Overhead Allocation: a Goal Programming Approach

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ABSTRACT

Overhead is an incurred cost which is matched against cost objects via an intervening base of allocation. The problem of overhead allocation is to come up with a cost allocation procedure which is objective, applies uniformly to all cost centers, is logically defensible, pays attention to the fact that no manager likes costs that are not under his control to be allocated to him, and makes use of the available data. This problem is solved in this paper via goal programming models. The procedures involved are illustrated through a detailed numerical example.

توزيع التكاليف العامة
غير المباشرة
بطريقة البرمجة الهدفية

يحيى بدران
جامعة الكويت

ملخص

يعالج هذا البحث مشكلة توزيع التكاليف العامة وغير المباشرة . المشكلة هنا تتركز في الحصول على نهج لتوزيع مثل هذه التكاليف بحيث ان يكون هذا النهج موضوعيا وان يطبق باستواء على جميع مراكز التكلفة في المؤسسة وان تكون حجة استعماله منطقية وأن يأخذ في الاعتبار ان مديري مراكز التكلفة لا يودون ان توزع عليهم نفقات لم تكن تحت رقابتهم وتحكمهم وأن يستعمل هذا النهج البيانات المتاحة .

توصل هذا البحث الى مثل هذا النهج عن طريق نماذج البرمجة الرياضية الهدفية . ولقد وضحنا استعمال هذا النهج عن طريق مثال عددي مطول .

Introduction

This section sets up the perspective in which the problem with which we are concerned is stated. To start with, a unit of the organization to which costs are assigned and within which costs are aggregated for purposes of control is called a *cost center.* Costs incurred entirely within a cost center are called *direct costs* (of that center). If the direct costs of a center are incurred for the benefit of that center alone, the center is called a *production center,* or a *production department.* A cost center which is not a production department is called a *service department.*

Costs assigned to a center which are not part of its direct costs are the center's *indirect costs.* Indirect costs of a center are also known as the center's *overhead.* Costs incurred that are not direct costs of *any* center will be referred to as the organization's *unstructured overhead.* Examples of unstructured overhead are property tax and depreciation of buildings.

A center's overhead which originates from other centers will be referred to as the center's *structured overhead.* A center's unstructured overhead, on the other hand, is that which originates from the unstructured overhead of the organization. The *semi-full costs* of a center are defined as the sum of its direct costs and its unstructured overhead, and a center's *full costs* are defined as the sum of its semi-full costs and its structured overhead.

An overhead is allocated through an intervening *base of allocation.* There are different bases for allocating overhead; one traditional method of allocation is to *use a different base for each type of overhead.* Bases of allocating different types of overhead which appear to be most typical in practice are *square footage* for rent, depreciation of buildings, property tax, heating and cooling, and fire insurance; *number of employees* for supervision, general administration, cafeteria, payroll, personnel, transportation, recreation, and computer services; and *prime cost* for research and development, and advertising (general) [3].

The problem with this method of allocation is that there are suitable alternatives for each base, and no general rule as to which base should be used in a particular case. Each organization wrestles constantly with these bases in an effort to achieve equity [3, p. 801].

Another traditional method of allocating overhead is to use *one base for all types of overhead.* Bases for this method which appears to be most typical in practice are *units of production, materials cost, direct labor cost,* and *machine hours* [9, pp. 186-243]. With this method of allocation, however, the question is which base is the correct one? It has been noted [3, p. 806] that there is no precise answer to this question. On the one hand it can be said that no base is the correct base, since the choice of any base is either arbitrary or a matter of managerial discretion. On the other hand, the choice of just *any* base does not appear to be satisfactory, because some appear to be better than others with respect to certain types of overhead, and different bases are likely to produce allocations that are materially different.

In regard to the problem of cost allocation, Professor Horngren [5, pp. 395-426] has noted that: 'cost allocation is an inescapable problem in nearly every organization, ... the choice of an allocation base is often necessary because there is no obvious or convenient direct link between a cost and the cost object, . . . questions of allocation are inevitably tough, so the answers often are not clearly right or wrong, . . . the entire methodology of reallocation (of indirect costs) is plagued by the frequent reliance on some arbitrary rules that are designed to charge (cost objects) in some "equitable" manner, . . . the argument anyway, and that managers generally do not get too concerned about such allocations as long as all departments are subject to uniform cost reallocation procedure, . . . the argument against the reallocation of such costs rests on the idea that no cost should be reallocated to a manager unless he has some direct influence over their amount, and to the extent that an allocation base is no more logically or empirically defensible than some other base, either do not allocate or allocate via a predetermined agreement.

The problem situation with which this paper is concerned can now be stated: An organization which decided on full costing of its cost centers via one base for the allocation of all types of overhead, discovered that there are several suitable alternative bases, no one of them is more logically or empirically defensible than the others, and each one of them is likely to produce a cost allocation that is materially different from the others. The problem facing this organization is to come up with a cost allocation procedure which is objective, applies uniformly to all cost centers, is logically defensible, pays attention to the fact that no manager likes costs that are not under his control to be allocated to his department, and make use of the available bases. The purpose of this paper is to construct such a procedure through goal programming models.

