Engineering Journal of the University of Qatar, Vol. 15, 2002, pp. 177-192

PARAMETRIC STUDY ON SINGLE MACHINE EARLY/TARDY PROBLEM UNDER FUZZY ENVIRONMENT

Mohamed S. A. Osman
Higher Technological Institute
Ramadan 10^{th} City
Egypt.

Omar M. O. Saad
Department of Mathematics
Faculty of Science
Qatar University
B. O. Box 2713.
Doha, Qatar,

Samir A. Abass
Atomic Energy Authority
Nuclear Research Center
Mathematics and Theoretical Physics Department
B. O. Box 13759.
Egypt

ABSTRACT

This paper presents a solution algorithm for solving the single machine early/tardy problem having fuzzy parameters in the constraints. There are several cost structures: job-independent, job-dependent and symmetric, and job-dependent and asymmetric. The last problem is shown to be NP-hard and it will be studied here. Some basic stability notions are defined and characterized for the problem of concern. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. A parametric study is carried out for the problem of concern. Finally, an illustrative numerical example is given to clarify the theory and the solution algorithm.

KEY WORDS : Fuzzy parameters, Single machine, Early/Tardy jobs, α-level set, NP-hard.

1. INTRODUCTION

Consider an n job single machine early/tardy problem. Let each job have a processing time, a due date, an earliness weight, and a tardiness weight (penalty cost). In a given schedule, let each job be penalized by the fixed individual earliness (tardiness) weight, if it is completed prior to (after) its due date. A schedule is sought that will minimize the total cost incurred for all penalized jobs. There are many applications to such a cost structure, e.g. in chemical or hi-tech industries, where often parts need to be early at specific times in order to meeting certain required conditions [5].

The single machine early/tardy problem (SMETP) is NP-hard and denoted by $n/1//ET$ problem. In this class of problems, some of them are polynomial and some other are known to be NP-hard. Hall and Posner [2] studied the minimization of weighted deviation of completion times when the common due date is not early enough to constrain the schedule and Hall et al [3] studied the same problem when the common due date is restrictive. Both are NP-hard problems. Kahlbacher [4] showed that some of the properties valid for these problems still hold for equal slack due date rule problems. Moore developed the problem of minimizing the number of tardy jobs on a single machine and introduce an $O(nlogn)$ algorithm to solve the problem [7]. Moore's problems have been extensively studied and clearly have numerous real-life applications in [6].

This paper is organized as follows: In section 2, some basic definitions on fuzzy set theory are introduced. In section 3, we formulate the SMETP having the processing times p_j as fuzzy parameters (FSMETP). Section 4 is devoted to a parametric study on the FSMETP. In section 5, we propose a solution algorithm to solve the problem of concern. In section 6, an illustrative numerical example is given to clarify the theory and the solution algorithm. Finally, section 7 contains the conclusions.

2. Fuzzy Concepts

L. A. Zadeh advanced the fuzzy theory at the university of California in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions.

The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al [9] in the framework of the fuzzy decision of Bellman and Zadeh [10]. Now, we present some necessary definitions [1].

DEFINITION 1

A real fuzzy number $\tilde{a}$ is a fuzzy subset of the real line R with membership function $\mu_{\tilde{a}}$ satisfies the following conditions:

(1) $\mu_{\tilde{a}}$ is a continuous mapping from R to the closed interval [0,1].

(2) $\mu_{\tilde{a}}(a) = 0 \ \forall\ a \in (-\infty, a_1]$.

(3) $\mu_{\tilde{a}}$ is strictly increasing and continuous on $[a_1, a_2]$.

(4) $\mu_{\tilde{a}}(a) = 1 \qquad \forall\ a \in [a_2, a_3]$.

(5) $\mu_{\tilde{a}}(a)$ is strictly decreasing and continuous on $[a_3, a_4]$.

(6) $\mu_{\tilde{a}}(a) = 0 \qquad \forall\ a \in [a_4, +\infty)$.

where a_1, a_2, a_3 and a_4 are real numbers, and the fuzzy number is denoted by $\tilde{a} = [a_1, a_2, a_3, a_4]$.

DEFINITION 2

The fuzzy number $\tilde{a} = [a_1, a_2, a_3, a_4]$ is a trapezoidal number, denoted by $[a_1, a_2, a_3, a_4]$, its membership function $\mu_{\tilde{a}}$ is given by (see Fig.1):

$$\mu_{\tilde{a}}(a) = \begin{cases} 0, & a \le a_1, \\ 1 - \left(\dfrac{a - a_2}{a_1 - a_2}\right)^2 & a_1 \le a \le a_2, \\ 1, & a_2 \le a \le a_3, \\ 1 - \left(\dfrac{a - a_3}{a_4 - a_3}\right)^2 & a_3 \le a \le a_4, \\ 0, & \text{otherwise} \end{cases}$$

