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THROUGHPUT-DELAY PERFORMANCE ANALYSIS OF SSMA PACKET RADIO NETWORK

Yousef G. El-Jaafreh
Electrical Engineering Department
Mutah University, Al-Karak , Jordan

ABSTRACT

Multiple access communication accommodates a large population of relatively uncoordinated users of a common spectral allocation in the same and neighboring geographical areas with the number of simultaneous users proportional to various variable parameters. A continuous-time Markov chain model for an asynchronous spectrum packet radio network is presented. The network consists of N fully connected nodes and the mobile users in all nodes are assumed identical. Packets arrive at each node and are re-transmitted when lost, according to Poisson processes with different rates and with packet lengths that are exponential in distribution. A simple threshold approximation is used to account for the multi-user interference and the preamble collision probability at receiving mobile users to account for the capture effect. Results obtained demonstrate the effects on throughput and packet delay performance of the network according to the variations of the network size, the packet re-transmission rate, the preamble collision probability at receiving mobile users, and the threshold value of the radio channel capacity. Further the approximate analysis results are very close to those obtained by computer simulations.

INTRODUCTION

Code division multiple access (CDMA) is a form of spread spectrum communications. The original applications of this technology were in military communications systems where principal appeal of spread spectrum signals are their immunity to interference from other signals. The earliest proposals to apply spread spectrum to cellular systems appeared in the late 1970s. These proposals stimulated theoretical work, which revealed some of the strengths and weaknesses of spread spectrum in cellular applications [1]. The first commercial systems were dual-mode cellular systems in the 850 MHz band then the 1,900 MHz band at the end of 1996.

The capacity of a multiple access network is measured by the average number of users receiving service at a given time with a given level of quality which includes requirements for both accuracy and service availability. Availability is defined as the complement of the probability that a user does not receive service at any given time because all slots (either frequency division or time division multiple access) are currently assigned to calls-a situation that evokes a busy signal.

In wireless systems, the total number of available slots depends on total bandwidth, data rate per user, and frequency reuse factor, all of which determine the quality of the call in terms of availability or accuracy or both [2]. Much research efforts [3]-[5] have concentrated on the slotted (synchronous) spread spectrum multiple access (SSMA) networks. The slotted transmission is possible in some systems such as those using the satellite channel where time is always referenced to the satellite. However, the synchronization of geographically dispersed mobile nodes is a difficult problem.

For unslotted systems employing spread spectrum multiple access, there are other variables including, (N) users,(L) simultaneous transmission, (λ) arrival rate, (S) packet throughput, and (D) average packet delay.

The importance of this study is due to the fact that this CDMA approach allocates all resources to all simultaneous users, yielding a considerably higher capacity than any other multiple access technique may achieve [6].

In this paper, analysis of the throughput and packet delay of an unslotted SSMA packet radio network will be investigated considering the network stability conditions.

COMMUNICATIONS SYSTEMS MODEL

A fully connected packet radio network consisting of N nodes is considered. Each node has a radio operating in a half duplex mode and the packet transmission of a radio is assumed to be asynchronous. Further, the radio receiver in each node is assigned a networkwide unique spreading code and transmitters use the same spreading code assigned to the receiver to which they are trying to transmit a packet. The radio channel is assumed noiseless, consequently, transmission failures are only due to preamble collisions, multi-user interference and the intended receiver being not ready to receive the incoming packet at the time transmission attempt starts. The preamble collisions result from the simultaneous presence of preambles using the same spreading code at a receiver in synchronization state. If a receiver is already in receiving mode and another packet is transmitted to that receiver, it is probable that the preamble collisions will occur and both packets will be lost. Let $P_{c|r}$ be the conditional probability that when a packet is transmitted to a receiver in the receiving mode, both packets are lost. With spread spectrum signaling, packets under transmission may be lost when more radio users are trying to access the channel than the channel capacity allows. As the number of simultaneous transmissions L increases due to active nodes in the network, the packet error probability increases. Of course, a larger packet error probability implies that a large number of re-transmission will be required, leading to increased delay D. Generally, for a large population of users, the packet arrival rate per user is small, but the total average arrival rate from the entire population, λ packet /s may be large. Arrivals occur randomly at Poisson distributed intervals. This is equivalent to modeling the arrival process as a sequence of independent binary variables, in successive infinitesimal time intervals Δt, with a single arrival per interval occurring with probability $\lambda\Delta t$ [7]. The packet service time per user is assumed to be exponentially distributed, so that the probability that service time τ exceeds T is given by: -