Formulation of the Model

The idea of goal programming, which was introduced by A. S. Charnes and W. W. Cooper, states that, whether goals are attainable or not, an objective may be stated in which optimization gives a result which comes 'as close as possible' to the indicated goals [4, p. 215].

In the context of our problem there are two sets of goals that are clearly in conflict with each other. The first set of goals pertains to the organization: its decision on full costing of its departments. The second set of goals pertains to the departments of the organization: cost allocation is a necessary evil, thus the smaller the amount allocated to a department the better it is.

Full costing of the departments will be achieved through two successive stages. The first stage allocates the unstructured overhead among all of the departments. This is the stage of semi-full costing. The second stage allocates the structural overhead, and is the stage of full costing.

Notations and Symbols

The importance of the choice of symbols (in mathematics) was recognized long ago by the German universal genius G. Leibniz. Here we list the symbols and describe the notations that are used in what follows. The principle of mnemonics is used as far as the choice of the majority of symbols is concerned. The notations for row-vectors, column-vectors, matrices, and cross-sections of matrices are generalized adaptations of those used in the PL1 computer programming language and E. Bodewig's 'Matrix Calculus' [2]. Let

K = (1, 2, . . .,k) , index set of bases of allocation,

N = (1, 2, . . .,n) , index set of production departments,

M = (1, 2, . . .,m) , index set of service departments,

a_{ij} = Base i data with respect to production department j,

b_{ij} = Base i data with respect to service department j,

d_{pj} = *D*irect cost of *p*roduction department j,

ds_j = *D*irect cost of *s*ervice department j,

u = *U*nstructured overhead of the organization,

up_{ij} = *U*nstructured overhead allocated by base i to *p*roduction department j,

us_{ij} = *U*nstructured overhead allocated by base i to *s*ervice department j,

$spijt$ = *S*tructured overhead originating from service department i to *p*roduction department j throught base t,

ss_{ijt} = *S*tructured overhead originating from service department i to *s*ervice department j through base t,

sfp_{ij} = *S*emi-*f*ull cost through base i to *p*roduction department j,

sfs_{ij} = *S*emi-*f*ull cost through base i to *s*ervice department j,

fp_{ij} = *F*ull cost through base i to *p*roduction department j,

fs_{ij} = *F*ull cost through base i to *s*ervice department j,

The following symbols stand for decision variables:

up_j = *U*nstructured overhead to be allocated to *p*roduction department j,

us_j = *U*nstructured overhead to be allocated to *s*ervice department j,

sfp_j = *S*emi-*f*ull cost of *p*roduction department j,

sfs_j = *S*emi-*f*ull cost of *s*ervice department j,

sp_{ij} = *S*tructured overhead to be allocated from service department i to *p*roduction department j,

ss_{ij} = *S*tructured overhead to be allocated from service department i to *s*ervice department j,

fp_j = *F*ull cost of *p*roduction department j,

fs_j = *F*ull cost of *s*ervice department j.

The following notations for row-vectors, matrices, and cross-sections of matrices are used in what follows:

X, X_{**} = The matrix whose element in the i-th row and j-th column is x_{ij},

X_{i*} = The i-th row of the matrix X,

X_{*j} = The j-th column of the matrix X,

$X_{i.}$ = The sum of the elements of the i-th row of X,

$X_{.j}$ = The sum of the elements of the j-th columns of X ,

$X_{*.}$ = The column vector whose elements are the sums of the rows of X ,

$X_{.*}$ = The row vector whose elements are the sums of columns of X,

Y_* = The vector whose components are $y_1, y_2, \ldots,$

$Y_.$ = The sum of the elements of Y_*.

Definitional and Structural Equations

It is clear that *any* allocation must satisfy the following definitional and structural equations. By definition, we have, for any department, that its semi-full cost equals the sum of its direct cost and the unstructured overhead allocated to it; and that its full cost is equal to the sum of its semi-full cost and costs allocated to it from service departments. Thus,

$$SFP_* = DP_* + UP_* ,$$

$$SFS_* = DS_* + US_* ,$$

$$FP_* = SFP_* + SP_{.*} ,$$

$$FS_* = SFS_* + SS_{.*} .$$

There are two structural relationships: The unstructured overhead of the organization is equal to the sum of the unstructured overhead allocated to the production departments and the unstructured overhead allocated to the service departments; and the full cost of a service department is equal to the sum of the structured overhead originating from it to the production departments and the structured overhead originating from it to the other service departments. Thus,

$$u = UP_. + US_. ,$$

$$SFS_* + SS_{.*} = SP_{*.} + SS_{*.} .$$

Finally, since the direct cost of a service department is incurred ultimately for the benefit of the other departments, we, then, must have that,

$$ss_{ii} = O \text{ , for all i in M .}$$

Semi-full Costing Under a Base

The unstructured overhead, u, is allocated by base i to the production and service departments according to the *preration* formulas,

