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Run Rules-Based EWMA Charts for Efficient Monitoring of Profile Parameters

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ABSTRACT In usual quality control methods, the quality of a process or product is evaluated by monitoring one or more quality characteristics using their corresponding distributions. However, when the quality characteristic is defined through the relationship between one or more response and independent variables, the regime is referred to as profiles monitoring. In this article, we improve the performance of the Exponentially Weighted Moving Average Range (EWMAR) control charts, which are implemented for monitoring linear profiles (i.e., intercept, slope and average residual between sample and reference lines) by integrating them with run rules in order to quickly detect various magnitudes of shifts in profile parameters. The validation of the proposed control chart is accomplished by examining its performance using the average run length (ARL) criteria. The proposed EWMAR chart with run rules exhibits a much better performance in detecting small and decreasing shifts than the other competing charts. Finally, an example from multivariate manufacturing industry is employed to illustrate the superiority of the EWMAR chart with run rules.

INDEX TERMS Control chart, linear profiles, phase II, profile monitoring, run rules scheme.

I. INTRODUCTION

Statistical Process Control (SPC) is widely utilized to monitor industrial processes and most of the academic researchers in SPC focus on charting techniques; see the introductory chapters of Montgomery [1] or.Chakraborti and Graham [2]. It is usually assumed that the quality of a process or product can be sufficiently demonstrated by distributing a qualitative characteristic. These quality characteristics are most monitored by some univariate control chart; however, when they are described by the distribution of multiple qualitative characteristics, then they are monitored by a multivariate control chart. Most of the control charts in the SPC literature are based on monitoring location, variability or the latter two jointly. Note though, when the quality of a process or product is described by the relationship between a response variable and one or more independent variables, this is referred to as profile monitoring; see Woodall *et al.* [3]. The exact time for the origin of profile monitoring is not known in the literature and some researchers drew on different terms to describe it; for instance, the term ''*signature*'' was introduced by Gardner *et al.* [4] while Jin and Shi [5] used the ''*waveform signals*'' term in monitoring functional relationships. In fact, these terms are equivalent to the profile monitoring method in a way. In various situations and applications, the profiles relationship can be represented by linear [6], multichannel profiles [7], Gaussian process [8], nonlinear [9] and even a complex relationship [10].

Like usual quality control methods, profile monitoring is conducted in two phases entailing Phases I and II. Evaluating process stability and estimating the in-control (IC) process parameters are the main goals in Phase I. In Phase II, on the other hand, it is important to monitor the process and identify the Out-of-Control (OC) situations as soon as possible. This goal is usually measured by the Average Run Length (ARL) which is defined as the number of samples taken to observe an OC signal; when the process is IC (OC), it is denoted by ARL_0 (ARL_1), respectively. For more details, the interested reader is referred to the review articles provided by Woodall [11] and Maleki *et al.* [12] on profile monitoring.

Generally speaking, linear profiles are classified into three main groups based on the number of response and independent variables, i.e., simple, multiple and multivariate. Simple linear profiles are the simplest one in which the relationship

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between a response and an independent variable is assumed linear. In the multiple linear profiles, there is a multiple linear relationship between one response variable and several independent variables. Note that in univariate models, there is one response variable; however, multivariate models have more than one response variable. The aim of this study is to investigate the process shifts in simple, multiple and multivariate linear profiles in Phase II. Note that the other types of profiles (including nonlinear profiles) will be considered in a different study in the future.

Stover and Brill [13] implemented Hotelling's T^2 and univariate charts based on the first principal component of the vectors of the regression parameters estimation for determining the response stability of a calibration tool and the optimal calibration frequency Kang and Albin [6] proposed two control charts for monitoring Phase II of the linear profiles, the first one is the multivariate T^2 chart and the second is a combination of the Exponentially Weighted Moving Average (EWMA) chart using a Range (R) charting statistic (denoted as EWMAR) Kim *et al.* [14] proposed a method based on a combination of three EWMA (EWMA3) charts to identify the shifts in intercept, slope and standard deviation. Simulation studies showed that the EWMA3 chart perform better in detecting sustained shifts in parameters than the T^2 & EWMAR charts proposed by Kang and Albin [6] in terms of ARL Zou *et al.* [15] introduced the multivariate EWMA (MEWMA) chart to monitor linear profiles in Phase II. Although the performance of this method seems to be similar to the one exhibited by EWMA3, it can additionally identify decreasing shifts in the error variance Saghaei *et al.* [16] proposed the CUSUM3 control chart which has a similar design as the EWMA3 chart Xu *et al.* [17] used the Generalized Likelihood Ratio (GLR) control chart to monitor linear profiles and showed that the GLR chart performs better than EWMA3, especially when the shifts are small Huwang *et al.* [18] introduced a control chart based on the simultaneous confidence set's concept and their chart provided a systematic diagnostic method for estimating the change point and identifying the shifted parameters in the process or profile diagnosis Riaz *et al.* [19] monitored linear profiles by implementing the EWMA structure under ranked set schemes which was also applied in monitoring quality characteristic (see for example Abbasi *et al.* [20]) similar to Mahmood *et al.* [21]. Riaz *et al.* [22] categorized the EWMA3 and CUSUM3 charts as memory-type, and then they proposed Shewhart-based charts (denoted as Shewhart3) using a modified successive sampling strategy instead of simple random sampling Motasemi *et al.* [23] presented a novel approach to leverage the information in the area formed between the sampled and IC profile to improve the monitoring scheme performance Saeed *et al.* [24] designed a memory-type control chart taking into account the progressive mean to detect changes in linear profile parameters. Some similar works can also be investigated in Abbas *et al.* [25] and Riaz *et al.* [26]. Bayesian control charts have also been developed for linear profiles, see [27]–[29].

For some earlier research works on these concepts, see [30]–[33]. It is evident that most of the abovementioned schemes can be utilized in both simple and multiple profiles; however, only few researchers focused on multiple linear profiles; this was attempted by Zou *et al.* [34], Amiri *et al.* [35], Mahmoud *et al.* [36] and Qi *et al.* [37]. Among these works, Amiri *et al.* [35] generated better results based on Monte Carlo simulations by dimension reduction method.

In many practical situations, the profiles cannot be represented adequately by a simple or multiple linear model, because there are more than one dependent quality characteristics as response variables, which are modeled as functions of one or more explanatory variables called multivariate linear profiles. The aforementioned process monitoring approaches could not be directly employed in multiple linear profiles because of the correlation between response variables and hence, some modifications are needed in this type of profile Noorossana *et al.* [38] were the first researchers, who studied control schemes for multivariate profiles in Phase II by proposing three control charts denoted as MEWMA, MEWMA/χ^2 and MEWMA3. Their results revealed that MEWMA and MEWMA/χ^2 outperformed MEWMA3 in most of the situations. In a similar research, Eyvazian *et al.* [39] proposed four monitoring schemes based on MEWMA statistics, parameter reduction, likelihood ratio test (LRT) and change point estimator Haq [40] proposed adaptive MEWMA charts for monitoring linear and multivariate profile parameters.

In addition, some other studies employed variable sampling interval (VSI) technique to increase the sensitivity of control charts in linear profiles (see [15], [30], [31], [41], [42]). Run rules seem to be an alternative supplementary approach, which can be used to increase the sensitivity of a control chart. That is, non-random patterns including existence of two or more points (samples) in the region nearby the OC limit of control charts may reveal different situations according to the nature of their root causes. In other words, these rules utilize historical data and look for a non-random pattern that can signify that the process is OC, before reaching the main limits usually by the aid of additional warning limits. Several researchers including [43]–[46] implemented different run rules in the Shewhart, non-parametric and CUSUM charts for monitoring location and variability parameters. However, not enough attention has been paid to this subject in profile monitoring; see for instance Riaz and Touqeer [47].

Recently, a heuristic and effective run rules approach has been proposed by Yeganeh and Shadman [48] and Yeganeh *et al.* [49]. This approach (different from previous researches, i.e. based on a simulation matrix) which is astonishingly matched with machine learning [32] and statistical [15] control charts considered the ratio of the samples in different prespecified IC regions with definition of a rule matrix. The new matrix runs rule-based approach was shown in [48] and [49] to improve the detection ability of the MEWMA and artificial neural network (ANN)-based control charts for monitoring *simple* linear profiles in Phase II.

It is worth mentioning that the MEWMA (by [15]) and ANN-based (by [32]) control charts for monitoring profiles have a single charting statistic. Note though, the EWMAR (by [6]) and EWMA3 (by [14]) control charts have two and three charting statistics, respectively; so these schemes are not directly applicable in combination with the MEWMA and ANN-based control charts. Therefore, in this article, the matrix runs rule-based approach is incorporated into the design of EWMAR and EWMA3 control charts to improve the detection ability of these charts in monitoring not only simple but also multiple and multivariate linear profiles.

The rest of the manuscript is structured as follows. The implementation of EWMAR scheme in simple, multiple and multivariate linear profiles are illustrated in Section II. In Section III, we present the structure of the run rules approach in monitoring linear profile and the design approach for the EWMAR scheme with the rule matrix is provided in Section IV. Section V depicts several performance comparisons of the proposed control charts versus existing ones. A real data application used to clarify the applicability and implementation of the proposed monitoring scheme is presented in Section VI. Section VII concludes this article with final remarks and future research suggestions.

II. MONITORING LINEAR PROFILES IN PHASE II

The general linear profile model entails n pairs of $(\boldsymbol{X}_j, \boldsymbol{Y}_j)$ at the j^{th} sampling time in phase II which is represented as:

$$Y_j = \boldsymbol{X}_j\boldsymbol{\beta} + E_j, \tag{1}$$

or equivalently,

$$\begin{pmatrix} y_{11j} & y_{12j} & \dots & y_{1pj} \\ y_{21j} & y_{22j} & \dots & y_{2pj} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ y_{n1j} & y_{n2j} & \dots & y_{npj} \end{pmatrix} = \begin{pmatrix} 1 & x_{11j} & \dots & x_{1qj} \\ 1 & x_{21j} & \dots & x_{2qj} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ 1 & x_{n1j} & \dots & x_{nqj} \end{pmatrix} \times \begin{pmatrix} \beta_{01} & \beta_{02} & \dots & \beta_{0p} \\ \beta_{11} & \beta_{12} & \dots & \beta_{1p} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ \beta_{q1} & \beta_{q2} & \dots & \beta_{qp} \end{pmatrix} + \begin{pmatrix} \varepsilon_{11j} & \varepsilon_{12j} & \dots & \varepsilon_{1pj} \\ \varepsilon_{21j} & \varepsilon_{22j} & \dots & \varepsilon_{2pj} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ \varepsilon_{n1j} & \varepsilon_{n2j} & \dots & \varepsilon_{npj} \end{pmatrix} \tag{2}$$

