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Some coincidence point results for generalized (ψ,φ)-weakly contractive mappings in ordered G-metric spaces

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Abstract
The aim of this paper is to present some coincidence point results for six mappings satisfying the generalized (ψ,φ)-weakly contractive condition in the framework of partially ordered G-metric spaces. To elucidate our results, we present two examples together with an application of a system of integral equations.
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1 Introduction and mathematical preliminaries

Let (X,d) be a metric space and f be a self-mapping on X. If $x = fx$ for some x in X, then x is called a fixed point of f. The set of all fixed points of f is denoted by $F(f)$. If $F(f) = \{z\}$, and for each x_0 in a complete metric space X, the sequence $x_{n+1} = f(x_n) = f^n(x_0)$, $n = 0,1,2,\dots$, converges to z, then f is called a Picard operator.

The function $\varphi : [0,\infty) \to [0,\infty)$ is called an altering distance function if φ is continuous and nondecreasing and $\varphi(t) = 0$ if and only if $t = 0$ [1].

A self-mapping f on X is a weak contraction if the following contractive condition holds:

$$d(fx,fy) \le d(x,y) - \varphi\bigl(d(x,y)\bigr),$$

for all $x,y \in X$, where φ is an altering distance function.

The concept of a weakly contractive mapping was introduced by Alber and Guerre-Delabrere [2] in the setup of Hilbert spaces. Rhoades [3] considered this class of mappings in the setup of metric spaces and proved that a weakly contractive mapping is a Picard operator.

Let f and g be two self-mappings on a nonempty set X. If $x = fx = gx$ for some x in X, then x is called a common fixed point of f and g. Sessa [4] defined the concept of weakly commutative maps to obtain common fixed point for a pair of maps. Jungck generalized this idea, first to compatible mappings [5] and then to weakly compatible mappings [6]. There are examples which show that each of these generalizations of commutativity is a proper extension of the previous definition.

Zhang and Song [7] introduced the concept of a generalized φ-weak contractive mapping as follows.

Self-mappings f and g on a metric space X are called generalized φ-weak contractions if there exists a lower semicontinuous function $\varphi : [0,\infty) \to [0,\infty)$ with $\varphi(0) = 0$ and $\varphi(t) > 0$ for all $t > 0$ such that for all $x, y \in X$,

$$d(fx, gy) \leq N(x, y) - \varphi\bigl(N(x, y)\bigr),$$

where

$$N(x, y) = \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{1}{2}\bigl[d(x, gy) + d(y, fx)\bigr]\right\}.$$

Based on the above definition, they proved the following common fixed point result.

Theorem 1.1 [7] *Let (X, d) be a complete metric space. If $f, g : X \to X$ are generalized φ-weak contractive mappings, then there exists a unique point $u \in X$ such that $u = fu = gu$.*

For further works in this direction, we refer the reader to [8–20].

Recently, many researchers have focused on different contractive conditions in complete metric spaces endowed with a partial order and studied fixed point theory in the so-called bistructural spaces. For more details on fixed point results, its applications, comparison of different contractive conditions and related results in ordered metric spaces, we refer the reader to [21–40] and the references mentioned therein.

Mustafa and Sims [41] generalized the concept of a metric, in which to every triplet of points of an abstract set, a real number is assigned. Based on the notion of generalized metric spaces, Mustafa *et al.* [42–49] obtained some fixed point theorems for mappings satisfying different contractive conditions. Chugh *et al.* [50] obtained some fixed point results for maps satisfying property P in G-metric spaces. Saadati *et al.* [51] studied fixed point of contractive mappings in partially ordered G-metric spaces. Shatanawi [52] obtained fixed points of Φ-maps in G-metric spaces. For more details, we refer to [21, 53–65].

Very recently, Jleli and Samet [66] and Samet *et al.* [67] noticed that some fixed point theorems in the context of a G-metric space can be concluded by some existing results in the setting of a (quasi-)metric space. In fact, if the contraction condition of the fixed point theorem on a G-metric space can be reduced to two variables instead of three variables, then one can construct an equivalent fixed point theorem in the setting of a usual metric space. More precisely, in [66, 67], the authors noticed that $d(x, y) = G(x, y, y)$ forms a quasi-metric. Therefore, if one can transform the contraction condition of existence results in a G-metric space in such terms, $G(x, y, y)$, then the related fixed point results become the known fixed point results in the context of a quasi-metric space.

The following definitions and results will be needed in the sequel.

Definition 1.2 [41] Let X be a nonempty set. Suppose that a mapping $G : X \times X \times X \to R^+$ satisfies:

(G1) $G(x, y, z) = 0$ if $x = y = z$;
(G2) $0 < G(x, y, z)$ for all $x, y, z \in X$, with $x \neq y$;
(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$, with $y \neq z$;

(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (symmetry in all three variables); and
(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$.

Then G is called a G-metric on X and (X,G) is called a G-metric space.

Definition 1.3 [41] A sequence $\{x_n\}$ in a G-metric space X is:
(i) a G-convergent sequence if there is $x \in X$ such that for any $\varepsilon > 0$, and $n_0 \in \mathbb{N}$, for all $n,m \geq n_0$, $G(x_n,x_m,x) < \varepsilon$.
(ii) a *G-Cauchy* sequence if, for every $\varepsilon > 0$, there is a natural number n_0 such that for all $n,m,l \geq n_0$, $G(x_n,x_m,x_l) < \varepsilon$.

A G-metric space on X is said to be G-complete if every G-Cauchy sequence in X is G-convergent in X. It is known that $\{x_n\}$ G-converges to $x \in X$ if and only if $G(x_m,x_n,x) \to 0$ as $n,m \to \infty$.

Lemma 1.4 [41] *Let X be a G-metric space. Then the following are equivalent*:
(1) *The sequence $\{x_n\}$ is G-convergent to x.*
(2) $G(x_n,x_n,x) \to 0$ *as* $n \to \infty$.
(3) $G(x_n,x,x) \to 0$ *as* $n \to \infty$.

Lemma 1.5 [68] *Let X be a G-metric space. Then the following are equivalent*:
The sequence $\{x_n\}$ is G-Cauchy.
For every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n,m \geq n_0$, $G(x_n,x_m,x_m) < \varepsilon$; that is, if $G(x_n,x_m,x_m) \to 0$ as $n,m \to \infty$.

Definition 1.6 [41] Let (X,G) and (X',G') be two G-metric spaces. Then a function $f: X \to X'$ is G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x; that is, whenever $\{x_n\}$ is G-convergent to x, $\{f(x_n)\}$ is G'-convergent to $f(x)$.

Definition 1.7 A G-metric on X is said to be symmetric if $G(x,y,y) = G(y,x,x)$ for all $x,y \in X$.

Proposition 1.8 *Every G-metric on X defines a metric d_G on X by*

$$d_G(x,y) = G(x,y,y) + G(y,x,x), \quad \forall x,y \in X. \tag{1.1}$$

For a symmetric G-metric space, one obtains

$$d_G(x,y) = 2G(x,y,y), \quad \forall x,y \in X. \tag{1.2}$$

However, if G is not symmetric, then the following inequality holds:

$$\frac{3}{2}G(x,y,y) \leq d_G(x,y) \leq 3G(x,y,y), \quad \forall x,y \in X. \tag{1.3}$$

Definition 1.9 A partially ordered G-metric space $(X,\preceq,G)$ is said to have the sequential limit comparison property if for every nondecreasing sequence (nonincreasing sequence) $\{x_n\}$ in X, $x_n \to x$ implies that $x_n \preceq x$ ($x \preceq x_n$).

Definition 1.10 Let f and g be two self-maps on a partially ordered set X. A pair (f,g) is said to be

(i) weakly increasing if $fx \preceq gfx$ and $gx \preceq fgx$ for all $x \in X$ [69],
(ii) partially weakly increasing if $fx \preceq gfx$ for all $x \in X$ [22].

