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# An Algorithm for the Consturction of Admissible Tests of the Weibull Distrbution

**By**

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***(Dedicated to the memory of Dr. H. Rawwash)***

نستعمل هنا طريقة حسين الرواش وج . ماردن [2] لتركيب اختبار مقبول سائد على اختبار غير مقبول من حيث اختبار الفرضيات لتوزيع وايبول . ونضمّن الخوارزمية ونتائجها العددية ، لكي نبين سيادة الاختبار الأول على الثاني .

**Key Words**: Admissible test, algorithm, Weibull distribution, negative definite matrices, positive definite matrices, domination.

## ABSTRACT

The method of H. Al-Rawwash and J. Marden [2] is used here for the construction of an admissible test which dominates a given inadmissible one for testing hyupothesis about the Weibull distribution. The algorithm and its mumerical results are included, in order to demonstrate this domination.

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## 1. INTRODUCTION

The Weibull distribution has the density function :

$$f(x/\theta)=\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^{\beta}}$$

where β and θ are positive parameters [5]. Notice that this generalizes the exponential distribution, because this corresponds to the case when β = 1. This case was completely discussed in this aspect in [1]. Our goal is to test the hypothesis.

$H_0$: $\theta = \theta_0$ against $H_1$: $\theta \neq \theta_0$ when β is fixed.

Burnbaum [3] and Mathes and Traux [4] give a comprehensive theory about complete classes of admissible tests of the exponential family model. A class of tests is called complete if given any test not in the class, there exists a test in the class which dominates the given test. A test $\phi_1$ dominates the test $\phi_2$ if the following two conditions are satisfied :

(i) $E_{\theta_0}\phi_1 = E_{\theta_0}\phi_2$
(ii) $E_{\theta}\phi_1 \geq E_{\theta}\phi_2$

A class of tests of the form:

$$\phi^*(x)=\begin{cases}1, if\ x^{\beta}\notin [u,v]\\ 0, if\ x^{\beta}\in [u,v]\end{cases} \quad \text{.........................(1.1)}$$

forms a complete class.

We are interested here in constructing an admissible test from the complete class that dominates a given inadmissible test (not in the complete class). This means that given a test $\phi$ of the form

$$\phi(x)=\begin{cases}1, if\ x^{\beta}\notin A\\ 0, if\ x^{\beta}\in A\end{cases} \quad \text{.................................(1.2)}$$

where $A = \bigcup_{i=1}^{n} [a_i, b_i]$, $0 \leq a_1 \leq b_1 \leq a_2 \leq ... \leq b_n \leq \infty$, we want to construct a test $\phi^*$ of the form (1.1) that dominates it.

Section 2 contains some preliminaries relevant to the problem, section 3 contains the main results, while section 4 contains the algorithm, and the numerical computations of this algorithm, which demonstrate the domination.

2. Preliminaries

First, if we make the change of variables $y = x^{\beta}$, then y has the exponential distribution $Exp(\theta^{\beta})$, which means that the density function $f(\gamma, \theta)$ will be of the form:

$$f(y,\theta)=\frac{1}{\theta^{\beta}}e^{-y/\theta^{\beta}}$$

Second, it is easy to show that when a test $\phi_1$ dominates a test $\phi_2$, than

(i) $E_{\theta_0}\phi_1 = E_{\theta_0}\phi_2$ and
(ii) $E_{\theta_0}x\phi_1 = E_{\theta_0}x\phi_2$

Since we have

$$\int_0^y f(y,\theta)\,dy = 1 - e^{-y/\theta^{\beta}}$$

and

$$\int_a^b f(y,\theta)\,dy = e^{-a/\theta^{\beta}} - e^{-b/\theta^{\beta}}$$

then for a test of the form (1.2), the following holds:

$$E_{\theta}\phi(y) = 1 - \sum_{i=1}^{n}\left\{e^{-a_i/\theta^{\beta}} - e^{-b_i/\theta^{\beta}}\right\}$$

and

$$E_{\theta}y\phi(y) = \theta^{\beta}E_{\theta}\phi(y) - \sum_{i=1}^{n}(a_ie^{-a_i/\theta^{\beta}} - b_ie^{-b_i/\theta^{\beta}})$$

