pricing, production quantity allocation in each market segment, and process (target) mean. Thus, this study presents an optimal way to interface the firm's operational decisions by controlling its production and process mean, along with its market decisions by determining its market segmentation and pricing decisions. We assume that the firm experiences leakage of price-dependent stochastic demand from the full price market segment to the discounted segment. A multi-objective mathematical model is developed for the problem which is identified as multi-objective nonlinear constrained optimization problem. We transform the mathematical model into a goal program. To avoid the limitations of a goal program as it produces only a single solution, we proposed a simulation-based optimization approach. Using this approach based on Monte-Carlo simulations, we replicate the goal program model under various goals settings. The goal of these simulations is to obtain a set of non-dominated (Pareto-optimal) solutions for a decision maker to choose from based on her informational criteria. The non-dominated solutions obtained are then analyzed to identify their closeness to the true Pareto-optimal frontier. This research also studies the performance of the proposed simulation-based optimization to the proposed goal program model.

We used an additive model for price-dependent stochastic demand with a linear deterministic demand curve which is the most widely used in the literature [39]. However, depending upon the modeling framework for stochastic price-dependent demands observed in the market segments by a firm as well as the type of deterministic demand curve, the proposed multi-objective optimization approach may vary. A detailed comparative analysis of the different modeling approaches is not the focus of this paper.

This work can be extended to consider several interesting problems. The present analysis has considered the firm in monopoly only, but an interesting avenue would be to consider a game theoretic approach to the problem at a firm's level in a duopoly or in an oligopoly. Also, this research only outlines a risk neutral analysis of the problem, future researchers may want to investigate the problem in the context of a risk averse firm.

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