$$P_{r|c}(\tau > T) = e^{-\mu T} \qquad , T > 0$$

From this it follows that average packet duration is $(1/\mu)$s. Continuing with the infinitesimal independent model, the probability that the packet terminates during an interval of duration Δt seconds is

$$P_{r|c}(T < \tau < T + \Delta t \mid_{\tau} > T = [\, e^{-\mu T} - e^{-\mu\,(T+\Delta t)}\,]/e^{-\mu T}$$

$$= 1 - e^{-\mu\Delta t} \sim \mu\,(\Delta T) + O(\Delta T),$$

Where $O(\Delta T) \rightarrow 0$ as $\Delta T \rightarrow 0$

There are two commonly used models for determining the occupancy distribution and the probability of lost packet using Markov chains, as shown in figure (1), where the states represent the number of users in the system.

For the SSMA networks, there is a pronounced threshold where the channel performance degradation as a function of the number of transmissions ceases to be gradual and performance degrades rapidly [8].

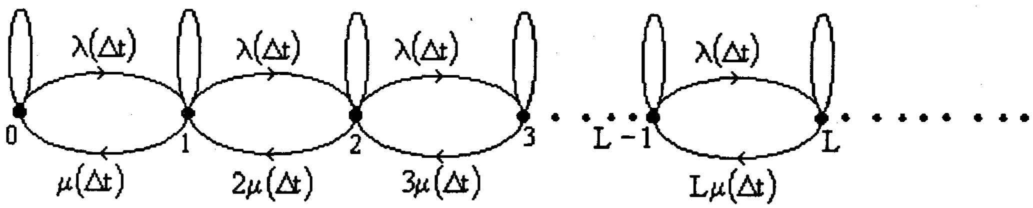

Fig. 1 Markov state diagram for determining the occupancy distribution and probability of lost packets

In this analysis, the threshold approximation is used, where it is assumed that all packet transmissions do not exceed some threshold value L, and that all packet transmissions fail otherwise. Thus, the threshold L implies the channel capacity of the network with respect to the number of simultaneous packet transmissions. Furthermore, perfectly power-controlled users are assumed, so that each is received by the base station at the same power level and during inactive periods, the user's signal power is suppressed. Receivers abort receiving as soon as the packet failures due to the preamble collisions or the multi-user interference are detected.

The model assumes that all radios are identical and propagation delay is zero. Hence, traffic requirements between any two nodes within the network are all alike, and each radio has identical arrival process, scheduling process for re-transmission, and packet length distribution.

The channel access protocol is similar to disciplined ALOHA [9]. The packets arrival from other mobile users are not allowed, not only when the radio is transmitting or receiving a packet as in the disciplined ALOHA, but also when it is in the backlogged mode,i.e., the radio can initiate receiving if a packet is transmitted to this radio before the next scheduling point of re-transmission, then, the radio enters the backlogged receiving mode, and returns to the backlogged mode when the packet reception is completed or aborted.

For Markovian network model, it is assumed that the arrival process and the scheduling process for re-transmission are Poisson processes, and that the packet lengths are exponentially distributed. The rates of the arrival process and scheduling process are λ_o and λ_r respectively, and the mean packet length is $1/\mu$. Finally, due to extreme difficulty for analytical tractability, it is assumed that all packet lengths and destination addresses are independably redistributed at each attempt for re-transmission.

PERFORMANCE ANALYSIS

The network model uses simplifying approximation by considering the time-continuous Markov chain state methods. Each radio may be in one of five operating modes; idle, transmitting, receiving, backlogged, and backlogged receiving. Since all radios are assumed identical, it is only required to keep track of the number of radios in each mode, rather than the exact mode of each individual radio for a sufficient network state description. To describe the state transition probabilities in order to evaluate throughput and packet delay, the state of the network model is assumed to be the four component vector (i,j,k,l), where i is the number of the radios transmitting packets successfully, j is the number of radios transmitting packets that are not being received, k is the number of radios in the backlogged mode, and l is the number of radios in the backlogged receiving mode.

