QATAR UNIVERSITY

COLLEGE OF ENGINEERING

MIXED-INTEGER PROGRAMMING BASED HEURISTIC FOR BUS ROUTING PROBLEM ARISING IN DOHA

BY

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A Thesis Submitted to

the College of Engineering

in Partial Fulfillment of the Requirements for the Degree of

Masters of Science in Engineering Management.

January 2023

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ABSTRACT

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Title: Mixed-Integer Programming Based Heuristic for Bus Routing Problem Arising in Doha

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School bus operation in Qatar is a very important element in the school system of Qatar. Studies conducted by Qatar Transportation and Traffic safety center shows that more than half of the students use the buses to get to their school, and so using the buses without proper study and planning will increase dramatically the number of buses used and put more pressure on the roads especially during rush hours Besides the economic effect generated from the bus numbers, the parents and the students are experiencing major problems with the bus routes that force the students to spend a lot of time on their trips to and from schools.

Also, the large number of buses used by the school in Qatar contributes to heavy traffic volumes, which increase the time spent by students on the buses

Our study aims to improve the bus system by proper planning and optimization to reach the most economical and social efficient solution considering the future growth and increasing number of students

The main objectives of this study are to produce better school bus routing planning, finding the optimized bus numbers, minimizing the time spent by students on the buses, and optimizing the bus route schedule to avoid the rush hours, taking into consideration that the bus can have more than one trip, the possibility of having different types of buses with different capacities for better optimization of operation cost and separation of gender

DEDICATION

To my parents, My wife and My kids .

ACKNOWLEDGMENTS

This publication was made possible by the Collaborative Grant [QUCG-CENG-21/22-5] from Qatar University. The statements made herein are solely the responsibility of the authors.

I want to share my gratitude to my supervisor Dr. Mohammed Haouari who gave me a great support in my educational and job career. Dr. Haouari guided me through this inspiring journey which opened my eyes to whole new level of thinking. Also I express my sincere gratitude to Dr. Mohamed Kharbeche for his valuable assistance, guidance and support over the past months.

And lastly I thank my lovely wife for her support and patience.

TABLE OF CONTENTS

DEDICATION IV

ACKNOWLEDGMENTS V

LIST OF TABLES IX

LIST OF FIGURES XI

CHAPTER 1: INTRODUCTION 1

1.1 Background 1

1.2 Problem Description 2

1.3 The Vehicle Routing Problem: An Overview 4

1.3.1 The Vehicle Routing Problem 4

1.3.2 School Bus Routing Problem as a special case of Vehicle Routing Problem 5

1.4 Proposed Methodology 7

1.5 Research Objectives 9

CHAPTER 2: LITERATURE REVIEW 11

CHAPTER 3: METHODS 26

3.1 Students School Allocation 26

3.2 Bus School Route Optimization 28

CHAPTER 4: EXPERIMENTATIONS AND RESULTS 32

4.1 Students-School Allocation 32

4.1.1 Experiments on Different Size Data 32

4.1.2 Experiments on Students-Schools Allocation 32

4.2 Bus Route Optimization 32

4.2.1 Experiments on Bus Route Optimization 34

4.2.2 Computational Experiments to Compare Results on Different Formulations 45

4.2.3 Sensitivity Analysis 47

CHAPTER 5: CONCLUSION AND FUTURE WORK 56

APPENDIX A: DATA COLLECTION FORMAT 62

APPENDIX B: HAFSA PREPARATORY SCHOOL FOR GIRLS DATA 65

APPENDIX C: JABER BIN HAYAN PRIMARY INDEPENDENT SCHOOL FOR BOYS DATA 67

APPENDIX D: BIRLA PUBLIC INTERNATIONAL SCHOOL DATA 68

LIST OF TABLES

4.1 Computational Experiments on small size data for Students-School allocation . . . 33
4.2 Solutions for Hafsa Preparatory School . . . 35
4.3 Weights changes on Solution for School 1 . . . 37
4.4 Solutions for Jaber Bin Hayan Primary Independent School for Boys . . . 38
4.5 Weights changes on Solution for School 2 . . . 39
4.6 Solutions of Groups for Birla School . . . 42
4.7 Weights changes on Solution for School 3 . . . 44
4.8 Computational Experiments 15-Stop Instances . . . 46
4.9 Computational Experiments on 20-Stop Instances . . . 46
4.10 Computational Experiments on 25-Stop Instances . . . 47
4.11 Change of Optimal solution with sample size 25 with different k . . . 47
4.12 Change of CPU Time with sample size 25 (sec) with different k . . . 48
4.13 Change of Solution with Sample Size with nearest two points . . . 49
4.14 CPU solution time for the nearest two points (sec) . . . 49
4.15 Change of Solution with Sample Size with nearest three points . . . 50
4.16 CPU solution time for the nearest three points (sec) . . . 51
4.17 Change of Optimal solution with T_{pref} . . . 52
4.18 Change of CPU Time with T_{pref} (sec) . . . 53
4.19 Change of Optimal solution with T_{max} . . . 54
4.20 Change of CPU Time with T_{max} (sec) . . . 54

5.1 Hafsa Preparatory School for Girls Data . . . 65
5.2 Jaber Bin Hayan Primary Independent School for Boys Data . . . 67
5.3 Birla Public International School Data . . . 79

LIST OF FIGURES

1.1 Schools Distribution in Qatar 3

1.2 Zones Distribution in Qatar 3

1.3 VRP with Depot 5

1.4 Travelling Salesman Problem 5

1.5 Simple SBRP 6

4.1 Hafsa Preparatory School Map 35

4.2 School 1 Change of Solution with Sample Size 36

4.3 School 1 Change of CPU Time 36

4.4 Jaber Bin Hayan Primary Independent School for Boys Map 37

4.5 School 2 Change of Solution with Sample Size 38

4.6 School 2 Change of CPU Time 38

4.7 Birla School Group 1 Map 40

4.8 Birla School Group 2 Map 40

4.9 Birla School Group 3 Map 41

4.10 School 3 Change of Solution with Sample Size 43

4.11 School 3 Change of CPU Time Group1 43

4.12 School 3: Change of CPU Time Group2 44

4.13 School 3 Change of CPU Time Group3 44

4.14 Change of CPU Time with sample size 25 (sec) 48

4.15 Change of Solution with Sample Size with nearest two points 50

4.16 Change of CPU Time with nearest two points (sec) 50

4.17 Change of solution with nearest three points (sec) 51

4.18 Change of CPU Time with nearest three points (sec) 51

4.19 Change of Solution with Sample Size 52

4.20 Change of Solution with T_{pref} 53
4.21 Change of CPU Time with T_{pref} 53
4.22 Change of Solution with T_{pref} 54
4.23 Change of CPU Time with T_{max} 55

5.1 Data Collection Format 62

CHAPTER 1: INTRODUCTION

1.1 Background

Public schools in Qatar are managed by the Ministry of Education, where all schools (Elementary, Primary, and Secondary) share the same bus fleet. The bus fleet is managed by the local transportation authority (Karwa) where it provides all schools with the necessary transportation requirements to cover all the required bus stops. In Qatar, the Ministry of Education allocates students to schools depending on two main criteria: gender separation, there will be separate schools for males and females, and distance from schools where students are allocated depending on the school capacity and the distance between the school and the student's house so that students are usually assigned to the nearest school available. Karwa is taking the responsibility of providing transportation from student homes to schools and vice versa for all government schools, which raises significant challenges to coordinate and maintain the best service level at a reasonable cost Due to the climate nature in Qatar, where temperatures are very high, especially during the summer season, all students ride vehicles to their schools in Qatar, either picked up by their parents or using the school bus system. The School Bus Routing Problem (SBRP) has been studied extensively through the years, each study has considered different methodology and objectives [Park and Kim, 2010]. A search on Web of Science using the key words (School Bus Optimization) shows 112 published papers in the last five years which reflects the importance of this subject. Moreover, poorly planned school trips may indirectly affect students' health as long duration at early morning hours or after a long school day may cause tiredness and fatigue leading to health problems and low academic performance. Lengthy bus trips are a cause of poor bus routing management that includes several issues such as including far located students in the same route and inappropriate route sequence between the students in the same bus resulting and extra route time. Therefore, the optimization of school bus routing plays a vital role in planning the school bus routing problem to minimize transportation costs and negative health impacts. Therefore, after intensive literature review we have considered a new approach in solving the SBRP where beside the economical objective of reducing the number of buses, we also considered to optimize the travel time of students which has a major effect on the social importance of using buses by adding two more constraint, a hard constraint where the trip time cannot exceed a predefined time limit and a soft constraint where we add a penalty on each time exceeds the preferred trip time. This will help in controlling the trips to the degree it is within acceptable limits of the students. We applied our model to a real case study data for one of the schools in Qatar to give an example of the effect will have on changing from private transportation to public bus fleet if it is done by proper planning.

1.2 Problem Description

In Qatar, there are 986 schools divided into 180 schools for Male and 181 schools for females, and 625 mixed. With a total of around 327,841 students divided into 66,235 in male schools,70,626 in female schools, and 190,980 in mixed schools. As per Annual Statistics of Education in the State of Qatar for the year 2019-2020, as shown in Figure 1.1.

Currently, Karwa is using its buses for school transportation, and all buses are dispatched from depots where they start their assigned trips and then return to the depot .

Schools are distributed over 8 municipalities with a total of 98 zones (Ad Dawhah Municipality with 62 zones, Al Rayyan Municipality with 10 zones, Al Daayen Municipality with 2 zones, Umm Salal Municipality with 1 zone, Al Khor Municipality with 3 zones, Al Shamal Municipality with 3 zones, Al Shahaniya Municipality with 10 zones, Al Wakrah Municipality with 7 zones), as shown in different zones in Figure(1.2).

Each school is assigned a certain number of buses according to the number of stu-

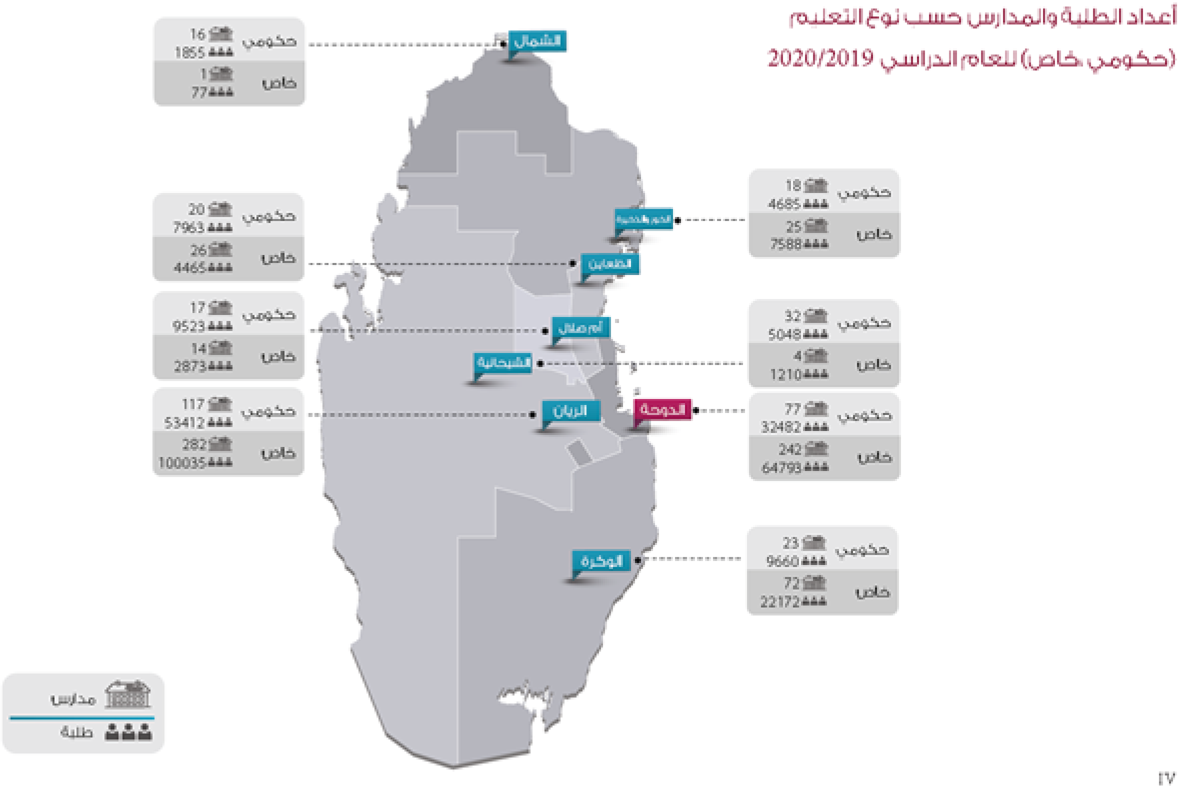

Figure 1.1: Schools Distribution in Qatar

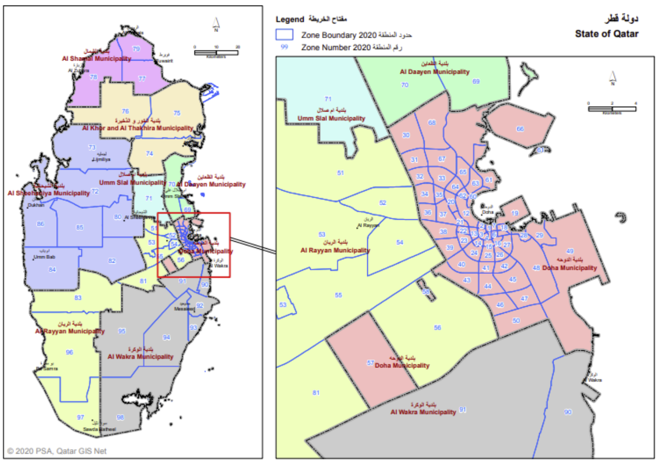

Figure 1.2: Zones Distribution in Qatar

dents in the school, and this is usually done manually depending on the planner experience; Each school has a supervisor who is responsible for assigning students to each bus, then the bus driver generates the route for his trip to cover all the required stops. Qatar authorities always recommend that students use public transportation to minimize the traffic, especially in the morning and afternoon hours.

Currently the usual planning is a standard manual approach which depends totally on the planner's experience at each stage. So the efficiency of using these resources depends on the planners, but with the increased usage of the public transportation needs emerge for more effective planning systems to better use the available resources.

1.3 The Vehicle Routing Problem: An Overview

1.3.1 The Vehicle Routing Problem

The classic Vehicle Routing Problem (VRP) as a general type has significant importance in transportation scheduling in several practical problems. In its simplest forms, the VRP is designed to find the optimal traveling route for a vehicle with at least one depot, and the vehicle should travel the shortest path between designated stops while satisfying capacity constraint of the vehicle as shown in Figure1.3.

