THE VOLTAGE COLLAPSE PROBLEM BASED ON THE POWER SYSTEM LOADABILITY

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ABSTRACT

This paper addresses the problem of voltage instability at which the power system reaches its maximum admissible load. A voltage collapse proximity indicator (VCPI) is derived and its performance is investigated. The VCPI is the ratio of the system equivalent impedance to the load equivalent impedance. In this paper, an algorithm for network equivalent impedance is proposed; it employs the PV-PQ sensitivity and "referencing" techniques to determine an equivalent impedance of a multi-machine multi-node power system. The validity and effectiveness of this method is demonstrated with the application of different network configurations ranging from the standard IEEE 14-bus to the realistic 116-bus Qatar power system.

Keywords: Voltage instabillity, Voltage collapse, Loadability, Sensitivity, Equivalent impedance.

INTRODUCTION

Power system loadability is becoming increasingly important as the overall system demand increases. When the load is increased, the distribution voltage will decrease, and in the worst scenario, the voltage drops rapidly to a point beyond which the voltage is uncontrollable (voltage instability). As a result, voltage collapses leading to blackout. This is of serious concern to the electric power utilities, which strive to make every effort to develop new planning criteria, off-line and on-line security monitoring, and control tools that may help in avoiding blackouts. Voltage stability problems normally occur in heavily stressed systems, and voltage collapse may be initiated by a variety of causes. Broadly speaking, the most commonly reported voltage collapse incidents appear to be the inability of the system to meet the load demand.

Voltage instability spans a range in time from a fraction of a second to tens of minutes. The time frame of transient voltage instability is in seconds whereas the time frame of longer-term voltage instability varies from tens of minutes to hours. Load pick-up associated with a heavily loaded system is an example of the longer-term voltage instability. Several voltage collapse indicators were proposed in the literature.

One of the earliest works on voltage collapse is probably by Weedy [1]. In his investigation of voltage collapse, Weedy indicated that the induction motor load was the critical constituent of system loads, which was modelled by polynomial equations. Venikov *et al.* [2] suggested the use of Newton-Raphson load flow divergence to estimate the stability limit. Tamura *et al.* [3] investigated the relationship between voltage instability and multiple load flow solutions. In a pair of multiple solutions, one, with the higher voltage magnitude, is assumed to be stable, and the other is unstable. Kessel and Glavitsch [4] developed a voltage stability index (called the L indicator). The bus with the largest index is said to be the critical bus in the network. Based on this indicator, a method was developed by Tuan *et al.* [5], which determined a relationship between the L indicator variations and load power to be shed in emergency load shedding to avoid risks of voltage instability. For comparison purposes, an explanation of the L indicator is given in the appendix. Flatabo *et al.* [6] presented a method in which the MVAR distance to voltage collapse is used as a quantitative measure for determining the voltage stability condition of the power system.

Based on the optimal impedance theory described by Calvaer [7], the proximity to voltage collapse can be estimated when the equivalent impedance of the receiving-end is equal to the Thevenin's equivalent impedance; this was demonstrated by Chebbo *et al.* [8]. Chebbo proposed a voltage collapse proximity indicator based on the optimal impedance of a two-bus system generalised to an actual system. For an N-bus system, however, the maximum power transferred to a load is reached when the impedance of the load equals the Thevenin's equivalent impedance of the network.

This paper investigates the problem of the longer-term voltage instability at which the system reaches its maximum admissible load. A voltage collapse proximity indicator (VCPI) is derived, which is an extension of that described by [8]. The VCPI is defined as the ratio of the system equivalent impedance to the equivalent load impedance. In this paper, an algorithm for network equivalent impedance is proposed; it employs the PV-PQ sensitivity and "referencing" techniques to determine an equivalent impedance of a multi-machine multi-node power system. Looking from a PQ node, the equivalent impedance of the power system network is constant regardless of the load level at the concerned PQ

node. An accurate prediction of the critical state from any operating point is also a feature of the proposed method and the repeated load-flow solution is avoided. The main extension of the proposed method in this paper to the Chebbo's method can be summarised in the following:

♦ In obtaining the power system equivalent impedance, the proposed algorithm introduces a "referencing" technique to maintain the accuracy of the equivalent impedance and source voltage which leads to the accuracy of system's power and voltage margins. This is not the case in Chebbo's method; besides it is time consuming, Chebbo's method is liable for divergence, and the equivalent voltage cannot be obtained for most cases.

