NON UNIFORM DIFFERENTIAL AMPLITUDE PHASE–SHIFT–KEY (DAPSK) MODULATION FOR MULTIMEDIA MULTICAST WIRELESS COMMUNICATIONS WITH PUNCTURED CONVOLUTIONAL CODING

Khalid Hassan Sayhood\textsuperscript{1} and Wu Le Nan\textsuperscript{2}

Radio Engineering Department
Southeast University
Nanjing 210096
P. R. CHINA
\textsuperscript{1}Email: khsay57@yahoo.com, \textsuperscript{2}Email: wuln@seu.edu.cn

ABSTRACT

Simple closed–form expressions for the probability of error for nonuniform M–ary Differential Amplitude Phase–Shift–Key (DAPSK) signaling over an Additive White Gaussian Noise (AWGN) and Rayleigh fading channels are presented. Nonuniform M–DAPSK is shown to be useful for multimedia communication and multicast transmission. The performance of nonuniform M–DAPSK with punctured convolutional coding is presented. The results show that the coding can be used with nonuniform M–DAPSK modulation to transmit additional message in mobile communication systems with little degradation to the basic message.

Keywords: multimedia transmission, multicast transmission, nonuniform Differential Amplitude Phase–Shift–Key (DAPSK), punctured convolutional coding, Additive White Gaussian Noise (AWGN) channel, Rayleigh fading channel

I. INTRODUCTION

The quality of a channel in a mobile wireless communication network is highly variable because of the mobility of the terminals, variations in the propagation characteristics and fluctuations in the interference environment. Fixed transmission methods that are designed to provide the required quality of service when channel conditions are poor are very inefficient when better channel conditions prevail. Many recent investigations have examined transmission methods that adapt the symbol rate, transmitted power, code rate and number of points in the signal constellation in response to variations in the channel [1]–[7].

In a wireless communication network, it is often necessary to transmit one or more messages to multiple destinations. For example, it is required in certain routing protocols that each radio broadcast status information to all of its neighbors in the network. Such a broadcast transmission may include information on the quality of various links into that radio or data from the radios routing table. More generally, it is advantage to have the ability to send a multicast transmission, which is a transmission that is intended for more than one (but not necessarily all) of the neighbors of the transmitting radio.

The capacities of the communication links from the transmitting radio to the intended recipients of a multicast transmission may differ greatly because of the variations in the communication range, fading and interference on these links. For a given transmission and a given requirement on the probability of error, some receivers require the transmitted energy per bit to be relatively large and this is called the less capable receivers. For other receivers, the transmitted energy per bit can be relatively small and they are called the more capable receivers. In spite of the typical differences in the capabilities among the receivers in a mobile radio network, the standard multicast transmission provides the same information to each recipient.

The network efficiency can be increased greatly if the multicast transmissions are designed to provide some basic information to less capable receivers and additional information to more capable receivers. These considerations suggest that some multimedia transmission capability can be added without requiring additional network resources.
Multicast transmissions can be modified to enable multimedia message delivery to the more capable receivers. It is envisioned that in some applications, such as compact network radio, the basic message may be a voice message, which must be delivered, to several radios and the additional capacity of the some of the links can delivered data simultaneously to the more capable receivers. Similarly, the more capable radios in a store–and–forward network could be used to advance a data packet toward its destination while they are also receiving voice messages. Also a voice packet can be forward to a more capable radio at the same time that a network control packet is being sent.

Nonuniform Phase–Shift–Key (PSK) constellations have been proposed to enhance the performance of trellis–coded modulation [8]. Such constellations can be used for unequal error protection of multiresolution source–encoded analog information such as speech where a nonuniform 4–PSK is employed for this purpose in [9]. Nonuniform Quadrature Amplitude Modulation (QAM) has also been proposed in [10],[11] for unequal error protection. In [12]–[18], the transmission schemes that use nonuniform PSK modulation to transmit two messages with differential parameters were described. The two messages are a basic message that is intended for all receivers and an additional message that is intended for more receivers that are capable. The transmission of the additional message along with the basic message provides additional throughput that can be employed to transmit multimedia messages (e.g., voice and data) to the more capable receivers, thereby providing multimedia transmission capability without requiring additional network resources.

