

# MATHEMATICAL MODEL TO SIMULATE THE SURFACE RUNOFF FOR UBAIYIDH VALLEY IN THE WESTERN DESERT

Abdulsattar R. Al-Dabbagh  
College of Engineering, Baghdad University  
Baghdad, Iraq

## ABSTRACT

Water plays an important role in human life and development. The evaluation, planning and management of water resources become important in sustaining human life, specially in arid and semi arid regions. This paper aims to develop a mathematical model to simulate the surface runoff for Ubaiyidh valley in the western Iraqi desert. The model also simulates the processes of interception storage, depression storage, infiltration percolation and inter flow. Two approaches are used for routing runoff. The first is time area curve which is applied to the wetted area, and the second is the Muskingum method for modeling the runoff in the main channel of the valley. The model contains ten parameters which have been optimized using Rosenbrcks method. The model parameters were calibrated by using available data.

## NOMENCLATURE

- $R_a$  : accumulated runoff (mm)
- $P_a$  : accumulated precipitation (mm)
- $I_a$  : interception storage (mm)
- $E_v$  : evapotranspiration or evaporation (mm)
- $D$  : depression storage (mm)
- $f$  : infiltration rate (mm/hr)
- $f_a$  : actual infiltration rate (mm/hr)
- $I_r$  : rainfall intensity (mm/hr)
- $SM$  : soil moisture storage (mm)
- $FC$  : field capacity (mm)
- $SW$  : sustained water above field capacity (mm)
- $Per$  : percolation (mm)
- $SSF$  : subsurfase flow (mm)

$s_c$	: interflow coefficient
$f_c$	: final infiltration rate (mm)
$\Delta F_a$	: accumulated infiltration depth (mm)
$E_v$	: evaporation (mm/hr)
$A$	: watershed area(km <sup>2</sup> )
$C$ & $f$	: constants
$L_c$	: length of channel reach m
$S_o$	: average slope of channel reach m/m
$q_n$	: outflow from watershed (m <sup>3</sup> /sec)
$R_i$	: excess rainfall (mm/hr)
$A_j$	: watershed subarea (km <sup>2</sup> )
$i$	: number of ordinate of excess rainfall
$j$	: number of subarea
$T_c$	: time of concentration (min)
$h(z)$	: the response of the system ( unit hydrograph )
$\alpha_j$	: selected roots
$p$	: the number of selected roots
$q_o$	: observed flow (m <sup>3</sup> /s)
$q_s$	: simulated flow (m <sup>3</sup> /s)
$\hat{F}$	: computed infiltration (mm)
$F_o$	: observed infiltration (mm)
$RVF$	: ratio computed to observed flow volumes

## INTRODUCTION

Water resources plays an important role in the development of arid and semi arid regions. In the western desert the surface runoff is affected directly by rainfall. The quantity of the rainfall is so little that the surface runoff resulting from it difficult to estimate because most of the rain falling is absorbed by the porous mantle of soil or is lost by evaporation. The lack of rainfall stations, stream flow gauging stations and the limited available data for the valley (Al-Sadder, 1995). Stresses the need to develop a mathematical model to simulate the surface runoff for the valley. The model is based on the water balance equation using rainfall, evaporation and the characteristics of the valley as inputs. The parameters of this model are estimated using an optimization procedure (Kuester and Mize, 1973). Moreover the root selection method is used to estimate the unit hydrograph of stream records of the Ubaiyidh-Ukhaider part of the valley (Dilamiy, 1994).

### Site Description and Model Development Watershed Area Description

Ubaiydh valley begins in Saudi Arabia and when it reaches the Nukhaib city, the valley expands and becomes wider. Surface water is lost and disappears in the flat territory, Figure (1). The length of the main channel of this part of the valley is 340 km with a slope of 0.0017 m/m and the catchment area is 6300 km<sup>2</sup>. The Ubaiydh- Ukhaider part begins 30 km east of Nukhaib city and continues until Al-Razaza lake. The total length of the main channel is 120 km, the slope along the valley is 0.0016 m/m and the catchment area is 1200 km<sup>2</sup> (Al-Sadder, 1995). The climate of the region is hot-dry in summer and cold-humid in winter. The mean annual rainfall is less than 115 mm, and occurs mostly during winter-spring season. The annual pan evaporation varies between 4222 mm at Bussaiya station to 2552 mm at Ana station. (Ministry of Transport and Communication, 1994).

