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A NOTE ON SEMI GENERALIZED CLOSED SETS

By

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المجموعات المغلقة شبه المعممة

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تم الحصول في هذا البحث على بعض خصائص وصفات المجموعات المغلقة شبه المعممة .

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ABSTRACT

In the present note authors obtain some characterizations and properties of semi generalized closed sets.

INTRODUCTION

Let S be a subset of a topological space X. The interior and the closure of S in X will be denoted by Int(s) and cl(s) respectively. A subset A of X is said to be semiopen [1] if there is an open set O such that $O \subset A \subset clO$. Every open set is semiopen but the converse may not be true. The complement of a semiopen set is called semiclosed [2]. A point x of space X is said to be semi limit point [2] of a subset A of X if every semiopen set containing x contains a point of a other than x. The set of all semi limits points of A is called semidrived set of A. It is denoted by 5d(A). The intersection of all semiclosed sets containing a set A is semiclosure [2] of A. It is denoted by scl(A). For any subset A of space X, $scl(A) = A \cup sd(A)$ [2]. A mapping $f:X \to Y$ is said to be irresolute [3] if the inverse image of every semiopen set in Y is semiopen in X. A mapping $f:X \to Y$ is presemiclosed [4] if the image of every semiclosed set of X is semiclosed in Y.

A subset F of X is said to be g-closed [5] if $cl(F) \subset O$ whenever $F \subset O$ and O is open. The complement of a g closed set is called g open [5]. Every closed (resp. open) set is g closed (resp. g open) but the converse may not be true.

SEMI GENERALIZED CLOSED SETS

Definition 1 [6]: A subset A of X is said to be semi generalized closed (written as s. g closed) if and only if $scl(A) \subset O$ whenever $A \subset O$ and O is semiopen.

Remark 1 [6]: Every semiclosed (resp. g closed) set is s g closed but the converse may not be true.

Theorem 1: In a topological space X the following conditions are equivalent:

(a) A is s.g closed.

(b) for each $x \in scl(A)$, $scl(\{x\}) \cap A \neq \emptyset$

(c) scl (A)-A contains no nonempty semiclosed subsets.

(d) $A = F - N$, where F is semiclosed and N contains no nonempty semiclosed subsets.

Proof

(a) $\Rightarrow$ (b) : Suppose $x \in scl(A)$ but $scl(\{x\}) \cap A = \emptyset$. Then $A \subset X - scl(\{x\})$ and so, $scl(A) \subset X - scl(\{x\})$, contradicting $x \in scl(A)$.

(b) $\Rightarrow$ (c) : Let $F \subset scl(A) - A$ with F is semiclosed. If there is an $x \in F$, then by (b), $\emptyset \neq scl(\{X\}) \cap A \subset scl(F) \cap A = F \cap A \subset (scl(A) - A) \cap A = \emptyset$, a contradiction. Hence $F = \emptyset$.

(c) $\Rightarrow$ (d) : Let $F = scl(A)$ and $N = scl(A) - A$, then $A = F - N$ and by (c), N contains no nonempty semiclosed subsets.

(d) $\Rightarrow$ (a) : Let $A = F - N$ and $A \subset O$ where F is semiclosed, O is semiopen and N contains no nonempty semiclosed subsets. Then $F \cap (X - O)$ is a semiclosed subset of N and thus empty. Hence $scl(A) \subset F \subset O$ and A is s. g closed.

Theorem 2: In a space X, either $\{x\}$ is semiclosed or $X - \{x\}$ is s. g closed for each $x \in X$.

Proof: If $\{x\}$ is not semiclosed then only semiopen superset of $X - \{x\}$ is X itself. Thus $scl(X - \{x\})$. Hence $X - \{x\}$ is s. g closed.

Definition 2 [7]: A space X is said to be semi T_1 if for each pair of distinct points x, y of X, there exist semiopen sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.

Theorem 3: In a semi T_1-space s. g closed sets are semiclosed.

Proof: Suppose A is s. g closed set in a semi T_1 space X. If x

$\in$scl (A) then by theorem 1(b), $\emptyset \neq$ scl {x} $\cap$ A = {x} $\cap$ A. Consequently x $\in$A, Hence A is semiclosed.

Lemma 1 [8]: Let (X,J) be a extremelly disconnected space and A and B be semiopen sets in X then A $\cap$ B is semiopen.

Lemma 2 : Let A be a subset of a extremely disconnected space X and suppose sd(A) $\subset$ O for O semiopen. Then sd(sd(A)) $\subset$ O.