Let X be a nonempty set and $f : X \to X$ be a given mapping. For every $x \in X$, let $f^{-1}(x) = \{u \in X : fu = x\}$.

Definition 1.11 Let $(X, \preceq)$ be a partially ordered set, and let $f, g, h : X \to X$ be mappings such that $fX \subseteq hX$ and $gX \subseteq hX$. The ordered pair (f,g) is said to be: (a) weakly increasing with respect to h if and only if for all $x \in X$, $fx \preceq gy$ for all $y \in h^{-1}(fx)$, and $gx \preceq fy$ for all $y \in h^{-1}(gx)$ [34], (b) partially weakly increasing with respect to h if $fx \preceq gy$ for all $y \in h^{-1}(fx)$ [32].

Remark 1.12 In the above definition: (i) if $f = g$, we say that f is weakly increasing (partially weakly increasing) with respect to h, (ii) if $h = I$ (the identity mapping on X), then the above definition reduces to a weakly increasing (partially weakly increasing) mapping (see [34, 40]).

The following is an example of mappings f, g, h, R, S and T for which all ordered pairs (f,g), (g,h) and (h,f) are partially weakly increasing with respect to R, S and T but not weakly increasing with respect to them.

Example 1.13 Let $X = [0,\infty)$. We define functions $f, g, h, R, S, T : X \to X$ by

$$f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty, \end{cases} \qquad g(x) = \begin{cases} \sqrt{x}, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty, \end{cases}$$
$$h(x) = \begin{cases} x^2, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty, \end{cases}$$

and

$$R(x) = \begin{cases} x^3, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty, \end{cases} \qquad S(x) = \begin{cases} \sqrt[4]{x}, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty, \end{cases}$$
$$T(x) = \begin{cases} \sqrt[3]{x}, & 0 \le x \le 1, \\ 1, & 1 \le x \le \infty. \end{cases}$$

Definition 1.14 [60, 62] Let X be a G-metric space and $f, g : X \to X$. The pair (f,g) is said to be compatible if and only if $\lim_{n\to\infty} G(fgx_n, fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$ for some $t \in X$.

Definition 1.15 (see, *e.g.*, [67]) A quasi-metric on a nonempty set X is a mapping $p : X \times X \to [0,\infty)$ such that (p1) $x = y$ if and only if $p(x,y) = 0$, (p2) $p(x,y) \le p(x,z) + p(z,y)$ for all $x, y, z \in X$. A pair (X,p) is said to be a quasi-metric space.

The study of unique common fixed points of mappings satisfying strict contractive conditions has been at the center of vigorous research activity. The study of common fixed point theorems in generalized metric spaces was initiated by Abbas and Rhoades [56] (see also [21, 53, 54]). Motivated by the work in [8, 13, 16, 17, 22, 32] and [40], we prove some coincidence point results for nonlinear generalized (ψ,φ)-weakly contractive mappings in partially ordered G-metric spaces.

2 Main results

Let $(X,\preceq,G)$ be an ordered G-metric space, and let $f,g,h,R,S,T:X\to X$ be six self-mappings. Throughout this paper, unless otherwise stated, for all $x,y,z\in X$, let

$$M(x,y,z)=\max\biggl\{G(Tx,Ry,Sz),\\ G(Tx,fx,fx),G(Ry,gy,gy),G(Sz,hz,hz),\\ \frac{G(Tx,Tx,fx)+G(Ry,Ry,gy)+G(Sz,Sz,hz)}{3}\biggr\}.$$

Let X be any nonempty set and $f,g,h,R,S,T:X\to X$ be six mappings such that $f(X)\subseteq R(X)$, $g(X)\subseteq S(X)$ and $h(X)\subseteq T(X)$. Let x_0 be an arbitrary point of X. Choose $x_1\in X$ such that $fx_0=Rx_1$, $x_2\in X$ such that $gx_1=Sx_2$ and $x_3\in X$ such that $hx_2=Tx_3$. This can be done as $f(X)\subseteq R(X)$, $g(X)\subseteq S(X)$ and $h(X)\subseteq T(X)$.

Continuing in this way, we construct a sequence $\{z_n\}$ defined by: $z_{3n+1}=Rx_{3n+1}=fx_{3n}$, $z_{3n+2}=Sx_{3n+2}=gx_{3n+1}$, and $z_{3n+3}=Tx_{3n+3}=hx_{3n+2}$ for all $n\geq 0$. The sequence $\{z_n\}$ in X is said to be a Jungck-type iterative sequence with initial guess x_0.

Theorem 2.1 *Let $(X,\preceq,G)$ be a partially ordered G-complete G-metric space. Let $f,g,h,R,S,T:X\to X$ be six mappings such that $f(X)\subseteq R(X)$, $g(X)\subseteq S(X)$ and $h(X)\subseteq T(X)$. Suppose that for every three comparable elements $Tx,Ry,Sz\in X$, we have*

$$\psi\bigl(2G(fx,gy,hz)\bigr)\leq\psi\bigl(M(x,y,z)\bigr)-\varphi\bigl(M(x,y,z)\bigr),\tag{2.1}$$

where $\psi,\varphi:[0,\infty)\to[0,\infty)$ are altering distance functions. Let f, g, h, R, S and T be continuous, the pairs (f,T), (g,R) and (h,S) be compatible and the pairs (f,g), (g,h) and (h,f) be partially weakly increasing with respect to R, S and T, respectively. Then the pairs (f,T), (g,R) and (h,S) have a coincidence point z^* in X. Moreover, if Rz^*, Sz^* and Tz^* are comparable, then z^* is a coincidence point of f, g, h, R, S and T.

Proof Let $\{z_n\}$ be a Jungck-type iterative sequence with initial guess x_0 in X; that is, $z_{3n+1}=Rx_{3n+1}=fx_{3n}$, $z_{3n+2}=Sx_{3n+2}=gx_{3n+1}$ and $z_{3n+3}=Tx_{3n+3}=hx_{3n+2}$ for all $n\geq 0$.

As $x_1\in R^{-1}(fx_0)$, $x_2\in S^{-1}(gx_1)$ and $x_3\in T^{-1}(hx_2)$, and the pairs (f,g), (g,h) and (h,f) are partially weakly increasing with respect to R, S and T, so we have

$$Rx_1=fx_0\preceq gx_1=Sx_2\preceq hx_2=Tx_3\preceq fx_3=Rx_4.$$

Continuing this process, we obtain $Rx_{3n+1}\preceq Sx_{3n+2}\preceq Tx_{3n+3}$ for $n\geq 0$.

We will complete the proof in three steps.

Step I. We will prove that $\lim_{k\to\infty}G(z_k,z_{k+1},z_{k+2})=0$.