♦ In their representation of system equivalency, the equivalent source voltage (no-load voltage) in Chebbo *et al.* (1992) varies with the load behaviour, this cannot be a valid model for the simulation of buses that experience large load variations, which is the theme of the loadability limit and voltage collapse.

♦ An accurate prediction of the critical state from any operating point is one of the features of the proposed method and the repeated load-flow solution is avoided.

In the following section, the procedure of the proposed method is explained, then a numerical example to determine the voltage collapse proximity indicator is given.

METHODOLOGY

Looking from any PQ load node in the network, the multi-machine multi-node power system network is reduced to an equivalent impedance connected to a voltage source from one end and to the load port from the other end. A voltage collapse proximity indicator is derived which is the ratio of the equivalent impedance of the network to the impedance of the load. The procedure to determine the voltage collapse proximity indicator (VCPI) can be divided into four sections, (i) calculation of the load admittance, (ii) identifying the dependency of the load power of the PQ on the PV buses, (iii) deriving the equivalent impedance of the network, and finally, the VCPI is the ratio of the system equivalent impedance to load equivalent impedance.

Calculation of the Equivalent Admittance/Impedance of the Load at a Node

To allow for representation in the overall solution method, the complex power of the load/generation at each node must be represented as admittances with the appropriate signs. The equivalent load admittance at node i can be written in terms of complex power and voltage magnitude as follows

$$Y_i = \frac{S_i^*}{|V_i|^2} \tag{1}$$

where S_i^* is the conjugate complex power at node i,
Therefore, the equivalent load impedance can be written as

$$Z_i = \frac{|V_i|^2}{S_i^*} \tag{2}$$

The column vector of the equivalent load/generation admittances is formed as follows,

$$[Y_{Load}]^t = [Y_1 \ Y_2 \ \ldots \ Y_N]$$

where N is the total number of the nodes in the system.

PV-PQ Sensitivity and Selection of Reference Nodes

In a multi-node power system, it is important to identify the sensitivity of the PV to the PQ nodes in terms of reactive power which determines the voltage stability of the system. However, sensitivity is defined as the ratio ??
relating small changes Dx of some dependent variable to small changes Dy of some independent or controllable variable y. This section derives a direct sensitivity solution in terms of reactive power between PV and PQ nodes; it can be derived from power flow equations as follows. The standard equations of the power injections at a node can be written as

$$P_i = \sum_{j=1}^{N} |V_i| |V_j| |Y_{ij}| \cos(\delta_i - \delta_j - \theta_{ij})$$

and the expression of the reactive power is

$$Q_i = \sum_{j=1}^{N} |V_i||V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij})$$

where

$|V_i|\angle\delta_i$ is the voltage at node i.

$|Y_{ij}|\angle\theta_{ij}$ is the admittance of the line i-j.

$|Y_{ii}|\angle\theta_{ii}$ is the summation of all admittances that are connected to node i.

and N is the number of total nodes in the network.

The partial derivative of the reactive power with respect to the voltage is

$$\frac{\partial Q_i}{\partial V_{j(j\neq i)}} = |V_i||Y_{ij}| \sin((\delta_i - \delta_j - \theta_{ij})$$

and

$$\frac{\partial Q_i}{\partial V_i} = \sum_{\substack{j=1 \\ j\neq i}}^{N} |V_j||Y_{ij}| \sin(\delta_i - \delta_j - \theta_{ij}) + 2|V_i||Y_{ii}| \sin(-\theta_{ii})$$

The proper expression for the reactive power sensitivity must now be developed. The standard system nodal equation in a decoupled load flow can be written in matrix form as

$$[\Delta Q] = \left[\frac{\partial Q}{\partial V}\right][\Delta V] \tag{3}$$

Separating the generator (G) and the load (L) of [DQ] the equation becomes as follows

$$\begin{bmatrix} \Delta Q_G \\ \Delta Q_L \end{bmatrix} = \begin{bmatrix} \dfrac{\partial Q_G}{\partial V_L} \\ \dfrac{\partial Q_L}{\partial V_L} \end{bmatrix} [\Delta V_L] \tag{4}$$