Proof: Suppose x $\in$sd(sd(A)) but x $\notin$O. Then x $\notin$sd(A) and so, for some semiopen set U, x $\in$ U and A $\cap$ U $\subset$\{x\}. But x $\in$sd(sd(A)) implies y $\in$ sd (A) $\cap$ U $\cap$ (X-{x}) for some y. Now y $\in$ O $\cap$ U, O X U is semiopen (by lemma 1) and y $\in$ sd(A) and so, $\emptyset \neq$ A$\cap$ O $\cap$ U $\cap$ (X-{y}) $\subset$ A $\cap$ U $\subset$\{x\}. It follows that x $\in$ O a contradiction.

Theorem 4: In any extremely disconnected space, semi drived sets are s. g closed.

Proof: Let A be a subset of a extremely disconnected space x with sd(A) $\subset$ O where O is semiopen. Then Lemma 1 implies scl (sd(A)) = sd(A) $\cup$ sd(sd(A)) $\subset$O. Hence A is s. g closed.

Theorem 5: If A is a s. g closed set in X and f:X$\rightarrow$ Y is presemiclosed and irresolute then f(A) is s. g closed in Y.

Proof: Let f (A) $\subset$ O, where O is semiopen in Y. Then A$\subset f^{-1}$(O) and f^{-1}(O) is semiopen in X because f is irresolute. Therefore scl(A) $\subset f^{-1}$(O). Thus f (scl(A))$\subset$O and f (scl(A)) is semiclosed because f is presemiclosed.

Theorem 6: If f:X$\rightarrow$ Y is continuous and pre-semiclosed and B is s. g closed subset of Y, then f^{-1}(B) is s. g closed in X.

Proof: Let B is s. g closed subset of Y and f^{-1} (B) $\subseteq$ O, where O is semiopen in X. Then, we have to show that:

scl (f^{-1} (B) $\subseteq$ O or scl (f^{-1} (B)) $\cap$ (X-O) = $\emptyset$. Now, f [scl (f^{-1} (B)) $\cap$ (X-O)] $\subseteq$ scl(B)-B, by theorem 1 (b), f [scl (f^{-1} (B)) $\cap$ (X-O)] = $\emptyset$. Thus, scl (f^{-1} (B)) $\cap$ (X-O) = $\emptyset$, Hence, f^{-1} (B) is s. g closed in X.

Theorem 7: If (X.J) = X{$X_\propto$, $T_\propto$) : $\propto$ $\in\Delta$} and if $A_\propto$ is s. g closed in $X_\propto$ for each $\propto$ in Δ, then X{$A_\propto$: $\propto\in\Delta$} is s. g closed in X.

Proof: Let Q = scl (X{$A_\propto$: $\propto$ $\in\Delta$}) - X{$A_\propto$: $\propto$ $\in\Delta$}. Then by theorem 1 (b) it is sufficient to show that Q contains no nonempty semiclosed sets. Suppose on the contrary that scl(x) $\subseteq$ Q for some x $\in$X where x = X{x $\propto$: $\propto$ $\in\Delta$}.

It follows that $scl_\propto$ ($X_\propto$) $\subseteq$ $scl_\propto$ ($A_\propto$) for each $\propto$ in Δ and since scl $\propto$ ($A_\propto$) - $A_\propto$ contains no nonempty semiclosed set by theorem 1(b). We have $scl_\propto$ ($X_\propto$) $\cap$ $A_\propto$) $\neq \emptyset$ for each $\propto$in Δ. Choose $x'_\propto$ $scl_\propto$ ($X_\propto$) $\cap$ $A_\propto$ for each $\propto$ and let x'=X{$x'_\propto$: $\propto\in\Delta$}. Then x' $\in$scl (x') = X {scl $\propto$ ($x'_\propto$) : $\propto$ $\in\Delta$} $\subseteq$ X{scl $\propto$($x_\propto$) : $\propto$ $\in\Delta$} = scl (x) $\subseteq$ Q $\subseteq$ [complement of X {$A_\propto$: $\propto\in\Delta$}]. There re x' $\notin$ X {$A_\propto$: $\propto\in\Delta$}. This is contrary to the fact that $x'_\propto$ $\in A_\propto$ for each $\propto$. Therefore X {$A_\propto$: $\propto$ $\in\Delta$} is s. g closed in X.

Theorem 8: If A is s. g open in x and B is s. g open in Y then (A x B) is s. g. open in (X x Y).

Proof: Let F is semiclosed in X x Y and F $\subseteq$ A x B. The by theorem 6 of [6]. It is sufficient to show that F $\subseteq$ sint (AXB) Let (x, y) $\in$F, then scl (x, y) = scl (x) X scl (y) $\subseteq$ F $\subseteq$ (AXB) and it follows then that scl (x) $\subseteq$ sint (A) and scl (y) $\subseteq$ sint (B). Thus (x,y) $\in$scl (x) X scl (y) $\subseteq$ sint (A) X sint (B) = sint (AXB). by lemma 2 of [9]. Therefore F $\subseteq$ sint (AXB).

Hence (A X B) is s. g open in (X X Y).

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