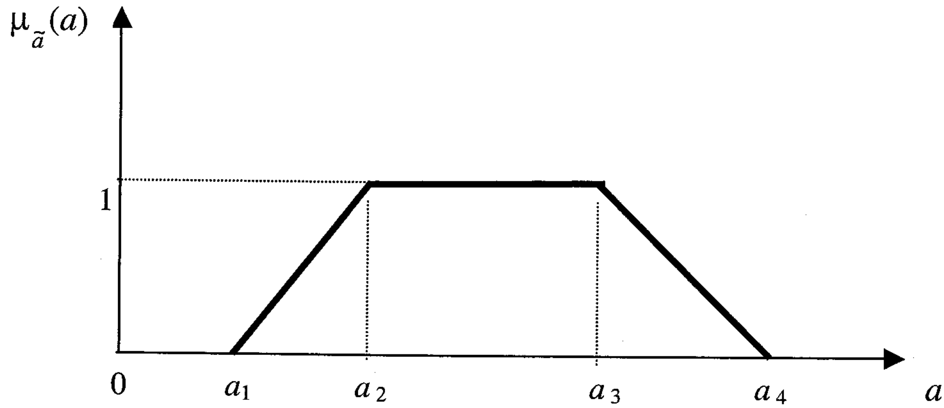

Fig.1 Membership function of a fuzzy number $\tilde{a}$.

DEFINITION 3

The α-level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level $\alpha \in [0,1]$:

$$L_\alpha(\tilde{a}) = \{a \in \mathrm{R} \mid \mu_{\tilde{a}}(a) \geq \alpha\}.$$

3. EARLY/TARDY SCHEDULING PROBLEM HAVING FUZZY PARAMETERS IN THE CONSTRAINTS

Lann and Mosheiov et al [5] introduced an integer programming formulation for the problem of concern in the deterministic case. We will extend this formulation where the processing times are fuzzy parameters.

Notations

We use the following notations:
n = number of jobs,
$\tilde{p}_j$ = fuzzy processing time of job j, $\quad j = 1,\ldots,n$,

D_j = due date of job j, $\quad j = 1,\dots,n,$

Y_j = starting time of job j, $\quad j = 1,\dots,n,$

λ_j^E = earliness weight of job j, $\quad j = 1,\dots,n,$

λ_j^T = tardiness weight of job j, $\quad j = 1,\dots,n,$

$$X_j^E = \begin{cases} 1 & \text{if job } j \text{ is early} \\ 0 & \text{otherwise} \end{cases}$$

$$X_j^T = \begin{cases} 1 & \text{if job } j \text{ is tardy} \\ 0 & \text{otherwise} \end{cases}$$

Let M be a large number. The problem of consideration is the following fuzzy single machine early/tardy problem (FSMETP):

$$\text{(FSMETP):} \quad \min \sum_{j=1}^{n} (\lambda_j^E X_j^E + \lambda_j^T X_j^T)$$

Subject to

$$\begin{aligned} & Y_j \geq Y_{j-1} + \tilde{p}_{j-1} && j = 2,\dots,n \\ & MX_j^E \geq D_j - (Y_j + \tilde{p}_j) && j = 1,\dots,n \\ & MX_j^T \geq (Y_j + \tilde{p}_j) - D_j && j = 1,\dots,n \\ & X_j^E, X_j^T \in \{0,1\} && j = 1,\dots,n \\ & Y_j \geq 0 && j = 1,\dots,n \end{aligned}$$

where $\tilde{p}_j$, $j = 1,\dots,n,$ represent fuzzy parameters involved in the constraints where their membership functions are $\mu_{\tilde{p}_j}$. For a certain degree α together with the concept of α-level set of the fuzzy numbers $\tilde{p}_j$, therefore problem (FSMETP) can be understood as the following nonfuzzy single machine early/tardy problem (P_1):

$$(P_1): \quad \min \sum_{j=1}^{n} (\lambda_j^E X_j^E + \lambda_j^T X_j^T)$$

Subject to

$$Y_j \geq Y_{j-1} + p_{j-1} \quad j = 2, ..., n$$

$$MX_j^E \geq D_j - (Y_j + p_j) \quad j = 1, ..., n$$

$$MX_j^T \geq (Y_j + p_j) - D_j \quad j = 1, ..., n$$

$$0 \leq X_j^E \leq 1, \ 0 \leq X_j^T \leq 1, \qquad j = 1, ..., n$$

$$p_j \in \mathrm{L}_\alpha(\tilde{p}_j) \qquad j = 1, ..., n,$$

$$Y_j \geq 0 \qquad j = 1, ..., n$$

where $\mathrm{L}_\alpha(\tilde{p}_j)$ are the α-level set of the fuzzy numbers $\tilde{p}_j$. The above problem can be written in the following equivalent form:

$$(\mathrm{P}_2): \quad \min \sum_{j=1}^{n} (\lambda_j^E X_j^E + \lambda_j^T X_j^T)$$

Subject to

$$Y_j \geq Y_{j-1} + p_{j-1} \qquad j = 2, ..., n$$

$$MX_j^E \geq D_j - (Y_j + p_j) \qquad j = 1, ..., n$$

$$MX_j^T \geq (Y_j + p_j) - D_j \qquad j = 1, ..., n$$

$$0 \leq X_j^E \leq 1, \ 0 \leq X_j^T \leq 1, \qquad j = 1, ..., n$$

$$h_j^{(0)} \leq p_j \leq H_j^{(0)}, \qquad j = 1, ..., n,$$

$$Y_j \geq 0 \qquad j = 1, ..., n$$

It should be noted that the constraint $p_j \in \mathrm{L}_\alpha(\tilde{p}_j)$ has been replaced by the constraint $h_j^{(0)} \leq p_j \leq H_j^{(0)}$, where $h_j^{(0)}$ and $H_j^{(0)}$ are lower and upper bounds on p_j.