A very well known special format of VRP called Travelling Salesman Problem is when we have one vehicle with no capacity constraint and the objective is to find the optimal minimum route between designated stops as shown in Figure1.4.

To have a better solution primarily related to the loading capacity of vehicles, a more complicated fleet can be designed for the VRP, including different capacities usually called Heterogeneous fleets.

The VRP has been extended to many real-life forms by introducing many characteristics. In logistics and delivery industries, the vehicles should pick up a package through the optimal route to deliver the package to the designated drop point. This means the exact vehicle should do pick-up and drop-off.

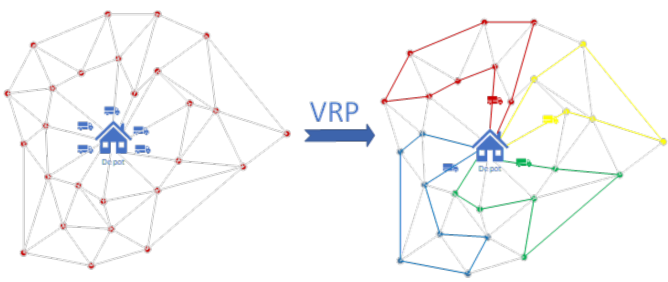

Figure 1.3: VRP with Depot

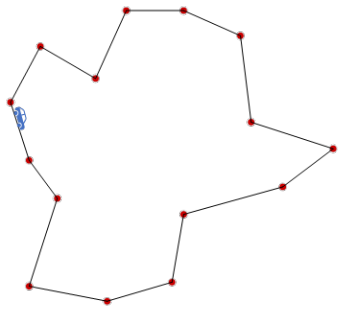

Figure 1.4: Travelling Salesman Problem

Adding the time constraints also emerged in many scenarios where the deliveries at specific locations must be done in certain time windows, which varies from location to location.

The VRP has many variants and solutions suggested for every situation, including optimal solutions using Integer programming and many heuristics and meta-heuristics to solve the increasingly complicated models.

1.3.2 School Bus Routing Problem as a special case of Vehicle Routing Problem

The School Bus Routing Problem (SBRP) is one of the many forms of VRP. Over the last years, it has become more popular to solve the increasing demand for public transportation as required by many local authorities worldwide to minimize Traffic and Environmental effect.

Many types of research were developed in different parts of the world, each one with unique characteristics and objectives depending on their particular needs.

Public schools are more and more under economic pressure to reduce their costs around the world, and one of the primary services provided by them is transportation, so over the years, many strategies were developed to improve the planning and reduce

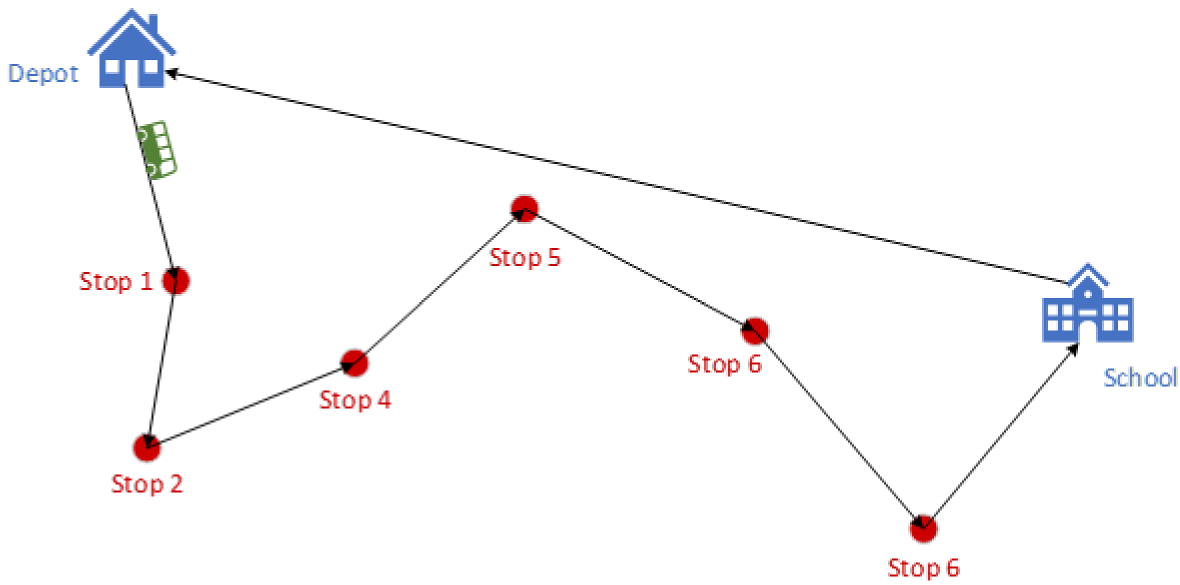

Figure 1.5: Simple SBRP

the costs of this cost.

Traditionally planning was done manually, but later with the increasing volume of this transportation problem and its related costs, especially in rural areas where a more significant number of vehicles are usually used, and more considerable distances are traveled daily, more and more need to have optimized ways to have a more efficient planning methodology emerged.

In SBRP, especially in the public sector, the buses are usually dispatched from central depots to serve the designated district.

The students gather at pre-allocated stops to be picked off, so each stop will have a different loading vector. The Buses take their designated routes, pass by student's stops to pick them, take the designated rout to drop them at the school, and then return to the depot awaiting the afternoon trip to bring the students back from schools to their house as shown in Figure 1.5. One of the primary essential characteristics of the SBRP is arrival timing, where students must arrive in schools before the bell time, so the planner should always keep the routes within the time windows given by schools. The SBRP has had a lot of variants through the years and areas, each one dependent on the real-life situations in the designated area where we will see in the Literature review

1.4 Proposed Methodology

The Qatari authorities always encourage students to use public transportation to minimize traffic, especially during the morning and afternoon rush hours.

Our study will divide this problem into two sub-problems

- Student- School Allocation:

  Where the students are allocated to the nearest school taking into consideration the available seats in each school.

  Each year new students joins the schools, and for the public schools in Qatar the Ministry of Education has put several rules to assign the new students to the schools, students are allowed only to join the schools near to their area as per their gender and not to exceed the school capacity, for this sub-problem we define the characteristics, the objectives, and the constraints as follows:

  Characteristics

  – Students in each zone are considered as one group,

  – The students can be assigned to correct gender school.

  Objective

  Minimizing the accumulated travelling distance.

  Constraints

  – Each school cannot accommodate more people than its seating capacity,

  – All students in each zone should be assigned to schools.

- Route Optimization:

The usual planning process is a standard manual approach, without the use of scientific approaches, which is therefore totally dependent on the experience of the planner at each stage, and therefore, the efficiency of the use of these resources depends on the planners. In this regard, each school is assigned a certain number of buses as per students number.

A supervisor is assigned to each school who is responsible for assigning students to each bus, and then each bus driver generates the route to cover all required stops. However, with the increase in public transport use, there is an emerging need for more efficient scheduling systems to make a better use of available resources.

Therefore, after a thorough analysis of the literature, we investigated a new school bus routing model where, in addition to the economic objective of reducing the number of buses, we also took into account the optimization of student journey time. We used our model to solve real cases arising in Qatari public schools. Over the past two decades, Qatar has experienced rapid growth in public and private education.To define our problem in detail, we shall successively describe the characteristics, the objectives, and the constraints.

Characteristics

- Each bus will carry students from one school only,
- They choose a homogeneous bus fleet with a fixed capacity for all buses,
- The departure point for all buses is the designated school. However, the total travel time is only accounted starting the first student stop to the time he/she arrives at the school,
- All students are being picked up from preassigned stops usually at their houses so we will consider that bus stops are already defined,
- Arrival times at schools are already defined and fixed and by considering a fixed maximum trip time we can identify the earliest start time of bus trips.

Objectives

- Minimize the number of buses. This would directly reduce the total cost of operating the bus fleet,
- Minimize the excess of student travel time over a predefined maximum travel time target. In practice, this goal amounts to minimizing the penalties for exceeding the preferred maximum target time. In this way, more students will be encouraged to use the school buses and comfortable travel times will be ensured.

In practice, we shall aggregate both objectives into a single objective function using appropriate weights.

Constraints

- Each bus cannot accommodate more students than its seating capacity,
- Each stop should be covered by one single bus,
- All bus routes start and end at the school,
- The time required for the first student to reach the school cannot exceed a predefined maximum time.

1.5 Research Objectives

The main objective of this research is to develop a systematic planning model to cover all government schools and their student's transportation requirements. Besides covering all current requirements, the suggested model should be able to optimize the bus fleet to minimize the costs and make sure all students reach their schools at the required bell time. The main objectives of the suggested model will be:

- Optimize Travel distance of bus routes.

- Optimize the bus routes to minimize the travel time to ensure all students reach their school destination at the targeted bell time.

CHAPTER 2: LITERATURE REVIEW

The first time School Bus Routing Problem (SBRP) was introduced by [Newton and Thomas, 1969] they have divided this problem into the geographical determination of a bus route and a time schedule for each bus, but they have considered a very limited problem where covered only one school,

SBRP can be categorized based on some main characters such as:

- Number of schools
- Starting and ending point
- Surrounding of services
- Mixed loads
- Fleet mix
- Special education students
- Gender separation
- Objectives
- Constraints

Also, the SBRP can be divided into four sub-problems :

- Bus stop selection
- Bus route generation
- School bell time adjustment
- Route scheduling

Further differences can be discussed through the solution methods used from the 1960s till today Review as pr the above classification :

Number of Schools: As per the schools number, we can divide the SBRP into two categories: Problems with a single school, and Problems with Multi-School.

[Newton and Thomas, 1969] have started the studies with one school to simplify the problem. They have a similar solution to the traditional Vehicle Routing Problem, and then [Angel et al., 1972] started considering the multi number of schools, [Newton and Thomas, 1974].

[Bennett and Gazis, 1972], [Gavish and Shlifer, 1979] have also re-considered the multi-school system where a school fleet serves them, and there is mainly two approaches for multi schools a school-based approach and a home-based approach. [Braca et al., 1997] considered the home-based approach by inserting a stop, and this insertion stop is determined concerning the cost related, this allows for mixed loads,

[Spada et al., 2005] has considered school-based approach where routes are generated for individual schools, and then these routes are assigned to the bus fleet, this approach doesn't allow for mixed load

Staring and Ending Point : The buses have two options to start (at the depot / at school) and the same two options to end [Newton and Thomas, 1969] have considered the buses will start at the school, visit every stop once drop at school, and park there. then at their paper,[Newton and Thomas, 1974] have considered different starting points for different buses, which is determined at the beginning of the problem, while [Bennett and Gazis, 1972] considered the starting point at the garage and the endpoint at the school same is for [Verderber, 1974] [Dulac et al., 1980] have considered the start and the drop point is the school [Gavish and Shlifer, 1979] have considered starting point at the depot, and the buses will return to it after completing their route [Chen et al., 1990] have considered the starting points at the Bus driver house, which usually applicable in the rural areas

Surrounding Services: The solution of SBRP will highly differ between urban and rural areas. In urban, the students will be walking from their homes to selected bus stops, while in rural areas, the bus will be picking up the students from their homes, so bus stop selection is not usually considered in the rural area's solution [Newton and Thomas, 1969] didn't specify any details about their choice as they have considered an artificial study; however, as they considered a different loading vector at each stop so we can consider this as a simulation for urban areas, real Urban problems are considered by [Bennett and Gazis, 1972], [Verderber, 1974], [Dulac et al., 1980], [Chen et al., 1990], [Thangiah and Nygard, 1992], [Ripplinger, 2005].

[Chen et al., 2015] has differentiated the rural problems from urban problems with the following characteristics: lower population and so lower student number, longer traveling distance, fewer alternative routes, less number of students per bus, and usually buses will stay overnight at the driver's homes.

[Howley et al., 2001] studied the difference between rural and suburban areas, while [Ripplinger, 2005] have found that the manual solution in rural areas can be optimal due to the small size of the problem. [Hargroves and Demetsky, 1981] have taken both cases of urban and rural areas, and as a result of this usually, the number of High school students is much less than elementary students so the buses can make more than one trip and this will reduce mush the bus numbers.

[Miranda et al., 2021] have completed their paper on 80 rural schools in Brazil and, [Howley et al., 2001] have made a comparison between the rural and the suburban in five different states in the United States where they found major differences, especially in the ride duration where it was found that the duration of the longest ride is at least 30 minutes longer in the rural areas.

Mixed Load: Early papers considered that buses could pick up students only from the same school, but later [Bodin and Berman, 1979] found that better solutions can be obtained by considering mixed loading where buses can pick up students from more than one school, where the bus start-up point will be the depot pick up the designated students, drop them in the first school, and then continue to the second school and so on, especially in rural areas If mixed loading is not allowed, then the SBRP is simplified to a single school problem, however as per [Chen et al., 1990], this will generate an excessive number of buses, especially when we have large distances between the bus stops.

[Spada et al., 2005] have considered an interactive tool for bus loading mix where the operator (the planner) can generate different routes with different loads, [Li and Chow, 2021] considered a mixed loading with a heterogeneous fleet. Still, they considered that each special-education student house location during bus stop selection is designated as a stop while general-education students can walk to the designated bus stop.

Fleet Mix: Bus fleets are one of the essential characteristics of the SBRP, and types of fleets have been studied thoroughly. It started with homogeneous fleets with [Newton and Thomas, 1969] as they have simplified the problem to one school each time, so they assumed for each school the buses will have the same capacity; also [Bennett and Gazis, 1972] have considered a homogeneous fleet, [Gavish and Shlifer, 1979] have considered all the vehicles are identical and used the available buses then considered adding new buses as per required capacity for better optimization and cost-saving.

[Newton and Thomas, 1974] have considered a unique solution where the bus capacity for each school is the same but can be different from school to school. A more general situation where a Heterogeneous fleet is considered first by [Hargroves and Demetsky, 1981]. however, the costs of having different buses capacities were not considered Most of the papers like [Ripplinger, 2005], [Chen et al., 2015] has considered unlimited heterogeneous fleets.

[Ripplinger, 2005] considered that fleet mix is more important in rural areas than urban ones because the heavy density of students in urban areas allows for maximum loading fulfillment for the buses, so the need for small buses or vehicles in urban areas is usually uncommon [Miranda et al., 2021] considered a heterogeneous fleet in rural areas, which helped to reduce the cost of the transportation

Special Education Students: This is one the recently added characteristic to the SBRP where buses with special access for special education students and these students are assigned specifically to certain schools that has special programs for such students, [Russell and Morrel, 1986] started presenting the special education problem by modifying the normal solution to include them in the same solution, while [Braca et al., 1997] discussed the issue for special education students but didn't present any solution.