In this paper, we examine the coupling of punctured convolutional coding and nonuniform M–ary Differential Amplitude PSK (M–DAPSK) modulation. The punctured convolutional coding and nonuniform DAPSK constellations can be used for multicast transmissions in a mobile wireless communication system to deliver a basic message to each intended receiver and an additional message to more capable receivers. The system employs the same code for the basic message and any encoding of the additional message, so that a single decoder can be employed in the receiver. The error probabilities and performance results presented here are for M–DAPSK transmission over an Additive White Gaussian Noise (AWGN) and Rayleigh fading channels.

II. SYSTEM DESCRIPTION

There are two binary input streams for the DAPSK modulator. The first is the output of a punctured convolutional encoder for which the input is the basic message stream. The second input is a binary sequence that represents the additional message. This sequence may/may not be encoded; if it is encoded, the encoder is the same as the one that is used for the basic message. The output of the encoder for the basic message is modulated onto a carrier using Gray–coded QuadriPhase–Shift–Key (QPSK) modulation. For each information bit at the input to the encoder, three binary symbols are mapped into one DAPSK symbol as shown in Fig. 1(a). The 8–DAPSK signal constellation is converted to a nonuniform 16–DAPSK signal constellation by splitting each point in the 8–DAPSK constellation into two points, each of which is rotated away from the original point by an amount \( \theta \) where \( \theta \) is called the offset angle as shown in Fig. 1(b). The result is a nonuniform 16–DAPSK signal constellation with signals at angles \( \theta, -\theta, (\pi/2)+\theta, (\pi/2)-\theta, \pi+\theta, \pi-\theta, (-\pi/2)+\theta \) and \( (-\pi/2)-\theta \). The encoded basic message is represented by a three bits of binary symbols that appear as labels on the points in the constellation of Fig. 1(a). This encoded basic message also appears as the first three bits of the labels of the points in the 16–DAPSK constellation of Fig. 1(b).

The basic message can be transmitted using either constellation. The fourth bit of each label in Fig. 1(b) represents the additional message that is being conveyed by that symbol. This assignment method satisfies the requirements for Gray coding, in which the bit patterns assigned to nearest neighbors differ in only one bit position. Varying the modulation parameter \( \theta \) controls the relative probabilities of error for the two message streams. If a given DAPSK symbol represents a portion of an additional message, the resulting error probabilities for the basic message and the additional message depend on \( \theta \). If \( \theta \) is small, it is much easier to demodulate the basic message than the additional message. An important feature of these constellations is that a receiver who attempts to demodulate the basic message does not even need to know whether an additional message has been included. The duration of each of the DAPSK symbols is referred to as the transmission interval and a block consists of \( n \) consecutive transmission intervals.
The nonuniform M–DAPSK modulation employs signals of the form
\[ s(t) = A(t) \cos(\omega c t + \psi) \]
for \( 0 \leq t \leq T \). The angle \( \psi \) represents the data symbol being transmitted and this data symbol is a combination of both the basic and additional messages. The parameters \( A(t) \) and \( \omega_c \) are the amplitude and the modulated radian frequency of the transmitted signal respectively. The parameter \( \psi \) is selected from the set \( S \in \{(4k\pi/N) \pm \theta : 0 \leq k \leq (N/2)–1\} \) where \( M = 2N \). The nonuniform M–DAPSK constellation is based on a uniform N–DAPSK constellation. The parameter \( \theta \) controls the shape of the nonuniform M–DAPSK constellation where \( M \) is a power of 2 and \( 0 \leq \theta \leq 2\pi/N \). Each point is splitting into two half points. Then, rotating the two half points away from the position of the original point by an equal amount around the circle defined by the original uniform N–DAPSK constellation. Nonuniform M–DAPSK constellations constructed in this way deliver messages at two error probabilities and the value of \( \theta \) controls the relative values of the two error probabilities. The original uniform N–DAPSK constellation can be used to send \( \log_2 N \) bits of basic information and the resulting nonuniform M–DAPSK constellation can be used to send \( \log_2 N \) bits of basic information and one bit of additional information.