$$UP_{i*} = u. B_{i*}/(A_{i.} + B_{i.}) \text{ , for all i in K,}$$

$$US_{i*} = u. A_{i*}/(A_{i.} + B_{i.}) \text{ , for all i in K.}$$

The semi-full cost allocated by base i to the production and service departments, then, is,

$$SFP_{i*} = DP_* + UP_{i*} ,$$

$$SFS_{i*} = DS_* + US_{i*} .$$

Full Costing Under a Base

Here again, the structured overhead originating from service department i to the production and service departments is allocated through base t according to the preration formulas,

$$SP_{i*t} = fs_{ti} . H_{i*t} \text{ , for all i in M , and all t in K,}$$

$$SS_{i*t} = fs_{ti} . G_{i*t} \text{ , for all i in M , and all t in K ;}$$

where,

$$H_{i*t} = B_{t*}/(A_{t.} + B_{t.} - a_{ti}) ,$$

$$G_{i*t} = A_{t*}/(A_{t.} + B_{t.} - a_{ti}) ,$$

$$g_{iit} = O ,$$

are the preration vectors for the production and service departments for a given service department i and a given base t. The above equations in conjunction with the definitional equations yield,

$$FP_{t*} = SFP_{t*} + FS_{t*} . H_{**t} ,$$

$$FS_{t*} = SFS_{t*} + FS_{t*} G_{**t} \text{ , for all t in K .}$$

From the above two equations the full costs of the production and service departments according to base t are,

$$FS_{t*} = SFS_{t*} . (I - G_{**t})^{-1} ,$$

$$FP_{t*} = SFP_{t*} + FS_{t*} . H_{**t} \text{ , for all t in K.}$$

Finally, the structured overhead allocations for the production and service departments according to base t can be written as,

$$SP_{**t} = \text{Diag}(FS_{t*}) . H_{**t} ,$$

$$SS_{**t} = \text{Diag}(FS_{t*}) . G_{**t} ,$$

where,

Diag (Y_*) = The diagonal matrix whose diagonal elements are the components of Y_* .

Goal Programming Semi-full Costing

First, we develop the goals of the departments. Let,

$$\underline{UP}_* = \underset{i \in K}{\text{minimum}} \ (UP_{i*}) ,$$

= The least unstructured overhead allocated by the bases to the production departments.

Similarly,

$$\underline{US}_* = \underset{i \in K}{\text{minimum}} \ (US_{i*}) .$$

The goals of the departments, least possible unstructured overhead, can be stated as,

$$UP_* \leqslant \underline{UP}_* \text{ , and } US_* \leqslant \underline{US}_* \text{ ,}$$

where order relations on aggregates (matrices and vectors) are element-wise.

That part of the organization goal that requires semi-full costing of its departments is the fulfilling of the structural relationship,

$$u = UP_. + US_. \text{ .}$$

It is clear, however, that the goals of the departments and that part of the organization goal are in conflict, and they cannot be attained simultaneously unless it is true that,

$$u = \underline{UP}_. + \underline{US}_. \text{ ;}$$

in which case the unstructured overhead allocation that comes 'as close as possible' to the departments' goals is,

$$UP^*_* = \underline{UP}_* \text{ , and } US^*_* = \underline{US}_* \text{ .}$$

The other possibility, namely,

$$u > \underline{UP}_. + \underline{US}_. \text{ ,}$$

means that the system,

$$\underline{UP}_* = UP_* + X_* \text{ ,}$$

$$\underline{US}_* = US_* + Y_* \text{ ,}$$

$$u = UP_. + US_. \text{ ,}$$

$$(UP_* \text{ , } US_* \text{ , } X_* \text{ , } Y_*) \geqslant 0 \text{ ,}$$

has *no* solution. In which case the basic idea of goal programming comes to play: *'create feasibility as necessary'.*

One interpretation of the above dictum with respect to the above system is to allow the 'ought to' be non-negative vectors X_* , and Y_* to be unrestricted in sign but in such a manner that a measure of their deviation from non-negativity be as small as possible. In which case, however, there are several ways by which such deviation can be measured. Two such measures are the so-called *weighted city block metric,* and the *weighted Euclidean metric.* At this point we note that most goal programming applications assume a weighted-city block metric for their objective functions [5, 6, 7, 10].

On the one hand, a goal programming model under a weighted city-block metric can be defined as [10]:

minimize $W^+_* \; D^+_* + W^-_* \; D^-_*$,

subject to,

$$A_{**} X_* + D^+_* - D^-_* = G_* \text{ , } B_{**} X_* \geqslant B_* \text{ ,}$$

$$(X_* \text{ , } D^+_* \text{ , } D^-_*) \geqslant 0.$$

Where W^+_* and W^-_* are row vectors of goal weights, D^+_* is a column vector of underachievement of goal levels, and D^-_* is a column vector of overachievement of goals. A_{**} is a matrix of coefficients, X_* is a column vector of decision variables, and G_* is a column vector of desired goal levels. The constraints defined by $B_{**} X_* \geqslant B_*$ are any additional constraints that are independent of goals. On the other hand, however, a goal programming model under a weighted Euclidean metric can be defined as

minimize $D_* \; \mathrm{Diag}(W_*) \; D^t_*$, subject to,

$$A_{**}X_* + D_* = G_* \ , \ B_{**}X_* \geqslant B_* \ , \ X_* \geqslant 0.$$

where Diag (W_*) is a diagonal matrix whose diagonal elements are the components of the vector W_*, D_* is a row vector of goals' deviations, and D_*^t is the transpose of D_*.