A Parametric Study on Problem (P_2)

Problem (P_2) can be rewritten in the following parametric form:

(P₃): $\min \sum_{j=1}^{n} (\lambda_j^E X_j^E + \lambda_j^T X_j^T)$

Subject to

$$Y_j - Y_{j-1} - p_{j-1} \geq 0 \qquad j = 2, \dots, n$$

$$MX_j^E - D_j + (Y_j + p_j) \geq 0 \qquad j = 1, \dots, n$$

$$MX_j^T - (Y_j + p_j) + D_j \geq 0 \qquad j = 1, \dots, n$$

$$X_j^E \geq 0, \quad 1 - X_j^E \geq 0, \qquad j = 1, \dots, n$$

$$X_j^T \geq 0, \quad 1 - X_j^T \geq 0, \qquad j = 1, \dots, n$$

$$p_j - h_j \geq 0, \quad H_j - p_j \geq 0, \qquad j = 1, \dots, n$$

$$Y_j \geq 0 \qquad j = 1, \dots, n$$

where h_j and H_j, $j = 1, \dots, n$ are assumed to be parameters rather than constants. Let $G_j(Y, p) = Y_j - Y_{j-1} - p_j$, $H_j(Y, p, D, X_j^E) = MX_j^E - D_j + Y_j + p_j$, $Q_j(Y, p, D, X_j^T) = MX_j^T + D_j - Y_j - p_j$, and $Z_j(Y) = Y_j$. Let $X(h, H)$ denotes the decision space of problem (P_3), defined by:

$$X(h, H) = \{(Y, X_j^E, X_j^T, p) \in R^{4n} \mid G_j(Y, p) \geq 0, H_j(Y, p, X_j^E) \geq 0, Q_j(Y, p, X_j^T) \geq 0, Z_j(Y) \geq 0, X_j^E \geq 0, 1 - X_j^E \geq 0, X_j^T \geq 0, 1 - X_j^T \geq 0, p_j - h_j \geq 0, H_j - p_j \geq 0, \ j = 1, \dots, n\}.$$

Some Basic Stability Notions for the Problem (P₃)

In what follows, we give the definitions of some basic stability notions for the problem (P_3). These notions are the set of feasible parameters, the solvability set, and the stability set of the first kind, (see [8]).

The set of feasible parameters

The set of feasible parameters of the problem (P_3), which is denoted by U, is defined by:

$$U = \{(h, H) \in R^{2n} \mid X(h, H) \neq \phi\}.$$

The solvability set

The solvability set of the problem (P_3), which is denoted by V, is defined by:

$$V = \{(h, H) \in U \mid \text{problem } (P_3) \text{ has } \alpha\text{-optimal solutions}\}.$$

The stability set of the first kind

Suppose that $h^*, H^* \in V$ with a corresponding α- optimal solution $(Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*)$ of problem (P_3). The stability set of the first kind of problem (P_3) corresponding to $(Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*)$, which is denoted by $S(Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*)$ is defined by:

$$S(Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*) = \{(h, H) \in V \mid (Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*) \text{ is } \alpha\text{-optimal solution of problem } (P_3)\}.$$

Utilization of the kuhn-tucker conditions corresponding to problem (P_3)

The Lagrange function of problem (P_3) can be written as follows:

$$\begin{aligned} L = & \sum_{j=1}^{n} (\lambda_j^E w_1 X_j^E + \lambda_j^T w_2 X_j^T) + \sum_{j=1}^{n} \nu_j \, G_j(Y, p) \\ & + \sum_{j=1}^{n} \eta_j \, H_j(Y, p, X_j^E) + \sum_{j=1}^{n} \delta_j \, Q_j(Y, p, X_j^T) + \sum_{j=1}^{n} \pi_j \, Z_j(Y) \\ & + \sum_{j=1}^{n} \rho_j X_j^E + \sum_{j=1}^{n} \sigma_j (1 - X_j^E) + \sum_{j=1}^{n} \gamma_j X_j^T + \sum_{j=1}^{n} \chi_j (1 - X_j^T) \\ & + \sum_{j=1}^{n} \beta_j (p_j - h_j) + \sum_{j=1}^{n} \theta_j (H_j - p_j) \end{aligned}$$