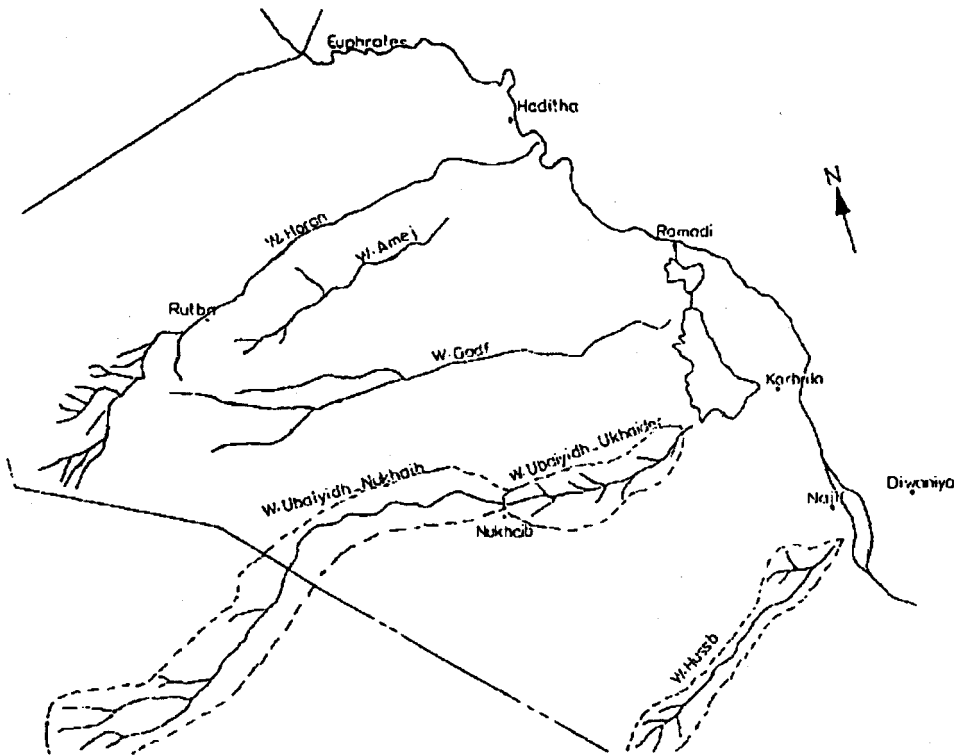


Fig. 1. Location map

The soil types of the valley are silty loam for Ubaiydh-Nukhaib part and sandy loam for the Ubaiydh-Ukhaider part, (ACSAD, 1983). There are no soil

investigations in this valley, and soil characteristics are estimated from the hydrological soil property, (Rawls et. al., 1982).

The available hydrological data is very limited where the only rainy seasons recorded are in the year (1975-1976) for Ubaiyidh-Nkhaib and Ubaiyidh-Ukhaider (Yugoslavian report, Minitisry of Agriculture, 1977). The basic data available are mass curve for rainfall-flood hydrograph and evaporation. The rainfall hyetograph was derived from the mass curves for each station. Due to limitation of monitoring stations for the region, the point rainfall is assumed as a real rainfall.

### Model Development

The model is based on conceptual deterministic representation. The data needed are rainfall, evaporation, watershed characteristics and observed runoff hydrograph. This model is based on the water balance equation that can be written as (Anderson and Burt, 1985).

$$R_a = P_a - F_a - I_a - Ev - D \quad (1)$$

where

$R_a$  : accumulated runoff (mm)

$P$  : accumulated precipitation (mm)

$F_a$  : accumulated infiltration (mm)

$I_a$  : interception storage (mm)

$Ev$  : evapotranspiration or evaporation (mm)

$D$  : depression storage (mm)

The model structure is shown in, Figure (2).