Now a goal programming model for semi-full costing under a weighted city-block metric is

$$\text{minimize ud} = \sum_{j \in N} (1/wp_j)(x_j^+ + x_j^-) + \sum_{j \in M} (1/ws_j)(y_j^+ + y_j^-),$$

subject to,

$$UP_. + US_. = u, \ ,$$
$$UP_* + X_*^+ - X_*^- = \underline{UP}_* \ ,$$
$$US_* + Y_*^+ - Y_*^- = \underline{US}_* \ ,,$$
$$(UP_* \ , US_* \ , X_*^+ \ , X_*^- \ , Y_*^+ \ , Y_*^-) \geqslant 0.$$

It is a simple linear programming model in which wp_j is the priority weight given to the goal of production department j, and ws_j is the priority weight given to the goal of service department j. The simple constraints of this model and the fact that $u > \underline{UP}_. + \underline{US}_.$, indicate the following 'black or white' characteristic of the model, namely, a goal is either achieved or is underachieved. Furthermore, the achievement or the extent of underachievement of a goal depends on the relative value of its preemptive priority weight ($1/wp_j$), or ($1/ws_j$). For example, the goal which is underachieved most is the one with the smallest preemptive priority weight, and the goal which is considered first for achievement is the one with the largest preemptive priority weight.

A goal programming model for semi-full costing under a weighted Euclidean metric, on the other hand, is

$$\text{minimize ud} = \sum_{j \in N} (1/wp_j)(up_j - \underline{up}_j)^2 + \sum_{j \in M} (1/ws_j)(us_j - \underline{us}_j)^2 \ ,$$

subject to,

$$UP_. + US_. = u \ ,$$
$$(UP_* \ , US_*) \geqslant 0 \ .$$

The solution of this model is,

$$UP_*^* = \underline{UP}_* + (eu/w)\, WP_* \ ,$$
$$US_*^* = \underline{US}_* + (eu/w)\, WS_* \ ,$$

where,

$$eu = u - (\underline{UP}_. + \underline{US}_.) \ ,$$
$$w = WP_. + WS_. \ .$$

We note that *all* the goals under this model are underachieved and that the extent of this underachievement for a goal is inversely proportional to its preemptive priority weight. The semi-full costing of the departments under this model is

$$SFP_*^* = DP_* + UP_*^* \ ,$$
$$SFS_*^* = DS_* + US_*^* \ .$$

The above solution of semi-full costing is not completely determined, and in fact it still has (m + n) degrees of freedom, namely, the weights WP_*, and WS_*. However, it is precisely this freedom in the solution that enables management of the organization to fulfill the rest of its goal, namely, equitable and guided costing of its departments! Values of these weights are completely in the hands of management. This managerial discretion, however, should be guided by considerations of objectivity and equity.

Goal Programming Full Costing

As in the above section, we start by specifying the goals of the departments. Let,

$$\underline{SP}_{**} = \underset{t \in K}{\text{minimum}} (SP_{**t}) ,$$

= The least structured overhead allocated by the bases to the production departments.

Similarly,

$$\underline{SS}_{**} = \underset{t \in K}{\text{minimum}} (SS_{**t}) .$$

The goals of the departments, least possible overhead, can now be stated as,

$$SP_{**} \leqslant \underline{SP}_{**} \text{ , and } SS_{**} \leqslant \underline{SS}_{**} .$$

That part of the organization goal that requires full costing of its departments is simply the fulfilling of the structural relationship,

$$SFS^*_* + SS_{.*} = SP_{*.} + SS_{*.} ,$$

where SFS^*_* is the semi-full costs of the service departments obtained from the goal programming model of the above section.

Similar to the above treatment of semi-full costing, a goal programming model under a weighted city-block metric is:

$$\text{minimize sd} = \sum_{i \in M} \sum_{j \in N} (1/wp_{ij}) (x^+_{ij} + x^-_{ij}) + \sum_{i \in M} \sum_{j \in M} (1/ws_{ij}) (y^+_{ij} + y^-_{ij}) ,$$

subject to,

$$SP_{**} + X^+_{**} - X^-_{**} = \underline{SP}_{**} ,$$

$$SS_{**} + Y^+_{**} - Y^-_{**} = \underline{SS}_{**} ,$$

$$SFS^*_* + SS_{.*} = SP_{*.} + SS_{*.} ,$$

$$(SP_{**}, SS_{**}, X^+_{**}, X^-_{**}, Y^+_{**}, Y^-_{**}) \geqslant 0 .$$

The goal programming model under the weighted Euclidean metric is,

$$\text{minimize sd} = \sum_{i \in M} \sum_{j \in N} (1/wp_{ij}) (\underline{sp}_{ij})^2 + \sum_{i \in M} \sum_{j \in M} (1/ws_{ij}) (ss_{ij} - \underline{ss}_{ij})^2 ,$$