In this model, $\boldsymbol{Y_j}$ is the matrix of j^{th} response variable, $\boldsymbol{X_j}$ is $n \times p$ with $(n > p)$ independent explanatory variables matrix, β is a $(q+1) \times p$ matrix of regression parameters and $\boldsymbol{E_j}$ is the $n \times p$ with $n > p$ matrix of error variances with an assumption of independent normally distributed with mean vector $\mathbf{0}$ and covariance matrix $\sum$, which is given by

$$\sum = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{p1} & \sigma_{p2} & \dots & \sigma_{pp} \end{pmatrix}. \tag{3}$$

It is also a customary assumption to neglect the variation of explanatory variables in each profile; hence, $\boldsymbol{X_j}$ hereafter is denoted by $\boldsymbol{X}$ as follows:

$$X = \begin{pmatrix} 1 & x_{11} & \dots & x_{1q} \\ 1 & x_{21} & \dots & x_{2q} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ 1 & x_{n1} & \dots & x_{nq} \end{pmatrix}. \tag{4}$$

The estimators of coefficients in (2) (i.e., $\hat{\beta}_j$) are obtained based on ordinary least square (OLS) estimation as follows:

$$\hat{\beta}_j = (\boldsymbol{X}\boldsymbol{X}')^{-1}\boldsymbol{X}'\boldsymbol{Y}_j, \tag{5}$$

where the vector form of $\hat{\beta}_j$ is shown as a $(q+1)p \times 1$ vector and is denoted by $\hat{\beta}_{Vj}$ as follows:

$$\hat{\beta}_{Vj} = (\hat{\beta}_{01j}, \hat{\beta}_{11j}, \dots, \hat{\beta}_{q1j}, \hat{\beta}_{02j}, \hat{\beta}_{12j}, \dots \\ \dots, \hat{\beta}_{q2j}, \dots, \hat{\beta}_{0pj}, \hat{\beta}_{1pj}, \dots, \hat{\beta}_{qpj}). \tag{6}$$

The IC model in (1) simplifies to multiple and simple linear profiles when considering $p = 1$, $q > 1$ and $p = q = 1$ respectively. The model consists of observations of the quality characteristic along an auxiliary information:

$$\begin{aligned} Y_{ij} &= A_0 + A_1x_1 + A_2x_2 + \dots + A_qx_q + \varepsilon_{ij}, \\ i &= 1, 2, \dots, n; \quad j = 1, 2, \dots, \\ \varepsilon_{ij} &\sim N(0, \sigma^2). \end{aligned} \tag{7}$$

where the subscript i shows the i^{th} observations within each profile and subscript j shows the j^{th} profile collected over time; with ($\beta = \{A_0, A_1, \dots, A_q\}$, $\boldsymbol{E_j} = (\varepsilon_{1j}, \varepsilon_{2j}, \dots, \varepsilon_{nj})$ and å $= \sigma_{1\times1}$). Note that the parameters of model in (7) assume that the intercept (A_0), slopes ($A_1, A_2, \dots, A_q$) and error variance (σ^2) are known in phase II.

To monitor simple and multiple linear profiles, Kang and Albin [6] defined two independent control charts entailing monitoring range and average of residuals. The weighted average of residuals at the j^{th} sample for the EWMA statistic ($\bar{e}_j = n^{-1}\sum_{i=1}^{n} e_{ij}$) is defined as follows

$$z_j = (1-\theta)z_{j-1} + \theta\bar{e}_j, \tag{8}$$

where θ is the weighting constant (as a common manner in other studies, θ is set at 0.2) and the proper initial value of z_0 may be *0*. The control limits for the first two-sided control chart are defined by:

$$LCL_z = -UCL_z = -L\sigma\sqrt{\frac{\theta}{(2-\theta)n}}. \tag{9}$$

The range of residuals ($r_j = \max_i(e_{ij}) - \min_i(e_{ij})$) is also monitored by the one-sided control chart with the limits defined as

$$\begin{aligned} LCL_R &= \sigma(d_2 - Ld_3) \\ UCL_R &= \sigma(d_2 + Ld_3). \end{aligned} \tag{10}$$

Hereafter, the two charts based on (9) and (10) are called Z and R, respectively; and the values of d_2 and d_3 are reported in Montgomery [1] and L is the multiple of the sample statistic standard deviation for both control charts determining the ARL_0. It is noteworthy that r_j is always positive and $LCL_R = 0$ in (10) for $n < 7$ based on the reported values of d_2 and d_3. In this method, an OC signal is given if at least one of the charting statistics falls beyond the control limits.

Equations (8), (9) and (10) cannot be utilized in multivariate linear profiles so to apply this scheme in multivariate linear profiles, the $1 \times n$ vector of residuals' average is calculated as follows [38]:

$$\bar{\boldsymbol{E}}_j = (\bar{e}_{1j}, \bar{e}_{2j}, \ldots, \bar{e}_{nj}). \tag{11}$$

It is followed by a multivariate normal distribution with mean $\boldsymbol{0}$ and known covariance matrix $\sum_{\bar{E}} = \frac{\sum}{n}$. The weighted average of residuals at the j^{th} sample is the same as that in (8) and is calculated as follows:

$$Z_j = (z_1, z_2, \ldots, z_p),$$
$$z_j = (1-\theta)z_{j-1} + \theta \bar{e}_j \quad j = 1, 2, \ldots, p, \tag{12}$$

where θ is the weighting constant and $Z_0 = (0, \ldots, 0)_{1\times p}$. The charting statistic is defined by:

$$T^2_{Zj} = \boldsymbol{Z}_j \times \sum\nolimits_{\bar{E}} \times \boldsymbol{Z}'_j. \tag{13}$$

Due to non-zero value of T^2_{Zj}, $LCL_Z = 0$ and UCL_Z can be obtained with chi square random variable [38] or simulations.

In a multivariate scenario, the same as Z chart, only the value of UCL_R is required and it is determined by simulations specifies the ARL and $LCL_R = 0$. To construct the control chart for the range of residuals, $R_j = (r_{1j}, r_{2j}, \ldots, r_{nj})$ where $\sum_R = \frac{\sum}{2}$, thus, the charting statistic is given by

$$T^2_{Rj} = \boldsymbol{R}_j \times \sum\nolimits_{\boldsymbol{R}} \times \boldsymbol{R}'_j. \tag{14}$$

This statistic is monitored with $LCL_R = 0$ and the UCL_R is obtained based on desired ARL_0. When both of these statistics (T^2_{Zj} and T^2_{Rj}) are located within their control limits, it means that there are no OC situations. The proposed EWMAR scheme in multivariate profiles is very similar to $MEWMA/\chi^2$ control chart in Noorossana *et al.* [38] and there is only a slight difference in the second statistic.

Because the EWMA3 approach can be directly applied only in simple linear profiles and for brevity, the details of this method are not provided here and the reader is referred to Kim *et al.* [14].

III. COMBINATION OF RUN RULES AND EWMAR METHOD IN MONITORING LINEAR PROFILES

The main application of run rules entails receiving previous samples information and involving them in the current sample for decision making and this has been shown to generally increase the control charts detection ability of OC situations, especially for small and moderate shifts. The proposed rule matrix is defined corresponding to the charts' control limit with a 3-columns rule matrix consisting of S rows (S equals to the number of rules) as follows:

$$\begin{pmatrix} o_1 & p_1 & m_1 \\ \ldots & \ldots & \\ o_k & p_k & m_k \\ \ldots & \ldots & \\ o_S & p_S & m_S \end{pmatrix}. \tag{15}$$

First column of rule matrix is the indicator of regions of each rule including o_k to UCL where $1 \le k \le S, o_1 \le o_k \le o_S$ and $p_k \le 1$ (Region 1 = $(o_1 - UCL)$, …, and Region $S = (o_S - UCL)$). The second column defines the maximum acceptable ratio of the number of points in each region to the total number of points until the current sample. The last column is the maximum number of points plotted (or minimum point with reverse definition) in each region for not firing rules ($m_k \ge 0$). For more details of this type of rule matrix procedure, the reader is referred to Yeganeh and Shadman [48] and Yeganeh *et al.* [49]. Note that the first column has been shown with 'r_k' in these articles but it has been changed here due to similarity with the range of residuals (r_j) of the EWMAR statistics.

For better illustration, the details of the proposed approach and designing are illustrated for EWMAR and the same can be extended for other control charts. To employ this approach in EWMAR as a control charts with more than one charting statistic, the rule matrix needs to be defined for each separate control chart corresponding to its control limits with definition of S_R and S_Z as the number of rules in each rule matrix. Hence, two rule matrices are designed (the designing procedure are described in the next section) for the EWMAR control chart.

Suppose there are two rule matrices for Z and R chart (see (9) and (10)), after computation of z_j and r_j in the j^{th} sample for the simple or multiple profiles, the charting statistics are firstly compared with their control limits. When the charting statistics are not located within the control limits (fall outside the limits), the chart signals naturally. In other words, this is not a pure run rule system in which there are no control limits (i.e. there are warning limits only) and a signal is triggered by run rules [50]; however, to declare the process as IC, all the conditions of the two rule matrices should not be satisfied in addition to locating statistics in IC region. The signaling procedure of EWMAR control chart in the j^{th} profile is illustrated in Figure 1. The procedure of EWMAR signaling can easily be applied in multivariate profiles by changing of z_j and r_j to T^2_{Zj} and T^2_{Rj}, respectively.

IV. DESIGNING OF THE PROPOSED SCHEME

When a control chart has more than one charting statistic, precautions need to be taken into account when designing the control limits because they have an impact in the chart performance. If the charts are independent, the overall type I error can be as expressed:

$$\alpha_{overall} = 1 - \prod_{g=1}^{G} (1 - \alpha_g), \tag{16}$$

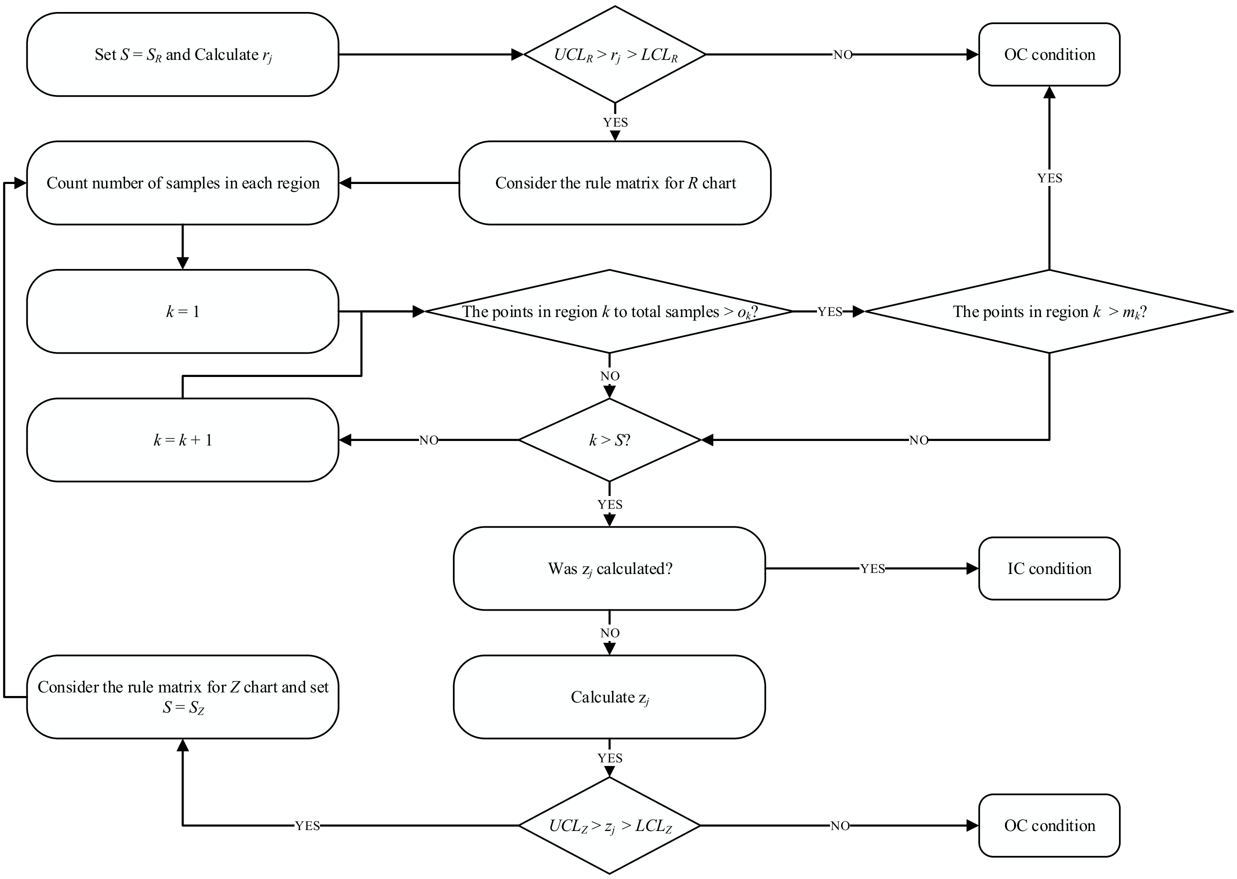

FIGURE 1. **The signaling procedure in combination of rule matrix and EWMAR control chart for the j^{th} sampling point.**

where G denotes the number of control charts (i.e., $G = 2$ for the EWMAR and $G = 3$ for the EWMA3). As a common approach in phase II applications, ARL is utilized instead of α_g; $g = 1, 2, \ldots, G$ in a way that $\alpha_{overall}$ denotes the inverse of the IC ARL ($\alpha_{overall} = \frac{1}{ARL_0}$).

Although the use of run rules violates the independency assumption, (16) with some modifications is employed in rule matrix designing in this article. Set the equal values of α_g for each control chart is a common manner in designing of control charts [24], [27], [51]. Note though Kang and Albin [6] did not set the IC limits in the same manner as done in the latter publications. That is, to reach a final ARL_0 equal to 200, they set $L = 3$ in (9) and by using this adjustment, the values of individual ARL_0 for the chart were obtained equal to 270 and 790. However, to be consistent with most of the previous researches, the rule matrix designing is conducted to reach the same value of ARL_0 in each control chart. So, firstly, without consideration of rule matrix, the limits of control charts are obtained with (9) and (10). Then, the IC region of control chart is divided into some predefined regions in each step (i.e. the same as Yeganeh *et al.* [49] one region (rule) is added to the rule matrix). One region has the best performance based on the minimizing ARL_1 for predefined shift(s) is selected as a fixed region from candidate regions in the rule adding procedure. In each steps of rule adding, there are $S_R - 1(S_Z - 1)$ fixed regions in the rule matrix of R (Z) chart, respectively. Because the IC region is naturally widened, by adding of run rules, the candidate and fixed region(s) are varied during the designing process.

To assign the ratio of each region, two main approaches can be used: (i) separate designing of each rule, and (ii) simultaneous designing. Because of spanning a bigger chunk of time in the first approach, the second approach is suggested in this article. In the second approach, the ratios of the fixed and candidate regions are set by four different definitions of ARL_0 which are defined as follows:

- ARL_0: Final value of the IC ARL for all of the G charts.
- ARL_{0R}, ARL_{0Z}: Final value of the IC ARL for R and Z charts in the EWMAR. Using (16), ARL_{0R} and ARL_{0Z} are given by:

$$ARL_{0R} = \frac{1}{1 - \sqrt{1 - \frac{1}{ARL_0}}}$$
$$ARL_{0Z} = \frac{1}{1 - \sqrt{1 - \frac{1}{ARL_0}}}. \quad (17)$$

- ARL_{0IR}, ARL_{0IZ}: Individual value of the IC ARL for each rule and control limits in each chart. It is calculated for each chart separately based on S_R, S_Z, ARL_{0R} and ARL_{0Z}, i.e.

$$ARL_{0IR} = \frac{1}{1-(1-\frac{1}{ARL_{0R}})^{\frac{1}{S_R+1}}}$$
$$ARL_{0IZ} = \frac{1}{1-(1-\frac{1}{ARL_{0Z}})^{\frac{1}{S_Z+1}}}. \quad (18)$$

- ARL_{0CR}, ARL_{0CZ}: The calculative value of IC ARL of both control charts for design of fixed rules in the designing procedure. It is a $(S_R-1)\times 1$ and $(S_Z-1)\times 1$ vector based on the fixed regions and are given by:

$$ARL_{0CR}(l) = \frac{1}{1-((1-\frac{1}{ARL_{0IR}})^{l+1}\times(1-\frac{1}{ARL_{0Z}}))};$$
$$l = 1, 2, \ldots, S_R$$
$$ARL_{0CZ}(l) = \frac{1}{1-((1-\frac{1}{ARL_{0IZ}})^{l+1}\times(1-\frac{1}{ARL_{0R}}))};$$
$$l = 1, 2, \ldots, S_Z. \quad (19)$$

In addition, to reach the same IC run length in run rules, reducing the value of ARL_1 for some specific and predefined shifts is an effective manner in designing of run rules. In this way, ARL_0 remains constant at the desired level while the ARL_1 is minimized by changing control limits based on simulations see for instance Riaz *et al.* [45] or Saeed *et al.* [24]. Considering this aim, the designing of proposed rule matrices will be performed under the following assumptions:

- For the ease of computations, the absolute value of z_j(i.e., $|z_j|$) is imported to the rule matrix procedure so the lower limit of Z chart is considered as zero ($LCL_Z = 0$). Also, the value of LCL_R is not changed during the designing steps.
- After the designing procedure, the values of type I error for each chart are the same ($ARL_{0R} = ARL_{0Z}$).
- The rule matrix for R chart is firstly designed and fixed then the designing of Z chart should be performed.
- In each step, one rule is added to rule matrix on the basis of relative reducing of ARL_1 in comparison with the previous state
- The control charts are divided to some equal regions based on the control limits. The regions are chosen based on the minimizing ARL_1 in predefined shift(s).
- The chosen regions are fixed in the subsequent adding procedure steps and selection of regions should be done among candidate regions. Because of changing the regions during the rule adding procedure, the closest region(s) to previous fixed region(s) are defined as a new fixed region(s).
- In designing of rule percentage in fixed regions, the rules are assumed to be independent, (16) can be used; however, in designing of the rule percentage in candidate regions, ARL_0 is the criteria.
- There is only one candidate region added to the rule matrix in each step.

In a nutshell, the ratio of fixed region(s) is tuned based on (19); in other words, the ratio of the l^{th} fixed region of the R (Z) chart are set to reach $ARL_{0CR}(l)$ ($ARL_{0CZ}(l)$), respectively. Then the rule percentage of candidate regions is set to reach ARL_0. Note that if S_R is equal to 0, the UCL_R should be calculated and go to step 4; else, the iterative suggested procedure from step 1 will be iterated. The same can be said for Z chart designing and to preserve writing space, it is not provided here. The following steps are provided for designing the run rule matrix of the R chart:

Step 1: **Investigation of rules adding condition**

The designing procedure is iterated by increasing the number of rules. The rule adding procedure is terminated when there is a significant decrease (i.e., relative distance to previous state) is not shown in ARL_1. The procedure is terminated when the relative difference to previous state is fewer than 2%. If relative distance is greater than 2%, the selected region in the current step is assumed as a fixed region.

Step 2: **Computing the ARL_{0IR} and UCL_R**

Considering S_R, ARL_{0IR} is calculated with (18) and then the UCL_R is increased to reach ARL_{0IR}.

Step 3: **Tuning rule percentage of fixed regions based on rule independency assumption**

The rule percentage of $S_R - 1$ fixed regions should be tuned with the sequence that they are selected. With the assumption of independence between rules, $ARL_{0CR}(l)$ is calculated based on (19) for l^{th}; $l = 1, 2, \ldots, S_R - 1$ fixed region with consideration of ARL_{0IR} of each rule and UCL_R.

Step 4: **Tuning rule percentage of candidate regions based on ARL_0**

In this step, from S_R rules in the rule matrix, $S_R - 1$ rules have been tuned and only one rule has remained. Because the rules are not independent, we can't use (16) in this step and we tune last rule based on ARL_0. The rule percentage is designed for all the candidate regions and the region with the lowest ARL_1 is selected as a fixed region for subsequent steps.

Step 5: **Assign maximum point for each rule in the rule matrix**

The same as Yeganeh *et al.* [49], the greater value of maximum point is assigned to the larger regions considering

$$m(k) = S - k + 1; \quad k = 1, 2, \ldots, S_R. \quad (20)$$

After designing of UCL_R and the rule matrix for R chart, these values are fixed and designing of Z chart is started. Note that UCL_R and the rule matrix for R chart do not change during designing of Z chart. The framework of the designing of each control chart is shown in Figure 2.