The goal is to make sure that each receiver (more capable and less capable) can demodulate with enough reliability to obtain at least the first three bits (the basic message). The less capable receiver will typically obtain only these bits; that is, it will usually be able to determine only whether the phase of the transmitted symbol is one of \( \{\psi_0, \psi_1\}, \{\psi_2, \psi_3\}, \{\psi_4, \psi_5\} \) or \( \{\psi_6, \psi_7\} \). Under normal conditions, the less capable receiver will not be able to distinguish between the symbols with phases \( (\psi_0 \text{ and } \psi_1), (\psi_2 \text{ and } \psi_3), (\psi_4 \text{ and } \psi_5) \) or \( (\psi_6 \text{ and } \psi_7) \). As a result, the less capable receiver normally obtains only three bits of information per symbol; these bits are the basic message. On the other hand, the more capable receiver is typically able to distinguish among all the 16 symbols, therefore giving four bits of information for each correct symbol.

Two performance measures are introduced that are beneficial in the characterization of the capabilities of multicast modulation schemes. These measures are defined in terms of the specified bit error probabilities \( P_B \) and \( P_A \) for the basic and additional messages respectively. Since the basic and additional messages may represent different
types of multimedia information, the two specified error probabilities might differ. The first performance measure is the degradation. In order to receive the basic message at a given error probability, the use of 16–DAPSK constellation requires more signal energy at the less capable receiver than if 8–DAPSK were used to send these bits. The amount of additional energy that is required to receive the first three bits at a given error probability is called the degradation [12]. The second performance measure is the capability disparity. It is defined to be the difference between the amount of energy at the more capable receiver to demodulate the additional message at probability of bit error \( P_A \) and the amount of energy required at the less capable receiver to achieve the probability of bit error \( P_B \) for the basic message. The disparity is a measure of how much more capable a receiver must be in order to demodulate the additional message at the required probability of error.

### III. DEMODULATION OF NONUNIFORM 16–DAPSK

For M–DAPSK, the phase differences between consecutive channel symbols convey the information, so taking an N–PSK constellation with parameter \( \theta \) and replacing the signal points with vectors that represent the allowable phase rotations can construct M–DAPSK constellation with parameter \( \theta \). Consider the maximum–likelihood differential demodulation that makes symbol decisions based on the phase difference between each pair of consecutive symbols. The error probabilities may be expressed in terms of the angle \( \theta \) and the energy to noise density ratio \( \varepsilon_i / N_o \), where \( i = 1, 2 \) for the less capable and the more capable receiver, respectively [12]. Let \( \Psi \) represents the phase difference between two consecutive received symbols. The error probabilities can be determined from a rotated signal diagram in which the phase difference for the received signal is \( 0^\circ \). A rotated signal diagram for 16–DAPSK is shown in Fig. 2(a). The shaded region indicates the error region for the basic message.