Define $G_k = G(z_k, z_{k+1}, z_{k+2})$. Suppose $G_{k_0} = 0$ for some k_0. Then $z_{k_0} = z_{k_0+1} = z_{k_0+2}$. If $k_0 = 3n$, then $z_{3n} = z_{3n+1} = z_{3n+2}$ gives $z_{3n+1} = z_{3n+2} = z_{3n+3}$. Indeed,

$$\begin{aligned}\psi\bigl(2G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) &= \psi\bigl(2G(fx_{3n}, gx_{3n+1}, hx_{3n+2})\bigr)\\ &\le \psi\bigl(M(x_{3n}, x_{3n+1}, x_{3n+2})\bigr) - \varphi\bigl(M(x_{3n}, x_{3n+1}, x_{3n+2})\bigr),\end{aligned}$$

where

$$\begin{aligned}&M(x_{3n}, x_{3n+1}, x_{3n+2})\\ &\quad= \max\biggl\{G(Tx_{3n}, Rx_{3n+1}, Sx_{3n+2}), G(Tx_{3n}, fx_{3n}, fx_{3n}),\\ &\qquad G(Rx_{3n+1}, gx_{3n+1}, gx_{3n+1}), G(Sx_{3n+2}, hx_{3n+2}, hx_{3n+2}),\\ &\qquad \frac{G(Tx_{3n}, Tx_{3n}, fx_{3n}) + G(Rx_{3n+1}, Rx_{3n+1}, gx_{3n+1}) + G(Sx_{3n+2}, Sx_{3n+2}, hx_{3n+2})}{3}\biggr\}\\ &\quad= \max\biggl\{G(z_{3n}, z_{3n+1}, z_{3n+2}), G(z_{3n}, z_{3n+1}, z_{3n+1}),\\ &\qquad G(z_{3n+1}, z_{3n+2}, z_{3n+2}), G(z_{3n+2}, z_{3n+3}, z_{3n+3}),\\ &\qquad \frac{G(z_{3n}, z_{3n}, z_{3n+1}) + G(z_{3n+1}, z_{3n+1}, z_{3n+2}) + G(z_{3n+2}, z_{3n+2}, z_{3n+3})}{3}\biggr\}\\ &\quad= \max\biggl\{0, 0, 0, G(z_{3n+2}, z_{3n+3}, z_{3n+3}), \frac{0 + 0 + G(z_{3n+2}, z_{3n+2}, z_{3n+3})}{3}\biggr\}\\ &\quad= G(z_{3n+2}, z_{3n+3}, z_{3n+3})\\ &\quad\le 2G(z_{3n+2}, z_{3n+2}, z_{3n+3})\\ &\quad= 2G(z_{3n+1}, z_{3n+2}, z_{3n+3}).\end{aligned}$$

Thus,

$$\psi\bigl(2G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) \le \psi\bigl(2G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) - \varphi\bigl(G(z_{3n+2}, z_{3n+3}, z_{3n+3})\bigr),$$

which implies that $\varphi(G(z_{3n+2}, z_{3n+3}, z_{3n+3})) = 0$; that is, $z_{3n+1} = z_{3n+2} = z_{3n+3}$. Similarly, if $k_0 = 3n+1$, then $z_{3n+1} = z_{3n+2} = z_{3n+3}$ gives $z_{3n+2} = z_{3n+3} = z_{3n+4}$. Also, if $k_0 = 3n+2$, then $z_{3n+2} = z_{3n+3} = z_{3n+4}$ implies that $z_{3n+3} = z_{3n+4} = z_{3n+5}$. Consequently, the sequence $\{z_k\}$ becomes constant for $k \ge k_0$.

Suppose that

$$G_k = G(z_k, z_{k+1}, z_{k+2}) > 0 \tag{2.2}$$

for each k. We now claim that the following inequality holds:

$$G(z_{k+1}, z_{k+2}, z_{k+3}) \le G(z_k, z_{k+1}, z_{k+2}) = M(x_k, x_{k+1}, x_{k+2}) \tag{2.3}$$

for each $k = 1, 2, 3, \ldots$.

Let $k = 3n$ and for $n \geq 0$, $G(z_{3n+1}, z_{3n+2}, z_{3n+3}) \geq G(z_{3n}, z_{3n+1}, z_{3n+2}) > 0$. Then, as $Tx_{3n} \preceq Rx_{3n+1} \preceq Sx_{3n+2}$, using (2.1), we obtain that

$$\begin{aligned}\psi\bigl(G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) &\leq \psi\bigl(2G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr)\\ &= \psi\bigl(2G(fx_{3n}, gx_{3n+1}, hx_{3n+2})\bigr)\\ &\leq \psi\bigl(M(x_{3n}, x_{3n+1}, x_{3n+2})\bigr) - \varphi\bigl(M(x_{3n}, x_{3n+1}, x_{3n+2})\bigr),\end{aligned} \tag{2.4}$$

where

$$\begin{aligned}&M(x_{3n}, x_{3n+1}, x_{3n+2})\\ &\quad= \max\biggl\{G(Tx_{3n}, Rx_{3n+1}, Sx_{3n+2}),\\ &\qquad G(Tx_{3n}, fx_{3n}, fx_{3n}), G(Rx_{3n+1}, gx_{3n+1}, gx_{3n+1}), G(Sx_{3n+2}, hx_{3n+2}, hx_{3n+2}),\\ &\qquad \frac{G(Tx_{3n}, Tx_{3n}, fx_{3n}) + G(Rx_{3n+1}, Rx_{3n+1}, gx_{3n+1}) + G(Sx_{3n+2}, Sx_{3n+2}, hx_{3n+2})}{3}\biggr\}\\ &\quad= \max\biggl\{G(z_{3n}, z_{3n+1}, z_{3n+2}),\\ &\qquad G(z_{3n}, z_{3n+1}, z_{3n+1}), G(z_{3n+1}, z_{3n+2}, z_{3n+2}), G(z_{3n+2}, z_{3n+3}, z_{3n+3}),\\ &\qquad \frac{G(z_{3n}, z_{3n}, z_{3n+1}) + G(z_{3n+1}, z_{3n+1}, z_{3n+2}) + G(z_{3n+2}, z_{3n+2}, z_{3n+3})}{3}\biggr\}\\ &\quad\leq \max\biggl\{G(z_{3n}, z_{3n+1}, z_{3n+2}), G(z_{3n+1}, z_{3n+2}, z_{3n+3}),\\ &\qquad \frac{2G(z_{3n}, z_{3n+1}, z_{3n+2}) + G(z_{3n+1}, z_{3n+2}, z_{3n+3})}{3}\biggr\}\\ &\quad= G(z_{3n+1}, z_{3n+2}, z_{3n+3}).\end{aligned}$$

Hence, (2.4) implies that

$$\psi\bigl(G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) \leq \psi\bigl(G(z_{3n+1}, z_{3n+2}, z_{3n+3})\bigr) - \varphi\bigl(M(x_{3n}, x_{3n+1}, x_{3n+2})\bigr),$$

which is possible only if $M(x_{3n}, x_{3n+1}, x_{3n+2}) = 0$; that is, $G(z_{3n}, z_{3n+1}, z_{3n+2}) = 0$, a contradiction to (2.2). Hence, $G(z_{3n+1}, z_{3n+2}, z_{3n+3}) \leq G(z_{3n}, z_{3n+1}, z_{3n+2})$ and

$$M(x_{3n}, x_{3n+1}, x_{3n+2}) = G(z_{3n}, z_{3n+1}, z_{3n+2}).$$

Therefore, (2.3) is proved for $k = 3n$.

Similarly, it can be shown that

$$G(z_{3n+2}, z_{3n+3}, z_{3n+4}) \leq G(z_{3n+1}, z_{3n+2}, z_{3n+3}) = M(x_{3n+1}, x_{3n+2}, x_{3n+3}), \tag{2.5}$$

and

$$G(z_{3n+3}, z_{3n+4}, z_{3n+5}) \leq G(z_{3n+2}, z_{3n+3}, z_{3n+4}) = M(x_{3n+2}, x_{3n+3}, x_{3n+4}). \tag{2.6}$$

Hence, $\{G(z_k, z_{k+1}, z_{k+2})\}$ is a nondecreasing sequence of nonnegative real numbers. Therefore, there is $r \geq 0$ such that

$$\lim_{k\to\infty} G(z_k, z_{k+1}, z_{k+2}) = r. \tag{2.7}$$

Since

$$G(z_{k+1}, z_{k+2}, z_{k+3}) \leq M(x_k, x_{k+1}, x_{k+2}) \leq G(z_k, z_{k+1}, z_{k+2}), \tag{2.8}$$

taking limit as $k \to \infty$ in (2.8), we obtain

$$\lim_{k\to\infty} M(x_k, x_{k+1}, x_{k+2}) = r. \tag{2.9}$$

Taking limit as $n \to \infty$ in (2.4), using (2.7), (2.9) and the continuity of ψ and φ, we have $\psi(r) \leq \psi(r) - \varphi(r) \leq \psi(r)$. Therefore $\varphi(r) = 0$. Hence,

$$\lim_{k\to\infty} G(z_k, z_{k+1}, z_{k+2}) = 0, \tag{2.10}$$

from our assumptions about φ. Also, from Definition 1.2, part (G3), we have

$$\lim_{k\to\infty} G(z_k, z_{k+1}, z_{k+1}) = 0, \tag{2.11}$$

and, since $G(x, y, y) \leq 2G(x, x, y)$ for all $x, y \in X$, we have

$$\lim_{k\to\infty} G(z_k, z_k, z_{k+1}) = 0. \tag{2.12}$$

Step II. We now show that $\{z_n\}$ is a G-Cauchy sequence in X. Because of (2.10), it is sufficient to show that $\{z_{3n}\}$ is G-Cauchy.