Now, if $a$ and $\gamma$ are given so that

$$E_{\theta}\phi = \alpha \text{ and } E_{\theta}y\phi = \gamma$$

then this leads to the following two equations :

$$\begin{aligned}&\sum_{i=1}^{n}\left\{e^{-a_i/\theta^{\beta}} - e^{-b_i/\theta^{\beta}}\right\} = 1-\alpha\\ &\sum_{i=1}^{n}\left\{a_ie^{-a_i/\theta^{\beta}} - b_ie^{-b_i/\theta^{\beta}}\right\} = \theta^{\beta}\alpha - \gamma\end{aligned} \quad \text{..................(2.1)}$$

To find a test of the form (1.1) such that :

$$E_{\theta}\phi^*(y) = \alpha \text{ and } E_{\theta}y\phi^*(y) = \gamma$$

we have

$$E_{\theta}\phi^*(y) = 1 - \left[e^{-u/\theta^{\beta}} - e^{-v/\theta^{\beta}}\right]$$

and

$$E_{\theta}y\phi^*(y) = \theta^{\beta}\alpha - \left[ue^{-u/\theta^{\beta}} - ve^{-v/\theta^{\beta}}\right]$$

thus, we obtain the following system :

$$\begin{aligned}&e^{-u/\theta^{\beta}} - e^{-v/\theta^{\beta}} = 1-\alpha\\ &ue^{-u/\theta^{\beta}} - ve^{-v/\theta^{\beta}} = \theta^{\beta}\alpha - \gamma\end{aligned} \quad \text{..............................(2.2)}$$

The first equation of this system implies the following two important results:

(i) $e^{-u/\theta^{\beta}} \geq 1-\alpha$, which yields

$$u \leq -\theta^{\beta}\ln(1-\alpha)$$

(ii) When this equation is solved for $v$ in terms of $u$, we obtain: $v(u) = -\theta^{\beta}\left[\ln\left\{e^{-v/\theta^{\beta}} + \alpha - 1\right\}\right]$ .................(2.3)

Thus, for $u \leq -\theta^{u}\ \text{In}\ (1-a)$, $v$ is a function of $u$ given by (2,3), and $v'(u) = \frac{e^{-u/\theta^{\beta}}}{e^{-v(u)/\theta^{\beta}}}$ ......................................(2.4)

### 3. Main Results

Lemma 3. 1: Let g be a finction from the interval $[0, -\theta^{\beta} \ln(1-a)]$ to $\Re$ given by : $\left[0, -\theta^{\beta} ln\ (1-\alpha)\right]$ to $\Re^{+}$ given by:

$$g(u) = ue^{-u/\theta^{\beta}} - v(u)\ e^{-v(u)/\theta^{\beta}}$$

then g' is a monotone function.

***Proof:***

$$g'(u) = -\frac{u}{\theta^{\beta}} e^{-u/\theta^{\beta}} + e^{-u/\theta^{\beta}} + \left(\frac{v}{\theta^{\beta}} e^{-v/\theta^{\beta}} - e^{-v/\theta^{\beta}}\right) \frac{e^{-u/\theta^{\beta}}}{e^{-v/\theta^{\beta}}}$$

$$= \frac{e^{-u/\theta^{\beta}}}{\theta^{\beta}} (v-u)$$

$> 0.$ ■

$> 0.$ ■

We want now to show that the system in (2.1) has a unique solution. To do that, we use the first equation to write down one of the variables as a function of the other variables, and then show that the quantity $\theta^{\beta} a - \gamma$ lies between the negative minimum and the positive maximum values of the monotone function on the left hand side of the second equation.

