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Gravitational Instability of a Radiating Gas Cloud with Variable streams

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استقرار انسياب سحابة غازية مشعة حرارياً ذات جاذبية ذاتية

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تمت دراسة استقرار انسياب سحابة غازية مشعة حرارياً تحت تأثير قوة الجاذبية الذاتية وقوى تدرج ضغط حركة المائع وضغط الإشعاع.

وجد أن قوة الإشعاع لها تأثير استقراري قوى بينما قوة الجذب الذاتي لها تأثير غير استقراري. في غياب الإشعاع وجدنا أن قوة القصور لها تأثير استقراري سواء كان الانسياب منتظم أم غير منتظم. أما في وجود تأثير الإشعاع فإن : (١) تأثير الانسياب المنتظم فمنعدم تماماً وكذلك تأثير الإشعاع على استقرار السحابة أقوى من تأثير الجذب الذاتي. (٢) الانسياب غير المنتظم أي المتغير له تأثير غير استقراري وبالتالي فإن السحابة المشعة تكون غير مستقرة. هذه النتائج لها دور أساسي في تكوين الضباب العاتم في طبقات الجو العليا وكذلك تحطم بعض من مادة الأجسام الفضائية.

Key Words: Radiation, Self-gravitation, Stability, Gas Cloud.

ABSTRACT

The self-gravitating instability of a radiating gas cloud possessing variable streams is investigated. The self-gravitating force is destabilizing according to restrictions. the radiation has a strong stabilizing influence for all short and long perturbation wavelengths.

In the presence of radiation, the uniform streaming has no influence at all on the instability of the model and the radiation overcomes the self-gravitating instability of gas cloud. Such study has correlation with a stellar cluster formation from fragmentation of interstellar matter.

INTRODUCTION

The oscillation of a homogeneous fluid under the effect of its pressure gradient force only was elaborated by Rayleigh (1945). Chandrasekhar and Fermi (1953) studied the instability of fluid medium under the influence of the self-gravitating force in addition to the fluid pressure gradient force.

The Nobel prize winner (1986), Chandrasekhar (1981) made several classical modifications. Moreover, he reviewed most of the published works in this domain, for related topics, see also Chandrasekhar (1957). The gravitational instability of a streaming fluid medium was discussed by Sengar (1981). Radwan and Elazab (1988) examined the effect of Lorentz force on the gravitational instability results obtained by Sengar (1981). Radwan (1996) investigated the capillary-instability of a liquid jet under the influence of the self-gravitating force.

It is well known that if a fluid medium is unstable under any kind of condition, it will be due to that the acting external forces are predominant over the fluid pressure gradient force. However, Shih-i, Pai (1966) proved, for Earth's atmosphere, that the pressure effect due to radiation cannot be neglected in comparison with that of gas pressure even at temperature about $10^4 K$, see also Vranjes and Cadez.

In this present work the effect of radiation on the self-gravitating instability of a gas cloud with streams of variables velocity distribution function of coordinate is discussed.

FORMULATION OF THE PROBLEM

Consider an unbounded gas cloud under the combined effect of self-gravitating variable inertia, gas pressure gradient and radiation forces. The gas medium is assumed to be homogeneous, inviscid and compressible. We are interested here to identify the inertia and radiation forces behaviors on the self-gravitational instability of the model.

The exchange instability principle occurs as the temperature is decreasing function of coordinate. So we may assume that the temperature of the gas medium in the unperturbed state is constant to ovoid the exchange instability principle. For the problem at hand, the governing equations are as follows.