[Ripplinger, 2005] have considered special education students in rural areas and presented two options, first where two problems are solved separately normal students and Special education students, and so there will separate routes, second solution combined solution were both normal and special education students are using the same buses and have the same route.

Gender separation: This is a very special characteristic where boys and girls are separated in the buses, and this is also reflected in the school stops as well.

[Rashidi Komijan et al., 2021]. As per research done, are the first to consider gender separation in Tehran with taking into consideration homogeneous fleets with fixed capacity

Objectives: Every author has considered different aspects to the main objectives needed from the SBRP,

- Minimize the bus numbers:

  [Newton and Thomas, 1969] started with the objective of generating the best bus routes to provide transportation for all the students and defining the schedule for each bus route, and this is one of the main objectives needed was the economic saving by minimizing the buses numbers and accordingly the cost of the buses, Most of the papers consider reducing the buses numbers as their main objective using different methodologies

- Minimize the travel time for the students:

  Besides the economic objectives, there are the social objectives where it is very important to minimize the traveling time for the students to help shorten their trip timing, [Bennett and Gazis, 1972] have considered this where they present the changes in travel time for buses and students as the main objectives of their paper, while [Gavish and Shlifer, 1979] have taken the cost function related to the number of the utilized vehicles.

  [Hargroves and Demetsky, 1981] considered the bus numbers and capacity is fixed and, [Spada et al., 2005] have considered an objective to minimize the time delay between Taxi riding and bus riding with a direct trip from their home to school.

Constraints:

- Bus Maximum load different types of buses are used by different schools and areas depending on the capacity of each school, so the maximum bus capacity should be taken into consideration [Dulac et al., 1980], [Hargroves and Demetsky, 1981], and all other authors used this to put an upper bound on the solution given, especially when considering the heterogeneous fleets

- Maximum student travel time Each student's travel time should not exceed a certain time as it will be tiring for the student's [Newton and Thomas, 1969] have considered this limit to avoid exceeding trips for the students. [Gavish and Shlifer, 1979] have considered this by putting an upper limit to the driving tome from each point to the school, same also considered by [Dulac et al., 1980], [Hargroves and Demetsky, 1981] considered a maximum of 1 hour, while [Chen et al., 1990] considered 75 minutes
- Bell Time: Another constraint taken into consideration is to optimize the bell time as it was observed that the very early school starting time could affect the final student's results negatively, and by optimizing the bell timing bus, Routs can be adjusted in a better way to avoid the rush hours and optimize the bell timing [Verderber, 1974] have considered the bell time to be assigned by the school as an input with some allowance, while [Newton and Thomas, 1974] have considered a different way by considering different bell periods for different schools and then solved the problem of determining when each bus is assigned to a certain school what will be the time period. [Hargroves and Demetsky, 1981] have considered the earliest pick-up time and bell can be moved up on one -half-hour only [Eguizábal et al., 2018] considered the allowed time window for bus arrivals is 2-10 mins only prior to class starts.
- Maximum Bus route time [Hargroves and Demetsky, 1981] have considered the minimum bus route distance of six miles instead of the maximum route time

The SBRP has been divided into main four problems for solution simplification

Bus Stop Selection: It is one of the most important points, especially in the rural areas, as this should take into consideration minimizing the walking distance for the students and optimizing the best bus route, while in Urban areas, students are usually picked up from their homes, so these points are already determined for each student.

[Dulac et al., 1980] considered the selection of the stop by the authorities or generated by the system, they have considered an upper bound for the student walking distance, and according to this, the bus stops are selected, which gives better routes with a smaller number of bus stops.

[Schittekat et al., 2013] has taken the possible bus stops selected by authorities and developed a solution to optimize these stops; however, this solution can be applied only on single school case, multiple schools were not studied

Bus Route Generation: After selecting the preliminary Bus stop points, the next step will be finding the bus routs from the Start point to the pick-up locations to the schools, [Newton and Thomas, 1969] have started with a predetermined pick-up location and moved directly to find the best routing.

[Verderber, 1974] considered to generate several options and then choose the best depending on the best travel time.

[Gavish and Shlifer, 1979] have considered solving the scheduling problem to find the maximum savings by combining the collection points into partial routs. This will help to minimize the number of the buses and shorten the routs and so minimizing the initial and operational costs, [Dulac et al., 1980] have considered this solution by having an upper limit to the length of a route.

[Bodin and Berman, 1979] and [Thangiah and Nygard, 1992] have considered a different solution method where they have specified two methods

- Route-First, Cluster-Second, by ignoring all constraints, they created a single route for each school that covers all the student's locations, then the route is divided into clusters that satisfy the constraints.

- Cluster-First, Route Second, by first combining students locations in groups that satisfy the constraints and then these groups are individually routed.

Route Scheduling: Route scheduling is used to define the exact start and end-point of the route, especially by multi-schools where we have different time periods Scheduling has been considered from the beginning by [Newton and Thomas, 1974], who assumed there are different time periods and schools will start at different times.[Fügenschuh, 2009] allowed the transshipment from one route to another to find the best bell time.

School Bell time Adjustment: [Miranda et al., 2021] have concentrated their study on bell time adjustment and used three different strategies, two used during the construction phase, and once applied on the local searches, one real problem in 56 cities in Brazil with multi loading.

Solution Methods: Taking into consideration the size and complexity of the problem-solution method will be changed, for small-sized problems, it can be solved by exact approaches, but for bigger and more complex problems, heuristic methods are more used.

The exact Approach by Mixed Integer Programming (MIP) is usually used to confirm the correctness of the heuristics by applying them on small-sized artificial samples and comparing them with the heuristic.

[Newton and Thomas, 1969] discussed a problem which is one the simplest simulation of SBRP. They have assumed a heuristic solution with two objectives: Bus route and Time schedule. For one school at a time, first the initial solution was generated similarly to the traveling salesman problem with the nearest city approach, the second step was to generate an algorithm to reduce the total time required to traverse the traveling salesman tour, the third step was to add the constraints like by adding the busloads at each stop point taking into consideration other constraints like (maximum riding time, maximum bus load, etc.. . . . .) and by this, they partitioned the solution generated from step two into several routes.

[Newton and Thomas, 1969] have applied their methodology on problems involving (50, 60, 70 and 80) bus stops and generated reasonable solutions in relatively reasonable computational time, but it was very limited and overlooked the real-life characteristics like optimizing the traveling costs, bus fleets sizes, different start and endpoints, and many other characteristics, later [Newton and Thomas, 1974] have extended their study into buses routes for a school district with different starting time and schedules, upgraded heuristic methodology was developed with objective of minimizing total sum of all routes completion time, and minimizing routes number and accordingly the related buses for each school separately, their heuristic consisted of main nine steps,

- Step 1, For individual school, the characteristics of all possible service routs,
- Step 2, For individual school, lower bound of route duration computation from origin departure point to each school,
- Step 3, Determining departure points of individual bus routs as per cost criteria for each time period,
- Step 4, selection of the best route for each school,
- Step 5, The first route selection based on the nearest city approach,
- Step 6, Partitioning routes of each school into individual routes,
- Step 7, Individual route improvement similarly to their earlier paper,
- Step 8, Generation of additional unlimited capacity trial routes,
- Step 9, Re-allocation of departure points to routes generated

This methodology was used in western New York in the suburban area , for four schools with 254 bus stops, 2894 students, and 9 origins, and managed to reduce traveling time between 25% to 61% and reduce the number of buses by 4, which showed the effectiveness of SBRP solution even in the early stages of discussion. [Bennett and Gazis, 1972] have also considered a type of heuristics to solve a single school routing using delivery vehicles routs scheduling method. First, enough buses are allocated to each stop enough to allocate all the students allocated in this stop, then whenever the bus is not fully loaded the routs of the buses are combined till reaches the full capacity of the bus, then improving the routs by the heuristic salesman problem solution to minimize the bus travel time.

This was tested on New Jersey district over fifteen different problems, with 256 bus stops, and found that in most cases the bus route time and student travel time was minimal,

[Gavish and Shlifer, 1979] have divided the SBRP into three different problems

- First, the Multi Traveling Salesman Problem to find the route for each school. They suggested an assignment method; however, it was not mathematically tested
- Second, the School bus problem, they have used the assignment method to minimize the number of the buses and their operational costs, which is related to the route length, with taking into consideration the capacity and time constraints
- Third, the bus scheduling problem top find the best timetable that complies with the changes of demand, also here assignment method is used

He tested this approach on artificial problems up to 1407 trips which calculated the need of 484 bused and saved up to 31 bus.

[Higginson and White, 1982] has suggested an improvement on [Gavish and Shlifer, 1979] by using the maximum matching problem instead of the assignment method or cost-saving, which improved the processing time significantly by obtaining better bounds. However, it didn't show any improvement in the optimization of the results.

[Ripplinger, 2005] used Mixed Integer Programming methodology with two objectives minimizing the cost of transporting the students to and from the school and minimizing the time needed for the students to reach the schools, then he suggested for the rural routing a heuristic with two phases, first phase which produces an initial route by sorting the homes depending on their distance from the school and choosing the farthest one as the starting point and so on, the second phase improving the solution by using Tabu search algorithm,

Other heuristics were used for comparison purposes like lin2-opt and modern randomized location-based heuristic (rLBH). This was tested on artificial data, considering 131 students, from 40 homes in an area of $25mi^2$, and found that using the rural heuristic was the most efficient by 13%

. [Spada et al., 2005] proposed a solution that starts with an automatic procedure to generate an initial solution then improve it using heuristics(Tabu search, simulated annealing). They have tested several heuristics on a set of 30 problems, and found that Simulated annealing gives the best solution. They considered two main objectives the first minimizing the sum of lost time for all children, the second is to minimize the maximum time loss. They applied their method on two different scenarios: first on two Swiss towns including 34 homes, 12 schools and 274 students with different start and finish time, second on a school project in Swiss, contains 151 nodes, 98 homes, and 2 schools.

[Chen et al., 2015] used two methodologies to solve a homogeneous and heterogeneous fleet problem of scheduling school bus. The first is exact method, using Mixed-integer for the objective to minimize both the number of the buses and the total travelling distance, the Second using the meta-heuristics such as simulated annealing and OPM(S, perm, rule) Using the MIP solution on CPLEX software on artificial data for schools instances of numbers between 1-100 for both homogeneous and heterogeneous where the number of buses was reduced dramatically from 562 to 135 and 127 respectively, then by using the heuristics, they could find the optimal solution for small size problems and a better solution for larger size problems.

[Schittekat et al., 2006] have studied a meta-heuristic to solve the SBRP, including three sub-problems: finding the stops to visit, determining the stop for each student, and finding the best route by minimizing the total travel distance by the buses, they have used (GRASP) greedy randomized adaptive search procedure, first it generate a solution where every stop is assigned to an individual rout, then assigning students to these stops by solving the location-allocation problem using the MIP programming to find the exact solution, then GRASP starts to generate different solutions using a roulette wheel solution procedure, after that thy used VND variable neighborhood descent to improve the solution.

112 instance generated from artificial experiments Artificial were ruined with size ranges from 5 stops with 25 students up to 80 stops and 800 students, and comparing it to the exact method using MIP, optimal solution was found for instances size up to 42, and the gap between the optimal solution and the heuristics was 1.4 percent.

[Rashidi Komijan et al., 2021] have used MIP and Genetic Algorithm for the SBRP for multiple schools with homogeneous bus fleet taking into consideration gender capacity and special students with the objective to minimize the total service cost They have done their study on 4 schools in Tehran, with 4 buses and 80 students, 40 students are boys, and 40 are girls By comparing the exact solution with the Genetic Algorithm solution, the heuristics solution values are 0.62 percent error from the exact solution, and decreasing the number of the buses from 44 buses at the current status to 40 buses proposed by this study, and traveling time reduced by 14% to 27%. [Li and Chow, 2021] did a study in New York City for special-education students, they have considered mixed loading (normal and special-education students uses the same bus) with heterogeneous fleets to minimize the total operating costs. First, they started with choosing the bus stops of each school separately, where Special-education student's homes should be considered a designated stop. Using MIP Bus stops were selected with an objective pf minimizing the total number of stops, and allocating the maximum number of General-education students to each stop, and improving it by minimizing the walking distance for each general-education student. Second, the route generation using MIP to minimize the operating cost by minimizing the travel distance for each bus. For comparison purposes, they have used Google OR tools They applied their methodology in New York City to three schools. 178 students, including 12 with a wheelchair, and found comparing the traditional non-mixed system with the mixed system a decrease in the bus number, number of stops, accumulated travel distance, and accumulated travel time.

[Eguizábal et al., 2018] used MIP to solve the school routing problem with two objectives minimizing the operating costs and minimizing the average travel time for routes They have done this in three phases, first phase, solving the routing problem for each school separately, the second phase optimizing the route combination by generating various routes for the same bus to serve several schools, the third phase analyzing all alternatives generated from the earlier phase to choose minimum cost route. They have applied their study on Cantabria region (Spain) with three primary schools who has a total of 384 students and got 71 feasible solutions, and the costs savings varied between 16.5% and 33.7%, and reduced 15 min of the average route time .

[Miranda et al., 2021] have done a study on rural schools in Brazil using iterated local search (ILS) for three sub-problems. bus stop selection, bus route generation, and school bell time adjustment They have concentrated their study on bell time adjustment and used two distinguished strategies in construction phase, and third one on local searches.They have done this study on 56 cities in Brazil, and the results showed around 2% improvement with the bell time modification with or without the multi loading.

[Bögl et al., 2015] did their study in Australia and have considered a very general case where students from different schools can share the buses and then move to a different to reach their designated school, and they have used heuristics with an objective of minimizing the total costs of the SBRP where first a preliminary solution is built using a mathematical algorithm and using a destroy and repair based optimization heuristic for improving the solution.

CHAPTER 3: METHODS

In this chapter, we describe the two mathematical programming models used for solving our two sub-problems:

- Stage one: We describe the linear programming model used to solve the Students-School allocation problem to provide the optimal distribution of students over the available seats at schools.

- Stage two: We describe the mixed-integer programming used model to solve the bus route problem to find the optimal routs of buses while minimizing the number of buses, and travel time.

Both models were solved using IBM-CPLEX software.

3.1 Students School Allocation

This is the first sub-problem and here we study the allocation of students to designated schools, For easy of modeling, we consider the number of students in each zone as one group located in the geographical center of the zone, and then these students are allocated to the schools depending on their distance, school capacity and gender, where we can divide our problem into two problems, one for male students with their designated gender schools and the other problem with female students.

we formulate our Mixed- Integer linear program as follows :

Sets and parameters

M: set of zones indexed by j,

β_j: Number of students in zone j,

N: Set of schools indexed by i,

α_i: Capacity of school i,

c_{ij}: Average distance between school i and zone j.