**Theorem 3.1** : Let $a_i$ and $b_i$ , $i = 1, \ldots, n$, be as specifien in (1.2), and suppose that they satosfy (2.1), then

$$\theta^{\beta}\alpha \ln \alpha \le \sum_{i=1}^{n}\left\{a_i e^{-a_i/\theta^{\beta}} - b_i e^{-b_i/\theta^{\beta}}\right\} \le -\theta^{\beta}(1-\alpha)\ln(1-\alpha)$$

***Proof*** : First, we prove that

$$\sum_{i=1}^{n}\left\{a_i e^{-a_i/\theta^{\beta}} - b_i e^{-b_i/\theta^{\beta}}\right\} \le -\theta^{\beta}(1-\alpha)\ln(1-\alpha)$$

The first equation in (2.1) implies that :

$$e^{-a_1/\theta^{\beta}} = e^{-b_1/\theta^{\beta}} + 1 - \alpha - \sum_{i=2}^{n}\left\{e^{-a_i/\theta^{\beta}} - e^{-b_i/\theta^{\beta}}\right\}$$

Thus,

$$\frac{\partial a_1}{\partial a_i} = -e^{(a_1 - a_i)/\theta^{\beta}}, i = 2, \ldots, n$$

and

$$\frac{\partial a_1}{\partial b_j} = e^{(a_1 - b_j)/\theta^{\beta}}, j = 1, \ldots, n$$

Define the function $f : \Re^{2n-1} \to \Re^{+}$ by

$$f(a_2, \ldots, a_n, b_1, \ldots, b_n) = \sum_{i=2}^{n}\left\{a_i e^{-a_i/\theta^{\beta}} - b_i e^{-b_i/\theta^{\beta}}\right\}, \text{ then}$$

$$\frac{\partial f}{\partial a_i} = \frac{a_1 - a_i}{\theta^{\beta}} e^{-a_i/\theta^{\beta}}, i = 2, \ldots, n$$

and

$$\frac{\partial f}{\partial b_j} = \frac{b_j - a_1}{\theta^{\beta}} e^{-b_j/\theta^{\beta}}, j = 1, 2, \ldots, n$$

For $x^*$ to be a maximizer of $f$ , we must have

$$\left.\frac{\partial f}{\partial a_i}\right|_{x=x^*} = 0 \qquad , i = 2, \ldots, n$$

and

$$\left.\frac{\partial f}{\partial b_j}\right|_{x=x^*} = 0 \qquad , j = 1, \ldots, n$$

and this happens only when

$a_i = a_1$ or $a_i = \infty$ for $i = 2, \ldots n$ and $b_i = a_1$ or $b_i = \infty$ for $J = 1, \ldots n$, But the assumption

$$a_1 \le b_1 \le a_2 \le b_2 \le \ldots a_n \le b_n$$

forces the existence of an integer $k$, $n > k \ge 1$ , such that $a_i = a_1$ for $i \le k$ and $a_i = \infty$ for $k < i \le n$, and $b_j = a_1$ for $J < k$ and $b_j = \infty$ for $k \le j \le n$. This implies that

$b_1 = a_2 = b_2 = \ldots a_{k-1} = b_{k-1} = a_k = a_1$, and

$b_k = a_{k+1} = b_{k+1} = \ldots = a_n = b_n = \infty$.

This reduces the first equation of (2.1) to :

$$e^{-a_1/\theta} = 1 - \alpha$$

which implies that

$$a_1 = -\theta^{\beta} \ln(1-\alpha) = y$$

Therefore, for x* to be a maximizer of $f$ , x* must have the form

$$x^* = [y, \ldots, y, \infty, \ldots, \infty, y, \ldots, y, \infty, \ldots, \infty]^T$$
$$k-1\downarrow \quad n-k \downarrow \quad k-1 \downarrow \quad n-k+1$$

Furthermore, we have to show that the matrix $\nabla^2 f(x^*)$ is negative definite. And to do that, we partition $\nabla^2 f(x^*)$ as follows:

$$\nabla^2 f(x^*) = \begin{bmatrix} C_{11} & \vdots & C_{12} \\ \cdots & \vdots & \cdots \\ C_{21} & \vdots & C_{22} \end{bmatrix}$$

where

$C_{11} = (c_{ij}), c_{ij} = \frac{\partial^2 f}{\partial a_i \partial a_j}$, $i = j = 2, \ldots, n$, $C_{12} = (c_{ij})$, $c_{ij} = \frac{\partial^2 f}{\partial b_i \partial a_j}$, $i = 1, \ldots, n, j = 2, \ldots, n$, $C_{21} = (c_{ij})$, $c_{ij} = \frac{\partial^2 f}{\partial a_i \partial b_j}$ $i = 2, \ldots, n, j = 1, \ldots, n$, and $C_{22} = (c_{ij}), c_{ij} = \frac{\partial^2 f}{\partial b_i \partial b_j}$ , $i, j = 1, \ldots, n$.