then, the Kuhn-Tucker necessary optimality conditions corresponding to the problem (P_3) at the solution $(Y^*, \overset{*}{X}{}_j^E, \overset{*}{X}{}_j^T, p^*)$, $j = 1, \ldots, n$, will take the form:

$$
(G_1)\quad \left\{\begin{array}{l}
\dfrac{\partial L}{\partial X_j^E} = \sum_{j=1}^{n} \dfrac{\partial((\lambda_j^E X_j^E + \lambda_j^T X_j^T)}{\partial X_j^E} + \sum_{j=1}^{n} \eta_j \dfrac{\partial H_j(Y,p,D,X_j^E)}{\partial X_j^E} \\
\qquad + \sum_{j=1}^{n} \dfrac{\partial}{\partial X_j^E}(\rho_j X_j^E) + \sum_{j=1}^{n} \dfrac{\partial}{\partial X_j^E} \sigma_j (1 - X_j^E) = 0, \\
\dfrac{\partial L}{\partial X_j^T} = \sum_{j=1}^{n} \dfrac{\partial(\lambda_j^E X_j^E + \lambda_j^T X_j^T)}{\partial X_j^T} + \sum_{j=1}^{n} \delta_j \dfrac{\partial Q_j(Y,p,D,X_j^T)}{\partial X_j^T} \\
\qquad + \sum_{j=1}^{n} \dfrac{\partial}{\partial X_j^T}(\gamma_j X_j^T) + \sum_{j=1}^{n} \dfrac{\partial}{\partial X_j^T} \chi_j (1 - X_j^T) = 0, \\
\dfrac{\partial L}{\partial p_j} = \sum_{j=1}^{n-1} \nu_j \dfrac{\partial G_j}{\partial p_j} + \sum_{j=1}^{n} \eta_j \dfrac{\partial H_j}{\partial p_j} + \sum_{j=1}^{n} \delta_j \dfrac{\partial Q_j}{\partial p_j} - \theta_j + \beta_j = 0, \\
\dfrac{\partial L}{\partial Y_q} = \sum_{j=1}^{n-1} \nu_j \dfrac{\partial G_j}{\partial Y_q} + \sum_{j=1}^{n} \eta_j \dfrac{\partial H_j}{\partial Y_q} + \sum_{j=1}^{n} \delta_j \dfrac{\partial Q_j}{\partial Y_q} \\
\qquad + \sum_{j=1}^{n} \pi_j \dfrac{\partial Z_j}{\partial Y_q} = 0, \quad q = 1,\dots,n,
\end{array}\right\} \quad (I)
$$

$$
\left.\begin{array}{ll}
Y_j - Y_{j-1} - p_{j-1} \geq 0, & j = 2, \dots, n \\
MX_j^E - D_j + Y_j + p_j \geq 0, & j = 1, \dots, n \\
MX_j^T - Y_j - p_j + D_j \geq 0, & j = 1, \dots, n \\
X_j^E \geq 0, \quad 1 - X_j^E \geq 0, & j = 1, \dots, n \\
X_j^T \geq 0, \quad 1 - X_j^T \geq 0, & j = 1, \dots, n \\
p_j - h_j \geq 0, \quad H_j - p_j \geq 0, & j = 1, \dots, n, \\
Y_j \geq 0, & j = 1, \dots, n \\
\nu_j(Y_j - Y_{j-1} - p_{j-1}) = 0, & j = 2, \dots, n
\end{array}\right\}
$$

$$
\left.
\begin{array}{ll}
\eta_j(MX_j^E - Y_j - p_j + D_j) = 0, & j = 1, \ldots, n \\
\delta_j\,(MX_j^T - Y_j - p_j + D_j) = 0, & j = 1, \ldots, n \\
\theta_j(H_j - p_j) = 0, & j = 1, \ldots, n \\
\beta_j(p_j - h_j) = 0, & j = 1, \ldots, n \\
\rho_j X_j^E = 0, & j = 1, \ldots, n \\
\sigma_j(1 - X_j^E) = 0, & j = 1, \ldots, n \\
\gamma_j X_j^T = 0, & j = 1, \ldots, n \\
\chi_j(1 - X_j^T) = 0, & j = 1, \ldots, n \\
\pi_j\, Y_j = 0, & j = 1, \ldots, n \\
\theta_j, \beta_j, \nu_j, \eta_j, \delta_j, \pi_j, \rho_j, \sigma_j, \gamma_j, \chi_j \le 0, & j = 1, \ldots, n,
\end{array}
\right\} \quad \text{(I)}
$$

where all the relations of system (I) are evaluated at (Y^*, $\overset{*}{X}{}_j^E$, $\overset{*}{X}{}_j^T$, p^*) and θ_j, β_j, ν_j ,η_j , δ_j, π_j, ρ_j, σ_j, γ_j, χ_j ($j = 1, \ldots, n$) are the Kuhn-Tucker multipliers. The set of constraints (G_1) together with the last ten relations of system (I) represent a polytope in $\beta\theta\nu\eta\delta\pi\rho\sigma\gamma\chi$-space. According to whether any of the vertices θ_j, β_j,ν_j, η_j, δ_j, π_j, γ_j, χ_j, ρ_j, σ_j are zero or negative, then the set T(Y^*, $\overset{*}{X}{}_j^E$, $\overset{*}{X}{}_j^T$, p^*) is the set of parameters h and H for which the Kuhn-Tucker necessary optimality conditions corresponding to problem (P_3) are utilized at (Y^*, $\overset{*}{X}{}_j^E$, $\overset{*}{X}{}_j^T$, p^*). Clearly, this set can be considered as a subset from the set S(Y^*, $\overset{*}{X}{}_j^E$, $\overset{*}{X}{}_j^T$, p^*), i.e.