From the state of the network, it is obvious that i-k represents the number of radios in the receiving mode, and N-2i-j-l represents the number of radios in the idle mode. Further, the state space of the network model ϕ, may by divided into two regions, the first is the normal area where the number of radios in the transmitting mode does not exceed L, therefore, it is possible to transmit packets successfully in the network model, and the second is the saturated area where the number of radios in the transmitting mode exceeds L, therefore, all packet transmission attempts result in failure.

For the state to be in the normal region and the saturated region, it must satisfy the following two sets of constraints, respectively:-

$0 \le i \le \min(\lfloor N/2 \rfloor, L)$
$0 \le j \le \min(N-2i, L-i)$
$0 \le k \le i$ and
$0 \le l \le N-2i-j$
$i = k = 0$
$L+1 \le j \le N$ and
$0 \le l \le N-j$

It should be stated that the number of values that l can take is N-2i–j+1 in the normal region and N-j+1 in the saturated region. Hence, the total number of states $|\phi|$ is given as

$$|\phi| = |\phi_{\text{normal}}| + |\phi_{\text{saturated}}|$$

$$= \sum_{i=0}^{min\left(\left\lfloor N/2 \right\rfloor, L\right)} \sum_{j=0}^{min(N-2i,L-i)} \sum_{k=0}^{i} (N-2i+j+1) + \sum_{j=L+1}^{N} (N-j+1)$$

Where $\lfloor x \rfloor$ is the largest integer that does not exceed x.

STATE TRANSITION RATES

The Markov state transition can occur by two events; the start of a packet transmission by any radio and the end of a packet transmission at any radio within the model network. The packet arrival process at each node is an independent Poisson process with rate λ_o, the scheduling process for packet re-transmission is an independent Poisson process with rate λ_r, and packet lengths are exponential in distribution with mean $1/\mu$.

The two Markov state transitions resulting from the start of a packet transmission when the current total number of simultaneous transmission is equal to or greater than L are as follows;

1. A state transition when an idle radio starts transmission, that is, $(i, j, k, l) \rightarrow (0, i+j+1, 0, l+k)$, then its state transition rate is given as

$$R_1(i, j, k, l) = \lambda_o(N-2i-j-l) \tag{1}$$

2. A state transition when a radio in the backlogged mode starts retransmission attempt, that is, $(i, j, k, l) \rightarrow (0, i+j+1, 0, l+k-1)$, then its state transition is given as

$$R_2(i, j, k, l) = \lambda_r l \tag{2}$$

However, the state transitions resulting from the end of a packet transmission are as follows;

1. A state transition when the radio transmitting a packet successfully to the radio in the receiving mode completes transmission, that is , $(i, j, k, l) \rightarrow (i-1, j, k,$

l). The number of these communicating pairs is $i = k$, then its transition rate is given as

$$R_3(i, j, k, l) = (i - k)\,\mu \tag{3}$$

2. A state transition when the radio transmitting a packet successfully to the radio in the backlogged receiving mode completes transmission, that is, $(i, j, k, l) \rightarrow (i-1, j, k-1, l)$ then its state transition rate is given as

$$R_4(i, j, k, l) = k\mu \tag{4}$$

3. A state transition when the radio transmitting a packet that is not being received completes transmission, the radio enters backlogged mode to re-transmit the packet at later time, that is, $(i, j, k, l) \rightarrow (i, j-1, k, l+1)$. The number of radios in this mode is j and its transition rate is given as

$$R_5(i, j, k, l) = j\mu \tag{5}$$

Further, the state transitions result from the start of a packet transmission when the number of simultaneous transmission is less than L. In this case, the new packet has a non-zero probability to be transmitted successfully as shown in following cases;

1. A state transition when an idle radio transmits a packet to another idle radio, this transmission starts successfully. This occurs at a rate $\lambda_o (N - 2i - j - l)$, and the probability that the packet is transmitted to one of the idle radios is $(N - 2i - j - l - 1)/(N-1)$. Thus, the state transition rate is given as

$$R_6(i, j, k, l) = [\lambda_o / N-1]\,(N - 2i - j - l)\,(N - 2i - j - l - 1) \tag{6}$$