### Model Operation

When the rainfall reaches the soil surface, the infiltration occurs immediately under the conditions:

IF  $(f > I_r)$ , Then,  $f_a = I_r$

IF  $(f \leq I_r)$ , Then,  $f_a = f$

where

$f$  : infiltration rate (mm/hr)

$f_a$  : actual infiltration rate (mm/hr)

$I_r$  : rainfall intensity (mm/hr)

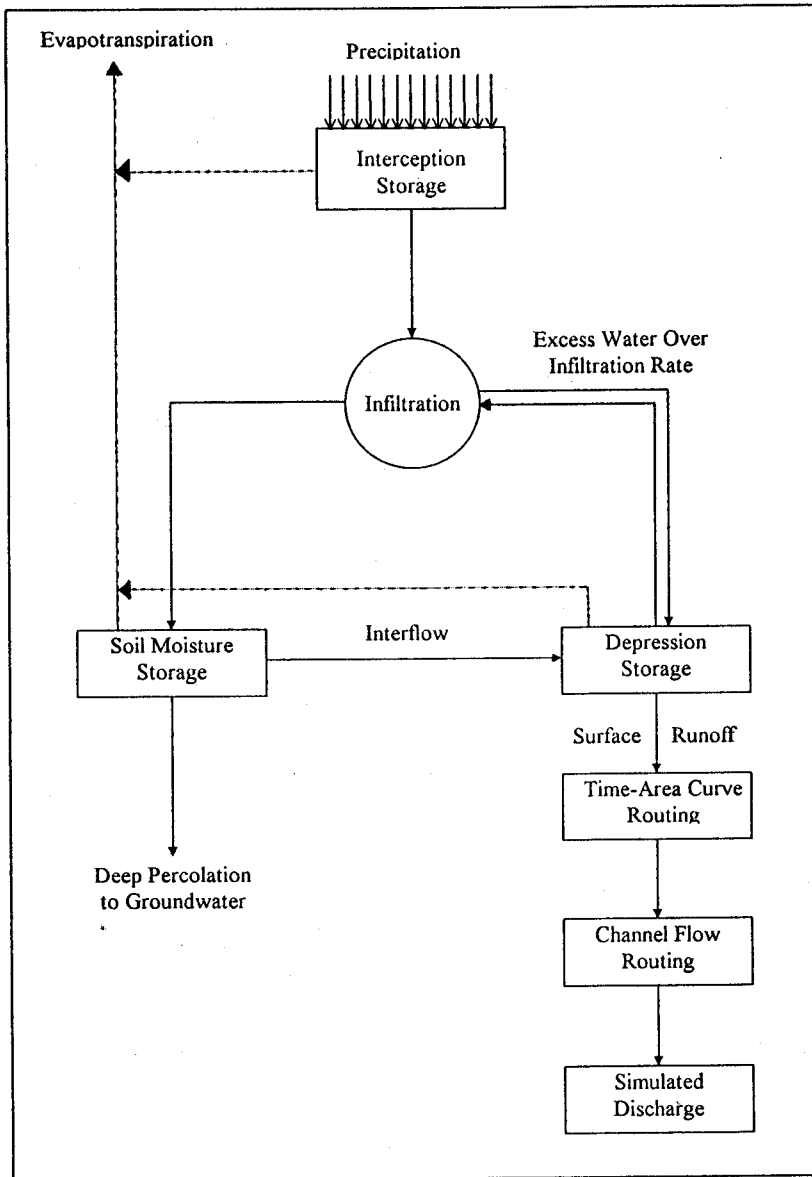


Fig. 2. Structure diagram of watershed model (After Al-Sadder, 1995)

When the rainfall intensity is larger than the infiltration rate, depression storage will be filled until it reaches the maximum depression storage, then the effective rainfall occurs (direct runoff).

The soil storage capacity  $S_a$  is depleted by evaporation and increased by infiltration. This process will continue until the soil storage capacity is filled. So, the percolation occurs with the rate not exceeding  $f_c$  (final infiltration rate).