subject to,

$$SFS^*_* + SS_{.*} = SP_{*.} + SS_{*.} ,$$

$$(SP_{**}, SS_{**}) \geqslant 0 .$$

The solution of the weighted Euclidean metric model is,

$$SP^*_{**} = \underline{SP}_{**} - \text{Diag}(C_*) \, WP_{**} ,$$

$$SS_{**} = \underline{SS}_{**} + WS_{**} . \text{Diag}(C_*) - \text{Diag}(C_*) \, . \, WS_{**} ,$$

where,

$$C_* = (WS_{**} + WS^t_{**} - \text{Diag}(WS_{*.} + WS_{.*} + WP_{*.}))^{-1} (SFS^*_* + \underline{SS}_{.*} - \underline{SS}_{*.} - \underline{SP}_{*.})$$

The full costing under the weighted Euclidean metric model is,

$$FP^{*}_{*} = SFP^{*}_{*} + SP^{*}_{\cdot *} ,$$
$$FS^{*}_{*} = SFS^{*}_{*} + SS^{*}_{\cdot *} .$$

Management's Choice of Priority Weights

It was suggested above that management should fulfill its directing and motivating roles through the judicious choice of the weights WP_{*} , WS_{*} , WP_{**} , and WS_{**} . Thus, in its choice of these weights, management should be guided by considerations of equity and objectivity.

Considerations of equity demand that equals be treated as equals, and unequals be treated as unequals. Based on this principle, then, it is reasonable to assume that if the *available data* corroborate one goal more than another one, the priority weight of the more corroborated goal be larger than that of the less corroborated one.

The goals involved and their available data are:

Goals	*Available Data*
$UP_{*} \leqslant \underline{UP}_{*}$	UP_{*t} for all t in K
$US_{*} \leqslant \underline{US}_{*}$	US_{*t} for all t in K
$SP_{**} \leqslant \underline{SP}_{**}$	SP_{**t} for all t in K
$SS_{**} \leqslant \underline{SS}_{**}$	SS_{**t} for all t in K

Corroboration accorded by the available data to the involved goals can be measured in terms of the deviations of the goals from the data. There are, however, many ways of measuring such deviations, and the simplest such measure, the averaged absolute deviations, can be used to make the following simple assignment for the weights:

$$WP_{*} = (1/k) \sum_{t \in K} \left| UP_{*t} - \underline{UP}_{*t} \right| ,$$
$$= \overline{UP}_{*} - \underline{UP}_{*} ,$$

where,

$$\overline{UP}_{*} = (1/k) \sum_{t \in K} UP_{*t} .$$

Similarly,

$$WS_{*} = \overline{US}_{*} - \underline{US}_{*} ,$$
$$WP_{**} = \overline{SP}_{**} - \underline{SP}_{**} ,$$
$$WS_{**} = \overline{SS}_{**} - \underline{SS}_{**}.$$

We note that this particular assignment of weights indicates that a goal will be satisfied only if there are no differences among the allocations of the bases with respect to that goal. The above assignment of weights is used in the illustrative numerical example of the next section.

Illustrative Numerical Example

To illustrate the above procedures, a hypothetical organization is assumed. This organization is made up of three production departments and two service ones. There are four bases of allocations that are available to the organization.

This section contains the given data and the overall picture of the overhead allocations made by each of the four available bases and the Euclidean goal programming models The numerical details of these allocations are presented in the Appendix.

The unstructured overhead of the organization, u, is £250,000. The direct costs of the three production and two service departments, DP_*, and DS_*, and the four bases' data with respect to the production and service departments, B, and A, arranged in the format of Fig. 1, are shown in Table 1.

Fig. 1.: Format of given data

Labels	*Departments*		*sums*
	Production *N*	*Service* *M*	
Direct costs	DP_*	DS_*	DP. + DS.
Bases K	B	A	$B_{*.} + A_{*.}$

TABLE 1

The given data.

Labels		*Departments*					*sums*
		Production			*Service*		
		1	*2*	*3*	*1*	*2*	
Direct costs		65,000	40,000	45,000	30,000	10,000	190,000
Bases	1	3,500	3,000	5,300	2,500	4,000	18,300
	2	2,200	970	1,500	650	1,000	6,320
	3	350	700	1,000	150	75	2,275
	4	500	800	1,500	670	470	3,920

The overall picture of overhead allocation produced by each one of the four bases of allocation and a Euclidean goal programming model, arranged in the format of Fig. 2, is obtained from Tables 7, 10, 9, 12, and 15 of the Appendix, and are shown in Tables 2, 3, 4, 5, and 6.

Fig. 2: Format of overhead allocation of base t

Labels	*Departments*		*sums*
	Production N	*Service* M	
Direct cost	DP_{*}	DS_{*}	$DP. + DS.$
Unstructured overhead	UP_{*t}	US_{*t}	$UP._{t} + US._{t}$
Semi-full cost	SFP_{*t}	SFS_{*t}	$SFP._{t} + SFS._{t}$
Structured overhead	SP_{**t}	SS_{**t}	$SP_{*.t} + SS_{*.t}$
Full cost	FP_{*t}	FS_{*t}	$FP_{.t} + FS_{.t}$

TABLE 2
Overhead allocations of base 1.