We assume on contrary that there exists $\varepsilon > 0$ for which we can find subsequences $\{z_{3m(k)}\}$ and $\{z_{3n(k)}\}$ of $\{z_{3n}\}$ such that $n(k) > m(k) \geq k$ and

$$G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) \geq \varepsilon, \tag{2.13}$$

and $n(k)$ is the smallest number such that the above statement holds; *i.e.*,

$$G(z_{3m(k)}, z_{3n(k)-3}, z_{3n(k)-3}) < \varepsilon. \tag{2.14}$$

From the rectangle inequality and (2.14), we have

$$\begin{aligned} &G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) \\ &\quad \leq G(z_{3m(k)}, z_{3n(k)-3}, z_{3n(k)-3}) + G(z_{3n(k)-3}, z_{3n(k)}, z_{3n(k)}) \\ &\quad < \varepsilon + G(z_{3n(k)-3}, z_{3n(k)}, z_{3n(k)}) \\ &\quad < \varepsilon + G(z_{3n(k)-3}, z_{3n(k)-2}, z_{3n(k)-2}) + G(z_{3n(k)-2}, z_{3n(k)-1}, z_{3n(k)-1}) \\ &\qquad + G(z_{3n(k)-1}, z_{3n(k)}, z_{3n(k)}). \end{aligned} \tag{2.15}$$

Taking limit as $k \to \infty$ in (2.15), from (2.11) and (2.13) we obtain that

$$\lim_{k\to\infty} G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) = \varepsilon. \tag{2.16}$$

Using the rectangle inequality, we have

$$\begin{aligned}
&G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) + G(z_{3n(k)}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+1}) + G(z_{3n(k)+1}, z_{3n(k)}, z_{3n(k)}) + G(z_{3n(k)}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+2}) + G(z_{3n(k)+1}, z_{3n(k)+2}, z_{3n(k)+2}) + G(z_{3n(k)+1}, z_{3n(k)}, z_{3n(k)}) \\
&\qquad + G(z_{3n(k)}, z_{3n(k)+1}, z_{3n(k)+2}).
\end{aligned} \tag{2.17}$$

Taking limit as $k \to \infty$ in (2.17), from (2.16), (2.10), (2.11) and (2.12) we have

$$\lim_{k\to\infty} G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+2}) = \varepsilon. \tag{2.18}$$

Finally,

$$\begin{aligned}
&G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}) \\
&\quad \le G(z_{3m(k)+1}, z_{3m(k)}, z_{3m(k)}) + G(z_{3m(k)}, z_{3n(k)+2}, z_{3n(k)+3}) \\
&\quad \le G(z_{3m(k)+1}, z_{3m(k)}, z_{3m(k)}) + G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) + G(z_{3n(k)}, z_{3n(k)+2}, z_{3n(k)+3}) \\
&\quad \le G(z_{3m(k)+1}, z_{3m(k)}, z_{3m(k)}) + G(z_{3m(k)}, z_{3n(k)}, z_{3n(k)}) \\
&\qquad + G(z_{3n(k)}, z_{3n(k)+1}, z_{3n(k)+1}) + G(z_{3n(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}).
\end{aligned} \tag{2.19}$$

Taking limit as $k \to \infty$ in (2.19) and using (2.16), (2.10), (2.11) and (2.12), we have

$$\lim_{k\to\infty} G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}) \le \varepsilon.$$

Consider

$$\begin{aligned}
&G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3m(k)+1}, z_{3m(k)+1}) + G(z_{3m(k)+1}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3m(k)+1}, z_{3m(k)+1}) + G(z_{3m(k)+1}, z_{3n(k)+3}, z_{3n(k)+3}) \\
&\qquad + G(z_{3n(k)+3}, z_{3n(k)+1}, z_{3n(k)+2}) \\
&\quad \le G(z_{3m(k)}, z_{3m(k)+1}, z_{3m(k)+1}) + G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}) \\
&\qquad + G(z_{3n(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}).
\end{aligned} \tag{2.20}$$

Taking limit as $k \to \infty$ and using (2.10), (2.11) and (2.12), we have

$$\varepsilon \le \lim_{k\to\infty} G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}).$$

Therefore,

$$\lim_{k\to\infty} G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3}) = \varepsilon. \tag{2.21}$$

As $Tx_{3m(k)} \preceq Rx_{3n(k)+1} \preceq Sx_{3n(k)+2}$, so from (2.1) we have

$$\begin{aligned}
&\psi\left(G(z_{3m(k)+1}, z_{3n(k)+2}, z_{3n(k)+3})\right) \\
&\quad = \psi\left(G(fx_{3m(k)}, gx_{3n(k)+1}, hx_{3n(k)+2})\right) \\
&\quad \leq \psi\left(M(x_{3m(k)}, x_{3n(k)+1}, x_{3n(k)+2})\right) - \varphi\left(M(x_{3m(k)}, x_{3n(k)+1}, x_{3n(k)+2})\right),
\end{aligned} \tag{2.22}$$

where

$$\begin{aligned}
&M(x_{3m(k)}, x_{3n(k)+1}, x_{3n(k)+2}) \\
&\quad = \max\Big\{ G(Tx_{3m(k)}, Rx_{3n(k)+1}, Sx_{3n(k)+2}), G(Tx_{3m(k)}, fx_{3m(k)}, fx_{3m(k)}), \\
&\qquad G(Rx_{3n(k)+1}, gx_{3n(k)+1}, gx_{3n(k)+1}), G(Sx_{3n(k)+2}, hx_{3n(k)+2}, hx_{3n(k)+2}), \\
&\qquad \frac{\begin{array}{c} G(Tx_{3m(k)}, Tx_{3m(k)}, fx_{3m(k)}) + G(Rx_{3n(k)+1}, Rx_{3n(k)+1}, gx_{3n(k)+1}) \\ + G(Sx_{3n(k)+2}, Sx_{3n(k)+2}, hx_{3n(k)+2}) \end{array}}{3} \Big\} \\
&\quad = \max\Big\{ G(z_{3m(k)}, z_{3n(k)+1}, z_{3n(k)+2}), G(z_{3m(k)}, z_{3m(k)+1}, z_{3m(k)+1}), \\
&\qquad G(z_{3n(k)+1}, z_{3n(k)+2}, z_{3n(k)+2}), G(z_{3n(k)+2}, z_{3n(k)+3}, z_{3n(k)+3}), \\
&\qquad \frac{\begin{array}{c} G(z_{3m(k)}, z_{3m(k)}, z_{3m(k)+1}) + G(z_{3n(k)+1}, z_{3n(k)+1}, z_{3n(k)+2}) \\ + G(z_{3n(k)+2}, z_{3n(k)+2}, z_{3n(k)+3}) \end{array}}{3} \Big\}.
\end{aligned}$$

Taking limit as $k \to \infty$ and using (2.11), (2.12), (2.18) and (2.21) in (2.22), we have

$$\psi(\varepsilon) \leq \psi(\varepsilon) - \varphi(\varepsilon) < \psi(\varepsilon),$$

a contradiction. Hence, $\{z_n\}$ is a G-Cauchy sequence.