V. SIMULATION RESULTS

Simulation results based on the performance of the proposed chart using ARL criteria are provided in this section for simple, multiple and multivariate linear profiles. Also, the polynomial profiles have been simulated for the proposed chart. The numerical examples, used by Kang and Albin [6], Amiri *et al.* [35], Noorossana *et al.* [38] and Huwang *et al.* [18] are utilized here for simple, multiple, multivariate linear

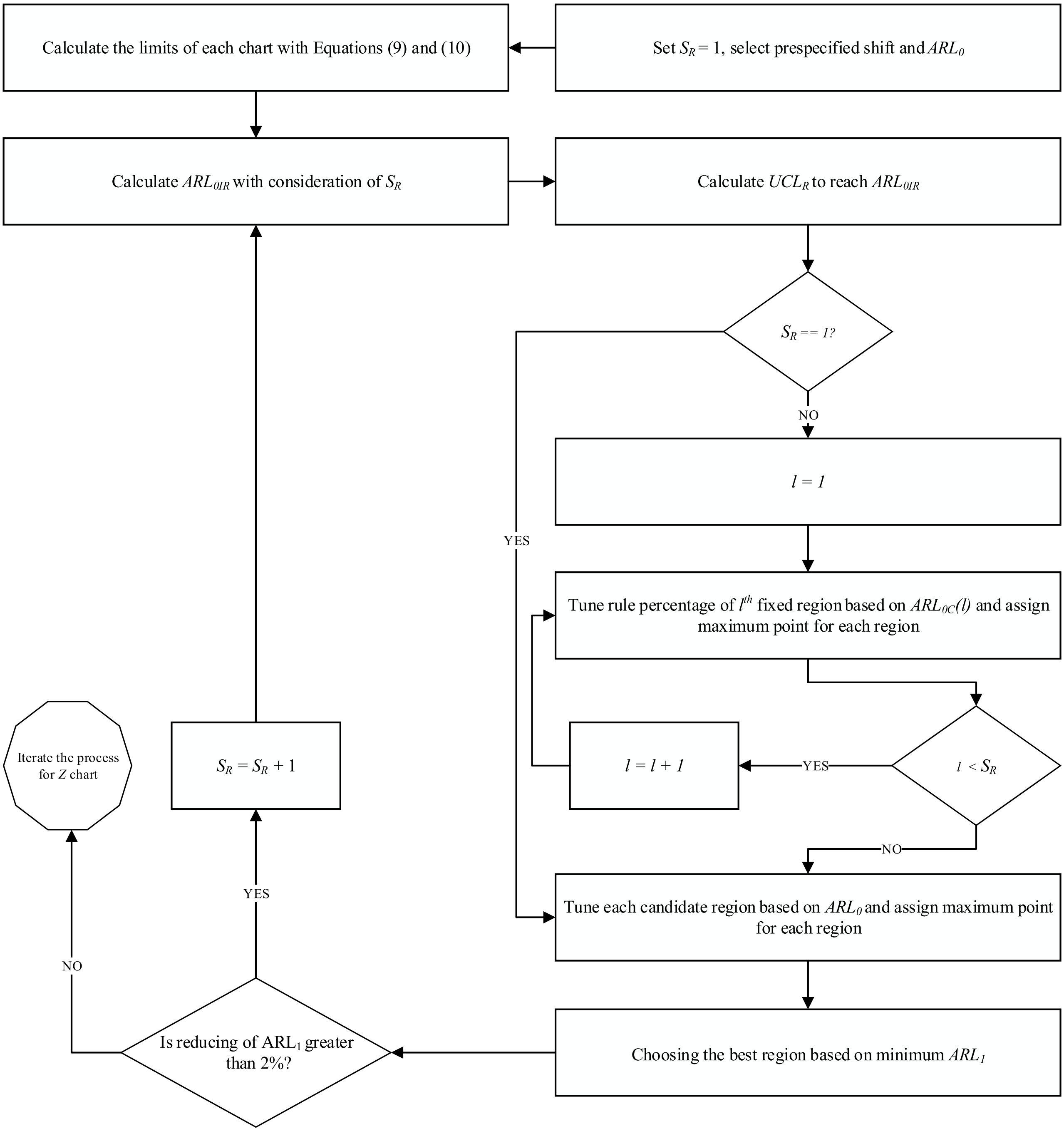

FIGURE 2. The designing framework of each control chart.

and polynomial profiles, respectively; with assumption of ARL_0 equal to 200 (so $ARL_{0R} = ARL_{0Z} = 400$ based on (17)). The IC profiles for these examples are defined in Table 1. The values of the control limits and the final rule matrices (for R and Z charts) after all designing steps are shown in two last columns.

A. NUMERIC EXAMPLE OF DESIGNING PROCEDURE OF THE RULE MATRIX IN SIMPLE LINEAR PROFILES

For better illustration of the proposed method, designing procedure of the R chart for the simple linear model is introduced here and the designing of the other profiles follow in a similar manner. The desired shifts for calculation of ARL_1 are $\lambda = 0.2$, $\eta = 1.1$ and $\gamma = 1.1$ i.e. shifts in intercept (A_0 to $\lambda\sigma$), slope (A_1 to $A_1 + \eta\sigma$) and standard deviation(σ to $\gamma\sigma$). Without the use of run rules, the conventional EWMAR control chart produces the average of ARL_1 in these shifts equal to 55.54 (the details are gathered in the first row of Table 5). Table 2 represents the main limit of each chart (UCL_R, UCL_Z) and individual run length of each rule (ARL_{0IR}, ARL_{0IZ}) based on independent rule assumption and (18). For example, if we want to use only one rule in R chart ($S_R = 1$ and $S_Z = 0$), we have two type I error (i.e., main limit and rule) in R chart so ARL_{0IR} for each of these rules should be equal to 800.

TABLE 1. The IC profiles used in performance comparisons.

IC model	n	$\boldsymbol{X}$	σ	(LCL_R, LCL_Z) (UCL_R, UCL_Z)	Final rule matrix
$Y = 3 + 2X$	4	$\begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \end{pmatrix}$	1	(0,0) (5.47,0.56)	$\begin{pmatrix} 4.65 & 0.05 & 1 \\ 4.1 & 0.19 & 2 \\ 3.01 & 0.5 & 3 \end{pmatrix} \begin{pmatrix} 0.32 & 0.12 & 1 \\ 0.16 & 0.51 & 2 \end{pmatrix}$
$Y = 3 + 2X_1 + X_2 + X_3$	8	$\begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 4 & 4 & 3 \\ 1 & 6 & 3 & 2 \\ 1 & 8 & 2 & 4 \\ 1 & 2 & 1 & 1 \\ 1 & 4 & 4 & 3 \\ 1 & 6 & 3 & 2 \\ 1 & 8 & 2 & 4 \end{pmatrix}$	1	(0,0.387) (6.02,0.41)	$\begin{pmatrix} 5.1 & 0.03 & 1 \\ 4.21 & 0.26 & 2 \\ 3.31 & 0.94 & 3 \end{pmatrix} \begin{pmatrix} 0.28 & 0.03 & 1 \\ 0.24 & 0.107 & 2 \\ 0.16 & 0.432 & 3 \end{pmatrix}$
$Y_1 = 3 + 2X_1$ $Y_2 = 2 + X_1$	4	$\begin{pmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \end{pmatrix}$	$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$	(0,0) (36.66,14.1)	$\begin{pmatrix} 26.47 & 0.05 & 1 \\ 19.10 & 0.19 & 2 \\ 14.01 & 0.80 & 3 \end{pmatrix} \begin{pmatrix} 8.04 & 0.04 & 1 \\ 5.24 & 0.20 & 2 \\ 3.16 & 0.51 & 3 \end{pmatrix}$
$Y = 1 + 2X + 3X^2$	20	$\begin{pmatrix} 1 & 0.025 \\ 1 & 0.075 \\ \vdots & \vdots \\ 1 & 0.925 \\ 1 & 0.975 \end{pmatrix}$	1	(0,1.54) (6.5,0.26)	$\begin{pmatrix} 5.9 & 0.03 & 1 \\ 5.33 & 0.18 & 2 \\ 4.44 & 0.94 & 3 \end{pmatrix} \begin{pmatrix} 0.18 & 0.03 & 1 \\ 0.11 & 0.24 & 2 \\ 0.08 & 0.46 & 3 \end{pmatrix}$

TABLE 2. The values of UCL_R, UCL_Z, ARL_{0IR} and ARL_{0IZ} to reach ARL_0 equal to 200.

R chart					
S_R	0	1	2	3	4
ARL_{0IR}	400	800	1200	1600	2000
UCL_R	4.997	5.24	5.39	5.47	5.59
Z chart					
S_Z	0	1	2	3	4
ARL_{0IZ}	400	800	1200	1600	2000
UCL_Z	0.478	0.515	0.54	0.556	0.567

TABLE 3. The values of ARL_{0CR} with different values of S_R.

S_R	l	$ARL_{0CR}(l)$
1	-	200
2	1	240
3	1	267
	2	229
4	1	285
	2	250
	3	223

Hence, ARL_{0R} is equal to 400 and ARL_0 become 200 ((I) $S_R = 1$ and $ARL_{0IR} = 800 \rightarrow ARL_{0R} = 400$. (II) $S_Z = 0 \rightarrow ARL_{0IZ} = ARL_{0Z} = 400$. (I) and (II) $\rightarrow ARL_0 = 200$). Note that Table 2 is only used in designing of fixed regions and in reality, the rules are not independent and, hence (16) cannot be used.

Tuning of the rule percentage of $S_R - 1$ fixed regions is done based on the values of ARL_{0CR}. Table 3 shows these values for different values of S_R. For example, in designing of rule matrix in R chart with $S_R = 3$, we have two fixed regions from previous steps so the values of $ARL_{0CR}(1)$ and $ARL_{0CR}(2)$ should be assigned. Tuning the first fixed region is done to reach the IC run length equal to 267 or $ARL_{0CR}(1) = 267$ (from (19) with $l = 1$, $S_R = 3$, $ARL_{0IR} = 400$ and $ARL_{0IR} = 1600$). The second fixed region is also designed with the same procedure with $ARL_{0CR}(2)$ equal to 229 after tuning the first fixed region (from (19) with $l = 2$, $S_R = 3$, $ARL_{0IR} = 400$ and $ARL_{0IR} = 1600$).

The rule adding procedure based on dividing IC region to 20 sections for R chart is shown in Table 4. The chart sections or candidate regions considering UCL_R is shown in the first column of each part of the chart. The candidate regions, selected candidate region and fixed region(s) are shown with white, green and yellow cells, respectively.

TABLE 4. Rule adding procedure for *R* chart based on ARL_0 equal to 200.

S_R													
0	1				2			3			4		
UCL_R	UCL_R	fixed region	rule percentage	Average of ARL_1	UCL_R	fixed regions	rule percentage	UCL_R	fixed regions	rule percentage	UCL_R	fixed regions	rule percentage
		4.98	0.014	49.64		5.12	0.005		5.20	0.003		5.31	0.007
		4.72	0.025	48.86		4.85	0.006		4.92	0.004		5.03	0.011
		4.45	0.039	47.11		4.58	0.04		4.65	0.05		4.75	0.032
		4.19	0.06	44.54		4.31	0.08		4.38	0.134		4.47	0.074
		3.93	0.11	41.24		4.04	0.12		4.10	0.185		4.19	0.145
		3.67	0.218	43.06		3.77	0.129		3.83	0.208		3.91	0.258
		3.41	0.365	44.04		3.50	0.401		3.56	0.271		3.63	0.475
		3.14	0.975	45.8		3.23	0.497		3.28	0.361		3.35	0.754
		2.88	0.999	46.87		2.96	0.549		3.01	0.5		3.07	0.955
4.997	5.24	2.62	-	-	5.39	2.70	0.639	5.47	2.74	0.95	5.59	2.80	-
		2.36	-	-		2.43	0.999		2.46	0.999		2.52	-
		2.10	-	-		2.16	-		2.19	-		2.24	-
		1.83	-	-		1.89	-		1.91	-		1.96	-
		1.57	-	-		1.62	-		1.64	-		1.68	-
		1.31	-	-		1.35	-		1.37	-		1.40	-
		1.05	-	-		1.08	-		1.09	-		1.12	-
		0.79	-	-		0.81	-		0.82	-		0.84	-
		0.52	-	-		0.54	-		0.55	-		0.56	-
		0.26	-	-		0.27	-		0.27	-		0.28	-
		0.00	-	-		0.00	-		0.00	-		0.00	-

The rule percentage of each candidate region for obtaining ARL_0 equal to 200 is shown in the second column. Also, for brevity, the average of ARL_1 for the selected shifts is only shown for $S_R = 1$. Note that all the calculations in Table 4 were done based on $S_Z = 0$ and UCL_Z equal to 0.478 (see Table 2).