When the soil moisture capacity increases more than the field capacity, the subsurface flow occurs under these conditions:

$$\begin{aligned} \text{IF } (SM > FC) \text{ AND } (SW > f_c \cdot \Delta t), \quad \text{then } SSF &= s_c(SW - Per) \\ \text{IF } (SM > FC) \text{ AND } (SW \leq f_c \cdot \Delta t), \quad \text{then } SSF &= 0 \end{aligned}$$

where

- SM : soil moisture storage (mm)
- FC : field capacity (mm)
- SW : sustained water above field capacity (mm)
- Per : percolation (mm)
- SSF: subsurface flow (mm)
- $s_c$  : interflow coefficient
- $f_c$  : final infiltration rate (mm)

Then the soil storage capacity  $S_a$  for the next time increment will be,

$$S_{a_t} = S_{a_{t-1}} - \Delta F_{a_{t-1}} + Per_{t-1} + SSF_{t-1} + Ev \cdot \Delta t \quad (2)$$

Where

- $\Delta F_a$  : accumulated infiltration depth (mm)
- Ev : evaporation (mm/hr)

Subsurface flow for each time is added to the surface depression until the maximum depression storage is exceeded, then excess is added to the direct runoff. In case rainfall is less than infiltration rate, the infiltration is supplied from depression storage.

The wetted area is estimated by optimization, using the following equation (Viessman, et. al., 1977).

$$A = C \cdot T_c^f \quad (3)$$

where

- A : watershed area (km<sup>2</sup>)
- C&f : constants
- $T_c$  : time of concentration (min)

The time of concentration is calculated by the equation suggested by Kirpic, (1940),

$$T_c = 0.02L_c^{0.77} S_o^{-0.385} \quad (4)$$

where

$L_c$  : length of channel reach (m)

$S_o$  : average slope of channel reach (m/m)

The direct runoff can be obtained by using the time area method as follows (Bedient and Huber, 1992):

$$q_n = \left[ R_i A_1 + R_{i-1} A_2 + \dots R_1 A_j \right] \frac{1}{3.6} \quad (5)$$

where

$q_n$  : outflow from watershed ( $m^3/sec$ )

$R_i$  : excess rainfall (mm/hr)

$A_j$  : watershed subarea ( $km^2$ )

$i$  : number of ordinate of excess rainfall

$j$  : number of subareas

$n = i + j - 1$

### Root Selection Method

This method was developed by (Turner, et. al. 1989) to derive the unit hydrograph from runoff records, where no rainfall record is available. The unit hydrograph roots can be selected from among the polynomial roots for the runoff from a single storm. If the runoff of the storm is

$$Q_1, Q_2, Q_3 \dots \quad (6)$$

Then using the Z - transformation, equation (6) becomes:

$$Q(z) = Q_1 Z^1 + Q_2 Z^2 + \dots + Q_p Z^p + \quad (7)$$

The complex roots of the resulting polynomial can be found by solving equation (7).

On the basis of the "skew circle" pattern when plotted on Argand diagram, which shows the real part of the roots as the abscissa and the imaginary part as the ordinates.

The roots of the polynomial are selected to construct the unit hydrograph using the following equation.

$$h(z) = \prod_{j=1}^{j=p} (z - \alpha_j) = 0 \quad (8)$$

where

$h(z)$  : response of the system ( unit hydrograph )

$\alpha_j$  : selected roots

$p$  : number of selected roots

### Results and Discussion

The valley is divided into two parts Ubaiyidh-Nakhaib and Ubaiyidh-Ukhaider. The deterministic model is applied to the first part of the valley, the available data for the rainy season (1975-1976) are used in this model. The input of this model are rainfall, evaporation, stream flow and basin characteristics. This model fits the ten parameters which represent the characteristics of the basin. The Rosenbrocks constrained method is applied using a computer program. The objective function is defined as the difference between the observed and simulated data,

$$F = \sum_{i=0}^N (qo_i - qs_i)^2 \quad (9)$$

where

$qo_i$  : observed flow

$qs_i$  : simulated flow

Table (1) shows the optimized parameters of the storm event for the Ubaiyidh valley.

These characteristics are then used in flow routing. Two routing techniques are used to route the flow. Time-area method (for watershed routing) and Maskingum routing method (for stream routing).