Step III. We will show that f, g, h, R, S and T have a coincidence point.

Since $\{z_n\}$ is a G-Cauchy sequence in the G-complete G-metric space X, there exists $z^* \in X$ such that

$$\begin{aligned}
\lim_{n\to\infty} G(z_{3n+1}, z_{3n+1}, z^*) &= \lim_{n\to\infty} G(Rx_{3n+1}, Rx_{3n+1}, z^*) \\
&= \lim_{n\to\infty} G(fx_{3n}, fx_{3n}, z^*) = 0,
\end{aligned} \tag{2.23}$$

$$\begin{aligned}
\lim_{n\to\infty} G(z_{3n+2}, z_{3n+2}, z^*) &= \lim_{n\to\infty} G(Sx_{3n+2}, Sx_{3n+2}, z^*) \\
&= \lim_{n\to\infty} G(gx_{3n+1}, gx_{3n+1}, z^*) = 0,
\end{aligned} \tag{2.24}$$

and

$$\begin{aligned}\lim_{n\to\infty} G(z_{3n+3}, z_{3n+3}, z^*) &= \lim_{n\to\infty} G(Tx_{3n+3}, Tx_{3n+3}, z^*) \\ &= \lim_{n\to\infty} G(hx_{3n+2}, hx_{3n+2}, z^*) = 0.\end{aligned} \tag{2.25}$$

Hence,

$$Tx_{3n} \to z^* \quad \text{and} \quad fx_{3n} \to z^* \quad \text{as } n \to \infty. \tag{2.26}$$

As (f, T) is compatible, so

$$\lim_{n\to\infty} G(Tfx_{3n}, fTx_{3n}, fTx_{3n}) = 0. \tag{2.27}$$

Moreover, from $\lim_{n\to\infty} G(fx_{3n}, fx_{3n}, z^*) = 0$, $\lim_{n\to\infty} G(Tx_{3n}, z^*, z^*) = 0$, and the continuity of T and f, we obtain

$$\lim_{n\to\infty} G(Tfx_{3n}, Tfx_{3n}, Tz^*) = 0 = \lim_{n\to\infty} G(fTx_{3n}, fz^*, fz^*). \tag{2.28}$$

By the rectangle inequality, we have

$$\begin{aligned}&G(Tz^*, fz^*, fz^*) \\ &\quad\le G(Tz^*, Tfx_{3n}, Tfx_{3n}) + G(Tfx_{3n}, fz^*, fz^*) \\ &\quad\le G(Tz^*, Tfx_{3n}, Tfx_{3n}) + G(Tfx_{3n}, fTx_{3n}, fTx_{3n}) + G(fTx_{3n}, fz^*, fz^*).\end{aligned} \tag{2.29}$$

Taking limit as $n \to \infty$ in (2.29), we obtain

$$G(Tz^*, fz^*, fz^*) \le 0,$$

which implies that $fz^* = Tz^*$, that is, z^* is a coincidence point of f and T.

Similarly, $gz^* = Rz^*$ and $hz^* = Sz^*$. Now, let Rz^*, Sz^* and Tz^* be comparable. By (2.1) we have

$$\begin{aligned}&\psi\big(G(fz^*, gz^*, hz^*)\big) \\ &\quad\le \psi\big(M(z^*, z^*, z^*)\big) - \varphi\big(M(z^*, z^*, z^*)\big),\end{aligned} \tag{2.30}$$

where

$$\begin{aligned}M(z^*, z^*, z^*) = \max\Big\{&G(Tz^*, Rz^*, Sz^*), \\ &G(Tz^*, fz^*, fz^*), G(Rz^*, gz^*, gz^*), G(Sz^*, hz^*, hz^*), \\ &\frac{G(Tz^*, Tz^*, fz^*) + G(Rz^*, Rz^*, gz^*) + G(Sz^*, Sz^*, hz^*)}{3}\Big\} \\ = {}&G(Tz^*, Rz^*, Sz^*) = G(fz^*, gz^*, hz^*).\end{aligned}$$

Hence (2.30) gives

$$\psi\left(G\left(fz^*,gz^*,hz^*\right)\right) \le \psi\left(G\left(fz^*,gz^*,hz^*\right)\right) - \varphi\left(G\left(fz^*,gz^*,hz^*\right)\right).$$

Therefore, $fz^* = gz^* = hz^* = Tz^* = Rz^* = Sz^*$. □

In the following theorem, the continuity assumption on the mappings f, g, h, R, S and T is in fact replaced by the sequential limit comparison property of the space, and the compatibility of the pairs (f,T), (g,R) and (h,S) is in fact replaced by weak compatibility of the pairs.

Theorem 2.2 *Let $(X,\preceq,G)$ be a partially ordered G-complete G-metric space with the sequential limit comparison property, let $f,g,h,R,S,T : X \to X$ be six mappings such that $f(X) \subseteq R(X)$, $g(X) \subseteq S(X)$, and let $h(X) \subseteq T(X)$, RX, SX and TX be G-complete subsets of X. Suppose that for comparable elements $Tx, Ry, Sz \in X$, we have*

$$\psi\left(2G(fx,gy,hz)\right) \le \psi\left(M(x,y,z)\right) - \varphi\left(M(x,y,z)\right), \tag{2.31}$$

where $\psi,\varphi : [0,\infty) \to [0,\infty)$ are altering distance functions. Then the pairs (f,T), (g,R) and (h,S) have a coincidence point z^* in X provided that the pairs (f,T), (g,R) and (h,S) are weakly compatible and the pairs (f,g), (g,h) and (h,f) are partially weakly increasing with respect to R, S and T, respectively. Moreover, if Rz^*, Sz^* and Tz^* are comparable, then $z^* \in X$ is a coincidence point of f, g, h, R, S and T.

Proof Following the proof of Theorem 2.1, there exists $z^* \in X$ such that

$$\lim_{k\to\infty} G\left(z_k,z_k,z^*\right) = 0. \tag{2.32}$$

Since $R(X)$ is G-complete and $\{z_{3n+1}\} \subseteq R(X)$, therefore $z^* \in R(X)$. Hence, there exists $u \in X$ such that $z^* = Ru$ and

$$\lim_{n\to\infty} G(z_{3n+1},z_{3n+1},Ru) = \lim_{n\to\infty} G(Rx_{3n+1},Rx_{3n+1},Ru) = 0. \tag{2.33}$$

Similarly, there exists $v,w \in X$ such that $z^* = Sv = Tw$ and

$$\lim_{n\to\infty} G(Sx_{3n+2},Sx_{3n+2},Sv) = \lim_{n\to\infty} G(Tx_{3n},Tx_{3n},Tw) = 0. \tag{2.34}$$

Now, we prove that w is a coincidence point of f and T. For this purpose, we show that $fw = gu$. Suppose opposite $fw \ne gu$. Since $Sx_{3n+2} \to z^* = Tw = Ru$ as $n \to \infty$, so $Sx_{3n+2} \preceq Tw = Ru$.