δ_j: The distance between students in zone j and nearest school i, where

$$\delta_j = Min_{1 \leq i \leq m}(c_{ij}). \tag{3.1}$$

γ_{ij}: Square of the excess distance over δ_j actually traveled by students in zone j when they are not assigned to the nearest school. Accordingly if any student is assigned to a school which is not the nearest, will be travelling a longer distance than δ_j, so any extra distance will be penalized to drive the model into choosing the nearest school as much as possible, and we choose non-linear penalization to increase the penalty when school is further from student as in formulation (3.2).

$$\gamma_{ij} = (c_{ij} - \delta_j)^2. \tag{3.2}$$

Decision variables

x_{ij}: Number of Students form zone j assigned to school i.

The formulation reads as follows:

$$Minimize \sum_{i=1}^{n} \sum_{j=1}^{m} \gamma_{ij} x_{ij}$$

$$\sum_{j=1}^{m} x_{ij} \leq \alpha_i, \forall i \in N, \tag{3.3}$$

$$\sum_{i=1}^{n} x_{ij} = \beta_j, \forall j \in M, \tag{3.4}$$

$$x_{ij} \in \{0,1\}, \forall i \in N, \forall j \in M \tag{3.5}$$

Where constraint (3.3) guarantee that students allocated to school i don't exceed seats capacity α_i, and constraint (3.4) guarantee that all students in zone j are assigned to schools, and constraint (3.5) is to ensure the decision variables are binary.

3.2 Bus School Route Optimization

This is the second sub-problem and here we study the route and bus optimization, we formulate our Mixed- Integer linear program as follows :

Sets and parameters

S: set of bus stops, indexed by i ,j . The depot is indexed by 0,

s: number of bus stops,

A: set of arcs,

Q: capacity of a bus,

T_{max}: maximum allowed route duration,

T_{pref}: target (maximum) route duration,

d_{ij}: duration of a trip from node i to node j, l_i: number of students to be picked up at node i,

α: weight on the number of buses,

β: weight on the sum of excesses of the buses' travel times.

Decision variables

m: number of buses,

x_{ij}: binary variables that takes value 1 if the bus travels from node i to node j, and 0 otherwise,

l_{ij}: number of passengers on the bus during the trip from i to j,

t_j: arrival time of the bus to node j,

Δ_j: excess time (over T_{pref}) of the bus whose last stop before arrival at the school is node j. In other words, if $x_{j0} = 1$ then $\Delta_j = \max(0, T_{pref} - t_j - d_{j0})$.

The formulation reads as follows:

$$(F1) : Minimize(\alpha m + \beta \sum_{j=1}^{s} \Delta_j)$$

$$\sum_{i=0}^{s} x_{ij} = 1, \forall j \in S, \tag{3.6}$$

$$\sum_{j=0}^{s} x_{ij} = 1, \forall i \in S, \tag{3.7}$$

$$\sum_{j=1}^{s} x_{0j} = m, \tag{3.8}$$

$$\sum_{j=1}^{s} x_{i0} = m, \tag{3.9}$$

$$\sum_{i \in S;(j,i) \in A} l_{ji} - \sum_{i \in S;(j,i) \in A} l_{ij} = l_i, \forall (i) \in A, \tag{3.10}$$

$$d_i x_{ij} \leq l_{ij} \leq (Q - d_j) x_{ij}, \ \forall (i,j) \in A, \tag{3.11}$$

$$0 \leq t_j \leq (T_{max} - d_{j0})(1 - x_{jo}), \ \forall (j) \in S, \tag{3.12}$$

$$t_i + d_{ij} - t_j \leq M_{ij}(1 - x_{ij}), \ \forall i,j \in S, i \neq j, \tag{3.13}$$

$$t_j - \Delta j + (T_{max} - T_{Pref}) x_{j0} \leq T_{max} - d_{j0}, \ \forall (j) \in S \tag{3.14}$$

$$x_{ij} \in \{0,1\}, \ \forall (i,j) \in A. \tag{3.15}$$

Constraints (3.6), and (3.7) guarantee that each location is visited only once. Constraints (3.8), and (3.9) are used to ensure that the number of busses departing from the depot and arriving is equal which is equal to the number of used busses. Constraint (3.10) ensures that all students at stop i are carried by the bus coming from i to j , and constraint (3.11) is the bound of loading variable l_{ji} between the loading required at stop i so it won't exceed the bus capacity.

Constraints (3.13) and (3.12) We make sure that the time to reach node j after node i is always less than the maximum allowed trip duration considering the duration to reach back to the depot. If bus travelled from node i to node j

$if\ x_{ij} = 1$ then $t_j \geq t_i + d_{ij}$

and $x_{ji} = 0$

$t_i + d_{ij} - t_j \leq M_{ij}$ where M_{ij} is a big number

by combining both we get M_{ij} as follows :

$M_{ij} = T_{max} - d_{i0} + d_{ij}$

In constraint (3.14) we aim to target a preferable travel time T_{pref} we penalize any delays from this time Δ_j where

$t_j + d_{j0} \leq T_{pref} + \Delta_j,$

by considering T_{delay} is the difference between T_{max} and T_{pref}, $T_{delay} = T_{max} - T_{pref}$,

when $x_{jo} = 1$ then

$t_j + d_{j0} \leq T_{pref} + \Delta_j + T_{delay}(1 - x_{j0}),$

$t_j - T_{pref} - \Delta_j \leq T_{delay},$

$t_j - d_{j0} \leq T_{pref} + \Delta_j + (T_{max} - T_{pref})(1 - x_{j0}),$

Constraint (3.15) is to ensure that the decision variables is binary.

Consequently, a lifted formulation can be formulated as follows where the objective is formulated as below:

$$(F2) : Minimize(\alpha m + \beta \sum_{j=1}^{s} \Delta j)$$

Subjected to:

(3.6) - (3.12), (3.14) -(3.15), and where we modify constrain(3.13) with constraint(3.16) as below:

$$t_i + d_{ij} - t_j + \lambda_{ji} x_{ji} \leq M_{ij}(1 - x_{ij}), \forall i, j \in S, i \neq j, \quad (3.16)$$

there are four options for x_{ij} and x_{ji}

- when $x_{ij} = x_{ij} = 0$ then

  $t_i - t_j \leq T_{max} - d_{i0}$

- when $x_{ij} = 1$ and $x_{ji} = 0$ then

  $t_j - \Delta j + (T_{max} - T_{Pref})x_{j0} \leq T_{max} - d_{j0}$

- when $x_{ij} = x_{ij} = 1$ is not possible

- when $x_{ij} = 0$ and $x_{ji} = 1$ then

  $t_i = t_j + d_{ji}$

  $t_i - t_j + \lambda_{ji} \leq M_{ij} - d_{ij}$

  $\lambda_{ji} \leq M_{ij} - d_{ij} + t_j - t_i$

  since M_{ij} is a large number

  $\lambda_{ji} = M_{ij} - d_{ij} - d_{ji}$ which is the largest value possible.

CHAPTER 4: EXPERIMENTATIONS AND RESULTS

4.1 Students-School Allocation

4.1.1 Experiments on Different Size Data

The objective of this section is to assess the (F1)tion and solutions for different size data. We consider instances of sizes (zones x schools) (5x5), (10x10), (25x25), (50x50), (100x100), with random generated data of 5 experiments at each size and we compare mainly the needed CPU time to solve We notice from Table (4.1) the following:

- All solutions are optimal,
- We got the minimal solution of zero for 6 cases out of 25 where all students are allocated to their nearest schools.

4.1.2 Experiments on Students-Schools Allocation

Artificial data was generated covering the same parameters in Qatar, where 98 zones were considered with 986 schools, with random number of students in each zone with a random number of vacancies in each school. considering 3211 students to be distributed to 6441 vacancies in schools as per table An optimal solution was obtained with zero delays, and all students were allocated to the nearest school, solution was obtained in around 4hours using IBM Cplex.

4.2 Bus Route Optimization

After solving the first sub-problem and allocating students to schools, we need to solve the second sub-problem of Bus Route optimization

Table 4.1: Computational Experiments on small size data for Students-School allocation

Size	Sample	Cpu Time	Solution
5	1	0.20	64
5	2	0.48	431
5	3	0.55	1,157
5	4	0.5	8
5	5	0.56	0
10	1	0.66	72
10	2	0.48	145
10	3	0.45	44
10	4	0.58	85
10	5	0.55	30
25	1	1.14	60
25	2	1.02	35
25	3	1.08	8
25	4	1.17	38
25	5	1.12	21
50	1	9.48	21
50	2	11.07	7
50	3	10.89	0
50	4	10.00	6
50	5	10.26	30
100	1	144.00	0
100	2	177.85	0
100	3	180.74	0
100	4	171.35	3
100	5	171.67	0

4.2.1 Experiments on Bus Route Optimization

Real data was gathered by an online survey by Ministry of Transport and Communication with the Ministry of Education and Higher Education to collect the feedback from the parents about their children existing transportation to schools.

Since using the MIP model for solving large-scale instances takes a prohibitive amount of time. A heuristic strategy was used to find a good solution within a reasonable time. This heuristic requires formulating the problem on a very sparse sub-graph instead of the original complete graph, we don't need to check all travel time between very distinct points, the longer the distance between two points is less likely this path will be part of the optimal solution,This sub-graph is obtained by connecting each node to the k-nearest stop nodes in addition to the school node. In our experimental study, we set $k = 2$, and 3, respectively..

We choose three schools:

- School 1 : Hafsa Preparatory School for Girls where the students participated in the survey are 133 students, with 46 stops from 18 different zones shown in Figure (4.1).

  We start modeling using the two formulations and taking into consideration the following :

  - Bus Capacity: 30 Students.
  - Maximum Travel Time allowed: 50 minutes.
  - Preferred Travel Time allowed: 45 minutes.
  - All travel time between stops and between stops and school is taken using Google Maps with a departure time of 06:30 am to take into consideration the morning rush hours.

  Also, we define a time limit of 20 minutes for running the model, and we run

Figure 4.1: Hafsa Preparatory School Map

the two formulations using the nearest two-point and the nearest three-point and summarize the results in Table (4.2).

Table 4.2: Solutions for Hafsa Preparatory School

Sn	Stops	Students	Nearest	Formulation	Optimality	Cpu Time	Solution	Buses	Delay
1	46	133	2 points	(F1)	Yes	1.79	5	10	0
				(F2)	Yes	4.61	5	10	0
2	46	133	3 Points	(F1)	Yes	135.28	4.5	9	0
				(F2)	Yes	117.31	4.5	9	0

As per the Table (4.2) and summarized in Figures (4.2), (4.3) we notice the following:

– Both formulations give a similar solution.

– All solutions are optimal.

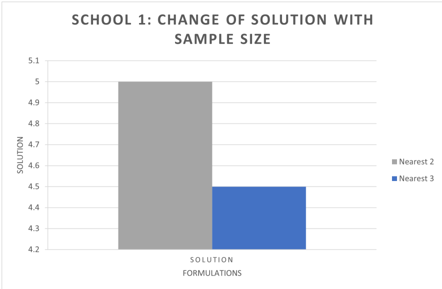

Figure 4.2: School 1 Change of Solution with Sample Size

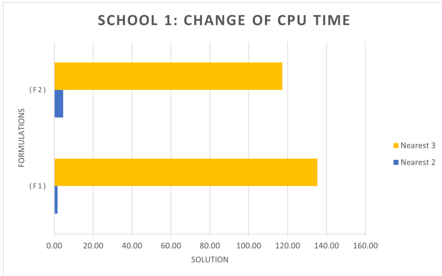

Figure 4.3: School 1 Change of CPU Time

- (F2) shows a better solution time than the (F1).
- The improvement of solutions between nearest two and nearest three is 10%, while the increase of time for (F1) from 2 seconds to 136 seconds and (F2) from 5 seconds to 118 seconds.
- Number of buses required reduced form 10 buses with solution obtained form nearest three stops to 9 buses with solution obtained form nearest two stops with 10% improvement.
- No delays were observed in any of the solutions.

Further experiments were done on weights to see the solution change, where three cases were tested as in Table (4.3).

- $\alpha = 0, \beta = 1$ only Time objective is considered.
- $\alpha = 1, \beta = 0$ only Bus objective is considered, with keeping the hard. constraints of the maximum allowed travel time
- $\alpha = 0.5, \beta = 0.5$ both objectives are considered equally.

As noticed weights of $\alpha = 0.5, \beta = 0.5$ shows a very good balancing between two objectives where no delays are foreseen and number of buses is minimized.

Table 4.3: Weights changes on Solution for School 1

Nearest Neighbour	**Description**	$(\alpha = 0, \beta = 1)$	$(\alpha = 1, \beta = 0)$	$(\alpha = 0.5, \beta = 0.5)$
N2	Delay	0	25	0
	No. Buses	46	9	10
N3	Delay	0	19	0
	No. Buses	46	8	9

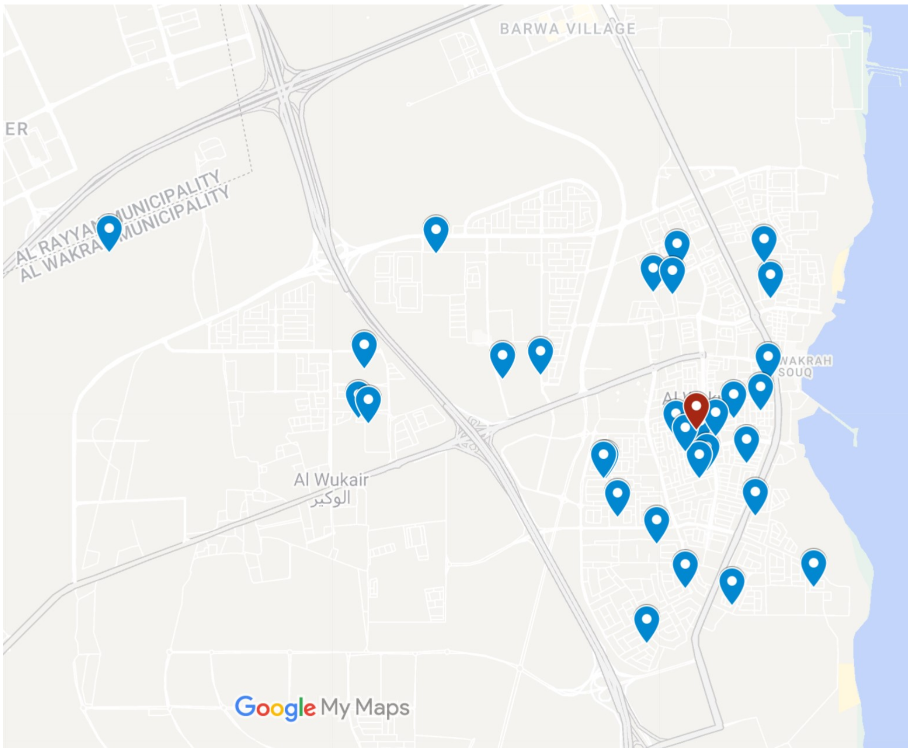

Figure 4.4: Jaber Bin Hayan Primary Independent School for Boys Map

- School 2 :Jaber Bin Hayan Primary Independent School for Boys, where the students participated in the survey are 117 students, with 31 stops from 2 different zones shown in Figure(4.4).