The values of UCL_R for different values of S_R are gathered from Table 2. Among candidate regions for adding the first rule, the region (3.93−5.24) has minimum ARL_1 hence this region is chosen as the first fixed region in further steps. It means that with UCL_R equal to 5.24, UCL_Z is equal to 0.478 and all the candidate regions in Table 4 with their rule percentage, ARL_0 becomes equal to 200 and the minimum ARL_1 is obtained with rule (3.92 0.11 1) and this region selected as the first fixed region. Note that for the regions greater than 2.88, it is impossible to design *R* chart based on ARL_0 equal to 200 with UCL_R equal to 5.24.

Because the reduction of ARL_1 is greater than 2%$(\frac{55.54-41.24}{41.24} \times 100)$, we continue adding rule procedure and set $S_R = 2$. From Table 2, UCL_R is equal to 5.39 and the closest region to 3.93 is 4.04 and is highlighted in yellow. The rule percentage of this region is designed based on ARL_{0C} *(1)* equal to 240 (from Table 3). It means that with UCL_R equal to 5.39, UCL_Z equal to 0.478 and the rule (4.04 0.12 1), the ARL_0 becomes 240. Then among candidate regions (19 regions), the rule percentage is tuned to reach ARL_0 equal to 200. The region (2.96 0.549 1) has the minimum ARL_1 and selected as fixed region for further calculations. Note that for the regions greater than 2.43, it is impossible to design *R* chart based on ARL_0 equal to 200 with UCL_R equal to 5.39 and fixed region (4.04 0.12 1). Other calculations for $S_R = 3$ and 4 can be explained in a similar manner.

The difference between reduction of ARL_1 in $S_R = 3$ and 4 is smaller than *2%* so the final rule matrix for *R* chart is obtained considering $S_R = 3$. This matrix and UCL_R equal to 5.47 are fixed and then the designing of *Z* chart starts. The designed rule matrix in each rule adding procedure is shown in Table 5. The final number of rules is 3 and 2 for the *R* and *Z* charts, respectively. Note that results of first row i.e. ARL_1 without consideration of run rules are different with the reported values in Kang and Albin [6] because of different values of control limits.

TABLE 5. The designed rule matrix and average of ARL_1 for predefined shifts based on adding rules.

Chart			(UCL_R, UCL_Z)	Rule matrix	ARL_1 $\lambda = 0.2$	$\eta = 0.05$	$\gamma = 1.1$	Average of ARL_1	Difference
Without run rules		-	(0.478, 4.997)	-	50.33	33.76	82.54	55.54	-
R	S_R	1	(0.478, 5.24)	$\begin{pmatrix} 3.93 & 0.11 & 1 \end{pmatrix}$	39.35	24.35	60.01	41.24	0.258
		2	(0.478, 5.39)	$\begin{pmatrix} 4.04 & 0.12 & 1 \\ 2.96 & 0.549 & 2 \end{pmatrix}$	36.09	24.78	59.83	40.23	0.024
		3	(0.478, 5.47)	$\begin{pmatrix} 4.65 & 0.05 & 1 \\ 4.1 & 0.185 & 2 \\ 3.01 & 0.5 & 3 \end{pmatrix}$	37.11	24.08	57.03	39.41	0.021
		4	(0.478, 5.59)	$\begin{pmatrix} 5.03 & 0.011 & 1 \\ 4.75 & 0.032 & 2 \\ 4.19 & 0.145 & 3 \\ 3.07 & 0.955 & 4 \end{pmatrix}$	37.32	24.11	57.64	39.69	-0.007
Z	S_Z	1	(0.515, 5.47)	$\begin{pmatrix} 4.65 & 0.05 & 1 \\ 4.1 & 0.185 & 2 \\ 3.01 & 0.5 & 3 \end{pmatrix} \begin{pmatrix} 0.309 & 0.118 & 1 \end{pmatrix}$	18.25	11.65	47.87	25.92	0.347
		2	(0.556, 5.47)	$\begin{pmatrix} 4.65 & 0.05 & 1 \\ 4.1 & 0.185 & 2 \\ 3.01 & 0.5 & 3 \end{pmatrix} \begin{pmatrix} 0.324 & 0.124 & 1 \\ 0.162 & 0.508 & 2 \end{pmatrix}$	14.76	9.74	45.95	23.48	0.094
		3	(0.567, 5.47)	$\begin{pmatrix} 4.65 & 0.05 & 1 \\ 4.1 & 0.185 & 2 \\ 3.01 & 0.5 & 3 \end{pmatrix} \begin{pmatrix} 0.33 & 0.068 & 1 \\ 0.25 & 0.301 & 2 \\ 0.16 & 0.627 & 3 \end{pmatrix}$	15.36	10.06	44.21	23.21	0.012

B. PERFORMANCE COMPARISON UNDER SIMPLE LINEAR PROFILES

Our proposed method (EWMAR-RULE, hereafter) is compared with some of the best competitive charts including EWMAR (see [6]), EWMA3 (see [14]), MEWMA (see [15]), MEWMARULE (see [49]) and ANNWR1 (see [48]). For purposes of easier visual inspection, best results are bold-faced. Note that all the simulations were obtained based on 10000 iterations with MATLAB 2018.

1) PERFORMANCE COMPARISONS UNDER UNIQUE POSITIVE SHIFTS IN THE SIMPLE LINEAR MODEL

Table 6 shows the results of ARL_1 for detecting unique positive shifts. The results clearly show that the proposed EWMAR-RULE chart significantly outperforms not only the EWMAR chart but also other competing methods in all of the shifts. As the shifts size increases, the difference between methods decreases. Note that in places within the tables where there is a '-', it means that the article that these values were taken from, did not consider that specific shift in λ, η or γ. It would be interesting to look how different control charts treat with the proposed rule matrix. Generally, combination of this scheme has better performance with EWMAR than other statistics.

2) PERFORMANCE COMPARISONS UNDER UNIQUE NEGATIVE SHIFTS IN THE SIMPLE LINEAR MODEL

Based on Table 7, we can conclude that the same as positive shifts, that is, our proposed approach is the best method in most of the shifts and the same discussion can be iterated here. It should be noted that some methods like MEWMA, can also detect decreasing shifts in error variance but the EWMAR does not have this ability; thus, the proposed chart is unable to detect decreasing error variance shifts.

3) PERFORMANCE COMPARISONS BETWEEN PROPOSED METHOD AND VSI SCHEME

As the proposed method is a tool for reducing the ARL_1 of the EWMAR control chart, one can compare it with

TABLE 6. Comparisons of ARL_1 for simple linear profile with unique positive shifts.

Chart	λ									
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMAR	66.5	17.7	8.4	5.4	3.9	3.2	2.7	2.3	2.1	1.9
EWMA3	59.1	16.2	7.9	5.1	3.8	3.1	2.6	2.3	2.1	1.9
MEWMA	59.9	17.2	8.5	5.8	4.1	-	-	-	-	2.0
MEWMARULE	20.75	8.06	5.70	4.70	4.31	3.97	3.69	3.48	3.27	3.11
ANNWR1	22.45	9.17	5.65	4.08	2.95	2.20	1.80	1.50	1.38	1.26
EWMAR-RULE	**14.76**	**4.38**	**2.57**	**1.78**	**1.53**	**1.23**	**1.09**	**1.02**	**1.02**	**1.01**
Chart	η									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMAR	119.0	43.9	19.8	11.3	7.7	5.8	4.7	3.9	3.4	3.0
EWMA3	101.6	36.5	17.0	10.3	7.2	5.5	4.5	3.8	3.3	2.9
MEWMA	99.0	35.0	16.4	9.8	6.9	5.3	-	3.7	-	2.9
MEWMARULE	48.7	12.8	7.8	6.1	5.2	4.8	4.4	4.1	3.9	3.7
ANNWR1	57.60	17.85	10.63	7.21	5.20	4.40	3.40	2.80	2.30	2.00
EWMAR-RULE	**46.7**	**9.74**	**4.68**	**2.94**	**2.26**	**1.89**	**1.53**	**1.33**	**1.25**	**1.17**
Chart	γ									
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMAR	34.3	12.0	6.1	3.9	2.9	2.3	1.9	1.7	1.4	1.4
EWMA3	33.5	12.7	7.2	5.1	3.9	3.2	2.8	2.5	2.3	2.1
MEWMA	76.2	33.2	12.1	7.0	4.9	-	3.1	-	2.3	1.9
MEWMARULE	11.66	6.69	5.20	4.48	3.98	3.69	3.57	3.38	3.16	2.97
ANNWR1	17.70	9.20	6.35	4.86	4.24	3.50	2.95	2.93	2.61	2.59
EWMAR-RULE	**10.54**	**2.86**	**1.47**	**1.24**	**1.14**	**1.10**	**1.09**	**1.05**	**1.05**	**1.02**

TABLE 7. Comparisons of ARL_1 for simple linear profile with unique negative shifts.

Chart	λ									
	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.8	-1	-1.5	-2
EWMAR	-	68.7	-	17.6	-	8.3	5.3	3.9	-	1.9
MEWMA	131.0	58.6	29.5	17.2	11.5	8.5	5.5	4.1	2.6	2.0
EWMA3	132.2	59.5	28.5	16.4	10.8	7.9	5.1	3.8	2.4	1.9
MEWMARULE	**76.03**	18.14	9.72	7.00	5.50	4.74	3.90	3.44	2.66	2.14
ANNWR1	-	-	-	-	-	-	-	-	-	-
EWMAR-RULE	88.00	**17.36**	**6.61**	**4.31**	**3.39**	**2.49**	**1.75**	**1.53**	**1.07**	**1.01**
Chart	η									
	-0.025	-0.0375	-0.05	-0.0625	-0.075	-0.10	-0.125	-0.15	-0.2	-0.25
EWMAR	118.7	-	44.2	-	20.0	11.4	7.8	5.9	3.9	3.3
MEWMA	99.6	56.8	35.2	22.8	16.5	9.8	6.9	5.3	3.7	2.9
EWMA3	102.9	59.1	36.5	24.0	17.0	10.3	7.2	5.5	3.8	2.9
MEWMARULE	**40.23**	**17.76**	10.96	8.28	6.68	5.11	4.36	3.86	3.26	2.84
ANNWR1	-	-	-	-	-	-	-	-	-	-
EWMAR-RULE	63.47	17.77	**9.50**	**6.50**	**4.29**	**2.99**	**2.13**	**1.80**	**1.39**	**1.28**

other chart integrated with the VSI approach. Both of this approach can increase the efficiency of detecting a shift in the process parameters. The EWMAR-RULE chart is compared with combination of VSI approach and the MEWMA by [15] denoted as VSI-MEWMA, LRT by [30] denoted as VSI-LRT and GLR by [41] denoted as VSI-GLR. Figure 3 shows the ARL_1 for shifts in intercept (b) and standard deviation (a) in simple linear profiles for comparing VSI enhanced schemes and our proposed scheme. It can be seen that the proposed scheme is almost uniformly superior to VSI enhanced schemes. It is observed in Figure 3(a) that the EWMAR-RULE and VSI-GLR charts do not have the ability to detect decreasing shifts in the error variance.