Therefore, from (2.31), we have

$$\psi\left(2G(fw,gu,hx_{3n+2})\right) \le \psi\left(M(w,u,x_{3n+2})\right) - \varphi\left(M(w,u,x_{3n+2})\right), \tag{2.35}$$

where

$$\begin{aligned}
&M(w,u,x_{3n+2}) \\
&\quad = \max\Big\{G(Tw,Ru,Sx_{3n+2}), G(Tw,fw,fw), \\
&\qquad G(Ru,gu,gu), G(Sx_{3n+2},hx_{3n+2},hx_{3n+2}), \\
&\qquad \frac{G(Tw,Tw,fw)+G(Ru,Ru,gu)+G(Sx_{3n+2},Sx_{3n+2},hx_{3n+2})}{3}\Big\}.
\end{aligned}$$

Taking limit as $n \to \infty$ in (2.35), as $G(z^*,z^*,z^*) = 0$ and from (G2) and the fact that $G(x,x,y) \leq 2G(x,y,y)$, we obtain that

$$\begin{aligned}
&\psi\big(2G(fw,gu,z^*)\big) \\
&\quad \leq \psi\left(\max\left\{G\big(z^*,fw,fw\big), G\big(z^*,gu,gu\big), \frac{G(z^*,z^*,fw)+G(z^*,z^*,gu)}{3}\right\}\right) \\
&\qquad - \varphi\left(\max\left\{G\big(z^*,fw,fw\big), G\big(z^*,gu,gu\big), \frac{G(z^*,z^*,fw)+G(z^*,z^*,gu)}{3}\right\}\right) \\
&\quad \leq \psi\big(2G(fw,gu,z^*)\big) \\
&\qquad - \varphi\left(\max\left\{G\big(z^*,fw,fw\big), G\big(z^*,gu,gu\big), \frac{G(z^*,z^*,fw)+G(z^*,z^*,gu)}{3}\right\}\right),
\end{aligned}$$

which implies that $fw = z^* = gu$, a contradiction, so $fw = gu$. Again from the above inequality it is easy to see that $fw = z^*$. So, we have $fw = z^* = Tw$.

As f and T are weakly compatible, we have $fz^* = fTw = Tfw = Tz^*$. Thus z^* is a coincidence point of f and T.

Similarly it can be shown that z^* is a coincidence point of the pairs (g,R) and (h,S).

The rest of the proof can be obtained from the same arguments as those in the proof of Theorem 2.1. □

Remark 2.3 Let (X,G) be a G-metric space. Let $f,R,S,T : X \to X$ be mappings. If we define functions $p,q : X \times X \to [0,\infty)$ in the following way:

$$p(x,y) = G(Tx,Ry,Sy)$$

and

$$q(x,y) = G(Tx,fy,fy)$$

for all $x,y \in X$, it is easy to see that both mappings p and q do not satisfy the conditions of Definition 1.15. Hence, Theorem 2.1 and Theorem 2.2 cannot be characterized in the context of quasi-metric as it is suggested in [66, 67].

Taking $T = R = S$ in Theorem 2.1, we obtain the following result.

Corollary 2.4 *Let $(X, \preceq, G)$ be a partially ordered G-complete G-metric space, and let $f, g, h, R : X \to X$ be four mappings such that $f(X) \cup g(X) \cup h(X) \subseteq R(X)$. Suppose that for every three comparable elements $Rx, Ry, Rz \in X$, we have*

$$\psi\big(2G(fx, gy, hz)\big) \le \psi\big(M(x, y, z)\big) - \varphi\big(M(x, y, z)\big), \tag{2.36}$$

where

$$M(x, y, z) = \max\bigg\{G(Rx, Ry, Rz), \\ G(Rx, fx, fx), G(Ry, gy, gy), G(Rz, hz, hz), \\ \frac{G(Rx, Rx, fx) + G(Ry, Ry, gy) + G(Rz, Rz, hz)}{3}\bigg\}$$

and $\psi, \varphi : [0, \infty) \to [0, \infty)$ are altering distance functions. Then f, g, h and R have a coincidence point in X provided that the pairs (f, g), (g, h) and (h, f) are partially weakly increasing with respect to R and either

a. *f is continuous and the pair (f, R) is compatible, or*
b. *g is continuous and the pair (g, R) is compatible, or*
c. *h is continuous and the pair (h, R) is compatible.*

Taking $R = S = T$ and $f = g = h$ in Theorem 2.1, we obtain the following coincidence point result.

Corollary 2.5 *Let $(X, \preceq, G)$ be a partially ordered G-complete G-metric space, and let $f, R : X \to X$ be two mappings such that $f(X) \subseteq R(X)$. Suppose that for every three comparable elements $Rx, Ry, Rz \in X$, we have*

$$\psi\big(2G(fx, fy, fz)\big) \le \psi\big(M(x, y, z)\big) - \varphi\big(M(x, y, z)\big), \tag{2.37}$$

where

$$M(x, y, z) = \max\bigg\{G(Rx, Ry, Rz), \\ G(Rx, fx, fx), G(Ry, fy, fy), G(Rz, fz, fz), \\ \frac{G(Rx, Rx, fx) + G(Ry, Ry, fy) + G(Rz, Rz, fz)}{3}\bigg\}$$

and $\psi, \varphi : [0, \infty) \to [0, \infty)$ are altering distance functions. Then the pair (f, R) has a coincidence point in X provided that f and R are continuous, the pair (f, R) is compatible and f is weakly increasing with respect to R.

Example 2.6 Let $X = [0, \infty)$ and G on X be given by $G(x, y, z) = |x - y| + |y - z| + |x - z|$ for all $x, y, z \in X$. We define an ordering '$\preceq$' on X as follows:

$$x \preceq y \iff y \le x, \quad \forall x, y \in X.$$

Define self-maps f, g, h, S, T and R on X by

$$\begin{aligned} fx &= \ln\bigl(\sqrt{x^2+1}+x\bigr) = \sinh^{-1} x, \qquad Rx = \sinh(3x), \\ gx &= \sinh^{-1}\left(\frac{x}{2}\right), \qquad Sx = \sinh(2x), \\ hx &= \sinh^{-1}\left(\frac{x}{3}\right), \qquad Tx = \sinh(6x). \end{aligned}$$

To prove that (f,g) are partially weakly increasing with respect to R, let $x, y \in X$ be such that $y \in R^{-1}fx$; that is, $Ry = fx$. By the definition of f and R, we have $\sinh^{-1} x = \sinh 3y$ and $y = \frac{\sinh^{-1}(\sinh^{-1} x)}{3}$. As $\sinh x \geq (\sinh^{-1} x)$ for all $x \geq 0$, therefore $6x \geq \sinh^{-1}(\sinh^{-1} x)$, or

$$fx = \sinh^{-1} x \geq \sinh^{-1}\left(\frac{1}{6}\sinh^{-1}\bigl(\sinh^{-1} x\bigr)\right) = \sinh^{-1}\left(\frac{1}{2}y\right) = gy.$$

Therefore, $fx \preceq gy$. Hence (f,g) is partially weakly increasing with respect to R. Similarly, one can show that (g,h) and (h,f) are partially weakly increasing with respect to S and T, respectively.

Furthermore, $fX = TX = gX = SX = hX = RX = [0,\infty)$ and the pairs (f,T), (g,R) and (h,S) are compatible. Indeed, if $\{x_n\}$ is a sequence in X such that for some $t \in X$, $\lim_{n\to\infty} G(t, fx_n, fx_n) = \lim_{n\to\infty} G(t, Tx_n, Tx_n) = 0$. Therefore, we have

$$\lim_{n\to\infty} \bigl|\sinh^{-1} x_n - t\bigr| = \lim_{n\to\infty} |\sinh 6x_n - t| = 0.$$

Continuity of $\sinh^{-1}$ and $\sinh$ implies that

$$\lim_{n\to\infty} |x_n - \sinh t| = \lim_{n\to\infty} \left|x_n - \frac{\sinh^{-1} t}{6}\right| = 0,$$

and the uniqueness of the limit gives that $\sinh t = \frac{\sinh^{-1} t}{6}$. But

$$\sinh t = \frac{\sinh^{-1} t}{6} \quad\Longleftrightarrow\quad t = 0.$$

So, we have $t = 0$. Since f and T are continuous, we have

$$\lim_{n\to\infty} G(fTx_n, fTx_n, Tfx_n) = 2\lim_{n\to\infty} |fTx_n - Tfx_n| = 0.$$

Define $\psi, \varphi : [0,\infty) \to [0,\infty)$ as $\psi(t) = bt$ and $\varphi(t) = (b-1)t$ for all $t \in [0,\infty)$, where $1 < b \leq 3$.