  We start modeling using the two formulations and taking into consideration the following :

  - Bus Capacity: 30 Students.
  - Maximum Travel Time allowed: 50 minutes.

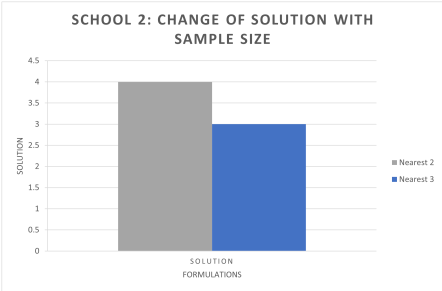

Figure 4.5: School 2 Change of Solution with Sample Size

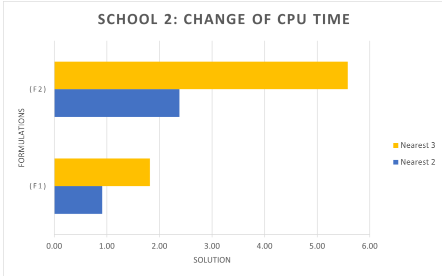

Figure 4.6: School 2 Change of CPU Time

- Preferred Travel Time allowed: 45 minutes.
- All travel time between stops and between stops and school is taken using Google Maps with a departure time of 06:30 am to take into consideration the morning rush hours.

Also, we define a time limit of 20 minutes for running the model, and we run the two formulations using the nearest two-point and the nearest three-point and summarize the results in Table (4.4).

Table 4.4: Solutions for Jaber Bin Hayan Primary Independent School for Boys

Sn	Stops	Students	Nearest	Formulation	Optimality	Cpu Time	Solution	Buses	Delay
1	31	117	2 points	(F1)	Yes	0.91	4	8	0
				(F2)	Yes	1.82	4	8	0
2	31	117	3 Points	(F1)	Yes	2.38	3	6	0
				(F2)	Yes	5.58	3	6	0

As per the Table (4.4) and summarized in Figures (4.5), (4.6) we notice the following:

- Both formulations give a similar solution.
- All solutions are optimal.

- (F2) shows a better solution time than the (F1).
- The improvement of solutions between nearest two and nearest three is 25%, while the increase of time for (F1) from 1 seconds to 2 seconds and (F2) from 2 seconds to 3 seconds.
- Number of buses required reduced form 8 buses with solution obtained form nearest three stops to 6 buses with solution obtained form nearest two stops with 25% improvement.
- No delays were observed in any of the solutions.

Further experiments were done on weights to see the solution change, where three cases were tested as in Table (4.5).

- $\alpha = 0, \beta = 1$ only Time objective is considered.
- $\alpha = 1, \beta = 0$ only Bus objective is considered, with keeping the hard. constraints of the maximum allowed travel time
- $\alpha = 0.5, \beta = 0.5$ both objectives are considered equally.

Table 4.5: Weights changes on Solution for School 2

Nearest Neighbour	**Description**	$(\alpha = 0, \beta = 1)$	$(\alpha = 1, \beta = 0)$	$(\alpha = 0.5, \beta = 0.5)$
N2	Delay	0	1	0
	No. Buses	31	8	8
N3	Delay	0	19	0
	No. Buses	31	5	6

As noticed weights of $\alpha = 0.5, \beta = 0.5$ shows a very good balancing between two objectives where no delays are foreseen and number of buses is minimized.

- School 3 : Birla Public International School, where the students participated in the survey are 496 students, with 281 stops form 40 different zones.

  also we consider to divide the students into three groups due to the bis size of the problem , where the stops are divided as

Figure 4.7: Birla School Group 1 Map

Figure 4.8: Birla School Group 2 Map

1. Group 1: With students in zones (13, 14, 15, 16, 17, 22, 23, 24, 25, 26, 27, 32, 33, 34) with 100 stop to serve 170 students as shown in Figure (4.7).
2. Group 2: With students in zones (35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48) with 104 stop to serve 178 students as shown in Figure (4.8).
3. Group 3: With students in zones (51, 52, 53, 54, 55, 56, 61, 65, 74, 90, 91, 92) with 77 stop to serve 148 students as shown in Figure (4.9).

We start modeling each group separately using the two formulations and taking into consideration the following :

- Bus Capacity: 30 Students.
- Maximum Travel Time allowed: 50 minutes.
- Preferred Travel Time allowed: 45 minutes.
- All travel time between stops and between stops and school is taken using Google Maps with a departure time of 06:30 am to take into consideration the morning rush hours.

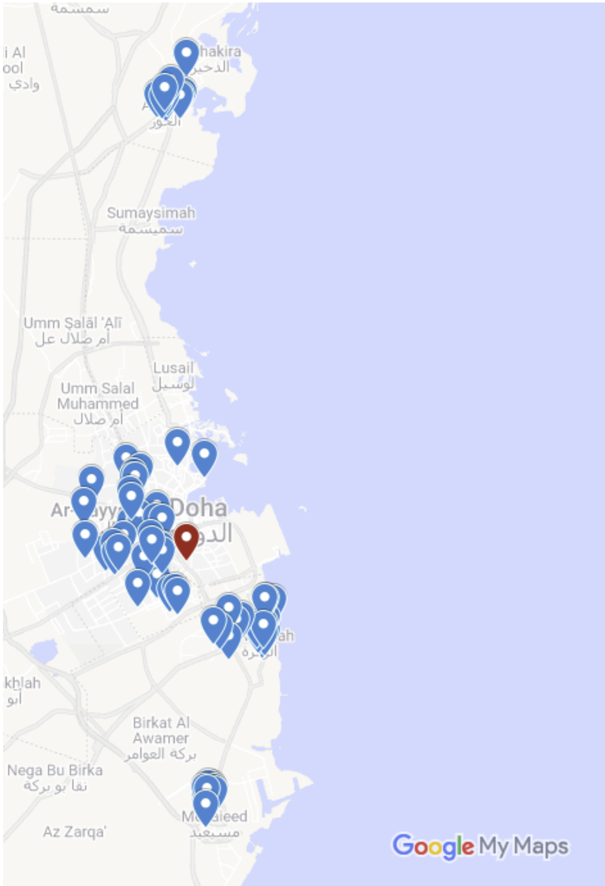

Figure 4.9: Birla School Group 3 Map

Also, we define a time limit of 20 minutes for running the model, and we run the two formulations using the nearest two-point and the nearest three-point and summarize the results in Table (4.6)

Table 4.6: Solutions of Groups for Birla School

Group	Stops	Students	Nearest	Formulation	Optimality	Cpu Time	Solution	Buses	Delay
Group 1	100	170	2 points	(F1)	Yes	5.34	13.5	27	0
				(F2)	Yes	14.09	13.5	27	0
Group 1	100	170	3 Points	(F1)	No	1,204.00	9.5	19	0
				(F2)	No	1279.14	9.5	19	0
Group 2	104	178	2 Points	(F1)	Yes	5.14	10.5	21	0
				(F2)	Yes	37.04	10.5	21	0
Group 2	104	178	3 Points	(F1)	No	1,204.27	6	12	0
				(F2)	No	1279.51	6	12	0
Group 3	77	148	2 Points	(F1)	Yes	3.39	10	20	0
				(F2)	Yes	15.43	10	20	0
Group 3	77	148	3 Points	(F1)	Yes	406.50	6.5	13	0
				(F2)	Yes	813.57	6.5	13	0

As per the Table (4.6) and summarized in Figures (4.10), (4.11), (4.12), (4.13) we notice the following:

- All formulations give a similar solution.
- All solutions with the nearest two stops are optimal while in the nearest three is not the case.
- In all solutions, there is a good improvement in average between the nearest two and nearest three stops.
- (F1) shows a better solution time than the (F2).
- The average improvement of solutions between nearest two and nearest three is 36%, while the increase of time for (F1) from 14 seconds to 2,815 seconds and (F2) from 67 seconds to 3,373 seconds.

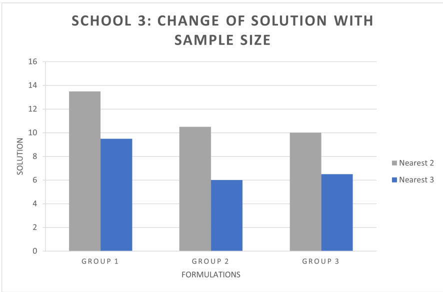

Figure 4.10: School 3 Change of Solution with Sample Size

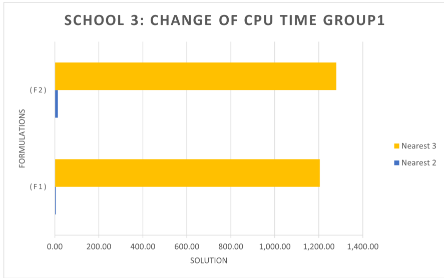

Figure 4.11: School 3 Change of CPU Time Group1

- Number of buses required reduced form 68 buses with solution obtained form nearest two stops to 44 buses with solution obtained form nearest two stops with 35% improvement.
- No delays were observed in any of the solutions.

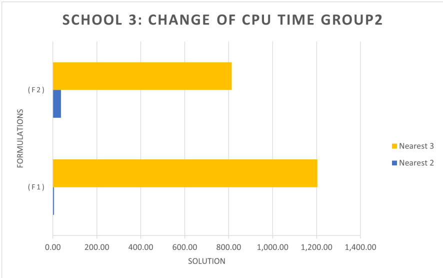

Figure 4.12: School 3: Change of CPU Time Group2

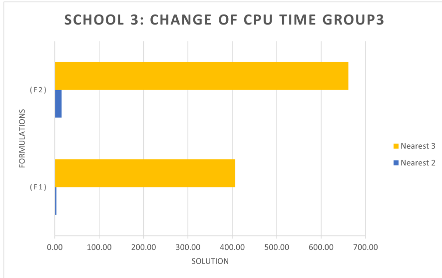

Figure 4.13: School 3 Change of CPU Time Group3

Further experiments were done on weights to see the solution change, where three cases were tested as in Table (4.7).

- $\alpha = 0, \beta = 1$ only Time objective is considered.
- $\alpha = 1, \beta = 0$ only Bus objective is considered, with keeping the hard. constraints of the maximum allowed travel time
- $\alpha = 0.5, \beta = 0.5$ both objectives are considered equally.

Table 4.7: Weights changes on Solution for School 3

Group	**Description**	$(\alpha = 0, \beta = 1)$	$(\alpha = 1, \beta = 0)$	$(\alpha = 0.5, \beta = 0.5)$
Group 1 ,N2	Delay	0	42	0
	No. Buses	100	26	27
Group 1 ,N3	Delay	0	0	0
	No. Buses	100	19	19
Group 2 ,N2	Delay	0	25	0
	No. Buses	104	21	21
Group 2 ,N3	Delay	0	38	0
	No. Buses	104	11	12
Group 3 ,N2	Delay	0	20	0
	No. Buses	77	19	20
Group 3 ,N3	Delay	0	44	0
	No. Buses	77	12	13

As noticed weights of $\alpha = 0.5, \beta = 0.5$ shows a very good balancing between two objectives where no delays are foreseen and number of buses is minimized.

4.2.2 Computational Experiments to Compare Results on Different Formulations

The objective of this section is to compare the efficiency of different formulation while solving several iterations of small sizes problem , for this we consider a problem of sizes 15, 20, 25 with random generated data of 10 experiments at each size and we compare mainly the needed CPU time to solve.

We start modeling each group separately using the different formulations and taking into consideration the following:

- Bus Capacity: 30 Students.
- Maximum Travel Time allowed: 50 minutes.
- Preferred Travel Time allowed: 45 minutes.
- Maximum CPU Time 3600 sec.

as per Tables (4.8) , (4.9) and (4.10) :

- (F1): All solutions are optimal within Time limit of 3,600 sec, Maximum CPU Time for optimal solutions 784 sec, Average CPU Time of optimal solutions 102.5 sec.
- (F2): All solutions are optimal within Time limit of 3,600 sec , Maximum CPU Time for optimal solutions 1,577 sec , Average CPU Time of optimal solutions 104.5 sec.
- By checking the average gap between the minimum CPU time for each instance, we find that average gap for (F1) is 30.7% and for (F2) is 88.4%
  , so (F1) presents a better CPU time.