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If we define $\quad [S_G] = \left[\dfrac{\partial Q_G}{\partial V_L}\right]$ and $[S_L] = \left[\dfrac{\partial Q_L}{\partial V_L}\right]$

then it follows

$$[\Delta Q_G] = [S_G][\Delta V_L] \tag{5}$$

$$[\Delta Q_L] = [S_L][\Delta V_L] \tag{6}$$

For a power system of N_G generator buses and N_L load buses, the dimensions of S_G and S_L are $N_G x N_L$ and $N_L x N_L$ respectively.
Combining and rearranging equations (5) and (6) yields,

$$[\Delta Q_G] = [S_G][S_L]^{-1}[\Delta Q_L] \tag{7}$$

If we define $[S] = [S_G][S_L]^{-1}$, then the matrix $[S]$ with a dimension of $N_G x N_L$ will contain the sensitivity of the reactive power of the generator buses to that of the PQ buses [9]. However, the sensitivity is a measure of the PQ load reactive power dependency on the PV nodes, and $[S]$ is the matrix which directly relates the reactive power generation with the reactive power load. Furthermore, if S_{ij} is an element in the matrix $[S]$, the bigger the value of the element S_{ij} the more sensitive is the PV node i to the PQ node j. The PV node which is the most sensitive to the variation of the load at a PQ node is called a "reference" to such node.

For each load, there must be at least one reference in the network, this is obvious since the system must run with at least one voltage controllable node (slack).

A voltage controllable node is considered to be a "reference" node when it satisfies one of the following two conditions:

1. It is directly connected to the PQ node. In other words, if $|Y_{Bus}(i,j)| > 0$, then node i is a reference to j, where i is a PV, and j is a PQ node. This condition is applied if more than one generator is connected to the PQ bus.

2. It is the most sensitive to the PQ node, i.e. its sensitivity S_{ij} is dominant, say, $|S_{ij}| > 50\%$. If no dominant S_{ij} in the set of PV nodes, then, there must be more than one PV node as a reference. In this case, the sensitivities are arranged from higher to lower, and then adding the elements S_{ij}, starting from the highest, until the summation becomes greater than or equal to 50%. The PV nodes of which their sensitivities are counted in this summation are taken as references.

Once the references have been defined, we can now determine the system equivalent impedance.

Calculation of the Equivalent Impedance of a Network

Looking from any PQ load node in the network, the multi-machine multi-node power system network is reduced to an equivalent impedance connected to a voltage source. The equivalent impedance of the power system network, looking from node i, is calculated in the following steps:

1. From the network line data, obtain the bus admittance matrix $[Y_{Bus}]$. The $[Y_{Bus}]$ building algorithm is as follows, the standard nodal equation in matrix form is

$$[I] = [Y_{Bus}][V]$$

Y_{ii} : The diagonal entries of $[Y_{Bus}]$ are called the self-admittances, and are found by summing all the admittance of the lines and ties connected to bus i and identified by repeated subscripts.
Y_{ij} : The off-diagonal entries are the negatives of the admittances of lines between buses i and j. If there is no line between i and j, this term is zero. $Y_{12} = Y_{21}$, $Y_{13} = Y_{31}$, $Y_{1N} = Y_{N1}$ and so on.
The matrix $[Y_{Bus}]$ is complex and symmetric, and it is sparse since each bus is connected to only a few nearby buses.

2. Add to the diagonal of $[Y_{Bus}]$ the admittances of the injected active and reactive power of the system nodes, excluding the load power at node i, to form the system admittance matrix $[Y_{System}]$; diag$[Y_{System}]$ = diag$[Y_{Bus}]+[Y_{Load-i}]$, where $[Y_{Load-i}]$ is the admittance of the injected active and reactive power at all nodes excluding the injected power at node i.