The vector equation of motion:

$$r(\frac{\partial u}{\partial t} = u.Du) = -DP - rD\phi \qquad (1)$$

Equation of continuity:

$$\frac{\partial \rho}{\partial t} + \Delta(\rho u) = 0 \qquad (2)$$

Newtonian self-gravitating equation:

$$\Delta^2 = \phi = 4\,p\,G\,\rho \qquad (3)$$

Radiating equation of state:

$$\rho(\frac{\partial T}{\partial t} - (T - 11)\,T\frac{\partial \rho}{\partial t} = 0 \qquad (4)$$

Here ρ is the gas cloud mass density, $\underline{u}$ the velocity vector, T the temperature at time t, ϕ the self-graviting potential, G the gravitational constant and $P\ (= p_g + p_r)$ the total pressure which is the sum of the gas kinetic pressure p_g and pressure p_r due to radiation:

$$\rho_g = R\ \rho\ T \qquad (5)$$

$$p_r = \frac{1}{3} a_R T^4 \qquad (6)$$

where a_R is the Boltzmann constant and R is the general constant of gas. Here equation (3) takes the form $\Delta^2 \phi = 0$ for empty space. Equation (4) is the equation of state that indicates adiabatic changes in an inclosure containing matter and radiation. For more details concerning equation (4) we may refer to Chandrasekhar (1957), where we find:

$$(T - 1)(4 - 3b) = T_1 - b \qquad (7)$$

with

$$T_1 - b = [b + 12(\gamma - 1)(1 - b)] = (\gamma - 1)\ (4 - 3b)^2 \qquad (8)$$

$$\frac{b = p_g}{P} \qquad (9)$$

where γ is the ratio of the specific heats. In the absence of radiation

$$p_r = 0 \qquad (10)$$

Therefore,

$$b = 1 \qquad (11)$$

$$T = T_1 = \gamma \qquad (12)$$

However, if the radiation is taken into account such that its pressure p_r is much greater than the gas pressure p_g (i.e. $pr >> pg$), we have

$$T = T_1 = \frac{4}{3} \qquad (13)$$

Let the gas cloud medium in the unperturbed state (i) possess streams moving in the x-direction with speeds $U_{(z)}$ varying along the z-direction of the Cartesian coordinates (x, y, z) (ii) be with constant T and constant temperature T_0 and (iii) be optically thick and the black-body radiation condition be assumed.

For small departure from the streaming unperturbed state, every physical quantity $H(x,y,z;t)$ may be expressed, up to second order, in the form

$$H = H_0 + H_1 + \dots\dots\dots \quad (14)$$

where H stands for any one of r, u, p_r, p_g, P, T and ϕ. Quantities with subscript "0" indicate their values in the unperturbed state while those with subscript "1" indicate their values in the perturbed state.

By inserting expansions (14) into the basic equations (1)-(6), the linearized perturbation equations relevant to the present problem are given by

$$\rho_0\left(\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x}\right) + \rho_0 w DU + RT_0\frac{\partial \rho_1}{\partial x} + R\rho_0(1+4Q)\frac{\partial T_0}{\partial x} + \rho_0\frac{\partial \phi_1}{\partial x} = 0 \quad (15)$$

$$\rho_0\left(\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x}\right) + RT_0\frac{\partial \rho_1}{\partial y} + R\rho_0(1+4Q)\frac{\partial T_1}{\partial y} + \rho_0\frac{\partial \phi_1}{\partial y} = 0 \quad (16)$$

$$\rho_0\left(\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x}\right) + RT_0\frac{\partial \rho_1}{\partial z} + R\rho_0(1+4Q)\frac{\partial T_1}{\partial z} + \rho_0\frac{\partial \phi_1}{\partial z} = 0 \quad (17)$$

$$\left(\frac{\partial \rho_1}{\partial t} + U\frac{\partial \rho_1}{\partial x}\right) + \rho_0\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}) = 0 \quad (18)$$

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2} = 4\pi G\rho_1 \quad (19)$$

$$\rho_0\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) T_1 = (\Gamma - 1)\, T_0\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\rho_1 \quad (20)$$