The overall efficiency of the model is obtained from the sum of squares of error of the hydrograph ordinates, according to criterion of Nash and Sutcliffe (1970),

Table 1. Optimized Parameters for Ubaiyidh Valley

Parameters	Value	Unit
a	0.396	
n	0.325	
f <sub>c</sub>	0.479	mm/hr
S <sub>ai</sub>	8.430	mm
D <sub>max</sub>	4.060	mm
S <sub>c</sub>	0.517	
K <sub>r</sub>	0.722	
k	0.729	
x	0.105	

a & n : the intercept and slope, respectively of logarithmic plotting of the quantity (f - f<sub>c</sub>) verses. S<sub>a</sub>  
 f<sub>c</sub> : final infiltration rate  
 S<sub>ai</sub> : available storage capacity in the surface layer  
 D<sub>max</sub> : maximum depression storage  
 S<sub>c</sub> : interflow coefficient  
 K<sub>r</sub> : recession constant  
 k : the storage time constant for the reach  
 x : weighting factor, which varies from 0 - 0.5 for a given reach  
 f : infiltration rate  
 S<sub>a</sub> : storage capacity at different (f-f<sub>c</sub>) values

$$R^2 = 1 - \frac{\hat{F}^2}{F_0^2} \quad (10)$$

where

- ^  
F : computed flow
- F<sub>0</sub> : observed flow

The ratio of computed to observed flow volumes (RFV) is also calculated using the following equation suggested by Khan (1989),

$$REV = \frac{\sum Q_s}{\sum Q_o} \quad (11)$$

Table (2) shows the comparison between the observed and simulated hydrographs for the valley, and indicates good agreement between the estimated and observed runoff volume, peak flow and time to peak in the three storm events.

Table 2. Comparison Between Some Characteristics of Observed and Simulated Hydrograph for Valley Ubaiyidh-Nukhaib

Storm Enents	Observed			Simulated			RFV
	Surface runoff volume (m <sup>3</sup> )	Peak flow (m <sup>3</sup> /s)	Time to peak (hr)	Surface runoff volume (m <sup>3</sup> )	Peak flow (m <sup>3</sup> /s)	Time to peak (hr)	
27 - 2 - 1976	662 * 10 <sup>3</sup>	12.7	3	662 * 10 <sup>3</sup>	11.22	3	1.00
15 - 5 - 1976	568 * 10 <sup>3</sup>	19.8	1	558 * 10 <sup>3</sup>	19.59	1	1.02
18 - 5 - 1976	875 * 10 <sup>3</sup>	35.8	1	886 * 10 <sup>3</sup>	37.47	1	1.01

The modified root selection method is used to estimate the unit hydrograph for the Ubaiyidh-Ukhaider part because the rainfall records are not available. Figure (3) shows 3hr unit hydrograph for Ubaiyidh-Ukhaider part using the stream flow record of 17-18/12/1975.

The model was used to compute the total runoff volume of storm events for the years (1987-1993), using the rainfall data from "Iraqi Meteorological Organization". Nukhaib station was used to calculate the runoff volume for Ubaiyidh valley. The runoff volume for each part of the valley is shown in tables (3) and (4). It is seen that storm events smaller than 7 mm did not produce runoff.

Regression equations are fitted to determine the relationship between rainfall depth and runoff volume for the two parts as shown in Figure (4). The correlation coefficient is found to be 0.880 and 0.869 for the Ubaiyidh-Nukhaib, Ubaiyidh-Ukhaider respectively. Accordingly it is possible to recommend to use these relationships for quick rainfall-runoff estimation.

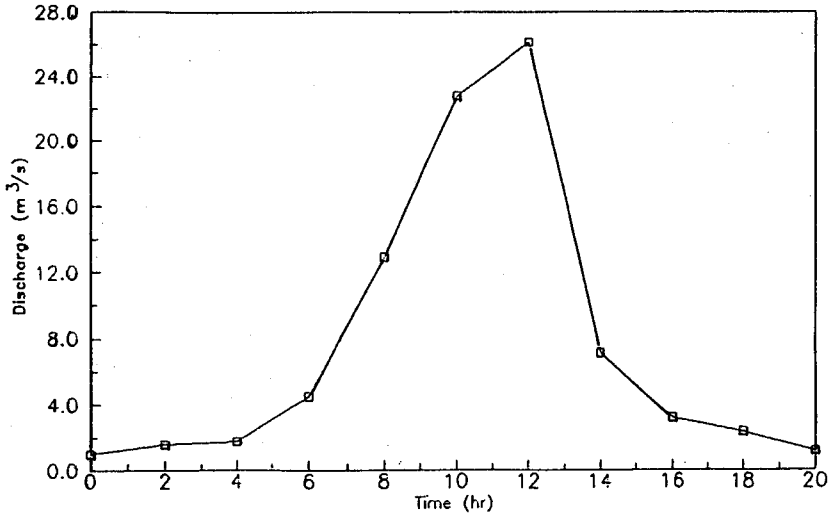


Fig. 3. Unit hydrograph for valley Ubaiyidh-Yukhaider (17-18/12/1975)