The probability of symbol error for the basic message of M–DAPSK at the ith receiver under an AWGN channel is

\[
P_{i,B} = \int_{-\pi}^{\pi} f_\psi(\psi) d\psi + \int_{-(2\pi/N)-\theta}^{(2\pi/N)-\theta} f_\psi(\psi) d\psi
\]

where \( f_\psi(\psi) \) is given in [19]

\[
F_i(\psi) = -\frac{\sin(\psi)}{4\pi} \int_{-\pi/2}^{\pi/2} \exp\left\{-\frac{\varepsilon_i}{N_o} \left[1 - \cos(\psi) \cos(t)\right]\right\} dt
\]

The probability of symbol error for the basic message can be expressed as

\[
P_{i,B} = -F_i\left(\frac{2\pi}{N} + \theta\right) - F_i\left(\frac{2\pi}{N} - \theta\right)
\]

For 16–DAPSK, the expression simplifies to

\[
P_{i,B} = \frac{\cos(\theta)}{2\pi} \int_{-\pi/4}^{\pi/4} \exp\left\{-\frac{\varepsilon_i}{N_o} \left[1 - \sin(\theta) \cos(t)\right]\right\} dt
\]
Similarly, error regions for the additional message and 16–DAPSK are shown in Fig. 2(b) that consists of multiple disjoint regions. For nonuniform M–DAPSK, an upper bound for the probability of bit error for the additional message is given by the probability of the received phase difference lying in the shaded region illustrated in Fig. 2(c). The probability of bit error for the additional message bit is

\[
P_{t,A} \leq P\{-\pi \leq \Psi < -\theta\} Y\{(4\pi/N) - \theta < \Psi < \pi\} v = 0
\]

\[
= P\{-\pi \leq \Psi < -\theta\} v = 0 + P\{(4\pi/N) - \theta < \Psi < \pi\} v = 0
\]

\[
= \int_{-\pi}^{0} f_{\psi}(\psi) d\psi + \int_{-(4\pi/N)-\theta}^{\pi} f_{\psi}(\psi) d\psi
\]

\[
= -F_{1}(\theta) - F_{1}\{(4\pi/N) - \theta\}
\]

\[
(5)
\]

**Fig. (2)** Rotated decision regions for the basic and additional messages with nonuniform 16–DAPSK constellation
For 16–DAPSK, this reduces to [16]

\[
P_{i,A} = \frac{1}{2\pi} \int_{0}^{\pi/2} \left[ \exp\left\{ -\frac{\varepsilon_i}{N_0} \left[ 1 - \cos(\theta) \cos(t) \right] \right\} + \exp\left\{ -\frac{\varepsilon_i}{N_0} \left[ 1 + \cos(\theta) \cos(t) \right] \right\} \right] dt
\]

(6)

With a Rayleigh fading and AWGN channels, the probability of bit error can be determined by integrating the product of the appropriate expression for \( P_{i,j} \) and the Rayleigh density function where \( P_{i,j} = P_{i,B} \) or \( P_{i,j} = P_{i,A} \) as given by eqns. (4) and (6) respectively. Let \( B_i \) represents the amplitude of the faded signal at receiver i. The energy \( \varepsilon_i = B_i^2 T \) is a random variable, because \( B_i \) is a Rayleigh random variable. From eqn. (2), \( F_i(\psi) \) depends on \( \varepsilon_i \), so \( F_i(\psi) \) is a random variable. Its mean value is \( R_i(\psi) \) which is given in [12]

\[
R_i(\psi) = \frac{\sin(\psi)}{4\pi} \left\{ M(1, t) \left[ \frac{\pi}{2} \right]_{\varepsilon_i} - M(y, t) \left[ \frac{\pi}{2} \right]_{\varepsilon_i} \right\}
\]

\[
= \frac{1}{\pi} \tan^{-1} \left( \frac{\sin(\psi)}{1 - \cos(\psi)} \right) \left( \frac{\sin(\psi)}{2} M(\pi/2) \right)
\]

(7)

where \( M(x, t) = \frac{2}{\sqrt{x^2 - \cos^2(\psi)}} \tan^{-1} \left( \frac{x + \cos(\psi)}{\sqrt{x - \cos(\psi)}} \tan(\pi/2) \right) \), \( y = 1 + \frac{1}{\varepsilon_i/N_0} \) and \( \varepsilon_i \) is the average value of \( \varepsilon_i \) which is the bit energy. For 16–DAPSK, the probability of bit error for the basic message at receiver i is \( P_{i,B} = R_i((\pi/4) + \theta) + R_i((\pi/4) - \theta) \) and the probability of bit error for the additional message at receiver i is \( P_{i,A} = R_i(\theta) + R_i((\pi/2) - \theta) \).