$$
\mathrm{T}(Y^*,\ \overset{*}{X}{}_j^E,\ \overset{*}{X}{}_j^T,\ p^*) \subseteq \mathrm{S}(Y^*,\ \overset{*}{X}{}_j^E,\ \overset{*}{X}{}_j^T,\ p^*).
$$

4. SOLUTION ALGORITHM

In this section, we describe a solution algorithm for solving the (FSMETP). In this algorithm, we assume that the jobs are sorted and scheduled in nondecreasing order of its due date.

Step 1 Determine the points (a_1, a_2, a_3, a_4) for the fuzzy number in the formulation problem (FSMETP).
Step 2 Convert the problem (P_1) in the form of the problem (P_2).

Step 3 Formulate the problem (P_2) in the parametric form of problem (P_3).

Step 4 Let G be the set of unscheduled jobs. From this set, schedule job j with smallest due

date. G := G - {j}.

Step 5 If G = ϕ, then go to step 6, else go to step 4.

Step 6 Solve the problem by any available integer software package.

Step 7 Determine the set T(Y^*, X_j^{E*}, X_j^{T*}, p^*), $j = 1,\ldots,n$, by utilizing the Kuhn-Tucker necessary optimality conditions. Stop.

5. AN ILLUSTRATIVE EXAMPLE

Consider $n = 4$, and $M = 100$. Table (1) contains the values of D_j, λ_j^E, λ_j^T and $\tilde{p}_j, (j = 1,\ldots,4)$ which are characterized by the following fuzzy numbers:

Table (1).

Jobs	D_j	λ_j^E	λ_j^T	$\tilde{P}_j$
1	3	3	1	$\tilde{p}_1 = (1,2,5,7)$
2	5	8	3	$\tilde{p}_2 = (1,3,4,6)$
3	6	4	5	$\tilde{p}_3 = (1,3,4,6)$
4	7	2	4	$\tilde{p}_4 = (0,2,3,5)$

The fuzzy problem can be written as follows:
Min $3X_1^E + X_1^T + 8X_2^E + 3X_2^T + 4X_3^E + 5X_3^T + 2X_4^E + 4X_4^T$

Subject to

$$Y_2 \geq Y_1 + \widetilde{P}_1, \qquad Y_3 \geq Y_2 + \widetilde{P}_2, \qquad Y_4 \geq Y_3 + \widetilde{P}_3,$$

$$100X_1^E \geq 3 - Y_1 - \widetilde{P}_1, \quad 100X_2^E \geq 5 - Y_2 - \widetilde{P}_2, \quad 100X_3^E \geq 6 - Y_3 - \widetilde{P}_3, \quad 100X_4^E \geq 7 - Y_4 - \widetilde{P}_4,$$

$$100X_1^T \geq Y_1 + \widetilde{P}_1 - 3, \quad 100X_2^T \geq Y_2 + \widetilde{P}_2 - 5, \quad 100X_3^T \geq Y_3 + \widetilde{P}_3 - 6, \quad 100X_4^T \geq Y_4 + \widetilde{P}_4 - 7,$$

$$X_1^E \geq 0,\ X_2^E \geq 0,\ X_3^E \geq 0,\ X_4^E \geq 0,\ X_1^T \geq 0,\ X_2^T \geq 0,\ X_3^T \geq 0,\ X_4^T \geq 0,$$

$$1 - X_1^E \geq 0,\ 1 - X_2^E \geq 0,\ 1 - X_3^E \geq 0,\ 1 - X_4^E \geq 0,\ 1 - X_1^T \geq 0,\ 1 - X_2^T \geq 0,\ 1 - X_3^T \geq 0,\ 1 - X_4^T \geq 0,$$

$$Y_1,\ Y_2,\ Y_3,\ Y_4 \geq 0.$$

Assume the membership function corresponding to the fuzzy numbers are in the form

$$\mu_{\tilde{p}}(p) = \begin{cases} 0, & p \leq p_1, \\ 1 - \left(\dfrac{p - p_2}{p_1 - p_2} \right)^2 & p_1 \leq p \leq p_2, \\ 1, & p_2 \leq p \leq p_3, \\ 1 - \left(\dfrac{p - p_3}{p_4 - p_3} \right)^2 & p_3 \leq p \leq p_4, \\ 0, & p \geq p_4. \end{cases}$$