2. A state transition when an idle radio starts transmission to any radio in the receiving mode, which results in the preamble collisions at the receiver, is, $(i, j, k, l) \rightarrow (i = 1, j+2, k, l)$. Thus, the state transition rate is given as

$$R_7(i, j, k, l) = [\lambda_o P_{c|r} / N-1]\,(i-k)(N-2i-j-l) \tag{7}$$

3. A state transition when an idle radio starts transmission to any radio in the backlogged mode, that is, $(i, j, k, l) \rightarrow (i+1, j, k+1, l-1)$. The state transition rate is given as

$$R_8(i, j, k, l) = [\lambda_o l / N-1](N-2i-j-l) \tag{8}$$

4. A state transition when an idle radio starts transmission to any radio in the backlogged receiving mode, that is, $(i, j, k, l) \rightarrow (i-1, j+2, k-1, l+1)$, which result in the preamble collisions at the receiver. The state transition rate is given as

$$R_9(i, j, k, l) = [\lambda_o P_{c|r} / N-1]\, k\, (N-2i-j-l) \tag{9}$$

5. A state transition when an idle radio starts transmission attempt to any radio in the transmitting mode, or to any receiving radio with no preamble collisions at the receiver, that is, $(i, j, k, l) \rightarrow (i, j+1, k, l)$.
The state transition rate is given as

$$R_{10}(i, j, k, l) = [\lambda_o / N-1]\, (N-2i-j-l)((i+j+i)(1-P_{c|r})) \tag{10}$$

6. A state transition when a radio in the backlogged mode starts re transmission to any radio in the receiving mode, which results in the preamble collisions at the receiver, that is, $(i, j, k, l) \rightarrow (i-1, j+2, k, l-1)$ The state transition rate is given as

$$R_{11}(i, j, k, l) = [\lambda_r P_{c|r} / N-1]\, l\, (i-k) \tag{11}$$

7. A state transition when a radio in the backlogged mode starts re transmission attempt to another radio in the backlogged mode, that is, $(i, j, k, l) \rightarrow (i+1, j, k+1, l-2)$. The state transition rate is given as

$$R_{12}(i, j, k, l) = [\lambda_r l / N-1](l-1) \tag{12}$$

8. A state transition when a radio in the backlogged mode starts retransmission attempt to any radio in the backlogged receiving mode, that is, $(i, j, k, l) \rightarrow (i-1, j+2, k-1, l)$. Which results in the preamble collisions of the receiver. The state transition rate is given as

$$R_{13}(i, j, k, l) = [\lambda_r P_{c|r} / N-1](kl) \tag{13}$$

9. A state transition when a radio in the backlogged mode starts retransmission attempt to any idle radio, in the backlogged mode, which can be considered successful, that is, $(i, j, k, l) \rightarrow (i+1, j, k, l-1)$. The state transition rate is given as

$$R_{14}(i, j, k, l) = [\lambda_r l / N-1](N-2i-j-l) \tag{14}$$

10. A state transition when a radio in the backlogged mode starts retransmission attempt to any radio in the transmitting mode, or to any receiving radio with no preamble collisions at the receiver, that is, $(i, j, k, l) \rightarrow (i, j+1, k, l-1)$.The state transition rate is given as :

$$R_{15}(i, j, k, l) = [\lambda_r l / N\text{-}1]\,[(i+j+i)\,(1\text{-}P_c|_r)] \tag{15}$$

The continuous-time Markov process considered is irreducible, homogeneous and has a finite state space [10].

THROUGHPUT PACKET DELAY

Maintaining the assumption of a uniform density of mobile users, the throughput of a network with infinite channel capacity increases without bound as N increases, therefore, the throughput normalized by the number of nodes, S, is useful and commonly defined as $S = S_N/N$ where S_N is the total throughput and can be obtained by taking expectation of i

$$S_N \cong \sum_{(i,j,k,l)\in\phi} i, \pi(i,j,k,l) \tag{16}$$

Where the component i in state (i,j,k,l) is the number of the radios transmitting packets successfully. However, since the transmission times of the packets whose transmission start successfully at the beginning but fail to be completed due to the occurrences of the preamble collisions or the multi-user interference are not excluded in (16), S_N is slightly overestimated.