4) EMPLOYING THE PROPOSED RULE MATRIX IN EWMA3 CONTROL CHART

To show the efficiency of the proposed method in control charts with more than one statistic, the designing approach

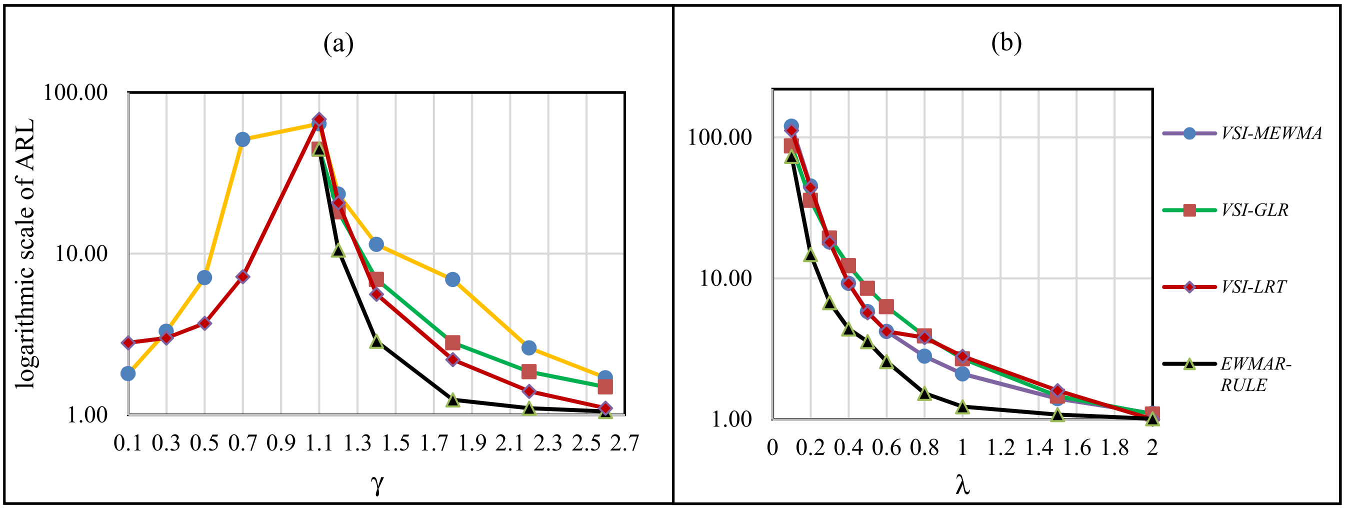

FIGURE 3. Comparisons of ARL_1 values for simple model for shifts in intercept and standard deviation based on VSI-MEWMA, VSI-GLR, VSI-LRT and EWMA-RULE charts.

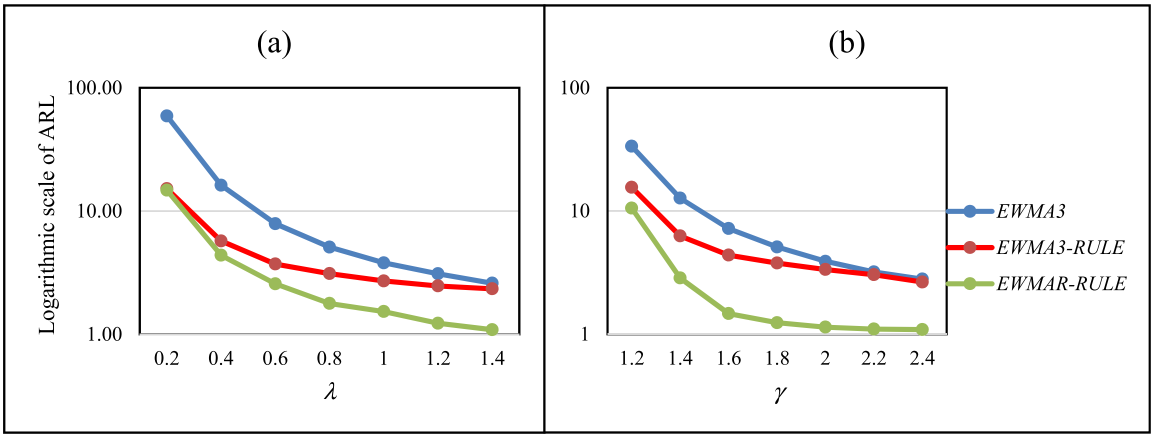

FIGURE 4. The results of ARL_1 for shifts in (a) intercept and (b) standard deviation of EWMA3, EWMAR-RULE and EWMA3-RULE.

has also been carried out on the EWMA3 chart in which transformation on explanatory variables is the main requirement [14]. Since transformation makes the estimators of each chart independent, the EWMA3 can only be implemented in simple linear profiles; hence, the integration of the run rules and the EWMA3 chart (denoted as EWMA3-RULE) can only be conducted in simple linear profiles. By using this approach, the IC intercept of simple linear model in Table 2 changes to 13 instead of 2 (for more details, see [14]). Equation (21) represents the obtained rule matrix for the intercept, slope and standard deviation of errrors, respectively; considering $UCL_I = 13.50$, $UCL_S = 2.32$ and $UCL_E = 0.62$. Note that the absouloute value of statistics are utilized here; hence, so $LCL_I = 13$, $LCL_S = 2$ and $LCL_E = 0.62$.

$$\begin{pmatrix} 13.15 & 0.32 & 1 \\ 13.10 & 0.69 & 2 \end{pmatrix} \begin{pmatrix} 2.15 & 0.08 & 1 \\ 2.11 & 0.25 & 2 \end{pmatrix} \begin{pmatrix} 0.44 & 0.05 & 1 \\ 0.18 & 0.49 & 2 \end{pmatrix} \quad (21)$$

The results of ARL_1 for shifts in the intercept and standard deviation of EWMA3, EWMAR-RULE and EWMA3-RULE are depicted in Figure 4. Although using of run rules improves the efficiency of the EWMA3 method; however, its performance is still inferior to that of the EWMAR-RULE. It is interesting that although EWMA3 has better performance than EWMAR (see Table 6), using of run rules has a reverse effect. We cannot state a clear reason for this; but this situation might be caused by the transformation in the explanatory variables of the EWMA3.

In the transformed model, the results of ARL_1 are dependent on the slope values in random profile generation of the slope shifts (for more details see Table 1 in Zou *et al.* [15]) . For a fair judgment, it is not apropos to compare EWMA3-RULE with EWMAR-RULE in this situation because EWMA-RULE has not been extended for the transformed model so the competitive methods are EWMA3, MEWMA and MEWMARULE in Figure 5. In this situation, EWMA3-RULE outperforms tangibly other competitive methods in all of the shifts.

TABLE 8. Comparisons of ARL_1 for multiple linear profiles with unique positive shifts.

Chart	λ									
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
LRT	44.4	12.4	6.5	4.4	3.3	2.6	2.1	1.8	1.5	1.3
LEWMA	29.9	8.7	4.8	3.4	2.6	2.2	1.9	1.8	1.5	1.3
PREDUCE	33.1	9.3	5.1	3.6	2.8	2.3	2.0	1.9	1.7	1.5
EWMAR-RULE	**7.73**	**2.77**	**2.02**	**1.67**	**1.24**	**1.11**	**1.05**	**1.02**	**1.00**	**1.00**
Chart	η									
	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.2
LRT	115.0	36.9	17.5	10.7	7.6	5.8	4.7	3.9	3.3	2.9
LEWMA	83.2	25.6	12.1	7.5	5.4	4.2	3.5	3.0	2.7	2.4
PREDUCE	91.3	28.6	13.4	8.1	5.9	4.5	3.7	3.2	2.8	2.5
EWMAR-RULE	**24.43**	**7.54**	**3.79**	**4.16**	**3.27**	**2.66**	**2.29**	**2.04**	**1.82**	**1.68**
Chart	γ									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
LRT	62.0	20.5	10.4	6.8	5.0	3.9	3.2	2.8	2.4	2.1
LEWMA	59.8	22.6	12.7	8.0	5.9	4.7	3.9	3.3	2.9	2.7
PREDUCE	50.7	17.0	9.2	6.4	4.7	3.8	3.2	2.9	2.5	2.3
EWMAR-RULE	**25.38**	**6.65**	**4.16**	**2.94**	**2.23**	**2.04**	**1.83**	**1.68**	**1.57**	**1.44**

TABLE 9. Comparisons of ARL_1 for multiple linear profiles with joint positive and negative shifts.

λ	Chart	η									
		0.02	0.04	0.06	0.08	0.1	-0.02	-0.04	-0.06	-0.08	-0.1
0.2	LRT	20.5	11.8	8.2	6.1	4.9	104.3	127.5	58.9	26.2	13.8
	LEWMA	13.7	8.2	5.8	4.5	3.7	89.0	137.0	49.6	18.9	10.2
	PREDUCE	15.3	8.8	6.3	4.8	4.0	88.2	**110.4**	47.8	19.6	10.5
	EWMAR-RULE	**4.87**	**3.15**	**2.25**	**1.85**	**1.74**	**29.53**	*250.65*	**28.45**	**7.63**	**4.56**
0.4	LRT	8.5	6.5	5.1	4.2	3.6	20.4	37.6	59.7	53.1	30.7
	LEWMA	6.1	4.7	3.8	3.2	2.8	14.6	30.0	62.9	59.3	28.1
	PREDUCE	6.6	5.1	4.1	3.5	3.0	15.1	28.4	47.5	**43.4**	23.5
	EWMAR-RULE	**2.37**	**1.96**	**1.67**	**1.44**	**1.38**	**4.63**	**7.45**	**24.16**	164.87	**23.41**
0.6	LRT	5.3	4.4	3.7	3.2	2.8	8.7	11.9	17.5	25.8	31.0
	LEWMA	3.9	3.3	2.9	2.6	2.3	6.3	8.8	13.9	23.8	32.8
	PREDUCE	4.2	3.5	3.1	2.7	2.5	6.6	9.0	13.4	19.7	23.8
	EWMAR-RULE	**1.67**	**1.48**	**1.30**	**1.22**	**1.17**	**2.49**	**3.07**	**4.38**	**8.14**	**23.16**

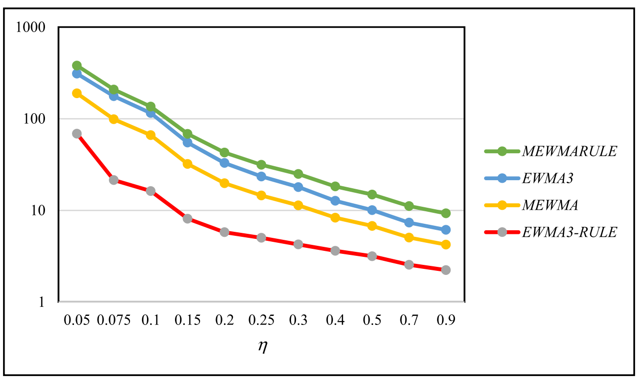

FIGURE 5. The results of ARL_1 for shifts in slope of EWMA3, EWMA3-RULE, MEWMA and MEWMA-RULE.