Let $\alpha = 0.36$, then we get

$1.2 \leq p_1 \leq 6.6, \qquad 1.4 \leq p_2 \leq 5.6, \qquad 1.4 \leq p_3 \leq 5.6\,, \qquad 0.4 \leq p_4 \leq 4.8$

The nonfuzzy problem can be written as follows:

Min $\quad X_1^E + 3X_1^T + 5X_2^E + 8X_2^T + 5X_3^E + 4X_3^T + 4X_4^E + 2X_4^T$

Subject to

$$Y_2 \geq Y_1 + p_1, \qquad Y_3 \geq Y_2 + p_2, \qquad Y_4 \geq Y_3 + p_3,$$

$$100X_1^E \geq 3 - Y_1 - p_1, \quad 100X_2^E \geq 5 - Y_2 - p_2, \quad 100X_3^E \geq 6 - Y_3 - p_3, \quad 100X_4^E \geq 7 - Y_4 - p_4,$$

$100X_1^T \geq Y_1 + p_1 - 3, \quad 100X_2^T \geq Y_2 + p_2 - 5, \quad 100X_3^T \geq Y_3 + p_3 - 6, \quad 100X_4^T \geq Y_4 + p_4 - 7,$

$X_1^E \geq 0,\ X_2^E \geq 0,\ X_3^E \geq 0,\ X_4^E \geq 0,\ X_1^T \geq 0,\ X_2^T \geq 0,\ X_3^T \geq 0,\ X_4^T \geq 0,$
$1-X_1^E \geq 0,\ 1-X_2^E \geq 0,\ 1-X_3^E \geq 0,\ 1-X_4^E \geq 0,\ 1-X_1^T \geq 0,\ 1-X_2^T \geq 0,\ 1-X_3^T \geq 0,\ 1-X_4^T \geq 0,$
$1.2 \leq p_1 \leq 6.6,\ 1.4 \leq p_2 \leq 5.6,\ 1.4 \leq p_3 \leq 5.6,\ 0.4 \leq p_4 \leq 4.8,\ Y_1, Y_2, Y_3, Y_4 \geq 0.$

So we can get the following results:

$Y_1^* = 0, Y_2^* = 2, Y_3^* = 4, Y_4^* = 6,,$
$\overset{*E}{X}_1 = \overset{*E}{X}_2 = \overset{*E}{X}_3 = \overset{*E}{X}_4 = \overset{*T}{X}_1 = \overset{*T}{X}_2 = \overset{*T}{X}_3 = \overset{*T}{X}_4 = 0$, with
α-optimal parameters $(p_1^*, p_2^*, p_3^*, p_4^*) = (3, 3, 2, 1)$.

The parametric form of the above problem is stated as follows:

$$\text{Min} \quad \left(X_1^E + 3X_1^T + 5X_2^E + 8X_2^T + 5X_3^E + 4X_3^T + 4X_4^E + 2X_4^T, \right)$$

Subject to

$$Y_2 - Y_1 - p_1 \geq 0, \qquad Y_3 - Y_2 - p_2 \geq 0, \qquad Y_4 - Y_3 - p_3 \geq 0,$$
$$100X_1^E - D_1 + Y_1 + p_1 \geq 0, \quad 100X_2^E - D_2 + Y_2 + p_2 \geq 0,$$
$$100X_3^E - D_3 + Y_3 + p_3 \geq 0, \quad 100X_4^E - D_4 + Y_4 + p_4 \geq 0,$$
$$100X_1^T - Y_1 - p_1 + D_1 \geq 0, \quad 100X_2^T - Y_2 - p_2 + D_2 \geq 0,$$
$$100X_3^T - Y_3 - p_3 + D_3 \geq 0, \quad 100X_4^T - Y_4 - p_4 + D_4 \geq 0,$$
$$H_1 - p_1 \geq 0, \quad H_2 - p_2 \geq 0, \quad H_3 - p_3 \geq 0, \quad H_4 - p_4 \geq 0,$$
$$p_1 - h_1 = 0, \quad p_2 - h_2 = 0, \quad p_3 - h_3 = 0, \quad p_4 - h_4 = 0,$$
$$X_1^E \geq 0, X_2^E \geq 0, X_3^E \geq 0, X_4^E \geq 0, X_1^T \geq 0, X_2^T \geq 0, X_3^T \geq 0, X_4^T \geq 0,$$
$$1 - X_1^E \geq 0, \quad 1 - X_2^E \geq 0, \quad 1 - X_3^E \geq 0, \quad 1 - X_4^E \geq 0,$$
$$1 - X_1^T \geq 0, \quad 1 - X_2^T \geq 0, \quad 1 - X_3^T \geq 0, \quad 1 - X_4^T \geq 0,$$
$$X_1^E \geq 0, \quad X_2^E \geq 0, \quad X_3^E \geq 0, \quad X_4^E \geq 0,$$
$$X_1^T \geq 0, \quad X_2^T \geq 0, \quad X_3^T \geq 0, \quad X_4^T \geq 0,$$
$$Y_1, Y_2, Y_3, Y_4 \geq 0.$$