Using the mean value theorem simultaneously for the functions $\sinh^{-1}$ and $\sinh$, we have

$$\begin{aligned} \psi\bigl(2G(fx,gy,hz)\bigr) &= 2b\bigl(|fx-gy| + |fx-hz| + |gy-hz|\bigr) \\ &= 2b\left(\left|\sinh^{-1} x - \sinh^{-1}\left(\frac{y}{2}\right)\right| + \left|\sinh^{-1}(x) - \sinh^{-1}\left(\frac{z}{3}\right)\right|\right. \\ &\quad \left. + \left|\sinh^{-1}\left(\frac{y}{2}\right) - \sinh^{-1}\left(\frac{z}{3}\right)\right|\right) \end{aligned}$$

$$\begin{aligned}
&\le 2b\left(\frac{1}{2}|2x-y| + \frac{1}{3}|3x-z| + \frac{1}{6}|3y-2z|\right)\\
&= b\frac{(|6x-3y| + |6x-2z| + |3y-2z|)}{3}\\
&\le \frac{b}{3}\bigl(|\sinh 6x - \sinh 3y| + |\sinh 3y - \sinh 2z| + |\sinh 2z - \sinh 6x|\bigr)\\
&\le |Tx - Ry| + |Ry - Sz| + |Sz - Tx|\\
&= G(Tx, Ry, Sz) \le M(x,y,z)\\
&= \psi\bigl(M(x,y,z)\bigr) - \varphi\bigl(M(x,y,z)\bigr).
\end{aligned}$$

Thus, (2.1) is satisfied for all $x, y, z \in X$. Therefore, all the conditions of Theorem 2.1 are satisfied. Moreover, 0 is a coincidence point of f, g, h, R, S and T.

The following example supports the usability of our results for non-symmetric G-metric spaces.

Example 2.7 Let $X = \{0, 1, 2, 3\}$ be endowed with the usual order. Let

$$A = \bigl\{(2,0,0), (0,2,0), (0,0,2)\bigr\}$$

and

$$B = \bigl\{(2,2,0), (2,0,2), (0,2,2)\bigr\}.$$

Define $G : X^3 \to \mathbb{R}^+$ by

$$G(x,y,z) = \begin{cases} 1 & \text{if } (x,y,z) \in A,\\ 2 & \text{if } (x,y,z) \in B,\\ 6 & \text{if } (x,y,z) \in X^3 - A \cup B,\\ 0 & \text{if } x = y = z. \end{cases}$$

It is easy to see that (X, G) is a non-symmetric G-metric space.

Also, (X, G) has the sequential limit comparison property. Indeed, for each $\{x_n\}$ in X such that $G(x_n, x, x) \to 0$ for an $x \in X$, there is $k \in \mathbb{N}$ such that for each $n \ge k$, $x_n = x$.

Define the self-maps f, g, h, R, S and T by

$$f = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 2 & 0 & 2 \end{pmatrix}, \qquad R = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 1 & 3 & 2 \end{pmatrix},$$

$$g = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 2 & 2 & 2 \end{pmatrix}, \qquad S = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 2 & 1 & 3 \end{pmatrix},$$

$$h = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 2 & 0 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 0 & 1 & 2 & 3\\ 0 & 2 & 3 & 1 \end{pmatrix}.$$

We see that

$$fX \subseteq RX = X,$$

$$gX \subseteq SX = X,$$

and

$$hX \subseteq TX = X.$$

Also, RX, SX and TX are G-complete. The pairs (f, T), (g, R) and (h, S) are weakly compatible.

On the other hand, one can easily check that the pairs (f, g), (g, h) and (h, f) are partially weakly increasing with respect to R, S and T, respectively.

Define $\psi, \varphi : [0, \infty) \to [0, \infty)$ by $\psi(t) = \frac{3}{2}t$ and $\varphi(t) = \frac{t}{2}$.

According to the definition of f, g, h and G for each three elements $x, y, z \in X$, we see that

$$G(fx, gy, hz) \in \{0, 1, 2\}.$$

But

$$G(Tx, Ry, Sz), G(Tx, fx, fx), G(Ry, gy, gy), G(Sz, hz, hz) \in \{0, 1, 2, 6\}$$

and

$$G(Tx, Tx, fx), G(Ry, Ry, gy), G(Sz, Sz, hz) \in \{0, 1, 2, 6\}.$$

Hence, we have

$$\psi\bigl(2G(fx, gy, hz)\bigr) \le 6 = M(x, y, z) = \psi\bigl(M(x, y, z)\bigr) - \varphi\bigl(M(x, y, z)\bigr).$$

Therefore, all the conditions of Theorem 2.2 are satisfied. Moreover, 0 is a coincidence point of f, g, h, R, S and T.

Let Λ be the set of all functions $\mu : [0, +\infty) \to [0, +\infty)$ satisfying the following conditions:

(I) μ is a positive Lebesgue integrable mapping on each compact subset of $[0, +\infty)$.

(II) For all $\varepsilon > 0$, $\int_0^{\varepsilon} \mu(t)\,dt > 0$.

Remark 2.8 Suppose that there exists $\mu \in \Lambda$ such that mappings f, g, h, R, S and T satisfy the following condition:

$$\int_0^{\psi(2G(fx,gy,hz))} \mu(t)\,dt \le \int_0^{\psi(M(x,y,z))} \mu(t)\,dt - \int_0^{\varphi(M(x,y,z))} \mu(t)\,dt. \tag{2.38}$$

Then f, g, h, R, S and T have a coincidence point if the other conditions of Theorem 2.1 are satisfied.

For this, define the function $\Gamma(x) = \int_0^x \mu(t)\,dt$. Then (2.38) becomes

$$\Gamma\bigl(\psi\bigl(2G(fx, gy, hz)\bigr)\bigr) \le \Gamma\bigl(\psi\bigl(M(x, y, z)\bigr)\bigr) - \Gamma\bigl(\varphi\bigl(M(x, y, z)\bigr)\bigr).$$

Take $\psi_1 = \Gamma o \psi$ and $\varphi_1 = \Gamma o \varphi$. It is easy to verify that ψ_1 and φ_1 are altering distance functions.

Taking $g = h$, $T = R = S = I_X$ and $y = z$ in Theorems 2.1 and 2.2, we obtain the following common fixed point result.

Theorem 2.9 *Let $(X, \preceq, G)$ be a partially ordered G-complete G-metric space, and let f and g be two self-mappings on X. Suppose that for every comparable elements $x, y \in X$,*

$$\psi\bigl(2G(fx, gy, gy)\bigr) \le \psi\bigl(M(x, y, y)\bigr) - \varphi\bigl(M(x, y, y)\bigr), \tag{2.39}$$

where

$$M(x, y, y) = \max\biggl\{G(x, y, y), G(x, fx, fx), G(y, gy, gy), \\ \frac{G(x, x, fx) + 2G(y, y, gy)}{3}\biggr\},$$

and $\psi, \varphi : [0, \infty) \to [0, \infty)$ are altering distance functions. Then the pair (f, g) has a common fixed point z in X provided that the pair (f, g) is weakly increasing and either

a. *f or g is continuous, or*
b. *X has the sequential limit comparison property.*

Taking $f = g$ in the above, we obtain the following common fixed point result.