C. PERFORMANCE COMPARISON UNDER MULTIPLE LINEAR PROFILES

The multiple linear model based on parameters mentioned in Table 1 has also simulated with proposed method. Competitive charts are parameter reduction (PREDUCE) proposed by Amiri *et al.* [35], MEWMA by [15], LRT by [30] and the LASSO-based EWMA chart by Zou *et al.* [34].

1) PERFORMANCE COMPARISONS UNDER UNIQUE POSITIVE SHIFTS IN THE MULTIPLE LINEAR MODEL

The multiple linear profiles model has also been simulated using the rule matrix method. The results of the unique positive shifts simulations in multiple profiles model are gathered in Table 8. Note that all the results of the LRT, LEWMA and PREDUCE are taken from Amiri *et al.* [35]. The results in Table 8 clearly shows that the EWMAR-RULE chart generate smaller ARL_1 than other competing charts in all of the shifts. Similarly, as in the simple linear profile scenario, as the shift sizes increase, the difference in performance for the different control charts also decreases.

2) PERFORMANCE COMPARISONS UNDER JOINT POSITIVE AND NEGATIVE SHIFTS IN THE MULTIPLE LINEAR MODEL

Based on the empirical analysis in Table 9, it can be concluded that the proposed EWMAR-RULE chart performs

TABLE 10. Comparisons of ARL_1 for multivariate linear profile with single positive shifts with $\rho = 0.1$.

Chart	λ_0						
	0.2	0.4	0.6	0.8	1	1.2	2
MEWMA	66.6	19.2	9.2	5.9	4.3	3.5	2.0
MEWMA / χ^2	64.7	17.6	8.5	5.5	4.0	3.2	1.6
MEWMA3	80.8	21.4	9.8	6.1	4.5	3.5	2.1
EWMAR-RULE	**25.63**	**6.47**	**3.56**	**2.66**	**2.25**	**1.76**	**1.09**
Chart	η_0						
	0.025	0.05	0.075	0.1	0.125	0.15	0.25
MEWMA	108.5	39.4	17.7	10.6	7.4	5.6	3
MEWMA / χ^2	116.2	43.1	20.2	11.6	7.9	5.9	2.9
MEWMA3	133.7	52.3	23.2	13.1	8.7	6.5	3.3
EWMAR-RULE	**82.9**	**11.5**	**6.5**	**4.6**	**3.5**	**2.6**	**1.7**
Chart	γ_0						
	1.2	1.4	1.6	1.8	2	2.2	3
MEWMA	75.0	35.4	20.5	14	10.1	7.9	4.1
MEWMA / χ^2	51.5	17.2	8.2	4.8	3.2	2.4	1.4
MEWMA3	33.1	11.4	6.4	4.4	3.4	2.9	1.9
EWMAR-RULE	**16.73**	**4.59**	**2.26**	**2.00**	**1.49**	**1.30**	**1.08**

TABLE 11. Comparisons of ARL_1 for multivariate linear profile with joint positive shifts with $\rho = 0.1$.

λ_0	δ_0					Chart
	0.1	0.2	0.3	0.4	0.5	
0.1	125.6	66.0	32.4	17.7	11.5	MEWMA
	124.3	64.9	30.3	16.7	10.5	MEWMA / χ^2
	143.8	82.3	39.1	20.2	12.3	MEWMA3
	53.12	**13.75**	**9.35**	**6.47**	**4.77**	EWMAR-RULE
0.2	66.6	52.7	32.6	19.4	12.4	MEWMA
	65.9	51.7	30.2	18.2	11.6	MEWMA / χ^2
	82.1	65.8	39	22	13.5	MEWMA3
	16.0	**11.7**	**7.1**	**5.4**	**4.6**	EWMAR-RULE
0.3	32	32.7	25.2	17.8	12.6	MEWMA
	30.3	30.5	24	16.3	11.5	MEWMA / χ^2
	39.3	37.5	29.9	19.9	13.6	MEWMA-3
	11.26	**7.34**	**6.48**	**4.91**	**3.97**	EWMAR-RULE
λ_0	λ_1					Chart
	0.02	0.04	0.06	0.08	0.1	
0.1	51.4	23.3	13.2	8.9	6.7	MEWMA
	50.9	24.0	13.6	9.1	6.8	MEWMA / χ^2
	64	28.9	15.7	10.2	7.5	MEWMA3
	16.25	**10.36**	**6.94**	**4.48**	**3.53**	EWMAR-RULE
0.2	24.6	13.8	9.3	6.9	5.5	MEWMA
	23.9	13.3	9	6.7	5.4	MEWMA / χ^2
	29.4	15.9	10.4	7.6	5.9	MEWMA3
	9.5	**6.8**	**4.7**	**3.5**	**3.1**	EWMAR-RULE
0.3	14.6	9.6	7.1	5.6	4.6	MEWMA
	13.7	9.0	6.9	5.4	4.4	MEWMA / χ^2
	16.1	10.4	7.6	5.9	4.9	MEWMA3
	6.44	**4.86**	**3.79**	**3.24**	**2.58**	EWMAR-RULE

very well in joint shifts in multiple linear profiles as compared to competing charts. All of the competitive methods have greater ARL_1 than proposed method in most of the shifts. Some outstanding results are generated in joint positive and negative shifts and in all of the joint positive and negative shifts, the same results can be obtained and for brevity, the results are not shown here. We also see the bias effect (i.e. greater ARL_1 than ARL_0) in one of the shifts (i.e. $\lambda = 0.2$

TABLE 12. Comparisons of ARL_1 for polynomial model with unique positive shifts.

Chart	λ									
	0.025	0.05	0.075	0.1	0.2	0.3	0.4	0.5	0.8	1
EWMAR	168.25	115.25	70.64	42.78	11.51	6.08	4.11	3.15	1.98	1.59
EWMAR-RULE	**164.75**	**52.56**	**19.75**	**9.12**	**3.49**	**2.17**	**1.64**	**1.32**	**1.02**	**1.00**
MEWMA	178.0	126.1	84.2	56.1	15.4	7.7	5.1	3.8	2.3	2.0
VSCS	171.6	123.0	80.0	50.6	13.5	6.3	3.8	2.7	1.4	1.1
Chart	η									
	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5	0.7	1
EWMAR	172.76	109.45	60.93	41.31	27.57	18.96	11.36	8.09	4.82	3.09
EWMAR-RULE	175.65	**57.86**	**19.54**	**10.28**	**8.63**	**5.29**	**3.54**	**2.51**	**1.76**	**1.31**
MEWMA	167.5	115.7	70.9	43.8	28.8	20.4	12	8.3	5.0	3.3
VSCS	**164.0**	108.6	65.0	39.9	26.2	18.0	10.2	6.7	3.8	2.2
Chart	κ									
	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.5	0.6	0.8
EWMAR	149.37	113.02	76.53	57.63	39.16	31.35	24.86	16.65	11.63	7.49
EWMAR-RULE	**121.45**	**42.27**	**22.39**	**13.98**	**9.96**	**7.62**	**5.88**	**4.43**	**3.38**	**2.34**
MEWMA	138.1	96.3	67.1	46.1	33.0	24.8	19.2	12.8	9.2	5.9
VSCS	135.3	96.2	63.5	43.7	30.8	22.5	17.0	11.1	7.9	4.7
Chart	γ									
	1.025	1.050	1.075	1.100	1.125	1.150	1.175	1.200	1.25	1.30
EWMAR	153.56	116.65	87.65	63.57	48.97	38.99	31.4	63.03	14.58	10.83
EWMAR-RULE	**118.65**	**65.75**	**34.43**	**16.98**	**9.19**	**5.89**	**4.38**	**3.35**	**2.36**	**1.72**
MEWMA	132.8	74.7	42.1	26.8	18.4	13.5	10.7	8.6	6.3	5.0
VSCS	133.1	75.5	42.9	25.8	17.0	12.3	9.3	7.3	5.0	3.8

and $\eta = -0.04$) and we cannot justify it. Huwang *et al.* [18] stated that the similarity to Shewhart-type charts usually make bias effect in machine learning based control charts in all the negative shifts; but for one shift value, it is not a resonable justification.

D. PERFORMANCE COMPARISONS UNDER SHIFTS IN THE MULTIVARIATE MODEL

Considering the IC model in Table 2 and Equations (11), 12 and (13), the EWMAR method is extended for multivariate profiles in this article. The three control charts in Noorossana *et al.* [38] i.e., MEWMA, $MEWMA/\chi^2$ and MEWMA3, are chosen as the competing charts. Different simulations setups can be defined for the multivariate models; but for brevity, the results of some single and joint shifts with $\rho = 0.1$ are analyzed in the following sections.

1) PERFORMANCE COMPARISONS UNDER SINGLE POSITIVE SHIFTS IN THE MULTIVARIATE LINEAR MODEL

Table 10 gathers the ARL_1 for the shifts in the first IC profile $(\lambda_0, \eta_0, \gamma_0)$. It is obvious that for all the shifts the EWMAR-RULE chart has superiority over the other competing charts.

2) PERFORMANCE COMPARISONS UNDER JOINT POSITIVE SHIFTS IN THE MULTIVARIATE LINEAR MODEL

The results of joint shifts in the multivariate profiles are illustrated in Table 11, where λ_0, δ_0 and λ_1 are the shift sizes in the first and second profiles intercept and the slope of the first profile, respectively. The same as single shifts, the EWMAR-RULE chart outperforms the other competitors.