The average packet delay D is defined as the time interval from the packet arrival at a node to the end of successful transmission of that packet. i.e,

$$D = \frac{\sum (i,j,k,l) \in \phi (i+j+k+l) \pi(i,j,k,l)}{\mu \sum (i,j,k,l) \in \phi^i . \pi(i,j,k,l)} \tag{17}$$

Since the network model is a four dimensional Markov chain, and the number of states can readily be shown to be $O(N^4)$from (1), then as the network size N increases, the computations become vast and involved. Therefore, for simplicity of analysis, the case where the channel capacity L is in proportion to N and large enough to ignore the effect of L on throughput will be considered.

The total throughput is approximately equal to the number of radios, which are transmitting packets that are being received successfully.

RESULTS AND DISCUSSION

The throughput per node as a function of the packed arrival rate λ_o for different network size N is plotted as shown in Fig.2. For N>10 the curves are almost the same. However, as N increases, the maximum throughput drops and approaches a threshold value of 0.171. Fig.3 shows the mean packet delay as a function of throughput per mode for different network size N. It should be noted that the curves are almost the same for N>10.

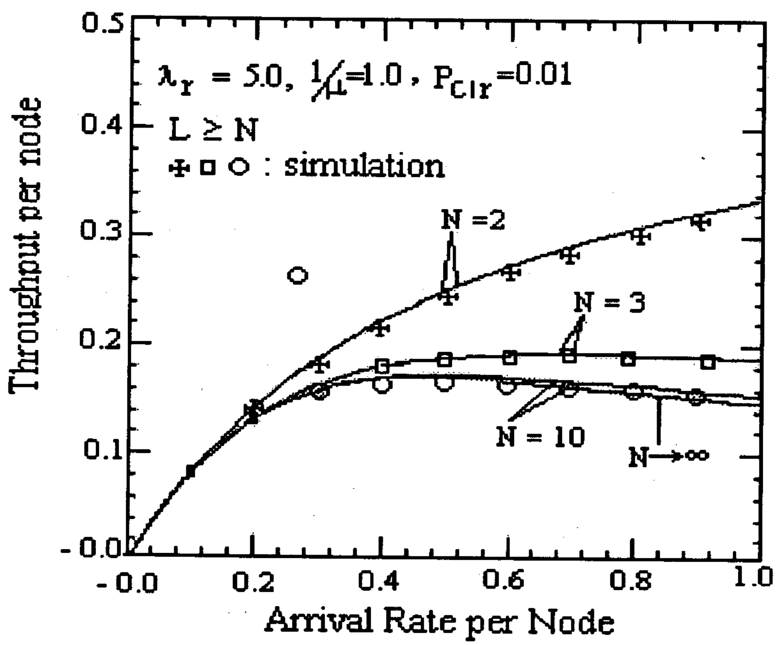

Fig. 2. Throughput versus arrival rate for different network size.