Here $D = d/dz$, (u,v,w) are the components of the velocity in perturbed state while $\rho 1$, $T1$ and $\phi 1$ are, respectively, perturbations in density, temperature and self gravitating potential of the gas cloud. The factor is $Q = (\frac{T_0^3 a_R}{3\rho_0 R})$ the ratio of the unperturbed radiation and gas pressure, viz,

$$Q = \frac{p_{ro}}{p_{go}} \quad (21)$$

with

$$p_{ro} = \rho_0 R\, T_0 \quad (22)$$

$$p_{go} = \frac{1}{3}a_R T_0^4 \quad (23)$$

EIGENVALUE RELATION

As it is usual for the stability problems, and based on the linearized theory of perturbation technique, we assume that the time space dependence of the wave propagation in the perturbation state is in the form

$$\rho_1, u, p_1, T_1, \phi_1 \sim \exp\left[\,i\,(k_x x + K_y y + k_z z) - st\,\right] \quad (24)$$

where kx, ky and kz are (any real values) the wave numbers in the three space directin of the Cartesian coordinates (x, y, z) while s is the oscillation frequency. By utilizing the space-time dependence (24) for the relevant perturbation equations (15) - (20), we get

$$\rho_0 Su + i\rho_0 w DU + RT_0 k_x \rho_1 + (1+4Q)\, k_x T_1 + \rho_0 k_x \phi_1 = 0 \quad (25)$$

$$\rho_0 Su + RT_0 k_y \rho_1 + R\rho_1 (1+4Q)\, k_y T_1 + \rho_0 k_y \phi_1 = 0 \quad (26)$$

$$S\rho_1 + \rho_0 (k_x u + k_y v + k_z w) = 0 \quad (27)$$

$$k^2\phi_1 + 4\pi G\rho_1 = 0 \quad (28)$$

$$\rho_0 T_1 - (\Gamma - 1)\, T_0 \rho_1 = 0 \quad (29)$$

$$\rho_0 T_1 - (\Gamma - 1)\, T_0 \rho_1 = 0 \quad (30)$$

$$S = U(z)\, k_x - s \quad (31)$$

where

$$k = (k_x^2 + k_y^2 + k_z^2)^{1/2} \qquad (32)$$

is the net wave number of the wave propagation. By simplifying the system of equations (25)-(30), we obtain the following set of linear homogeneous equations

$$\begin{bmatrix} S & 0 & -iDU(z) & Ak_x & k_x \\ 0 & S & 0 & Ak_y & k_y \\ 0 & 0 & S & Ak_z & k_z \\ k_x r_0 & k_y r_0 & k_z r_0 & S & 0 \\ 0 & 0 & 0 & 4\pi G & k^2 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ r_1 \\ \phi_1 \end{bmatrix} = 0 \qquad (33)$$

where $$A = \left(\frac{RT}{r_0}\right)[1 + (\Gamma - 1)(1 + 4Q)], \qquad DU(z) = \frac{dU(z)}{dz} \qquad (34)$$

It is worthwhile to mention here that the set of linear homogeneous equations (33) in the absence of radiation, coincide with those given by Radwan (1988 eqs. (29) as we neglect the magnetic field influence there) and with those given by Sengar (1981 eqs. (8) there).

Now, for non-trivial solution setting the determinant of the equations (33) equal to zero, following eigenvalue relation is obtained

$$k^2 S^3 + k^2 \rho_0 S (4\pi G - Ak^2) - i\rho_0 k_x k_z (4\pi G - Ak^2)\, DU = 0 \qquad (35)$$

In order to transform this complex equation to a real one, let us write

$$S = is = i\sigma(z) k_x - is = s + i\sigma(z) k_x \qquad (36)$$

with

$$s = -i\sigma \qquad (37)$$

Then equation (35) becomes

$$k^2 \sigma^3 + k^2 \rho_0 (Ak^2 - 4\pi G)\sigma - \rho_0 k_x k_z (Ak^2 - 4\pi G)\, DU = 0 \qquad (38)$$

DISCUSSIONS

Equation (38) is the desired eigenvalue relation of radiating self-gravitating gas cloud possessing variable streams as function of the coordinate z. It relates the oscillation frequency or rather the growth rate with the different parameters of the problem. The relation (38) is a general eigenvalue relation from which we could deduce other published works under appropriate simplification.