Table 4.8: Computational Experiments 15-Stop Instances

Size	Sample	Formulation	Cpu Time	Solution	No. Of Buses	Total Delay
15	1	(F1)	0.48	3.5	7	0
		(F2)	1.32	3.5	7	0
15	2	(F1)	0.77	3	6	0
		(F2)	2.02	3	6	0
15	3	(F1)	0.71	2.5	5	0
		(F2)	2.05	2.5	5	0
15	4	(F1)	0.53	3.5	6	1
		(F2)	1.54	3.5	6	1
15	5	(F1)	1.9	2.5	5	0
		(F2)	2.37	2.5	5	0
15	6	(F1)	0.9	3	6	0
		(F2)	1.63	3	6	0
15	7	(F1)	0.9	3	6	0
		(F2)	2.2	3	6	0
15	8	(F1)	0.7	3.5	7	0
		(F2)	1.77	3.5	7	0
15	9	(F1)	0.86	2.5	5	0
		(F2)	2.01	2.5	5	0
15	10	(F1)	1.8	2.5	5	0
		(F2)	3.02	2.5	5	0

Table 4.9: Computational Experiments on 20-Stop Instances

Size	Sample	Formulation	Cpu Time	Solution	No. Of Buses	Total Delay
20	1	(F1)	19.08	3	6	0
		(F2)	18.67	3	6	0
20	2	(F1)	3.33	3	6	0
		(F2)	10.6	3	6	0
20	3	(F1)	18.44	3	6	0
		(F2)	25.88	3	6	0
20	4	(F1)	0.45	5	10	0
		(F2)	3.67	5	10	0
20	5	(F1)	17.04	3	6	0
		(F2)	5.59	3	6	0
20	6	(F1)	343.07	3	6	0
		(F2)	262.81	3.5	7	0
20	7	(F1)	752	3	6	0
		(F2)	478.65	3	6	0
20	8	(F1)	172.98	3	6	0
		(F2)	80.27	3	6	0
20	9	(F1)	30.84	3.5	7	0
		(F2)	29.86	3.5	7	0
20	10	(F1)	110.28	3	6	0
		(F2)	39.28	3	6	0

Table 4.10: Computational Experiments on 25-Stop Instances

Size	Sample	Formulation	Cpu Time	Solution	No. Of Buses	Total Delay
25	1	(F1)	73.88	6.5	13	0
		(F2)	42.81	6.5	13	0
25	2	(F1)	35.97	6.5	13	0
		(F2)	41.21	6.5	13	0
25	3	(F1)	528.25	5.5	9	2
		(F2)	328.51	5.5	9	2
25	4	(F1)	35.97	5	9	1
		(F2)	27.99	5	9	1
25	5	(F1)	63.44	5	9	1
		(F2)	28.16	5	9	1
25	6	(F1)	784.84	4.5	9	0
		(F2)	1,577	4.5	9	0
25	7	(F1)	26.62	6.5	13	0
		(F2)	18.58	6.5	13	0
25	8	(F1)	10.9	6.5	13	0
		(F2)	23.85	6.5	13	0
25	9	(F1)	17.49	6.5	13	0
		(F2)	26.67	6.5	13	0
25	10	(F1)	21	5.5	9	2
		(F2)	35.84	5.5	9	2

4.2.3 Sensitivity Analysis

Sensitivity Analysis to the nearest neighbor points for each stop ”K”

To determine the sensitivity to the nearest neighbor points for each stop K we choose a small problem size of 25 stops and we run sensitivity analysis where we check the changes in optimal solution against changing the number of nearest points picked for each stop, for two, three, four and five nearest stops, and we see improvement of optimal solution as shown in Table 4.11 Optimal solution with against increase of solution time as shown in Table 4.12 and Figure 4.14

Table 4.11: Change of Optimal solution with sample size 25 with different k

Formula	Nearest 2	Nearest 3	Nearest 4	Nearest 5
(F1)	4	3	2.5	2.5
(F2)	4	3	2.5	2.5

Table 4.12: Change of CPU Time with sample size 25 (sec) with different k

Formula	**Nearest 2**	**Nearest 3**	**Nearest 4**	**Nearest 5**
(F1)	0.81	2.59	5.35	7.03
(F2)	1.38	3.21	6.41	12.16

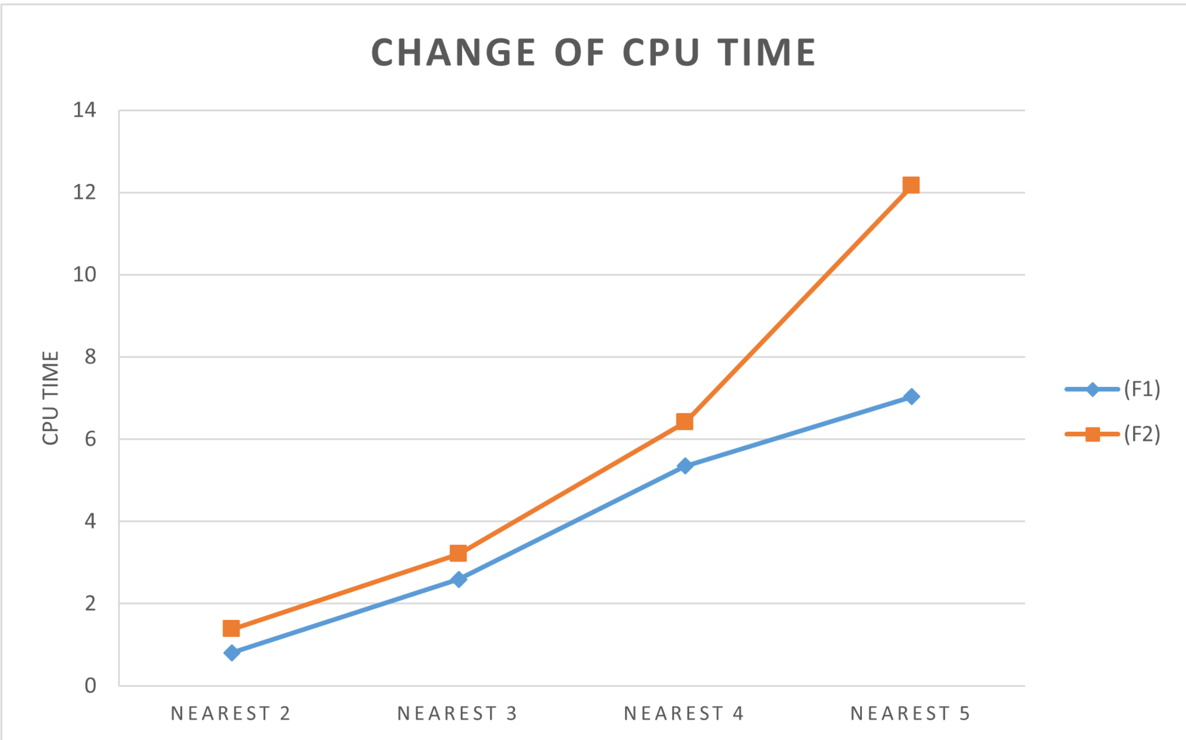

Figure 4.14: Change of CPU Time with sample size 25 (sec)

we notice the following :

- The average increase of CPU time between the nearest two and nearest three is 37 % with an improvement of solution of 25 %
- The average increase of CPU time between the nearest three and nearest four is 49 % with an improvement of solution of 16 %
- The average increase of CPU time between the nearest four and nearest five is 64% with no improvement in the solution

Sensitivity Analysis to the Sample Size

Determining the optimal sample size, to find the suitable sample size we run a sensitivity analysis to check the changes in optimal solution and solution time against the changes in sample size, we take samples of sizes 25, 50, 75, 100, 125, 150 with the nearest two and three points, a time limit is set for 3600 seconds on all runs.

- Samples with the nearest two points , where we find solutions as in Table (4.13) and Figure (4.15) against the CPU time of solution as shown in Table (4.14) and Figure (4.16)

Table 4.13: Change of Solution with Sample Size with nearest two points

Size	**25**	**50**	**75**	**100**	**125**	**150**
(F1)	4	6.5	7.5	13.5	15	17.5
(F2)	4	6.5	7.5	13.5	15	17.5

Table 4.14: CPU solution time for the nearest two points (sec)

Size	**25**	**50**	**75**	**100**	**125**	**150**
(F1)	0.86	2.47	46.18	8.76	11.03	15.35
(F2)	0.74	3.72	20.36	33.14	65.31	112.09

- Samples with the nearest three points where we find solutions as in Table (4.15) and Figure (4.17) against the CPU time of solution as shown in Table (4.16) and Figure (4.18)

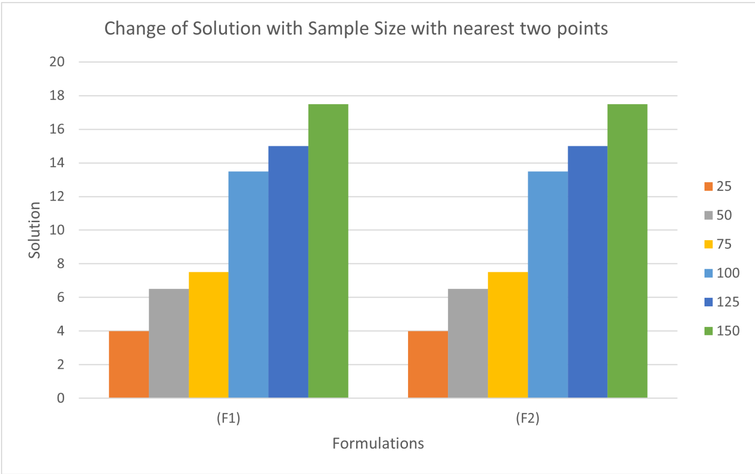

Figure 4.15: Change of Solution with Sample Size with nearest two points

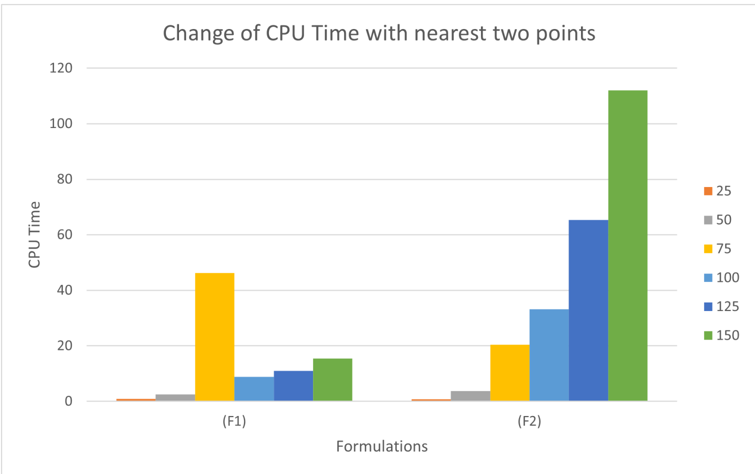

Figure 4.16: Change of CPU Time with nearest two points (sec)

Table 4.15: Change of Solution with Sample Size with nearest three points
* : not optimal solution stopped at time limit

Size	**25**	**50**	**75**	**100**	**125**	**150**
(F1)	3	5	7	9.5*	12*	14*
(F2)	3	5	7	9.5*	12*	14*

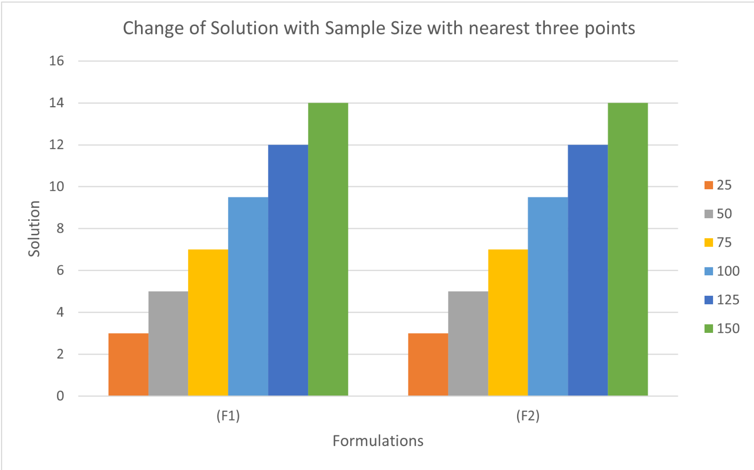

Figure 4.17: Change of solution with nearest three points (sec)

Table 4.16: CPU solution time for the nearest three points (sec)
* : not optimal solution stopped at time limit

Size	**25**	**50**	**75**	**100**	**125**	**150**
(F1)	3.43	688.79	1888.79	3600*	3600*	3600*
(F2)	4.37	607.19	163.29	3600*	3600*	3600*

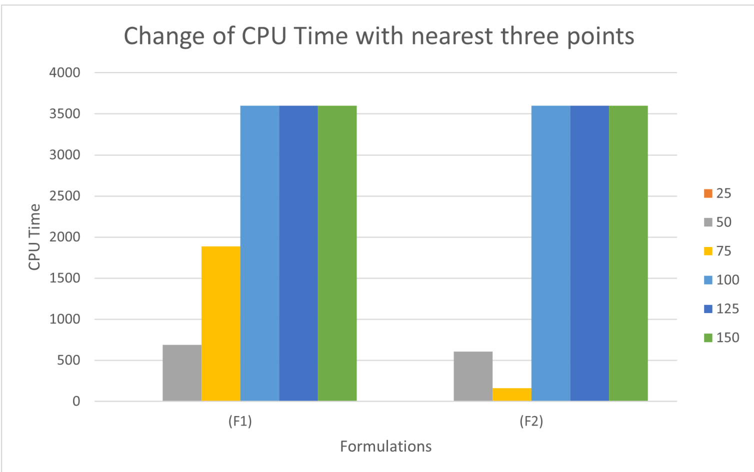

Figure 4.18: Change of CPU Time with nearest three points (sec)

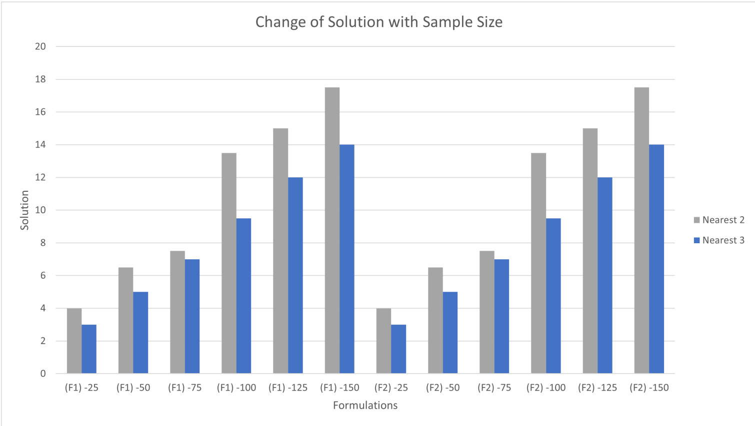

Figure 4.19: Change of Solution with Sample Size

It is observed that for sizes above 100 no optimal solution could be obtained with the nearest three points however with a long time limit suggested of one hour, we still get an improvement over the solution obtained with the nearest two points with an overall 21 % average improvement.

Sensitivity Analysis to the Time Bound

we check the sensitivity of the model to the preferred, we take a sample size of 25 with $K = 3$ and the time as follows:

Sensitivity to T_{pref}

T_{pref} = 30, 40, 50, 60 with T_{max}= 100

Table 4.17: Change of Optimal solution with T_{pref}

T_{pref}	**30**	**40**	**50**	**60**
Optimal Solution	13	3.5	2.5	2

It is observed that the higher the T_{pref} the better is the solution and the CPU time is shorter, specially at the change form 30 to 40 where there is an improvement of 73% in solution as shown in Table (4.17) and Figure (4.20) and average 75% in CPU Time as shown in Table (4.18) and Figure (4.21).