3. The reference nodes, determined by the sensitivity technique, are represented by voltage sources reduced to zero to determine the equivalent impedance.

4. Invert the system admittance matrix to obtain the system impedance matrix $[Z_{System}]$; the system admittance matrix $[Y_{System}]$ cannot be singular since its diagonal elements are much larger than its corresponding off-diagonal. The element Z_{ii} in the diagonal of $[Z_{System}]$ represents the system equivalent impedance for node i. Hence-forth, Z_{ii} will be called Z_{ie} to indicate the equivalent impedance for node i.

The power received by the load depends on the load impedance $|Z_i|$. When $|Z_{ie}| < |Z_i|$ the line is operating at the upper portion of the famous power-voltage curve, however, the upper part of the curve is the stable region. On the contrary, when $|Z_{ie}| > |Z_i|$, it is operating at the lower half of the curve which is unstable. The critical point which is also the maximum power transfer point is reached when the ratio $|Z_{ie}|/|Z_i| = 1.0$.

RESULTS AND ANALYSIS

The proposed method to determine a voltage collapse proximity indicator can easily be applied to a system of any number of buses. The method is applied to a number of networks with different configurations. It is applied to the standard IEEE-30 and 14-bus test systems, their bus and line data can be found in [10]. It is also applied to the Qatar power system [11]; a realistic system of 116 buses and 215 lines. For very large systems, however, the speed of calculation can be enhanced by utilising the well-known sparsity technique with any recent personal computer.

In this section, the IEEE 14-bus and 30-bus systems, and the 116-bus Qatar power system are used to demonstrate the capability of the proposed method and to investigate the voltage collapse proximity indicator.

The test involves a gradual increase of load (MW/MVAR) in a single bus while other loads in the system remain unchanged. To comply with the space provided in this paper, nodes 14 and 4 from the IEEE 14-bus system, nodes 30 and 26 from the IEEE 30-bus system, and buses Umm Said B and QAFCO from

the 116-bus Qatar power system are selected. The selection, however, is based on the fact that a wide range of location is covered to show the capability of the method, from remote buses such as node 30 to buses that are close to generation such as node 4; the results are shown in Figures 1 to 6.

The ratio of the system equivalent impedance to the equivalent load impedance ($|Z_{ie}|/|Z_i|$) is used as a voltage collapse proximity indicator (VCPI). As the load at the concerned node is increased, the equivalent load impedance is decreased while the system equivalent impedance maintains its constant value, as a result the VCPI increases. However, when the system reaches its loadability limit, the magnitude of the equivalent load impedance of the node equals the value of the system equivalent impedance. Consequently, the voltage collapse proximity indicator reaches the value of 1.0 beyond which the voltage collapses. For comparison purposes, the L indicator proposed by Kessel and Glavitsch [4] is also included in the plots.

Furthermore, node 7 of the IEEE 30-bus system is tested using Chebbo's [8] and the proposed method as shown in Figures 7 and 8 respectively. Figure 7 is shown in reference [8] as Fig. 6; it can be inferred from the graph that the predicted critical voltage and power are varying with the load, this cannot be a
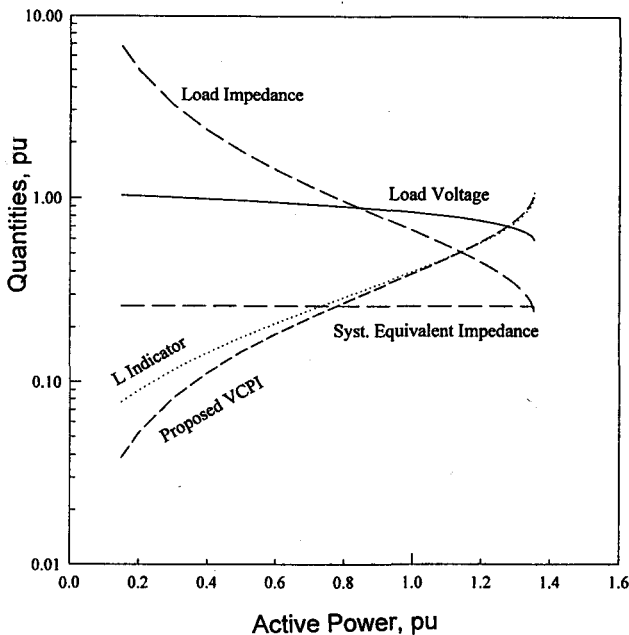