The Kuhn-Tucker necessary optimality conditions corresponding to the above parametric problem will have the form:

$\lambda_1^E + \eta_1 M + \rho_1 - \sigma_1 = 0, \qquad \lambda_2^E + \eta_2 M + \rho_2 - \sigma_2 = 0,$

$\lambda_3^E + \eta_3 M + \rho_3 - \sigma_4 = 0, \qquad \lambda_4^E + \eta_4 M + \rho_4 - \sigma_4 = 0,$

$\lambda_1^T + \delta_1 M + \gamma_1 - \chi_1 = 0, \qquad \lambda_2^T + \delta_2 M + \gamma_2 - \chi_2 = 0,$

$\lambda_3^T + \delta_3 M + \gamma_3 - \chi_3 = 0, \qquad \lambda_4^T + \delta_4 M + \gamma_4 - \chi_4 = 0,$

$-\nu_1 + \eta_1 + \pi_1 - \delta_1 = 0, \qquad \nu_1 - \nu_2 + \eta_2 + \pi_2 - \delta_2 = 0,$

$\nu_2 - \nu_3 + \eta_3 + \pi_3 - \delta_3 = 0, \qquad \nu_3 + \eta_4 + \pi_4 - \delta_4 = 0,$

$-\nu_1 + \eta_1 - \delta_1 - \theta_1 + \beta_1 = 0, \qquad -\nu_2 + \eta_2 - \delta_2 - \theta_2 + \beta_2 = 0,$

$-\nu_3 + \eta_3 - \delta_3 - \theta_3 + \beta_3 = 0, \qquad \eta_4 - \delta_4 - \theta_4 + \beta_4 = 0,$

$Y_2 - Y_1 - p_1 \geq 0, \quad Y_3 - Y_2 - p_2 \geq 0, \quad Y_4 - Y_3 - p_3 \geq 0,$

$100X_1^E - D_1 + Y_1 + p_1 \geq 0, \quad 100X_2^E - D_2 + Y_2 + p_2 \geq 0,$

$100X_3^E - D_3 + Y_3 + p_3 \geq 0, \quad 100X_4^E - D_4 + Y_4 + p_4 \geq 0,$

$100X_1^T - Y_1 - p_1 + D_1 \geq 0, \quad 100X_2^T - Y_2 - p_2 + D_2 \geq 0,$

$100X_3^T \geq Y_3 + p_3 - D_3, \quad 100X_4^T - Y_4 - p_4 + D_4 \geq 0,$

$1 - X_1^E \geq 0, \quad 1 - X_2^E \geq 0, \quad 1 - X_3^E \geq 0, \quad 1 - X_4^E \geq 0,$

$1 - X_1^T \geq 0, \quad 1 - X_2^T \geq 0, \quad 1 - X_3^T \geq 0, \quad 1 - X_4^T \geq 0,$

$X_1^E \geq 0, \quad X_2^E \geq 0, \quad X_3^E \geq 0, \quad X_4^E \geq 0, \quad X_1^T \geq 0, \quad X_2^T \geq 0, \quad X_3^T \geq 0, \quad X_4^T \geq 0,$

$$Y_1, Y_2, Y_3, Y_4 \geq 0,$$

$\eta_1\left(100X_1^E - D_1 + Y_1 + p_1\right) = 0, \quad \eta_2\left(100X_2^E - D_2 + Y_2 + p_2\right) = 0,$

$\eta_3\left(100X_3^E - D_3 + Y_3 + p_3\right) = 0, \quad \eta_4\left(100X_4^E - D_4 + Y_4 + p_4\right) = 0,$

$\delta_1\left(100X_1^T - Y_1 - p_1 + D_1\right) = 0, \quad \delta_2\left(100X_2^T - Y_2 - p_2 + D_2\right) = 0,$

$\delta_3\left(100X_3^T - Y_3 - p_3 + D_3\right) = 0, \quad \delta_4\left(100X_4^T - Y_4 - p_4 + D_4\right) = 0,$

$\nu_1 (Y_2 - Y_1 - p_1) = 0, \; \nu_2 (Y_3 - Y_2 - p_2) = 0, \; \nu_3 (Y_4 - Y_3 - p_3) = 0,$

$\theta_1(H_1 - p_1) = 0, \; \beta_1(p_1 - h_1) = 0, \; \theta_2(H_2 - p_2) = 0, \; \beta_2(p_2 - h_2) = 0,$

$\theta_3(H_3 - p_3) = 0, \; \beta_3(p_3 - h_3) = 0, \; \theta_4(H_4 - p_4) = 0, \; \beta_4(p_4 - h_4) = 0,$

$\rho_1 X_1^E = 0, \quad \rho_2 X_2^E = 0, \quad \rho_3 X_3^E = 0, \quad \rho_4 X_4^E = 0,$