Table 4.18: Change of CPU Time with T_{pref} (sec)

T_{pref}	**30**	**40**	**50**	**60**
(F1)	14.49	4.93	1.63	0.53
(F2)	9.85	4.4	3.14	1.62

Figure 4.20: Change of Solution with T_{pref}

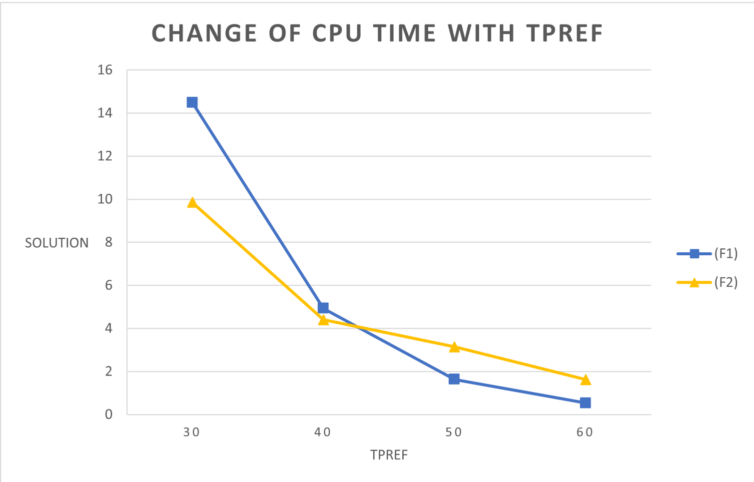

Figure 4.21: Change of CPU Time with T_{pref}

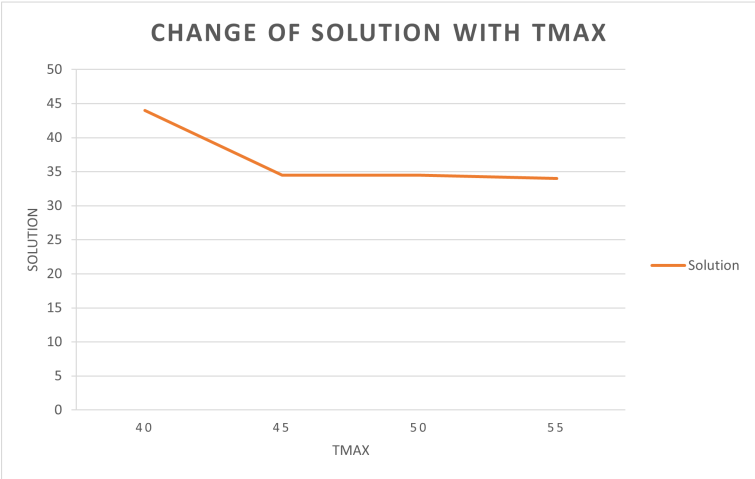

Figure 4.22: Change of Solution with T_{pref}

Sensitivity to T_{pref}

$T_{max} = 40, 45, 50, 55$ with $T_{pref} = 25$

Table 4.19: Change of Optimal solution with T_{max}

T_{max}	**40**	**45**	**50**	**55**
Optimal Solution	44	34.5	34.5	34

It is observed that the higher the T_{max} the better is the solution as shown in Table (4.19) and Figure (4.22 however these improvement are smaller than shown in T_{pref} comparison, but CPU Time shows a great increase , as shown in Table (4.20) and Figure (4.23).

From above we see that T_{pref} has bigger effect on the solution with much lower effect on the CPU Time however it is very important for the objectives of our study to maintain it within acceptable limits as it will be a very important criteria for the students

Table 4.20: Change of CPU Time with T_{max} (sec)

T_{max}	**40**	**45**	**50**	**55**
(F1)	0.6	3.17	12.07	9.47
(F2)	3.09	27.11	293.47	160.5

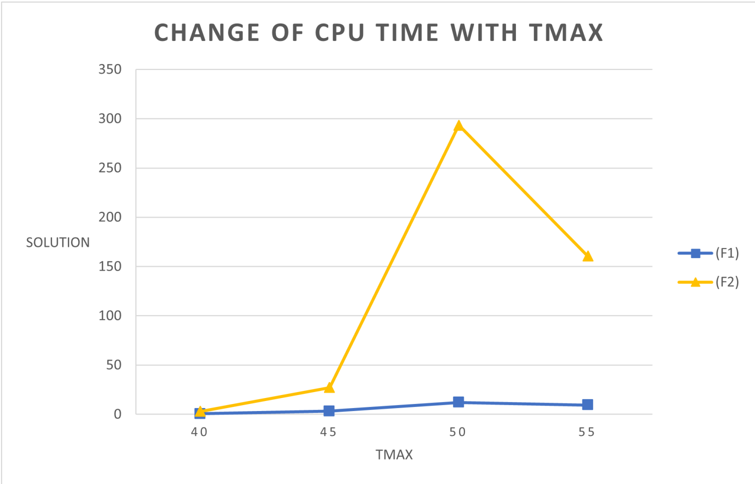

Figure 4.23: Change of CPU Time with T_{max}

and parents to take the decision to choose the bus transportation instead of private cars.

CHAPTER 5: CONCLUSION AND FUTURE WORK

The research provided an effective systematic methodology for planning and optimizing a Public Transportation system to support the schools and the students. It provided solutions that balanced between the economical and social requirements where it proofed that costs can be minimized while maintaining a reasonable acceptable travel time for each student, this should encourage the parents and the students to use more public transportation system to the schools.

Using Heuristic with Mixed Integer Programming with IBM Cplex software has given an optimal solution in most of cases, with very reasonable solution time, this will be a big leap comparing to the current manual planning methodology, where there is no records and no measurements could be done to understand how efficient being done.

Three real schools examples were thoroughly studied, it took into consideration the morning rush hours that increases significantly the travel time so all travel times were taken in real scenario from Google Maps at 06:30 am to make sure the limits on travel times are not exceeded in real life application, the examples testing showed a significant improvement on using private car transportation which is mostly now used with a very reasonable solution time and can be summarized as follows:

- School 1 (Hafsa Preparatory School for Girls): With 133 students, with 46 stops from 18 different zones currently using 46 vehicles and by implementing suggested methodology reduced to 9 buses with solution time of 118 seconds.
- School 2 (Jaber Bin Hayan Primary Independent School for Boys): With 117 students, with 31 stops from 2 different zones currently using 31 vehicles and by implementing suggested methodology reduced to 6 buses with solution time of 3 seconds.
- School 3 (Birla Public International School): With 496 students, with 281 stops

from 40 different zones currently using 281 vehicles and by implementing suggested methodology reduced to 44 buses with solution time of 2,815 seconds.

These examples showed the importance of proper planning and should help encouraging the students and parents to use more and more the public buses.

current study, only morning trips were considered, so for future researchers, both morning and afternoon trips should be considered to confirm routes and buses required would be enough as traffic might change from morning to afternoon. In addition, bus capacity is not fully occupied, and this can be solved by considering future research fleets with different bus capacities to assign a suitable bus to each route. Further multiple loading where students from different schools can use the same bus or the same bus can do more than one trip if the first trip is short and second trip can be completed before the school bell time.

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APPENDIX A: DATA COLLECTION FORMAT

Data was gathered by an online survey by Ministry of Transport and Communication with the Ministry of Education and Higher Education, the survey included data on school details, and Parents work details, as shown in Figure(5.1).

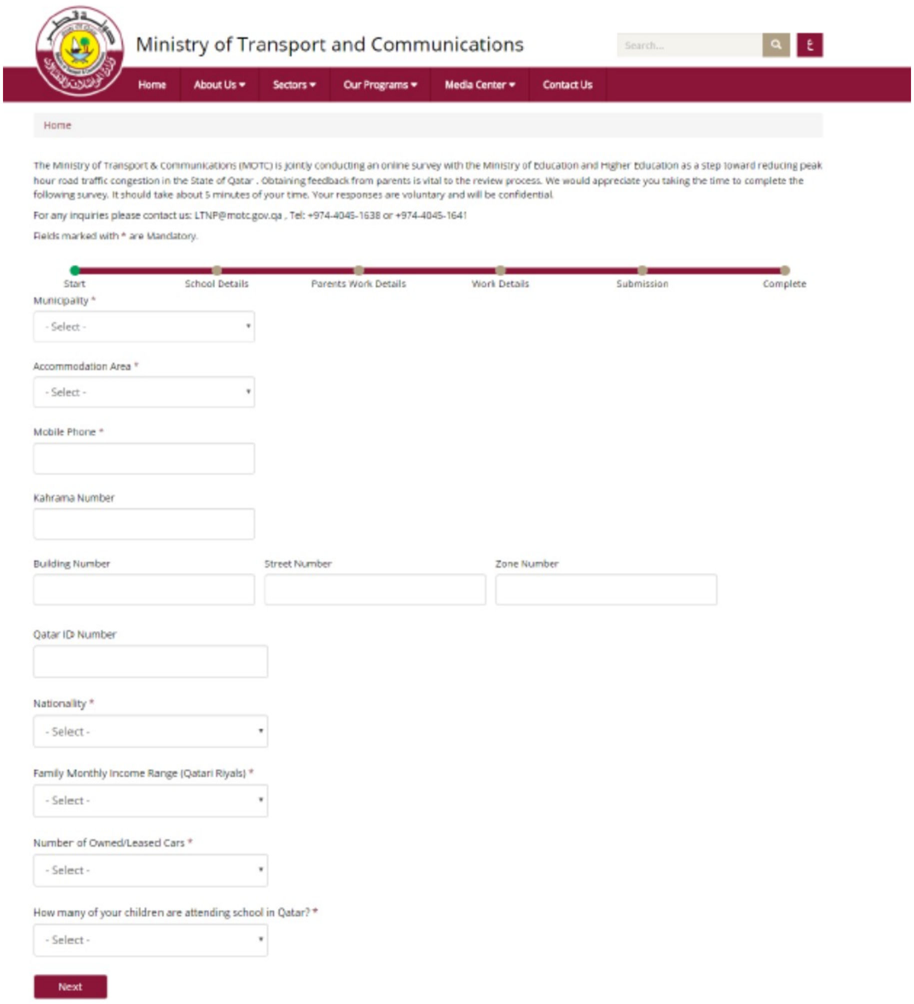

Figure 5.1: Data Collection Format

APPENDIX B: HAFSA PREPARATORY SCHOOL FOR GIRLS DATA

Data was collected for Hafsa school including each stop address, coordination, and number of students to be picked up form this stop, as shown in Table(5.1).

Table 5.1: Hafsa Preparatory School for Girls Data

Sn	Zone	Street	Building	No. of Students	Latitude	Longitude
1	13	830	14	2	25.285546	51.522328
2	15	820	5	3	25.326334	51.533591
3	16	220	46	3	25.278227	51.541603
4	16	820	38	8	25.279733	51.536768
5	23	834	39	3	25.281653	51.512269
6	23	847	14	4	25.280009	51.508963
7	24	846	6	3	25.267728	51.521615
8	24	876	4	3	25.267752	51.521836
9	24	941	12	6	25.274426	51.520603
10	25	810	9	3	25.273449	51.531628
11	25	853	6	2	25.268498	51.534117
12	25	878	16	3	25.264678	51.528304
13	25	885	33	3	25.263573	51.532507
14	25	950	47	3	25.268569	51.533439
15	26	880	39	2	25.265458	51.541742
16	26	952	8	3	25.266323	51.541523
17	27	840	54	2	25.28125	51.547813
18	27	930	68	1	25.27666	51.549836

Sn	Zone	Street	Building	No. of Students	Latitude	Longitude
19	34	847	2	2	25.316257	51.477792
20	38	904	27	1	25.285593	51.504164
21	38	960	57	4	25.287928	51.496184
22	39	811	67	2	25.274408	51.491266
23	39	820	149	1	25.27733	51.498231
24	39	934	4	2	25.275154	51.500073
25	39	970	22	2	25.277661	51.496217
26	40	829	1	4	25.262717	51.511264
27	40	850	31	4	25.261384	51.521343
28	40	905	22	4	25.262631	51.52251
29	40	915	9	1	25.262791	51.517488
30	40	949	26	4	25.256589	51.514611
31	40	995	38	4	25.262472	51.500392
32	42	850	63	4	25.259573	51.54653
33	43	750	102	2	25.2512	51.500889
34	43	879	20	3	25.241323	51.519039
35	44	915	8	2	25.242973	51.538683
36	45	620	11	3	25.250353	51.540685
37	45	703	52	1	25.243179	51.550967
38	45	703	52	1	25.243181	51.550967
39	45	851	11	1	25.250083	51.557299
40	45	851	12	3	25.249284	51.554851
41	45	851	12	5	25.250302	51.557375
42	45	891	3	3	25.243122	51.555873
43	45	965	2	1	25.257402	51.553203

Sn	Zone	Street	Building	No. of Students	Latitude	Longitude
44	54	514	4	5	25.276745	51.478934
45	56	411	61	3	25.253251	51.484173

APPENDIX C: JABER BIN HAYAN PRIMARY INDEPENDENT SCHOOL FOR BOYS DATA

Data was collected for Jaber Bin Hayan school including each stop address, coordination, and number of students to be picked up form this stop, as shown in Table(5.2).

Table 5.2: Jaber Bin Hayan Primary Independent School for Boys Data

Sn	**Zone**	**Street**	**Building**	**No. of Students**	**Latitude**	**Longitude**
1	90	748	4	2	25.1622	51.5939
2	90	509	5	5	25.1528	51.6045
3	90	647	6	3	25.1645	51.6015
4	90	960	8	15	25.1692	51.6062
5	90	868	9	3	25.1791	51.6066
6	90	1016	10	4	25.1526	51.5859
7	90	658	12	11	25.1612	51.5972
8	90	678	13	2	25.1581	51.598
9	90	1113	13	2	25.1372	51.5899
10	91	710	15	4	25.1645	51.5511
11	91	921	16	3	25.1707	51.552
12	91	908	19	3	25.1639	51.5524
13	90	1026	22	3	25.1493	51.5913
14	90	674	24	6	25.1592	51.6032
15	90	688	26	4	25.1572	51.597
16	90	747	31	1	25.1604	51.595
17	90	948	31	1	25.1835	51.6056
18	90	628	32	2	25.1655	51.6052

Sn	Zone	Street	Building	No. of Students	Latitude	Longitude
19	90	993	32	4	25.1624	51.5992
20	90	716	46	3	25.1828	51.594
21	90	534	47	4	25.1418	51.6014
22	90	786	55	3	25.18	51.5908
23	90	900	224	1	25.1847	51.5176
24	90	786	94	2	25.1796	51.5934
25	90	321	98	2	25.1441	51.6125
26	90	1037	142	4	25.1439	51.5951
27	90	1014	192	4	25.1572	51.5844
28	91	300	193	3	25.1846	51.5615
29	90	1014	196	8	25.1572	51.5841
30	91	200	244	4	25.1699	51.5756
31	91	212	429	1	25.1693	51.5705

APPENDIX D: BIRLA PUBLIC INTERNATIONAL SCHOOL DATA

Data was collected for Birla Public International School including each stop address, coordination, and number of students to be picked up form this stop, as shown in Table(5.3).