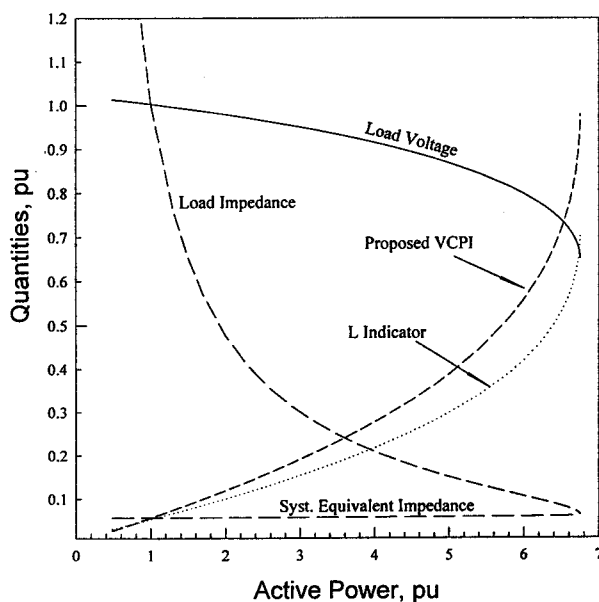**Fig. 1. Variations of node quantities with load (Node14, IEEE 14-bus system)**

Fig. 2. Variations of node quantities with load (Node 4, IEEE 14-bus system). Note for this particular node: the proposed VCPI reaches 0.99 while the L indicator reaches 0.71 at the voltage stability limit
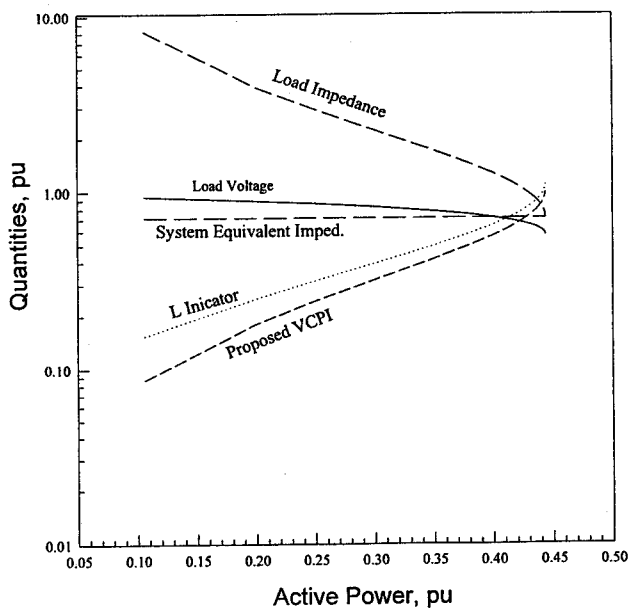

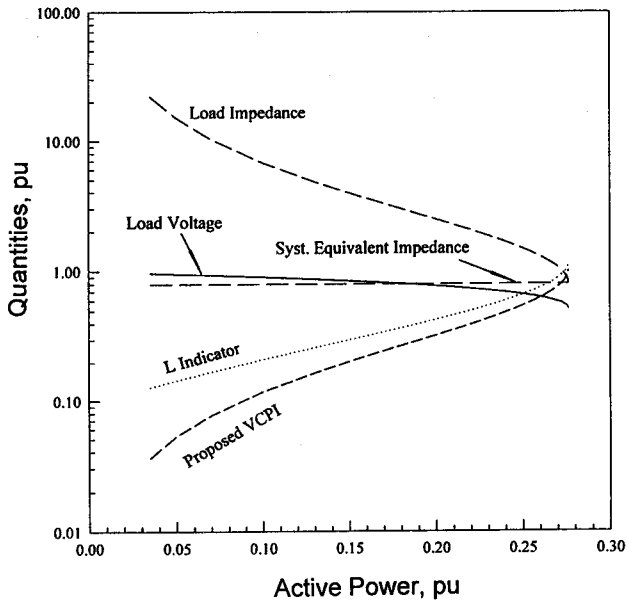**Fig. 3.** Variations of node quantities with load (Node30, IEEE 30-bus system)