E. PERFORMANCE COMPARISONS UNDER SHIFTS IN THE POLYNOMIAL MODEL

The polynomial profiles are very important in several practical applications such as deep reactive ion etching (DRIE) and semiconductor manufacturing [15, 18, 49]. To examine the performance of the proposed chart, the simulaion results of the IC model shown in the last row of Table 2 are reported in Table 12. It is shown that the EWMAR-RULE scheme outperforms other competitive methods entailing commom EWMAR (it has not been reported in any research yet), MEWMA [15] and VSCS (the results of Huwang *et al.* [18]'s control chart are shown with VSCS abbreviation) in the polynomial model for most shift values. To describe precisely, in the lowest values of η, the performance of the EWMAR-RULE scheme is in the second place but other positive shifts in η indicate that the performance of the proposed scheme is better than other competitive methods.

VI. ILLUSTRATIVE EXAMPLE

In a calibration experiment associated with body of an automotive industrial group, Noorossana *et al.* [38] considered a multivariate linear relationship between a set of responses and one explanatory variable. In this application, a hydraulic

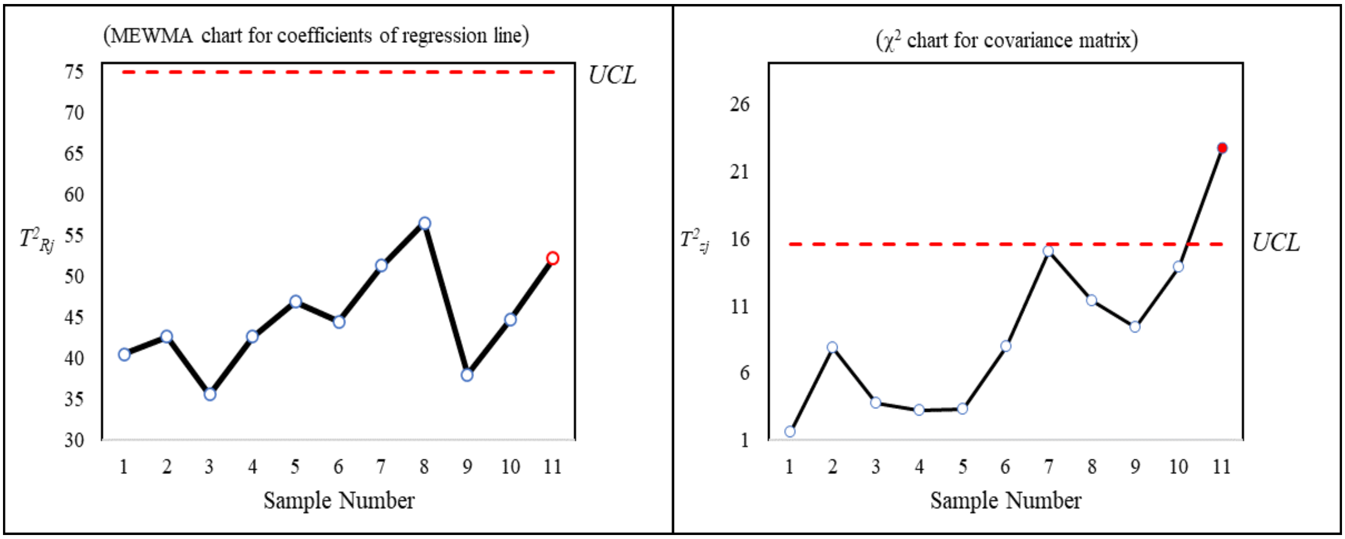

FIGURE 6. The statistics of proposed EWMAR method without rule matrix consideration in the illustrative example.

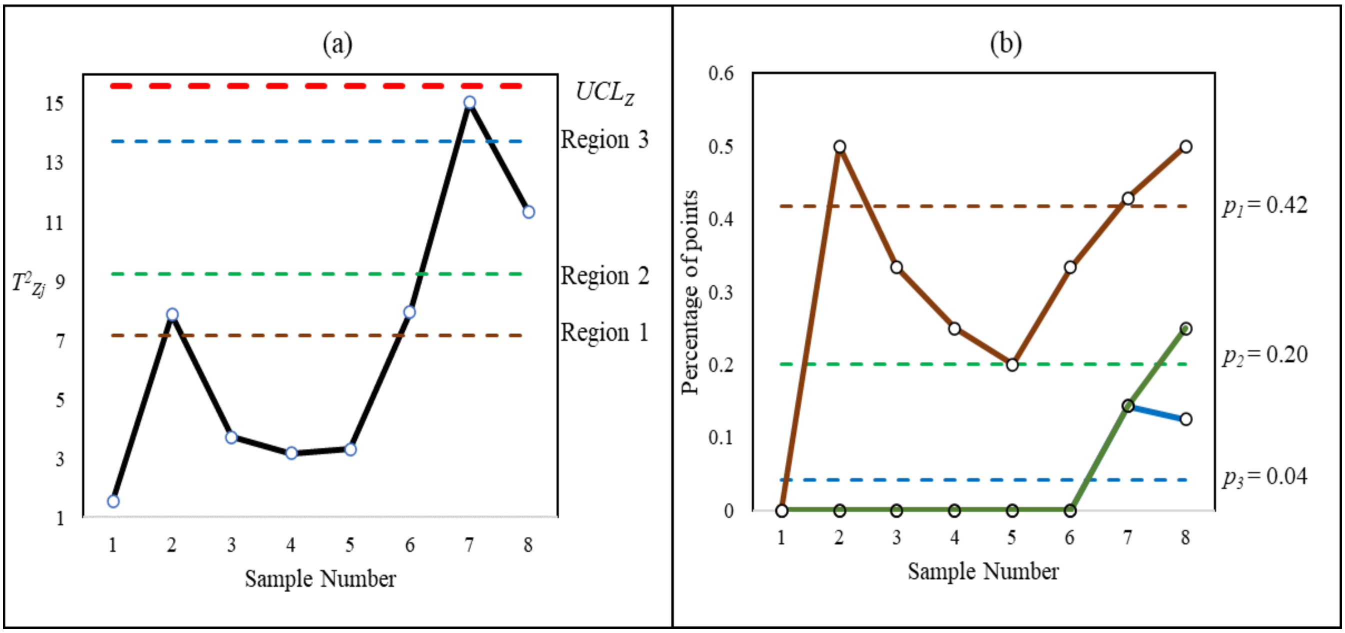

FIGURE 7. The location of the charting statistics in each region (a) and the percentage of points in each region (b).

press machine generated a real force which should be calibrated with the prespecified desired (designed) forces. In the components of the press machines entailing a set of cylinders, pistons and the hydraulic pipe, the first one has the most important function because of its effect on quality of outputs. In other words, the nominal forces that should be exerted by cylinders on the metal plates to give the desired parts are very important in the machine. Therefore, the multivariate IC profile, shown in (22), has been defined for each value of nominal force (explanatory variable) and a set of real forces (response variables) including four cylinders based on the historical data. These data measured by the controlling system of the press machine should be established to be close enough relationship to the IC model and the purpose of the profiles monitoring in this problem is to check whether it is violated or not. Violation of this IC model may happen due to oscillation in oil temperature, variation in oil volume, daily set-up changes, etc., and naturally lead to low quality outputs. It is noteworthy to mention that the response variables or real forces are correlated in this problem, so only the multivariate model can be used.

$$\begin{aligned} y_{i1j} &= -8.5 + 0.87x_i + \varepsilon_{1j}, \\ y_{i2j} &= -5.8 + 0.95x_i + \varepsilon_{2j}, \\ y_{i3j} &= 3.2 + 1.04x_i + \varepsilon_{3j}, \\ y_{i4j} &= 13.6 + 1.09x_i + \varepsilon_{4j}, \\ i &= 1, 2, \ldots, 11. \end{aligned} \tag{22}$$

In (22), the independent variable has the matrix form of

$$X' = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 50 & 80 & \cdots & 320 & 350 \end{pmatrix}$$

while the covariance matrix of the error terms is

$$\sum = \begin{pmatrix} 80 & 89.6 & 45.1 & 25.3 \\ 89.6 & 122.1 & 71.5 & 29.1 \\ 45.1 & 71.5 & 189 & -28.8 \\ 25.3 & 29.1 & -28.8 & 84.4 \end{pmatrix}.$$

To show the capability of the proposed EWMAR-RULE chart, an artificial shift in the first profile intercept (-8.5 to -7) was employed and the statistics of each control chart are shown in Figure 6. It is obvious that the proposed EWMAR chart without run rules in this article (or $MEWMA/\chi^2$) detected this shift in the 11^{th} sample (this is similar to Figure 2 in Noorossana *et al.* [38]).

As shown in Figure 6, to reach ARL_0 equal to 200 (or equivalently 400 for each chart) the control limits are 74.92 and 15.6, respectively. Considering the proposed designing approach and ARL_0 equal to 200, the following rule matrix has been obtained with $UCL_R = 78.66$ and $UCL_Z = 16.9$.

$$\begin{pmatrix} 60.46 & 0.05 & 1 \\ 45.11 & 0.19 & 2 \\ 41 & 0.30 & 3 \end{pmatrix} \begin{pmatrix} 13.74 & 0.04 & 1 \\ 9.23 & 0.2 & 2 \\ 7.16 & 0.42 & 3 \end{pmatrix} \quad (23)$$

Taking the rule matrix into account, the OC signal is triggered in the 8^{th} sample due to firing [49] of the first rule of the Z chart. In this sample, there are 4 samples in the first region and the percentage of points (0.5) is greater than 0.42 so the rule matrix shows an OC situation. The location of the charting statistics in each region are shown withbrown (Region 1), green (Region 2) and blue (Region 3) colors in Figure 7(a). Also, the percentages of points in each of these regions are depicted in Figure 7(b).

VII. CONCLUSION

In general, the aim of this article is to provide effective control charts for monitoring linear profiles in Phase II. As mentioned, the study used an EWMAR control chart to monitor the linear profiles with a combination of run rules to enhance the performance of the chart in detecting OC conditions. In order to evaluate the performance of the proposed control chart in comparison with other competitive methods including the conventional EWMAR, EWMA3 and etc., the numerical example of Kang and Albin [6] was used in simple linear profiles. The results indicate better performance of the proposed control chart in identifying OC shifts in almost nearly all of the shifts considered. Instead of simple linear profiles, performance evaluations of the proposed approach were also performed in multiple, multivariate and polynomial models. The results indicate the similar pattern as those observed when the simple model was implemented. Finally, a performance's appraisal of the proposed scheme leads to the conclusion that it seems to be almost superior to the performance of other existing charts based on the VSI scheme. According to all the numerical comparisons and evaluations carried out, a significant improvement in the performance of the EWMAR control chart is observed, when it is enhanced with run rules. Therefore, the implementation of run rules schemes in alternative control charts or Phase I applications is also recommended. Also, it seems to be evident that the VSI approach can be combined with the proposed scheme to boost its performance. Finally, we intend to incorporate the rule matrix to the adaptive MEWMA chart (proposed by Haq [40]) to improve its detection ability in monitoring univariate and multivariate simple linear profiles.

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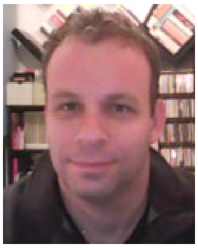

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