Labels	*Departments*					*sums*
	Production			*Service*		
	1	2	3	*1*	2	
Direct cost	65,000	40,000	45,000	30,000	10,000	190,000
Unstructured ovrhd	47,814	40,984	72,404	34,153	54,645	250,000
Semi-full cost	112,814	80,984	117,404	64,153	64,645	440,000
Structured overhead 1	17,363	14,995	26,833	00.000	19,730	78,921
Structured overhead 2	20,672	17,719	31,219	14,766	00.000	84,376
Full cost	150,848	113,698	175,456	78,921	84,375	603,298

TABLE 3

Overhead allocations of base 2.

Labels	Departments					sums
	Production			Service		
	1	2	3	1	2	
Direct cost	65,000	40,000	45,000	30,000	10,000	190,000
Unstructured ovrhd	87,026	38,370	59,335	25,712	39,557	250,000
Semi-full cost	152,026	78,370	104,335	55,712	49,557	440,000
Structured 1	24,487	10,792	16,725	00.000	11,108	63,112
overhead 2	25,115	11,041	17,108	7,401	00.000	60,665
Full cost	201,629	100,203	138,167	63,112	60,665	563,776

TABLE 4

Overhead allocations of base 3.

Labels	Departments					sums
	Production			Service		
	1	2	3	1	2	
Direct cost	65,000	40,000	45,000	30,000	10,000	190,000
Unstructured ovrhd	38,462	76,923	109,890	16,483	8,242	250,000
Semi-full cost	103,462	116,923	154,890	46,483	18,242	440,000
Structured 1	7,893	15,739	22,532	00.000	1,674	47,838
overhead 2	3,167	6,334	9,062	1,354	00.000	19,917
Full cost	114,522	138,996	186,484	47,839	19,917	507,758

TABLE 5
Overhead allocations of base 4.

Labels	Departments					sums
	Production			*Service*		
	1	*2*	*3*	*1*	*2*	
Direct cost	65,000	40,000	45,000	30,000	10,000	190,000
Unstructured ovrhd	31,888	51,020	95,663	42,730	28,699	250,000
Semi-full cost	96,888	91,020	140,663	72,730	38,699	440,000
Structured 1	12,689	20,270	38,067	00.000	11,371	82,397
overhead 2	7,210	11,567	21,631	9,664	00.000	50,072
Full cost	116,787	122,856	200,362	82,397	50,072	572,474

TABLE 6
Overhead allocations of a Euclidean goal programming model.

Labels	Departments					sums
	Production			*Service*		
	1	*2*	*3*	*1*	*2*	
Direct cost	65,000	40,000	45,000	30,000	10,000	190,000
Unstructured ovrhd	51,297	51,824	84,323	29,770	32,786	250,000
Semi-full cost	116,297	91,824	129,323	59,770	42,786	440,000
Structured 1	15,964	15,664	26,469	00.000	2,449	60,549
overhead 2	13,636	11,467	19,357	775	00.000	45,235
Full cost	145,897	118,955	175,149	60,545	45,235	545,781

Regarding the overhead allocation tables above we note that the row of semi-full costs in Table 6 is equal to the simple average of the semi-full cost rows in Tables 2, 3, 5, and 4. This result could have been predicted in advance given the simplicity of the solution of the Euclidean goal programming model for semi-full costing and the special nature of the preemptive priority weights used. The structured overhead rows in Table 6, however, are not equal to the simple average of the corresponding rows in Tables 2, 3, 4, and 5. The deviation of the structured overhead produced by the Euclidean goal programming model from the bases' averages is especially pronounced in the interaction among service departments. For example, $775 << (14{,}766 + 7{,}401 + 1{,}354 + 9{,}664)/4$.

Summary and Conclusion

Overhead is an incurred cost which is matched against cost objects via an intervening base (of cost reallocation). Usually, there are several suitable alternative bases, no one of them is more logically or empirically defensible than the others, and each one of them is likely to produce cost allocation that is materially different from the others. The problem of overhead allocation is to come up with a cost allocation procedure which is objective, applies uniformly to all cost centers, is logically defensible, pays attention to the fact that no manager likes costs that are not under his control to be allocated to him, and make use of the available data. This problem is solved in this paper via goal programming models. The procedures involved are illustrated through a numerical example whose numerical details are presented in the Appendix.

The goal programming setting is the most natural one for the problem of overhead allocation. The solution obtained is flexible, it does not abrogate the managerial prerogative of exercising discretion, and it accentuates the directing and motivating roles of management [1] through the judicious choice of the preemptive priority weights of goal programming models.

APPENDIX

This appendix contains most of the intermediate numerical results that are required for the production of Tables 2, 3, 4, 5, and 6 of the illustrative numerical example. The notations and symbols used here are the ones introduced in the body of the paper. The formulas required for the calculations are reproduced here for the convenience of the reader.