Fig. 4. Variations of node quantities with load (Node26, IEEE 30-bus system)
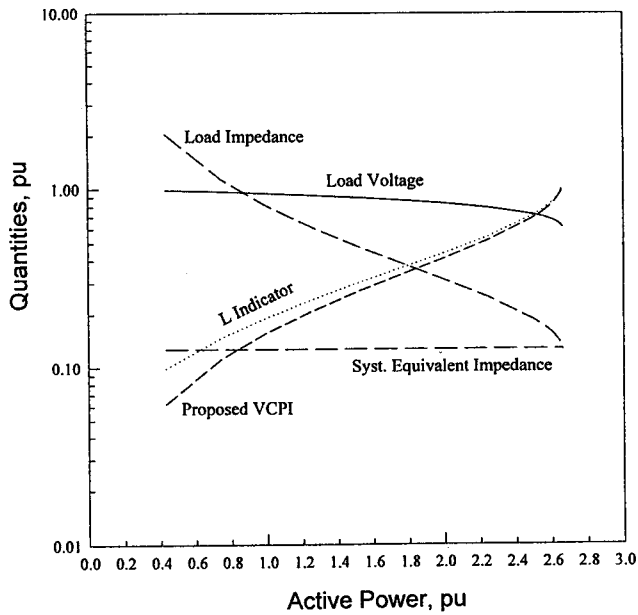

**Fig. 5.   Variations of node quantities with load
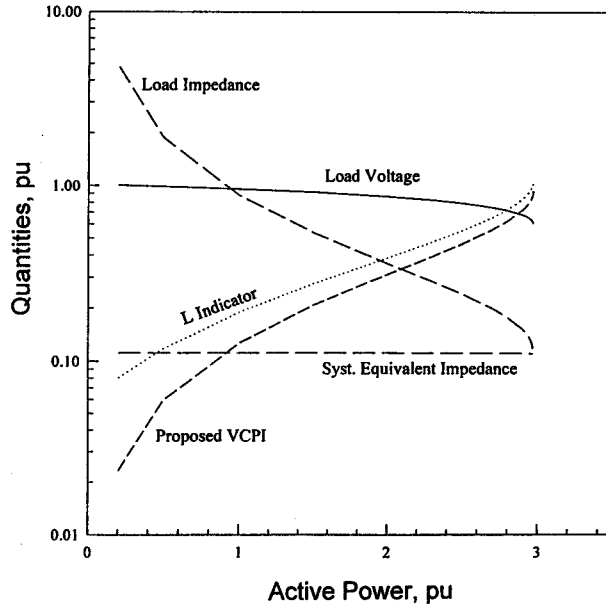(Umm Said B 66kv, Qatar 116-bus system)**

Fig. 6. Variations of node quantities with load active power (QAFCO 66 kV, Qatar power network).
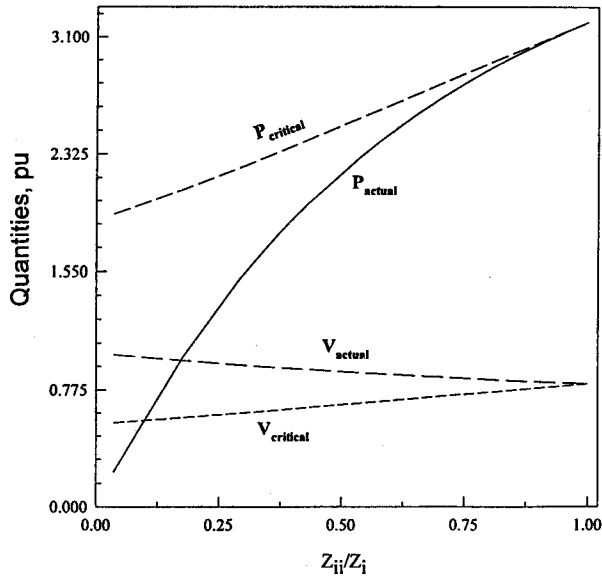