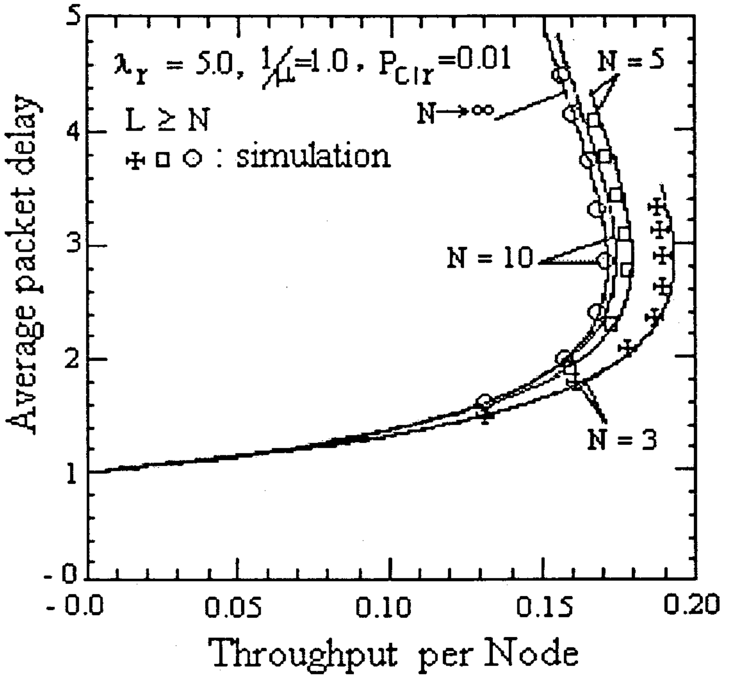

Fig. 3. Average packet delay versus throughput for different network size

The effect on throughput of the packet re-transmission rate λ_r is shown in Fig.4. It is obvious from the plot that as λ_r increases, the maximum throughput decreases, but at lower arrival rates, fast re-tarnsmission of the backlogged packet increases the throughput. Fig.5, shows the mean packet delay as a function of throughput per node for various λ_r values. It is clear that although the maximum throughput is slightly larger for low values of λ_r, smaller delay is obtained for large values of λ_r

In Fig.6, the effect of imperfect capture on throughput is demonstrated. The maximum throughputs are 0.174, 0.166, and 0.164 for $P_{c|r}$ =0, 0.1, and 0.2 respectively. Fig.7, shows the mean packet delay as a function of throughput per node for some values of $P_{c|r}$.

In all results presented so far, the approximations to the throughput yield slight over estimations of the simulation results and maximum error of less than 3%, for a mean packet length $1/\mu$ of 1 (in unit of time).