Semi-full Costing Under the Bases

The unstructured overhead, u = S250,000, allocated to the production departments UP_{**}, and the service department US_{**} through the bases, arranged in the format of Fig. 3, is shown in Table 7, where,

$$UP_{t*} = u.\, B_{t*}/(A_{t.} + B_{t.}) \quad \text{, for all t in K ,}$$

$$US_{t*} = u.\, A_{t*}/(A_{t.} + B_{t.}) \quad \text{, for all t in K ,}$$

Fig. 3 : Format of unstructured overhead

labels	*N*	*M*	*sums*
K	UP_{**}	US_{**}	$UP_{*.} + US_{*.}$

TABLE 7
Unstructured overhead allocations.

Labels	*1*	*2*	*3*	*1*	*2*	*sums*
1	47,814	40,984	72,404	34,153	54,645	250,000
2	87,026	38,370	59,335	25,712	39,557	250,000
3	38,462	76,923	109,890	16,483	8,242	250,000
4	31,888	51,020	95,663	42,730	28,699	250,000

The semi-full costs are now easily obtained by adding the row of direct costs in Table 1 to each row in Table 7.

Full Costing Under the Bases

The full costs of the service and production departments according to base t are,

$$FS_{t*} = SFS_{t*} \cdot (I - G_{**t})^{-1} \ ,$$

$$FP_{t*} = SFP_{t*} + FS_{t*} \cdot H_{**t} \ , \text{for all t in K} \ ,$$

where,

$$H_{i*t} = B_{t*}/(A_{t.} + B_{t.} - a_{ti}) \ ,$$

$$G_{i*t} = A_{t*}/(A_{t.} + B_{t.} - a_{ti}) \ , g_{iit} = 0 \ .$$

The preration distribution matrices H_{**t} and G_{**t} , t = 1, 2, 3, 4, arranged in the format of Fig. 4, are presented in Table 8.

Fig. 4 : Format of proration matrices

labels	*N*	*M*
1 M	H_{**1}	G_{**1}
2 M	H_{**2}	G_{**2}
3 M	H_{**3}	G_{**3}
4 M	H_{**4}	G_{**4}

TABLE 8
Proration matrices.

Labels		1	2	3	1	2
1	1	0.220	0.190	0.340	0.000	0.250
	2	0.245	0.210	0.370	0.175	0.000
2	1	0.388	0.171	0.256	0.000	0.176
	2	0.414	0.182	0.282	0.122	0.000
3	1	0.165	0.329	0.471	0.000	0.035
	2	0.159	0.318	0.455	0.068	0.000
4	1	0.154	0.246	0.462	0.000	0.138
	2	0.144	0.231	0.432	0.193	0.000

The full costs allocated to the production and service departments through the bases, FP_{**} and FS_{**}, arranged in the format of Fig. 5, are shown in Table 9.

Fig. 5 : Bases' full costs allocations, format.

labels	*N*	*M*	*sums*
K	FP_{**}	FS_{**}	$FP_{*\cdot} + FS_{*\cdot}$

TABLE 9
Bases' full costs allocations.

Labels	1	2	3	1	2	sums
1	150,848	113,698	175,456	78,921	84,375	603,298
2	201,629	100,203	138,167	63,112	60,665	563,776
3	114,522	138,996	186,484	47,839	19,917	507,758
4	116,787	122,856	200,362	82,397	50,072	572,474

The structured overhead allocated to the production and service departments through base t is,

$$SP_{**t} = \text{Diag}\,(FS_{t*}) \,.\, H_{**t}\,, \text{ for all t in K },$$

$$SS_{**t} = \text{Diag}\,(FS_{t*}) \,.\, G_{**t}\,, \text{ for all t in K }.$$

These structured overhead allocation matrices SP_{**t} and SS_{**t} for t = 1, 2, 3, 4, arranged in the format of Fig. 6, are presented in Table 10.

Fig. 6: Structured overhead allocation matrices, format.

labels	*N*	*M*	*sums*
1 M	SP_{**1}	SS_{**1}	$SP_{*.1} + SS_{*.1}$
2 M	SP_{**2}	SS_{**2}	$SP_{*.2} + SS_{*.2}$
3 M	SP_{**3}	SS_{**3}	$SP_{*.3} + SS_{*.3}$
4 M	SP_{**4}	SS_{**4}	$SP_{*.4} + SS_{*.4}$

TABLE 10
Structured overhead allocation matrices.