Thus, the matrix $\nabla^2 f(x^*)$ has the following structure:

$$\nabla^2 f(x^*) = \begin{bmatrix} -\delta & 0 & \delta & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \delta & 0 & -\delta & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where $\delta = \frac{1}{\theta^\beta} e^{-a_1/\theta^\beta}$.

Let $\tau = (t_2, \ldots t_n, w_1, \ldots w_n)^T \in \Re^{2n-1}$ be a nonzero vector, then

$$\tau^T (\nabla^2 f)\tau = -\delta \left\{ \sum_{i=1}^{n} t_i - \sum_{i=1}^{n} w_i \right\}^2 < 0$$

Therefore, $\nabla^2 f(x^*)$ is a negative definite matrix, and consequently, $x^*$ maximizes the function $f$, and the maximum value of $f$ is :

$$f(x^*) = a_1 e^{-a_1/\theta^\beta} = -\theta^\beta (1-\alpha)\ ln\ (1-\alpha)$$

Second, we prove that :

$$\sum_{i=1}^{n} \left[ a_i e^{-a_i/\theta^\beta} - b_i e^{-b_i/\theta^\beta} \right] \geq \theta^\beta \alpha ln\alpha$$

To do this, we notice that the first equation of (2.1) can be written as follows :

$$e^{-b_n/\theta^\beta} = \sum_{i=1}^{n-1} \left[ e^{-a_i/\theta^\beta} - e^{-b_i/\theta^\beta} \right] + e^{-a_n/\theta^\beta} + \alpha - 1$$

Define the function $g : \Re^{2n-1} \rightarrow \Re^+$ by

$$g(a_1, \ldots, a_n, b_1, \ldots, b_{n-1}) = \sum_{i=1}^{n} \left[ a_i e^{-a_i/\theta^\beta} - b_i e^{-b_i/\theta^\beta} \right]$$

and let x, be a minimizer of g. Then using a similar approach as in the first part of this proof, we find that $x_*$ must have the form :

$$x_* = [0, \ldots, 0, y, \ldots, y, 0, \ldots, 0, y, \ldots\ldots y]^T$$

$$n-k \downarrow \ k-1 \ \downarrow n-k+1 \downarrow k-1$$

where $y = -\theta^\beta ln\alpha$, and

$$g(x_*) = -ye^{-y/\theta^\beta} = \theta^\beta \alpha ln\alpha$$

This completes the proof of the theorem . ■

**Theorem (3.2)** : If $u_o$ is the solution of the equation

$$ue^{-u/\theta^\beta} - v(u)e^{-v(u)/\theta^\beta} = \sum_{i=1}^{n} \left[ a_i e^{-a_i/\theta^\beta} - b_i e^{-b_i/\theta^\beta} \right] \quad \ldots\ldots (3.1)$$

then the test $\phi^*$ given by :

$$\phi^*(x) = \begin{cases} 1, if & x \notin [u_0, v(u_0)] \\ 0, & \text{otherwise} \end{cases} \quad \ldots\ldots (3.2)$$

dominates the test $\phi$ given by (1.2), where

$$v(u) = -\theta^\beta \left[ ln \left\{ e^{-u/\theta^\beta} + \alpha - 1 \right\} \right] \quad \ldots\ldots (3.3)$$

Before we prove this theorem, the following lemma is now in order.

Lemma (3.3) : (Existence and uniqueness) There exists $u_0 \in (0, \theta^\beta \text{In}(1-a))$ which solves (3.1), $u_0$ is unique whenever $a \neq 1$.