The capability disparity and degradation for noncoherent reception of 16–DAPSK on the AWGN and Rayleigh channels depend on the desired error probabilities. The tradeoff between the allowed degradation and the required capability disparity is shown in Fig. 3 for \( P_{1,B} = P_{2,A} = 10^{-2} \). In order to have an acceptably large communication range for the basic message, it is necessary to limit the degradation. A limitation on the degradation limits the value of \( \theta \), which in turn sets a lower limit on the additional capability required to demodulate the fourth bit with the specified probability of error. As the allowed degradation is permitted to increase, the value of \( \theta \) can be increased, which permits the disparity between more capable receiver and the less capable receiver to be reduced.

**IV. PUNCTURED CONVOLUTION CODING WITH MULTICAST SIGNALING**

Consider a system in which the basic message is encoded with a punctured convolutional code of rate \( r = k/n = 8/24 \), constraint length 7 and the octal forms of the generator polynomials are 133, 171, 145 and 133 [20]. The additional message is either uncoded or encoded with the same punctured convolutional code. Viterbi decoding is used for this purpose. The channel is a memoryless binary symmetric channel that is denoted by BSC(p), where p is the crossover probability. If only the basic message is sent, the channel is BSC(p_b). If portion of both basic and additional messages are sent, nonuniform 16–DAPSK is employed, in which the error probabilities for the binary digits of the basic and the additional messages are denoted by \( p_1 \) and \( p_2 \) respectively. Thus, the channel seen by the binary digits of the basic message is BSC(p_1) and the channel seen by the additional message is BSC(p_2). For the
applications, it is $p_0 < p_1 < p_2$. For messages encoded with a rate $k/n$ punctured convolutional code and transmitted over a BSC($p$), an upper bound for the probability of an error event is given in [21]

$$P_e \leq \sum_{d=d_f}^{\infty} a(d) P_d$$

where $a(d)$ is the number of paths of Hamming weight $d$, $P_d$ is the probability of choosing a path of weight $d$ instead of the all–zero path and $d_f$ is the free distance of the code. An upper bound for the probability of bit error is given by

$$P_b \leq \sum_{i=1}^{\infty} \sum_{d=d_f}^{\infty} a(d,i) P_d$$

where $a(d,i)$ is the number of paths with Hamming weight $d$ that produce $i$ bit errors in the output. For a BSC($p$), $P_d$ is a function of $p$ that is given in [21]

$$P_d(p) = \begin{cases} S\left(\frac{d+1}{2}, d; p\right), & d = \text{odd} \\ \frac{1}{2} \left(\frac{d}{d/2}\right) p^{d/2} (1-p)^{d/2} + S\left(\frac{d}{2} + 1, d; p\right), & d = \text{even} \end{cases}$$

where

$$S(c,d;p) = \sum_{k=c}^{d} \binom{d}{k} p^k (1-p)^{d-k}$$

Then, an upper bound for the probability of bit error is given by

$$P_b(p) \leq \sum_{i=1}^{\infty} \sum_{d=d_f}^{\infty} a(d,i) P_d(p)$$

Three different signaling methods are described below. They deliver the additional message at the same rate of the basic message. The distinctions among the three methods are the way in which the additional message is sent.

**Method 1**: The additional message is not encoded and the transmission of each bit of the additional message requires $n$ consecutive transmission intervals. That is, the $k$th bit of the additional message is sent in each of the DAPSK symbols in the $k$th block.