Labels		*1*	*2*	*3*	*1*	*2*	*sums*
1	1	17,363	14,995	26,833	0.000	19,730	78,921
	2	20,672	17,719	31,219	14,766	0.000	84,376
2	1	24,487	10,792	16,725	0.000	11,108	63,112
	2	25,115	11,041	17,108	7,401	0.000	60,665
3	1	7,893	15,739	22,532	0.000	1,674	47,838
	2	3,167	6,334	9,062	1,354	0.000	19,917
4	1	12,689	20,270	38,067	0.000	11,371	82,397
	2	7,210	11,567	21,631	9,664	0.000	50,072

Goal Programming Semi-full Costing

The semi-full costs of the production and service departments are,

$$SFP^*_* = DP_* + UP^*_* ,$$

$$SFS^*_* = DS_* + US^*_* ,$$

where,

$$UP^*_* = \underline{UP}_* + (su/w)\, WP_* ,$$

$$US_* = \underline{US}_* + (eu/w)\, WS_* ,$$

$$eu = u - (\underline{UP}_. + \underline{US}_.) ,$$

$$w = WP_. + WS_. ,$$

$$WP_* = \overline{UP}_* - \underline{UP}_* ,$$

$$WS_* = \overline{US}_* - \underline{US}_* .$$

The goals boundaries and weights, $\underline{UP}_*$, $\underline{US}_*$, and WP_* , WS_* arranged in the format of Fig. 7, are displayed in Table 11.

Fig. 7 : Goals boundaries and weights, format

labels	*N*	*M*	*sums*
gls. bndrs weights	$\underline{UP}_*$ WP_*	$\underline{US}_*$ WS_*	$\underline{UP} + \underline{US}$ $WP + WS$

TABLE 11

Goals boundaries and weights.

Labels	*1*	*2*	*3*	*1*	*2*	*sums*
gls. bndrs weights	31,888 19,409	38,370 13,454	59,335 24,988	16,483 13,287	8,242 24,544	154,318 95,682

The goals boundaries in Table 11 are in the minima of the columns of table
From Table 11 we find that,

eu = 250,000 - 154,318 ,

w = 95,682

Finally, the semi-full costs for the three production and two service departments, SFP_*^* and SFS_*^* are as shown in Table 12.

TABLE 12

Semi-full costs.

Labels	*1*	*2*	*3*	*1*	*2*	*sums*
semi-full cost	116,297	91,824	129,323	59,770	42,786	440,000

Goal Programming Full Costs

Full costs of the production and service departments are,

$$FP_*^* = SFP_*^* + SP_{.*}^* ,$$

$$FS_*^* = SFS_*^* + SS_{.*}^* ,$$

where,

$$SP_{**}^* = \underline{SP}_{**} - \text{Diag}(C_*)\, WP_{**} ,$$

$$SS_{**}^* = \underline{SS}_{**} + WS_{**}\, \text{Diag}(C_*) - \text{Diag}(C_*)\, WS_{**} ,$$

$$WP_{**} = \overline{SP}_{**} - \underline{SP}_{**} ,$$

$$WS_{**} = \overline{SS}_{**} - \underline{SS}_{**} ,$$

$$C_* = (WS_{**} + WS^t_{**} - \text{Diag}\,(WS_{*.} + WS_{.*} + WP_{*.}))^{-1} \,.\, (SFS_* + \underline{SS}_{.*} - \underline{SS}_{*.} - \underline{SP}_{*.})$$

From Table 10, the goals boundaries and weights, $\underline{SP}_{**}$ $\underline{SS}_{**}$, and WP_{**}, WS_{**}, arranged in the format of Fig, 8, are obtained and are presented in Table 13.

Fig. 8 : Goals boundaries and weights, format

labels	*N*	*M*	*sums*
K	$\underline{SP}_{**}$	$\underline{SS}_{**}$	$\underline{SP}_* + \underline{SS}_{*.}$
K	WP_{**}	WS_{**}	$WP_{*.} + WS_{*.}$

TABLE 13

Goals boundaries and weights.

Labels	*1*	*2*	*3*	*1*	*2*	*sums*
1	7,893	10,792	16,725	0.000	1,674	37,084
2	3,167	6,334	9,062	11,354	0.000	19,917
1	7,715	4,657	9,314	0.000	9,297	30,983
2	10,874	5,331	10,693	6,942	0.000	33,839

The vectors entering in the definition of C_* and C_* itself, are shown in Table 14.

TABLE 14

C_* and its defining vectors.

$WS_{*.}$	$WS_{.*}$	$WP_{*.}$	$\underline{SS}_{.*}$	$\underline{SS}_{*.}$	$\underline{SP}_{*.}$	C_*
9,297	6,942	21,686	1,354	1,674	35,410	-1.04613
6,942	9,297	26,898	1,674	1,354	18,563	-0.96277

The structured overhead matrices and full costs of the production and service departments, SP^*_{**}, SS^*_{**}, and FP^*_*, FS^*_*, arranged in the format of Fig. 9, are shown in Table 15.

Fig. 9: Structured overhead and full costs allocations, format

Labels	*N*	*M*	*sums*
M	SP^{*}_{**}	SS^{*}_{**}	$SP^{*}_{*\cdot} + SS^{*}_{*\cdot}$
Full cost	FP^{*}_{*}	FS^{*}_{*}	$FP^{*}_{\cdot} + FS^{*}_{\cdot}$

TABLE 15
Structured overhead and full costs allocations.

Labels	*1*	*2*	*3*	*1*	*2*	*sums*
1	15,964	15,664	26,469	0.000	2,449	60,546
2	13,636	11,467	19,357	775	0.000	45,235
full cost	145,897	118,955	175,149	60,545	45,235	545,781

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