***Proof*** : Notice that (3.1) can be written as :

h (u) = g (u) - d

where g (u) is a defined in lemma (3.1), and

$$d = \sum_{i=1}^{n} \left[ a_i e^{-a_i/\theta^\beta} - b_i e^{-b_i/\theta^\beta} \right]$$

Now, $h(0) = -v(0)e^{-v(0)/\theta^\beta} - d = -\theta^\beta \alpha\ ln\alpha - d \leq 0$

and $h\{-\theta^\beta ln(1-\alpha)\} = -\theta^\beta (1-\alpha) ln(1-\alpha) - d \geq 0$

Since h is continuous, the intermediate value theorem guarantees the existence of $u_o$. The uniqueness comes from the monotonicity of h . ■

Proof of theoorem 3.2 : The Completeness of the class of tests of the form (1.1) guarantees the existence of a test $\phi^*$ given in (3.2) that dominates a test $\phi$ of the form (1.2). is $\phi^*$ is unique because

$$E_{\theta_0} \phi^* = \alpha \ \ and \ \ E_{\theta_0} x \theta^* = \gamma$$

and we have shown that any test dominates $\phi$ should satisfy two conditions. Thus, $\phi^*$ is the required test. ■

## 4. Numerical Results

### 4.1 The Algorithm

1. Input $n$, $a_1$, $a_2$, ... $a_n$, $b_1$, ... $b_n$ and $\beta$.
2. Compute

$$\alpha = 1 - \sum_{i=1}^{n} \left[ e^{-a_i/\theta^\beta} - e^{-b_i/\theta^\beta} \right]$$

$$\gamma = \theta^\beta \alpha - \sum_{i=1}^{n} \left[ a^{-a_i/\theta^\beta} - b^{-b_i/\theta^\beta} \right]$$

3. Solve the equation g (u) = d for u, where

$$g(u) = ue^{-u/\theta^\beta} - v(u)\, e^{-v(u)/\theta^\beta},\ v(u) = -\theta^\beta \left[ ln \left\{ e^{-u/\theta^\beta} + \alpha - 1 \right\} \right],$$

$$d = \sum_{i=1}^{n} \left[ a_i e^{a_i/\theta^\beta} - b_i e^{-b_i/\theta^\beta} \right]$$

4. Compare the power function for the given test and the constructed one.

### 4. 2. The Results

***Table 1***

| | inadmissible tests | | | | | | dominating tests | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\phi_l$ | $a_1$ | $b_1$ | $a_2$ | $b_2$ | $a_3$ | $b_3$ | $y$ | $z$ | $y$ | $z$ | $y$ | $z$ |
| $\phi_1$ | 0.20 | 0.60 | 1.00 | 2.00 | | | 0.25 | 1.03 | 0.36 | 1.63 | 0.41 | 1.77 |
| $\phi_2$ | 0.50 | 1.00 | 1.50 | 2.50 | | | 0.53 | 1.32 | 0.65 | 1.97 | 0.72 | 2.17 |
| $\phi_3$ | 0.20 | 0.50 | 0.80 | 1.10 | 1.50 | 1.90 | 0.26 | 0.86 | 0.38 | 1.27 | 0.45 | 1.42 |
| $\phi_4$ | 0.10 | 0.40 | 0.70 | 1.50 | 2.00 | 2.50 | 0.17 | 0.97 | 0.29 | 1.67 | 0.36 | 1.88 |

***Table 2***

***The Power Function For The Inadmissible Tests $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$, and The Corresponding Dominating Tests.***