Theorem 2.10 *Let $(X, \preceq, G)$ be a partially ordered complete G-metric space, and let f be a self-mapping on X. Suppose that for every comparable elements $x, y \in X$,*

$$\psi\bigl(2G(fx, fy, fy)\bigr) \le \psi\bigl(M(x, y, y)\bigr) - \varphi\bigl(M(x, y, y)\bigr), \tag{2.40}$$

where

$$M(x, y, y) = \max\biggl\{G(x, y, y), G(x, fx, fx), G(y, fy, fy), \\ \frac{G(x, x, fx) + 2G(y, y, fy)}{3}\biggr\}$$

and $\psi, \varphi : [0, \infty) \to [0, \infty)$ are altering distance functions. Then f has a fixed point z in X provided that f is weakly increasing and either

a. *f is continuous, or*
b. *X has the sequential limit comparison property.*

3 Existence of a common solution for a system of integral equations

Motivated by the work in [21] and [32], we consider the following system of integral equations:

$$\begin{aligned} x(t) &= \int_a^b K_1\bigl(t, s, x(s)\bigr)\, ds + k(t), \\ x(t) &= \int_a^b K_2\bigl(t, s, x(s)\bigr)\, ds + k(t), \\ x(t) &= \int_a^b K_3\bigl(t, s, x(s)\bigr)\, ds + k(t), \end{aligned} \tag{3.1}$$

where $b > a \geq 0$. The aim of this section is to prove the existence of a solution for (3.1) which belongs to $X = C[a,b]$ (the set of all continuous real-valued functions defined on $[a,b]$) as an application of Corollary 2.4.

The considered problem can be reformulated as follows.

Let $f, g, h : X \to X$ be defined by

$$fx(t) = \int_a^b K_1\big(t,s,x(s)\big)\,ds,$$
$$gx(t) = \int_a^b K_2\big(t,s,x(s)\big)\,ds,$$

and

$$hx(t) = \int_a^b K_3\big(t,s,x(s)\big)\,ds$$

for all $x \in X$ and for all $t \in [a,b]$. Obviously, the existence of a solution for (3.1) is equivalent to the existence of a common fixed point of f, g and h.

Let

$$d(u,v) = \max_{t\in[a,b]}\big|u(t) - v(t)\big|.$$

Equip X with the G-metric given by

$$G(u,v,w) = \max\big\{d(u,v), d(v,w), d(w,u)\big\}$$

for all $u, v, w \in X$. Evidently, (X,G) is a complete G-metric space. We endow X with the partial ordered $\preceq$ given by

$$x \preceq y \quad \Longleftrightarrow \quad x(t) \leq y(t)$$

for all $t \in [a,b]$. It is known that $(X,\preceq)$ has the sequential limit comparison property [37].

Now, we will prove the following result.

Theorem 3.1 *Suppose that the following hypotheses hold*:

(i) $K_1, K_2, K_3 : [a,b] \times [a,b] \times R \to R$ *and* $k : [a,b] \to R$ *are continuous*;

(ii) *For all* $t, s \in [a,b]$ *and for all* $x \in X$, *we have*

$$K_1\big(t,s,x(s)\big) \leq K_2\left(t,s,\int_a^b K_1\big(t,s,x(s)\big)\,ds + k(t)\right),$$
$$K_2\big(t,s,x(s)\big) \leq K_3\left(t,s,\int_a^b K_2\big(t,s,x(s)\big)\,ds + k(t)\right),$$

and

$$K_3\big(t,s,x(s)\big) \leq K_1\left(t,s,\int_a^b K_3\big(t,s,x(s)\big)\,ds + k(t)\right).$$

(iii) *For all $s,t \in [a,b]$ and for all $x,y \in X$ with $x \preceq y$, we have*

$$\bigl|K_i\bigl(t,s,x(s)\bigr) - K_j\bigl(t,s,y(s)\bigr)\bigr| \le \sqrt{p(t,s)\ln\bigl(1+\bigl|x(s)-y(s)\bigr|^2\bigr)},$$

where $p : [a,b] \times [a,b] \to [0,\infty)$ is a continuous function satisfying

$$\sup \int_a^b p(s,t)\,ds < \frac{1}{4(b-a)}.$$

Then system (3.1) *has a solution $x \in X$.*

Proof From condition (ii), the ordered pairs (f,g), (g,h) and (h,f) are partially weakly increasing.

Now, let $x,y \in X$ be such that $x \succeq y$. From condition (iii), for all $t \in [a,b]$, we have

$$\begin{aligned}
4\bigl|fx(t)-gy(t)\bigr|^2 &\le 4\biggl(\int_a^b \bigl|K_1\bigl(t,s,x(s)\bigr) - K_2\bigl(t,s,x(s)\bigr)\bigr|\,ds\biggr)^2 \\
&\le 4\biggl(\int_a^b 1^2\,ds\biggr)\biggl(\int_a^b \bigl|K_1\bigl(t,s,x(s)\bigr) - K_2\bigl(t,s,x(s)\bigr)\bigr|^2\,ds\biggr) \\
&\le 4(b-a)\biggl(\int_a^b p(t,s)\ln\bigl(1+\bigl|x(s)-y(s)\bigr|^2\bigr)\,ds\biggr) \\
&\le 4(b-a)\biggl(\int_a^b p(t,s)\ln\bigl(1+d(x,y)^2\bigr)\,ds\biggr) \\
&\le 4(b-a)\biggl(\int_a^b p(t,s)\ln\bigl(1+G(x,y,z)^2\bigr)\,ds\biggr) \\
&= 4(b-a)\biggl(\int_a^b p(t,s)\,ds\biggr)\ln\bigl(1+G(x,y,z)^2\bigr) \\
&= 4(b-a)\biggl(\int_a^b p(t,s)\,ds\biggr)\ln\bigl(1+M(x,y,z)^2\bigr) \\
&< \ln\bigl(1+M(x,y,z)^2\bigr) \\
&= M(x,y,z)^2 - \bigl(M(x,y,z)^2 - \ln\bigl(1+M(x,y,z)^2\bigr)\bigr).
\end{aligned}$$

Hence,

$$\begin{aligned}
\bigl(2d(fx,gy)\bigr)^2 &= \Bigl(2\sup_{t\in[a,b]}\bigl|fx(t)-gy(t)\bigr|\Bigr)^2 \\
&\le M(x,y,z)^2 - \bigl(M(x,y,z)^2 - \ln\bigl(1+M(x,y,z)^2\bigr)\bigr). \qquad (3.2)
\end{aligned}$$

Similarly, we can show that

$$\begin{aligned}
\bigl(2d(gy,hz)\bigr)^2 &= \Bigl(2\sup_{t\in[a,b]}\bigl|gy(t)-hz(t)\bigr|\Bigr)^2 \\
&\le M(x,y,z)^2 - \bigl(M(x,y,z)^2 - \ln\bigl(1+M(x,y,z)^2\bigr)\bigr), \qquad (3.3)
\end{aligned}$$

and

$$\begin{aligned}\big(2d(hz,fx)\big)^2 &= \Big(2\sup_{t\in[a,b]}\big|hz(t)-fx(t)\big|\Big)^2\\ &\le M(x,y,z)^2-\big(M(x,y,z)^2-\ln\big(1+M(x,y,z)^2\big)\big).\end{aligned}\tag{3.4}$$

Therefore, from (3.2), (3.3) and (3.4), we have

$$\begin{aligned}\big(2G(fx,gy,hz)\big)^2 &= \big(2\max\big\{d(fx,gy),d(gy,hz),d(hz,fx)\big\}\big)^2\\ &= \max\big\{\big(2d(fx,gy)\big)^2,\big(2d(gy,hz)\big)^2,\big(2d(hz,fx)\big)^2\big\}\\ &\le M(x,y,z)^2-\big(M(x,y,z)^2-\ln\big(1+M(x,y,z)^2\big)\big).\end{aligned}$$

Putting, $\psi(t)=t^2$, $\varphi(t)=t^2-\ln(1+t^2)$ and $R=I_X$ in Corollary 2.4, there exists $x\in X$, a common fixed point of f and g and h, which is a solution of (3.1). □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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