$\sigma_1(1 - X_1^E) = 0, \ \sigma_2(1 - X_2^E) = 0, \ \sigma_3(1 - X_3^E) = 0, \ \sigma_4(1 - X_4^E) = 0,$

$\gamma_1 X_1^T = 0, \quad \gamma_2 X_2^T = 0, \quad \gamma_3 X_3^T = 0, \quad \gamma_4 X_4^T = 0,$

$\chi_1(1 - X_1^T) = 0, \ \chi_2(1 - X_2^T) = 0, \ \chi_3(1 - X_3^T) = 0, \ \chi_4(1 - X_4^T) = 0,$

$\pi_1 Y_1 = 0, \quad \pi_2 Y_2 = 0, \quad \pi_3 Y_3 = 0, \quad \pi_4 Y_4 = 0,$

$\theta_1, \theta_2, \theta_3, \theta_4, \beta_1, \beta_2, \beta_3, \beta_4, \nu_1, \nu_2, \nu_3, \eta_1, \eta_2, \eta_3, \eta_4, \delta_1, \delta_2, \delta_3, \delta_4, \pi_1, \pi_2, \pi_3, \pi_4,$

$\rho_1, \rho_2, \rho_3, \rho_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \chi_1, \chi_2, \chi_3, \chi_4 \leq 0.$

Where all relations of the above system are evaluated at the solution (Y^*, $\overset{*}{X_j^E}$, $\overset{*}{X_j^T}$, p^*) = (0, 2, 4, 6, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 2, 1). From this system, it can be shown that:

$\eta_1 = \eta_2 = \eta_3 = \eta_4 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \nu_1 = \nu_2 = \nu_3 = \chi_1 = \chi_2 = \chi_3 = \chi_4 = \pi_1 = \pi_2 = \pi_3 = \pi_4 = \sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0$, $\rho_1 = -3$, $\rho_2 = -8$, $\rho_3 = -4$, $\rho_4 = -2$, $\gamma_1 = -1$, $\gamma_2 = -3$, $\gamma_3 = -5$, $\gamma_4 = -4$, $\theta_1 = \beta_1$, $\theta_2 = \beta_2$, $\theta_3 = \beta_3$, $\theta_4 = \beta_4$. Therefore the set S(Y^*, $\overset{*}{X_j^E}$, $\overset{*}{X_j^T}$, p^*), $j = 1, \ldots, n$, is given by:

S(0, 2, 4, 6, 0, 0, 0, 0, 0, 0, 0, 0, 3, 3, 2, 1) = $\{(h_1, h_2, h_3, h_4, H_1, H_2, H_3, H_4) \in R^8 \mid h_1 = H_1 = 3, h_2 = H_2 = 3, h_3 = H_3 = 2, h_4 = H_4 = 1\}$.

The set of feasible parameters is expressed as:

$$U = \{(h_j, H_j) \in R^8, j = 1, \ldots, 4 \mid H_1 \geq 2h_1 - 3, H_2 \geq h_2 - 2, H_3 \geq 2h_3 - 2, H_4 \geq h_4\}.$$

6. CONCLUSIONS

In this paper, we have proposed a solution algorithm for solving the early/tardy scheduling problem with fuzzy parameters in the constraints. Some basic stability notions for the problem of concern have been defined. These notions are the set of feasible parameters, the solvability set and the stability set of the first kind. A parametric study has been carried out for this problem. An illustrative example has been given to clarify the theory and the solution algorithm.

REFERENCES

1. **Dubois D.** and **Prade H., (1980),** Fuzzy sets and systems, theory and applications, Academic press, New York.

2. **Hall N. G.** and **Posner M. E., (1991),** Earliness-tardiness scheduling problems, I: weighted deviation of completion times about a common due date, Operations Research, 39, 836-846.

3. **Hall N. G., Kubiak W.** and **Sethi S. P., (1991),** Earliness-tardiness scheduling problems, II: deviation of completion times about a restrictive common due date, **Operations Research, 39, 847-856.**

4. **Kahlbacher H. G., (1993),** Scheduling with monotonous earliness and tardiness penalties, European J. of Operational Research, 64, 258-277.

5. **Mosheiov G.** and **Lann A., (1996),** Single machine scheduling to minimize the number of early and tardy jobs, Computers Oper. Res., vol. 23, No. 8, 769-781.

6. **Lawler E. L., Lenstra J. K., Rinnooy A. H.** and **Shmoys D. B., (1993),** Sequencing and scheduling: algorithm and complexity, A handbook on operations research.

7. **Moore J. M., (1968),** An n job one machine sequencing algorithm for minimizing the number of tardy jobs, Management. Science, 15, 102-109.

8. **Osman M. S. A., (1977),** Qualitative analysis of basic notions in parametric convex programming, II (parameters in the objective function), Aplikace Mathematiky, 22, 333-348.

9. **Tanaka H., Okuda T.** and **Asoi K., (1973),** Fuzzy mathematical programming, Trans. SICE, 9, 109-115.

10. **Zadeh L.** and **Bellman R., (1970),** Decision making in a fuzzy environment, Management Science, 17, 141-164.