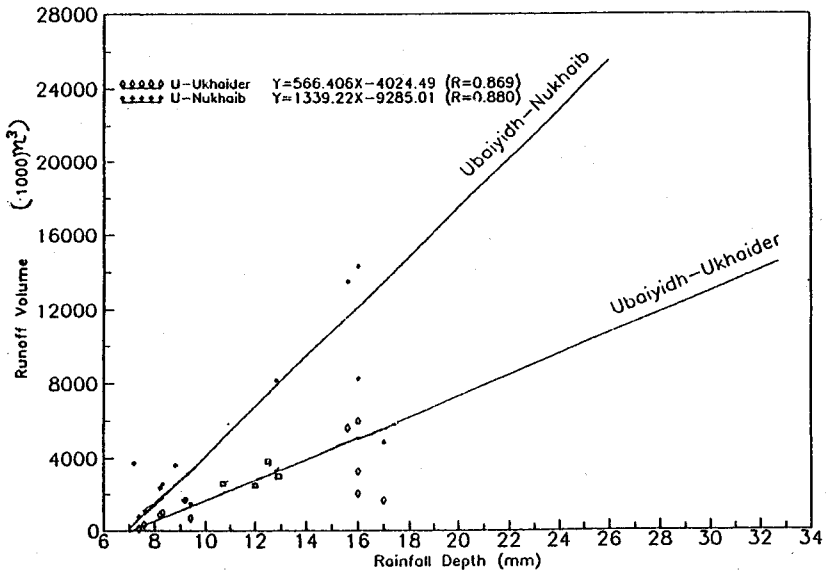


Fig. 4. Rainfall-runoff relationship for valleys under study

Table 3. Results for Valley Ubaiyidh-Nukhaib (1987-1993)

Storm Event	Rainfall (mm)	Rainfall Duration (hr)	Average Intensity (mm/hr)	Runoff (m <sup>3</sup> x 1000)	Evaporation (m <sup>3</sup> x 1000)	Infiltration (m <sup>3</sup> x 1000)	Peak Discharge (m <sup>3</sup> /s)
15 - 2 - 1987	9.4	3	3.13	1491	3288	14021	5.07
3 - 3 - 1987	17	4	4.25	4839	5022	24139	16.83
5 - 1 - 1988	7.2	2	3.60	3761	2053	8586	12.79
7 - 2 - 1988	16	4	4	8249	1888	21863	28.76
23 - 4 - 1988	16	4	4	5038	1104	25858	17.34
14 - 3 - 1989	8.8	2	4.40	3636	2568	11396	12.43
14 - 11 - 1989	20.4	3	6.80	23237	362	17201	80.55
9 - 2 - 1990	7.4	2	3.70	763	1548	12489	2.62
4 - 3 - 1991	8.3	2	4.15	2573	2584	11443	8.94
14 - 10 - 1991	7.6	2	3.80	1159	3657	10384	4.03
3 - 11 - 1991	8.2	2	4.10	2371	3509	10521	8.24
17 - 2 - 1992	16	3	5.33	14339	94	17567	49.71
23 - 2 - 1992	15.6	3	5.20	13513	4402	13286	46.91
2 - 4 - 1993	12.8	3	4.27	8145	5730	11724	27.83
24 - 4 - 1993	24	4	6	26451	5645	15903	91.59
28 - 4 - 1993	15.6	3	5.20	13513	4501	13186	46.91

Table (4) Results for Valley Ubaiyidh-Ukhaider (1987-1993)