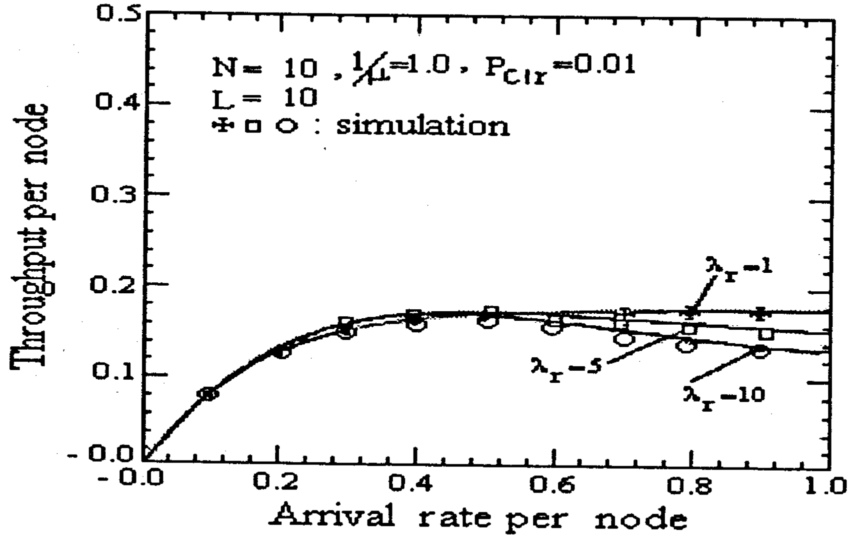

Fig. 4. Throughput versus arrival rate for different retransmission rate

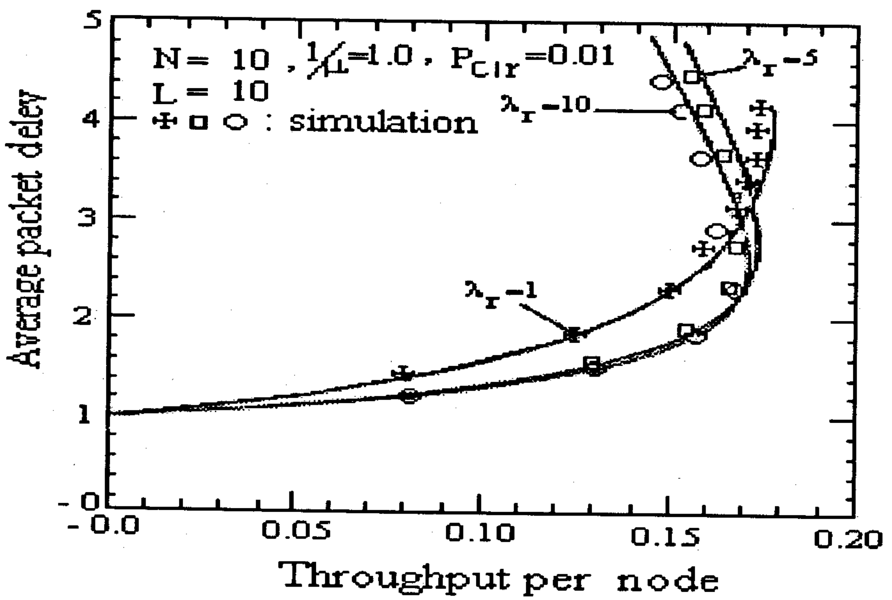

Fig. 5. Average packet delay versus throughput for different retransmission rate

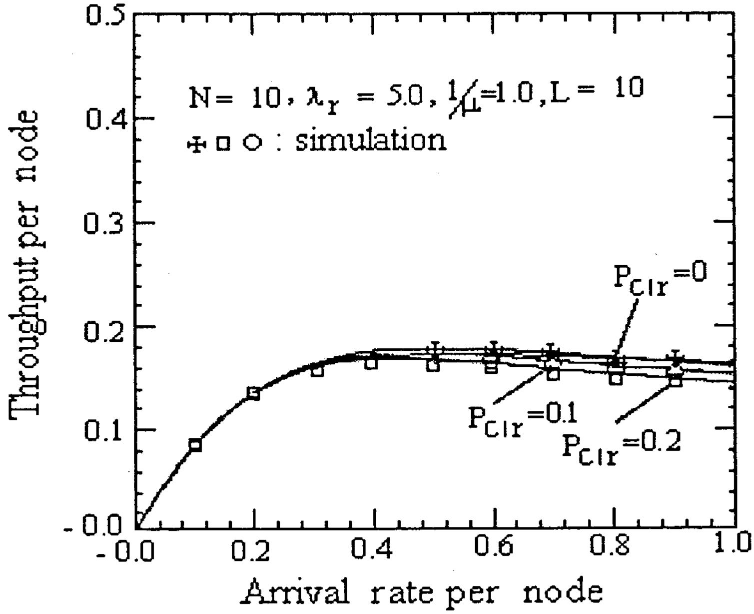

Fig. 6. Throughput versus arrival rate for different collision probability

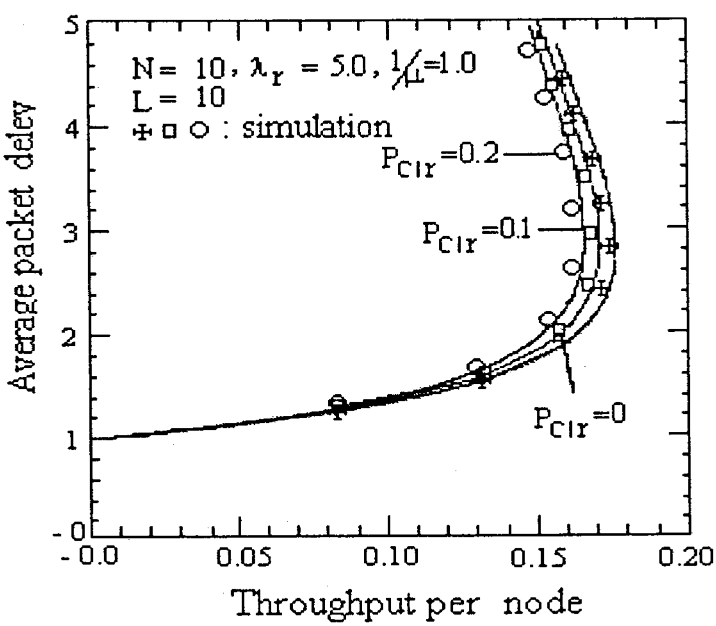

Fig. 7. Average packet delay versus throughput for different collision probability

The effect of the channel capacity L on throughput is plotted in Fig.8. As L decreases, the throughput decreases as expected. Since the number of radios in the transmitting mode exceeds L frequently for small values of L, the approximation analysis yields more overestimated results than those for L=N. Note that for N=10, the throughput curves do not vary for $L \geq 9$.Generally, the performance characteristics are all the same for $L \geq N-1$. At small values of L, the throughput curve decreases rapidly as λ_o increases beyond some threshold value. In Fig.9, average packet delay as a function of throughput per node is plotted for several values of L. The catastrophic failure of the network performance for small values of L is shown very clearly.