**Method 2**: The additional message is not encoded and one bit of the additional message is sent in every $n$th transmission interval. Thus, for the combination of the basic and additional messages, $n-1$ symbol in each block is uniform 8–DAPSK symbols and the remaining symbol is a nonuniform 16–DAPSK symbol.

**Method 3**: The additional information is encoded and one code symbol is sent in each transmission interval. Nonuniform 16–DAPSK is used in each transmission interval.
V. RESULTS

The performance of the signaling methods that produce the smallest disparity for a fixed degradation is considered to provide the best performance. The smallest one of the disparities corresponds to the smallest amount of additional energy required to achieve the desired probability of error in the reception of the additional message. The basic and additional messages may present different types of multimedia information. It is common that $P_A \neq P_B$.

Let $\mu = P_A/P_B$ be a multimedia factor, which is a measure of the dissimilarity in the bit error probability for the two types of the messages. The capability disparity between the less capable and more capable receivers as a function of bit error probability is shown in Fig. 4 for $\mu = 1$ and for a degradation of (0.1 and 1) dB in AWGN channel. The results indicate that if the desired probability of error is greater than $10^{-3}$, method 2 should be used to send the additional message. If the message types for both the basic and additional messages require a bit error probability of less than $10^{-3}$, method 3 achieves the lowest required capability disparity. From the results, it is shown that a higher degradation allows a lower required capability disparity to be achieved.
Fig. (4) Capability disparity for nonuniform 16-DAPSK in an AWGN channel with $\mu = 1$

The three methods are also compared for $\mu \neq 1$. The results are shown in Fig. 5 for $\mu = 0.1$ and 10, where the values of $P_B$ are shown along the bottom axis and the values of $P_A$ are shown along the top axis. Note that for $\mu \neq 1$, the disparity is not constant for signaling method 3. The results in Fig. 5(a) indicate that each of the disparities for the three methods is reduced if the required $P_A >> P_B$. The disparities may be negative for some ranges of parameters, such as in Fig. 5(a) for $P_B \geq 9.2 \times 10^{-2}$. A comparison of Figs. 4(b) and 5(b) shows that the disparities are increased if $\mu << 1$. For $\mu = 1$, the least disparity among the three methods is between 6 and 8.4 dB; for $\mu = 0.1$, the smallest disparity is between 13.8 and 15.2 dB.

In a Rayleigh fading channel, the results are shown in Fig. 6 for a degradation of $0.1 \text{ dB}$ and $\mu = 1$. The disparity for method 1 at low error probabilities is larger than for method 2. At most error rates of interest, method 2 produces a significantly smaller disparity than method 1. For voice communications, $P_B = 10^{-2}$ might be desired.

In this case, signaling method 2 has the least disparity of the three methods for the rate $1/3$, $K_c = 7$ code and by employing method 2, the additional message can be sent without coding. Method 2 has less complexity and processing delay than method 1, so, method 2 may be preferred in some applications. The results also show that for a Rayleigh fading channel at high error probabilities, method 1 is superior to method 2.
Fig. (5) Capability disparity for nonuniform 16–DAPSK in an AWGN channel with a degradation of 1 dB
(a) for $\mu = 10$ (b) for $\mu = 0.1$

Fig. (6) Capability disparity for nonuniform 16–DAPSK in a Rayleigh fading channel with degradation = 0.1 dB and $\mu = 1$
VI. CONCLUSION

The coupling of nonuniform DAPSK with a punctured convolutional coding is an effective method for dealing with variations in the received signal strength and interference in multicast transmission of multimedia data. The performance of nonuniform 16–DAPSK on both an AWGN and Rayleigh fading channels is analyzed. The results demonstrate that the way in which the additional message should be modulated and encoded depends on the type of additional message inserted. Different performance characteristics are shown for these methods. It is noted that a change in the degradation can be made to decrease the amount of required capability disparity between the less capable and more capable receivers.

REFERENCES

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