If the homogeneous and non streaming gas medium is perturbed in the absence of self-gravitating and radiating forces, relation (38) becomes

$$k^2 \sigma^3 + k^2 \rho_0 (Ak^2 - 4\pi G)\sigma - \rho_0 k_x k_z (Ak^2 - 4\pi G)\, DU = 0 \qquad (39)$$

Relation (39) shows that the gas medium is stable for all short and long wavelengths, and is true in the real situation.

If the gas cloud is not radiating and non-streaming in the unperturbed state, i.e. \$A=0\$, \$U(z)=0\$, the dispersion relation (38) reduces to

$$s^2 = k^2 C^2 - 4\pi G \rho_0 \qquad (40)$$

This relation coincides with that derived by Chandrasekhar (1981). The discussions of equation (40) reveal that the gas medium becomes unstable for all perturbations of wave number less than a critical value

$$k_j^* = (4\pi G\rho_0 / C^2)^{1/2} \tag{41}$$

called Jean's wave number, with

$$C = (g\rho_{go} / \rho_0)^{1/2}, \qquad \Gamma = g, \qquad \rho_{go} = \rho_0 R T_0 \tag{42}$$

In the absence of radiation force, while the gas medium streams uniformly (with velocity $\underline{u}0$ = (U,0,0)$ i.e. (DU=0) and acted upon by the combined effect of the self-gravitating and gas pressure gradient forces, relation (38) yields

$$k_x U - S)^2 = k^2 C^2 - 4\pi G\rho_0 \tag{43}$$

This relation shows that the streaming has strong destabilizing effect. This means that the inertia force increases the self-gravitating unstable domains and does enlarge Jean's critical wave number.

If the non-streaming gas cloud medium is acted upon by the combined effect of the self-gravitating, gas pressure gradient and radiating pressure gradient forces, relation (38) reduces to

$$s^2 - \rho_0 A k^2 + 4\pi G\rho_0 = 0 \tag{44}$$

Based on the time dependence given by equation (24), the radiating self-gravitating gas cloud is stable as

$$s^2 \geq 0 \tag{45}$$

In view of equation (44), nothing that $k_r - 2\pi$, $/ \lambda_4, C^2 - \gamma R T_0 = g\rho_{go} / \rho_0, \lambda_j^2 - \pi C_s^2 (\rho_0 G)$

restriction (45) reveals that the radiation wave number k_r satisfies the condition

$$k_r \geq (\frac{1}{\gamma}[1 + (\Gamma - 1)(1 + 4Q)])^{1/2} k_j \tag{46}$$

This condition leads to a very important result in the theory of stability that the radiation has strong stabilizing effect and could suppress completely the self-gravitating instability of the gas cloud.

As the radiating self-gravitating gas cloud streams uniformly with velocity $\underline{u}_0 = (U,0,0)$ the general eigenvalue relation (38) becomes

$$s^2 - 2Uk_x s + (U^2 k_x^2 + 4\pi G\rho_0 - \rho_0 A k^2) = 0 \tag{47}$$

This quadratic equation (47) has imaginary roots iff

$$(-2Uk_x s)^2 - 4(U^2 k_x^2 + 4\pi G\rho_0 - \rho_0 A k^2) < 0 \tag{48}$$

from which we obtain

$$4\pi G\rho_0 > A\rho_0 k^2 \tag{49}$$

The restriction (49) for streaming gas cloud is exactly the same as that which could be obtained from (44) for non-streaming gas cloud. Therefore, we conclude that the inertia force due to uniformly streaming gas cloud has no influence, at all, on the instability of a self-gravitating gas cloud in the presence of radiation.

Now, let us return to the general eigenvalue relation (38), which gives the stability criteria for radiating self-gravitating gas cloud possessing variable streams as function of coordinate z in x-direction. By using Cardan's method (cf. Abramowitz and Stegum 1970) for solving algebraic cubic equation, it is found that equation (38) gives, at least, one positive real root. Therefore, σ is real positive showing that the medium is unstable. Consequently, we deduce that the variable stream has strong destabilizing influence, which overpowers the stabilizing effect of the radiation. If the streaming effect together with that of the self gravitation is equivalent to the radiating stabilizing effect, then the model will be marginally stable.

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