Fig. 7 Voltage collapse proximity indicator using the method in [8] (node 7 of the IEEE 30-bus system, single-load change)
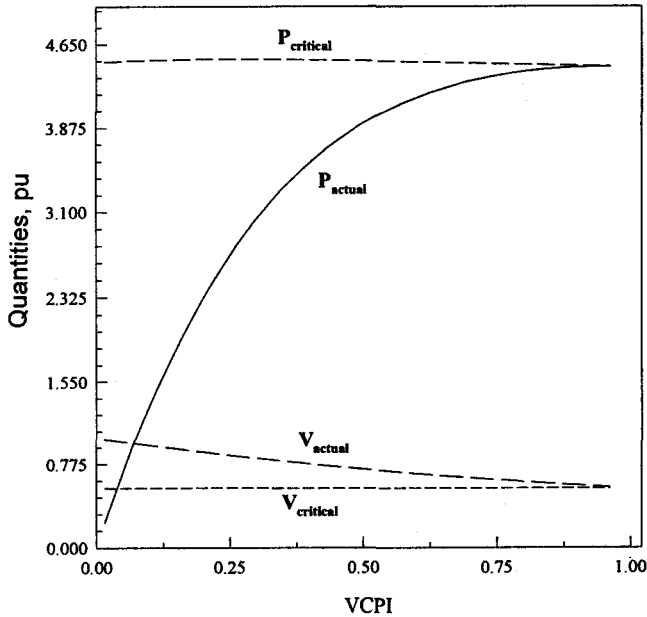
Fig. 8 Voltage collapse proximity indicator using the proposed method (node 7 of the IEEE 30-bus system, single-load change)

valid model for the simulation of buses that experience large load variations. However, the accuracy of the predicted power and voltage are shown in Fig. 8 as predicted by the proposed method.

CONCLUSION

This paper provided a method to determine a voltage collapse proximity indicator (VCPI) in the context of maximum loadability. The VCPI is the ratio of the system equivalent impedance to the load equivalent impedance. The value of the VCPI varies from zero at no load to 1.0 at the maximum loadability. For lightly load buses, the VCPI is almost linear with the load variation; the voltage variation is small for lightly loaded systems and hence the load equivalent impedance variation is marginal. However, for a heavy loaded system, any small increase of load demand induces a severe voltage drop which in turn causes a large increase in the VCPI.

The method introduces a network equivalency technique in which the sensitivity of the voltage controllable nodes to the load is employed. The validity

and effectiveness of this method is demonstrated with the application of different network configurations. The applicability of this method is verified by its ability to predict the buses which will experience voltage collapse when the load changes. Sets of results have been obtained for different cases of load variations for different systems which clearly demonstrate the validity and effectiveness of the method.

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APPENDIX

The L Indicator [4]

A method called the L indicator, aimed at the detection of voltage instability is proposed by Kessel and Glavitsch (1986). It uses the information of a normal load flow and varies in the range between zero (no load) and one (voltage collapse). The method was derived from a two-bus network where one of the nodes is the slack and the other is a PQ node. The model and the method are extended to a multi-machine power system. The L stability indicator can be computed for each node; the maximum value (closest to one) is an indication of proximity to voltage collapse.

The voltage stability indicator at bus j as proposed by Kessel and Glavitsch is expressed as

$$L_j = \left| 1 - \frac{\sum\limits_{i \in \alpha_G} F_{ji} V_i}{V_j} \right| \qquad (4.2.1)$$

where a_G is the set of generator buses and F_{ji} is an element in matrix [F] which is determined by

$$[F] = - [Y_{LL}]^{-1} [Y_{LG}] \qquad (4.2.2)$$

where $[Y_{LL}]$ and $[Y_{LG}]$ are sub-matrices in the bus admittance matrix which connects the injected currents I and voltages V of different buses in the system as in the following relationship

$$\begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} \qquad (4.2.3)$$

where the subscripts L and G indicate the load and generator buses respectively.