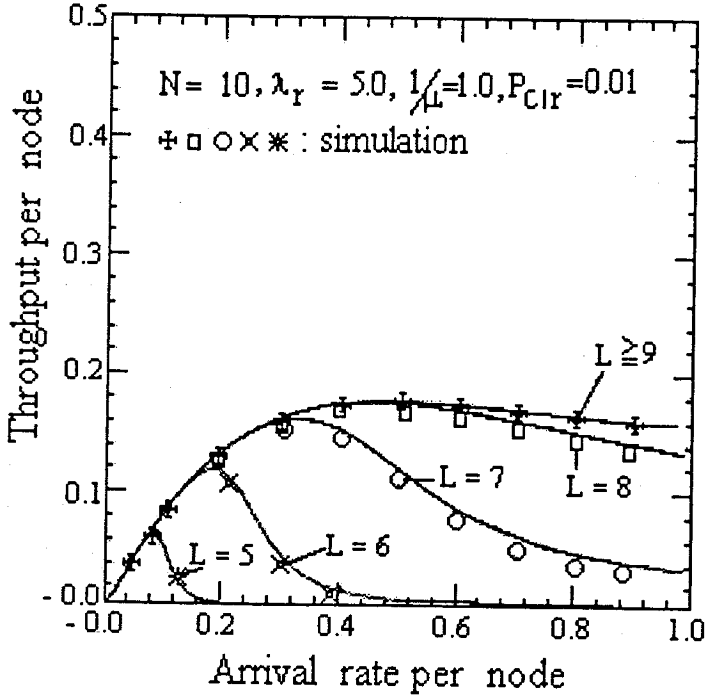

Fig. 8. Throughput versus arrival rate for different channel capacity

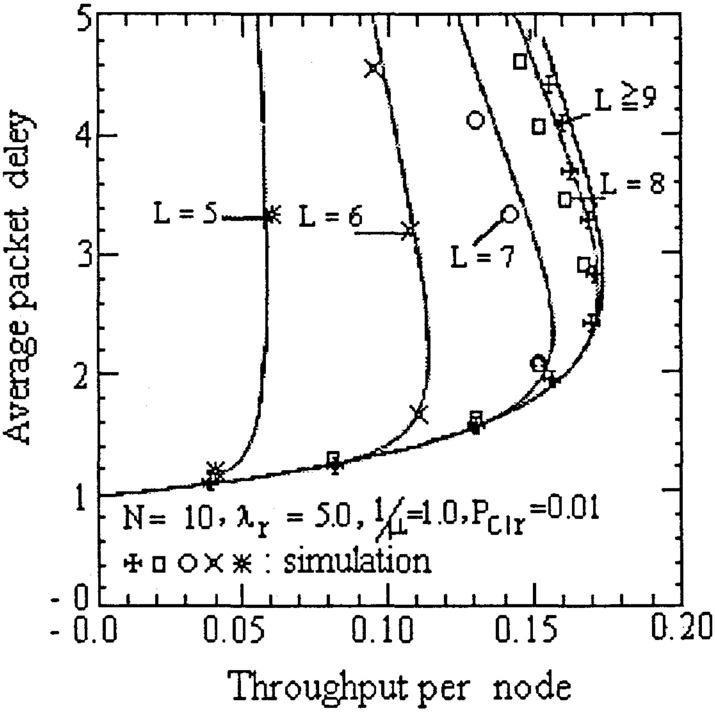

Fig. 9. Average packet delay versus throughput for different channel capacity

CONCLUSIONS

The performance of an asynchronous unslotted spread spectrum packet radio network has been analyzed. The network was modeled as a continuous-time Markov chain because it is more realistic for mobile users and because it more closely resembles the case for unslotted multiple access. A threshold approximation was introduced to account for the multi-user interference, and the preamble collision probability to account for the capture effect. Close agreements between the approximate analysis and simulations were obtained in most cases, that is, the results obtained for the two approaches are very similar.

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