Storm Event	Rainfall (mm)	Rainfall Duration (hr)	Average Intensity (mm/hr)	Runoff (m <sup>3</sup> x 1000)	Evaporation (m <sup>3</sup> x 1000)	Infiltration (m <sup>3</sup> x 1000)	Peak Discharge (m <sup>3</sup> /s)
15 - 2 - 1987	9.4	3	3.1.3	680	1257	6523	4.38
3 - 3 - 1987	17	4	4.25	1659	2119	11522	11.06
5 - 1 - 1988	7.2	2	3.60	1716	919	3845	11.04
7 - 2 - 1988	16	4	4	3280	735	10385	21.26
23 - 4 - 1988	16	4	4	2045	459	11896	13.4
14 - 3 - 1989	8.8	2	4.40	1413	1154	5353	9.16
14 - 11 - 1989	20.4	3	6.80	9981	162	8217	65.95
9 - 2 - 1990	7.4	2	3.70	115	697	5848	0.75
4 - 3 - 1991	8.3	2	4.15	971	1152	5347	6.14
14 - 10 - 1991	7.6	2	3.80	303	1641	4896	1.92
3 - 11 - 1991	8.2	2	4.10	876	1565	4939	5.54
17 - 2 - 1992	16	3	5.33	5954	42	8404	39.37
23 - 2 - 1992	15.6	3	5.20	5577	1977	6485	36.96
2 - 4 - 1993	12.8	3	4.27	3463	2564	5493	22.43
24 - 4 - 1993	24	4	6	11112	2525	7964	73.19
28 - 4 - 1993	15.6	3	5.20	5577	2022	6441	36.96

## CONCLUSIONS

1. Storm events smaller than 7 mm do not produce runoff in Ubaiyidh valley.
2. The runoff volume variation from year to year is inconsistent. Factors such as rainfall depth, duration, characteristics of stream and soil moisture play an important role in runoff production.
3. The unit hydrograph derived using modified root selection method, when applied to other storm events produced runoff hydrograph which agreed well with observed runoff.

## REFERENCES

- Al-Sadder, S.Z., 1995.** Mathematical Models for Determining Runoff Volume for Selected Valleys in the Western Desert. Thesis Presented to the College of Engineering , University of Baghdad.
- Anderson, M.G. and Burt , T. P., 1985.** Hydrological Forecasting, A Wiley - Interscience publication.
- Bedient, P.B. and Huber, W.C., 1992.** Hydrology and Flood plain Analysis, Addison - Weseley Publishing Company.
- Khan, M.H., 1989.** Evaluation of Rainfall-Discharge Models with Discrimination, J. of Hydrology , Vol. 108, pp. 63-78.
- Kirpich, Z F., 1940.** Time of Concentration of Small Agricultural Watersheds, ASCE , Vol. 10, p. 362.
- Kuester, J. L. and Mize, J.H., 1973.** Optimization Techniques with Fortran, Mc Graw-Hill Book Company .
- Ministry of Transport and Communication, 1994.** Iraqi Metrological Organization Rainfall Records.
- Nash, J. E. and Sutcliffe, J. V., 1970.** River Flow Forecasting Through Conceptual Models, J. of Hydrology Vol. 10, pp. 282-290.
- Rawls, W. J. Brakensiek, D. L. and Saxton , K. E., 1982.** Estimation of Soil Water Properties. ASAE Vol. 25 No. 3, pp. 1316-1325.

**Rawls, W. J. Brakensiek, D. L. and Saxton, K. E., 1982.** Estimation of Soil Water Properties. ASAE Vol. 25 No. 3, pp. 1316-1325.

**Turner, J. E., Dooge J. C. and Bree, T., 1989.** Deriving the Unit Hydrograph by Root Selection, J. of Hydrology Vol. 110, pp. 137-152.

**Vicssman, W., Knupp J. W., Lewis, G. L. and Harbaugh, T. E., 1971.** Introduction to Hydrology, Harper and Row Publishers.

اكساد (ACSAD) ؛ المركز العربي لدراسات المناطق الجافة والأراضي القاحلة (١٩٨٣) "دراسة حوض الحماد" الجزء الأول / الموارد الطبيعية ملحق رقم (٣) دمشق.

الدليمي ؛ احمد عامر (١٩٩٤) "تطوير طريقة انتقاء الجذور لاستنباط الهيدروغراف القياسي" رسالة ماجستير - جامعة الموصل - الموصل.

وزارة الزراعة (١٩٧٧) تقرير الشركة اليوغسلافية حول الصحراء الغربية في العراق.