Table 5.3: Birla Public International School Data

Sn	**Group**	**Zone**	**Street**	**Building**	**No. of Students**	**Latitude**	**Longitude**
1	1	13	802	9	1	25.28848	51.51839
2	1	13	805	35	2	25.28774	51.52084
3	1	13	840	19	1	25.28305	51.51949
4	1	13	851	14	1	25.28293	51.51903
5	1	13	860	42	1	25.28182	51.52107
6	1	13	950	7	3	25.28448	51.52002
7	1	14	815	8	2	25.27908	51.52383
8	1	14	828	12	1	25.2776	51.52081
9	1	14	910	47	2	25.27796	51.52631
10	1	14	910	68	2	25.27633	51.52672
11	1	14	915	41	2	25.27533	51.52614
12	1	14	920	86	2	25.27579	51.52514
13	1	15	820	33	1	25.27789	51.53333
14	1	15	850	63	1	25.27637	51.53365
15	1	15	860	19	1	25.27535	51.53682
16	1	15	910	9	1	25.27618	51.53754
17	1	16	310	21	2	25.27734	51.53803
18	1	16	810	1	1	25.28124	51.54376

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
19	1	16	850	11	2	25.27916	51.54191
20	1	16	930	67	2	25.27722	51.53965
21	1	16	980	19	1	25.28021	51.53816
22	1	16	985	4	1	25.27951	51.53793
23	1	17	820	58	4	25.28581	51.54613
24	1	17	950	26	1	25.28384	51.54544
25	1	22	835	6	1	25.2876	51.51312
26	1	22	895	6	2	25.28488	51.51626
27	1	22	930	56	2	25.28502	51.51411
28	1	22	933	8	3	25.28665	51.51323
29	1	22	980	3	2	25.28672	51.50813
30	1	23	940	19	2	25.27887	51.51329
31	1	23	940	23	1	25.27855	51.51338
32	1	23	945	2	3	25.2841	51.51149
33	1	23	960	66	1	25.28063	51.50982
34	1	23	975	47	3	25.27648	51.51067
35	1	24	873	22	2	25.26821	51.5202
36	1	24	873	25	1	25.26802	51.52019
37	1	24	905	44	1	25.27331	51.52638
38	1	24	930	12	2	25.26959	51.52227
39	1	25	810	41	1	25.27365	51.52905
40	1	25	828	7	1	25.27106	51.53156
41	1	25	834	7	2	25.27003	51.52796
42	1	25	845	11	1	25.2695	51.52865
43	1	25	848	26	1	25.26905	51.53185

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
44	1	25	851	3	1	25.26879	51.53566
45	1	25	870	8	1	25.26727	51.53739
46	1	25	874	51	1	25.26602	51.52753
47	1	25	885	25	2	25.26376	51.53196
48	1	25	885	33	1	25.26357	51.5325
49	1	25	889	1	1	25.26476	51.52792
50	1	25	898	8	3	25.26423	51.53725
51	1	25	950	3	2	25.2742	51.53313
52	1	25	950	68	1	25.2669	51.53346
53	1	25	964	15	3	25.26917	51.53147
54	1	25	970	49	1	25.27105	51.53084
55	1	25	976	27	2	25.26634	51.53021
56	1	25	984	31	6	25.26629	51.52958
57	1	25	984	32	2	25.26629	51.52936
58	1	25	985	15	2	25.26752	51.52889
59	1	25	985	23	2	25.26692	51.52892
60	1	26	320	1	2	25.28124	51.5438
61	1	26	875	23	1	25.26554	51.53934
62	1	26	880	70	3	25.26513	51.53949
63	1	26	882	10	1	25.26714	51.54582
64	1	26	890	28	1	25.26497	51.54418
65	1	26	890	69	1	25.26385	51.54128
66	1	26	890	90	3	25.2646	51.54017
67	1	26	931	14	2	25.2735	51.53776
68	1	26	931	23	1	25.2742	51.53745

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
69	1	26	932	9	1	25.2742	51.53869
70	1	26	935	46	2	25.26816	51.54514
71	1	26	940	30	1	25.26679	51.54518
72	1	26	950	140	1	25.26634	51.54257
73	1	26	970	80	1	25.26614	51.54063
74	1	26	986	30	2	25.2708	51.53819
75	1	26	990	56	1	25.26551	51.53817
76	1	27	810	109	1	25.28127	51.54614
77	1	27	820	39	2	25.28233	51.54881
78	1	27	830	90	1	25.28137	51.54544
79	1	27	880	70	2	25.27736	51.54719
80	1	27	898	29	4	25.27582	51.55164
81	1	27	898	55	1	25.27592	51.55053
82	1	27	925	18	1	25.28182	51.55006
83	1	27	930	72	1	25.27635	51.54989
84	1	27	939	1	2	25.28123	51.54878
85	1	27	939	4	1	25.28097	51.54866
86	1	27	950	10	1	25.28107	51.55016
87	1	27	950	24	1	25.28088	51.54902
88	1	27	990	5	3	25.27653	51.54335
89	1	32	829	3	1	25.32978	51.47419
90	1	32	831	20	2	25.33076	51.47242
91	1	32	935	27	2	25.33252	51.47487
92	1	32	958	46	2	25.32883	51.472
93	1	33	834	7	3	25.32627	51.4898

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
94	1	34	370	313	1	25.32203	51.47681
95	1	34	804	46	2	25.32186	51.4844
96	1	34	804	54	4	25.32203	51.48368
97	1	34	811	9	2	25.31981	51.47613
98	1	34	811	10	1	25.31995	51.4761
99	1	34	811	62	3	25.31876	51.47225
100	1	34	860	127	1	25.31361	51.4768
101	2	35	945	12	2	25.31319	51.49108
102	2	35	962	1	2	25.31305	51.4896
103	2	36	830	7	1	25.30321	51.48806
104	2	37	244	138	2	25.30392	51.48922
105	2	37	361	241	2	25.29608	51.49861
106	2	37	804	46	1	25.3075	51.49282
107	2	37	853	23	1	25.30194	51.49889
108	2	37	854	7	1	25.30081	51.4971
109	2	37	970	14	1	25.30718	51.49176
110	2	37	970	61	1	25.30397	51.49333
111	2	38	803	29	2	25.28945	51.50298
112	2	38	804	4	1	25.28996	51.50382
113	2	38	810	51	2	25.28968	51.49775
114	2	38	818	23	1	25.28763	51.50407
115	2	38	902	4	2	25.28741	51.50482
116	2	39	829	6	4	25.2747	51.49239
117	2	39	933	4	1	25.27738	51.50383
118	2	39	972	5	1	25.27624	51.49691

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
119	2	40	330	126	1	25.26094	51.52393
120	2	40	830	65	1	25.26319	51.51046
121	2	40	831	8	2	25.2637	51.51259
122	2	40	835	14	2	25.26393	51.51663
123	2	40	878	15	2	25.25737	51.51526
124	2	40	915	49	2	25.26235	51.51772
125	2	40	926	20	2	25.26398	51.51442
126	2	40	927	9	2	25.25854	51.51515
127	2	40	927	11	4	25.25843	51.51524
128	2	40	969	38	2	25.26178	51.50912
129	2	40	983	23	6	25.25994	51.50486
130	2	40	992	29	2	25.27582	51.55164
131	2	40	993	3	1	25.26005	51.50219
132	2	41	815	10	2	25.26219	51.53236
133	2	41	855	5	1	25.25793	51.53169
134	2	41	940	50	4	25.2594	51.5309
135	2	42	820	26	2	25.26537	51.55021
136	2	42	828	21	1	25.2597	51.54179
137	2	42	880	64	2	25.25723	51.54821
138	2	42	915	19	4	25.26471	51.55172
139	2	42	952	4	2	25.2591	51.54625
140	2	43	601	26	1	25.26072	51.49559
141	2	43	608	19	1	25.25924	51.49833
142	2	43	761	47	1	25.25332	51.49505
143	2	43	763	15	2	25.25736	51.49412

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
144	2	43	801	17	2	25.25317	51.5233
145	2	43	820	144	4	25.25153	51.51388
146	2	43	875	28	1	25.24421	51.51828
147	2	43	906	63	1	25.24229	51.52151
148	2	43	960	92	1	25.24689	51.52366
149	2	44	812	7	1	25.2511	51.52707
150	2	44	865	2	1	25.24462	51.53629
151	2	44	894	22	1	25.23922	51.53091
152	2	44	917	37	1	25.24097	51.542
153	2	44	962	36	2	25.24295	51.53105
154	2	44	970	13	2	25.25309	51.53094
155	2	45	675	67	1	25.24232	51.54663
156	2	45	720	22	2	25.24868	51.54931
157	2	45	770	97	2	25.24598	51.54584
158	2	45	810	9	1	25.25783	51.5602
159	2	45	814	5	1	25.25639	51.55418
160	2	45	818	11	1	25.25447	51.548
161	2	45	821	75	1	25.25395	51.55467
162	2	45	825	13	1	25.2547	51.55821
163	2	45	831	3	2	25.25446	51.55084
164	2	45	837	28	2	25.25233	51.55462
165	2	45	842	35	1	25.25159	51.55797
166	2	45	843	91	2	25.24971	51.55426
167	2	45	846	14	1	25.25144	51.56006
168	2	45	850	30	1	25.24964	51.55723

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
169	2	45	851	13	2	25.25	51.55712
170	2	45	853	14	2	25.24938	51.55413
171	2	45	855	3	2	25.24948	51.55851
172	2	45	855	13	2	25.24914	51.55777
173	2	45	855	28	1	25.24927	51.55769
174	2	45	860	28	1	25.24918	51.56338
175	2	45	861	32	1	25.24794	51.55802
176	2	45	865	16	2	25.24811	51.55707
177	2	45	914	42	1	25.25159	51.56306
178	2	45	915	21	2	25.25248	51.56241
179	2	45	915	42	2	25.2509	51.563
180	2	45	922	16	1	25.25548	51.56038
181	2	45	925	31	3	25.25131	51.56165
182	2	45	925	39	1	25.25081	51.56197
183	2	45	927	21	1	25.24924	51.56094
184	2	45	931	8	3	25.25469	51.5598
185	2	45	937	1	3	25.25822	51.55942
186	2	45	949	4	2	25.25506	51.55791
187	2	45	950	79	2	25.25443	51.5564
188	2	45	951	9	2	25.24868	51.55834
189	2	45	986	38	2	25.24809	51.55238
190	2	45	998	5	2	25.25312	51.54916
191	2	46	801	12	3	25.23751	51.54212
192	2	46	814	40	1	25.23433	51.54302
193	2	46	832	6	2	25.22978	51.54799

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
194	2	46	855	7	1	25.22714	51.53892
195	2	46	856	6	1	25.2266	51.53881
196	2	46	925	39	2	25.23296	51.53951
197	2	47	819	13	1	25.23729	51.55625
198	2	47	897	24	2	25.22912	51.56274
199	2	47	915	32	1	25.23605	51.56917
200	2	47	928	16	2	25.23269	51.56851
201	2	47	944	34	2	25.23027	51.56569
202	2	47	964	4	1	25.23922	51.55134
203	2	48	935	7	2	25.2684	51.55204
204	2	48	935	9	1	25.26821	51.55185
205	3	51	693	4	3	25.32254	51.45245
206	3	52	845	5	1	25.31453	51.46875
207	3	52	861	15	2	25.30939	51.46083
208	3	52	964	60	1	25.30585	51.45955
209	3	52	967	45	2	25.30608	51.46213
210	3	53	521	6	1	25.30195	51.41608
211	3	53	611	53	2	25.28055	51.40717
212	3	54	364	466	1	25.26239	51.45668
213	3	54	527	14	1	25.2708	51.46946
214	3	54	543	65	6	25.27663	51.48576
215	3	54	776	29	3	25.26931	51.46631
216	3	54	839	3	1	25.29168	51.45788
217	3	54	929	29	1	25.28518	51.45853
218	3	55	19	32	1	25.2653	51.48492

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
219	3	55	25	57	1	25.26438	51.49183
220	3	55	83	95	2	25.24849	51.45055
221	3	55	280	438	2	25.23634	51.43055
222	3	55	410	71	2	25.24135	51.43981
223	3	55	480	72	2	25.23196	51.44012
224	3	55	482	3	1	25.23559	51.44503
225	3	55	875	86	2	25.2479	51.40945
226	3	56	115	12	1	25.21024	51.4861
227	3	56	367	4	2	25.22785	51.4729
228	3	56	422	45	1	25.24359	51.48318
229	3	56	432	7	2	25.24103	51.48529
230	3	56	520	36	1	25.23863	51.4927
231	3	56	543	7	1	25.23845	51.49015
232	3	56	561	23	1	25.24997	51.47893
233	3	56	569	103	6	25.23336	51.49139
234	3	56	570	132	1	25.24331	51.48111
235	3	56	952	24	3	25.20108	51.46565
236	3	56	1146	2	1	25.19657	51.49934
237	3	56	1151	3	2	25.1978	51.50515
238	3	56	1170	5	3	25.1941	51.5082
239	3	61	803	4	2	25.32685	51.53753
240	3	65	970	37	6	25.33832	51.50845
241	3	74	213	239	2	25.71411	51.51746
242	3	74	617	47	2	25.67711	51.51514
243	3	74	631	126	2	25.67185	51.49622

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
244	3	74	634	3	2	25.67502	51.51263
245	3	74	635	41	2	25.67412	51.51117
246	3	74	645	24	1	25.67597	51.49764
247	3	74	651	15	1	25.67531	51.48633
248	3	74	746	49	1	25.67318	51.49126
249	3	74	825	20	2	25.68808	51.5005
250	3	74	869	118	3	25.68061	51.4945
251	3	90	201	50	2	25.17204	51.60321
252	3	90	313	76	6	25.15637	51.60475
253	3	90	392	7	2	25.18638	51.61144
254	3	90	405	15	1	25.15214	51.60035
255	3	90	406	29	3	25.15194	51.59913
256	3	90	624	52	2	25.16831	51.60409
257	3	90	647	10	1	25.16447	51.6012
258	3	90	666	12	1	25.16072	51.59947
259	3	90	678	13	2	25.15814	51.59801
260	3	90	689	29	2	25.15542	51.59579
261	3	90	697	14	2	25.15404	51.60148
262	3	90	749	93	1	25.15703	51.59458
263	3	90	750	20	2	25.16204	51.59278
264	3	90	843	3	2	25.18794	51.60215
265	3	90	843	14	1	25.18739	51.60205
266	3	90	992	63	1	25.16175	51.60029
267	3	91	200	241	1	25.16987	51.57622
268	3	91	212	471	2	25.1665	51.57127

Sn	Group	Zone	Street	Building	No. of Students	Latitude	Longitude
269	3	91	231	36	1	25.17702	51.56348
270	3	91	900	166	2	25.14979	51.56285
271	3	91	908	87	1	25.16073	51.55376
272	3	91	965	1	1	25.16554	51.54607
273	3	92	401	51	1	24.99868	51.53897
274	3	92	410	7	1	24.99853	51.53749
275	3	92	428	32	1	25.00589	51.54532
276	3	92	448	19	6	25.00688	51.5414
277	3	92	482	7	2	25.00337	51.54646
278	3	92	501	46	2	25.00102	51.54847
279	3	92	515	6	4	25.00382	51.53747
280	3	92	523	29	2	25.00234	51.5392
281	3	92	633	29	1	24.98683	51.53869