***( $\beta = 1$ )***

| $\theta$ | $\phi_1$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_2$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_3$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_4$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.200 | 0.676 | 0.720 | 0.834 | 0.871 | 0.924 | 0.932 | 0.962 | 0.973 | 0.699 | 0.739 | 0.854 | 0.896 | 0.499 | 0.580 | 0.762 | 0.833 |
| 0.400 | 0.541 | 0.541 | 0.610 | 0.633 | 0.774 | 0.774 | 0.813 | 0.840 | 0.594 | 0.594 | 0.657 | 0.706 | 0.434 | 0.434 | 0.528 | 0.600 |
| 0.600 | 0.498 | 0.520 | 0.516 | 0.547 | 0.688 | 0.701 | 0.702 | 0.727 | 0.574 | 0.590 | 0.591 | 0.624 | 0.417 | 0.444 | 0.443 | 0.493 |
| 0.800 | 0.489 | 0.544 | 0.492 | 0.510 | 0.642 | 0.680 | 0.644 | 0.662 | 0.581 | 0.619 | 0.584 | 0.602 | 0.422 | 0.488 | 0.426 | 0.456 |
| 1.000 | 0.498 | 0.578 | 0.498 | 5.506 | 0.620 | 0.682 | 0.620 | 0.629 | 0.598 | 0.633 | 0.598 | 0.606 | 0.439 | 0.534 | 0.439 | 0.453 |
| 1.200 | 0.514 | 0.611 | 0.516 | 0.518 | 0.613 | 0.693 | 0.615 | 0.617 | 0.618 | 0.684 | 0.619 | 0.621 | 0.461 | 0.577 | 0.462 | 0.466 |
| 1,400 | 0.535 | 0.642 | 0.538 | 0.536 | 0.615 | 0.708 | 0.619 | 0.616 | 0.639 | 0.711 | 0.642 | 0.640 | 0.484 | 0.613 | 0.490 | 0.486 |
| 1.600 | 0.556 | 0.669 | 0.562 | 0.556 | 0.622 | 0.723 | 0.628 | 0.622 | 0.659 | 0.735 | 0.664 | 0.659 | 0.508 | 0.645 | 0.517 | 0.509 |
| 1.800 | 0.577 | 0.693 | 0.585 | 0.577 | 0.631 | 0.738 | 0.640 | 0.631 | 0.678 | 0.755 | 0.685 | 0.678 | 0.532 | 0.673 | 0.544 | 0.532 |
| 2.000 | 0.597 | 0.714 | 0.607 | 0.597 | 0.642 | 0.752 | 0.653 | 0.642 | 0.695 | 0.773 | 0.703 | 0.695 | 0.554 | 0.696 | 0.569 | 0.554 |

***Table 3***

***The Power Function For The Inadmissible Tests $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$, and The Corresponding Dominating Tests.***

***( $\beta = 2$ )***

| $\theta$ | $\phi_1$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_2$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_3$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ | $\phi_4$ | $\theta_0 = 0.4$ | $\theta_0 = 1$ | $\theta_0 = 1.8$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.200 | 0.993 | 0.993 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.993 | 0.994 | 1.000 | 1.000 | 0.918 | 0.927 | 0.999 | 1.000 |
| 0.400 | 0.735 | 0.735 | 0.894 | 0.937 | 0.958 | 0.958 | 0.983 | 0.992 | 0.752 | 0.752 | 0.908 | 0.956 | 0.534 | 0.534 | 0.834 | 0.924 |
| 0.600 | 0.557 | 0.601 | 0.642 | 0.713 | 0.798 | 0.813 | 0.842 | 0.884 | 0.604 | 0.647 | 0.683 | 0.766 | 0.441 | 0.525 | 0.560 | 0.685 |
| 0.800 | 0.494 | 0.643 | 0.508 | 0.556 | 0.676 | 0.752 | 0.687 | 0.729 | 0.574 | 0.694 | 0.587 | 0.640 | 0.417 | 0.633 | 0.436 | 0.519 |
| 1.000 | 0.498 | 0.714 | 0.498 | 0.517 | 0.620 | 0.761 | 0.620 | 0.641 | 0.598 | 0.759 | 0.598 | 0.619 | 0.439 | 0.726 | 0.439 | 0.474 |
| 1.200 | 0.539 | 0.775 | 0.543 | 0.545 | 0.616 | 0.793 | 0.621 | 0.623 | 0.643 | 0.813 | 0.646 | 0.649 | 0.489 | 0.793 | 0.495 | 0.500 |
| 1,400 | 0.593 | 0.822 | 0.603 | 0.595 | 0.640 | 0.825 | 0.651 | 0.641 | 0.692 | 0.853 | 0.700 | 0.693 | 0.549 | 0.840 | 0.564 | 0.552 |
| 1.600 | 0.647 | 0.857 | 0.660 | 0.648 | 0.674 | 0.854 | 0.690 | 0.674 | 0.736 | 0.882 | 0.747 | 0.737 | 0.608 | 0.873 | 0.628 | 0.609 |
| 1.800 | 0.696 | 0.883 | 0.709 | 0.696 | 0.710 | 0.877 | 0.728 | 0.710 | 0.775 | 0.904 | 0.787 | 0.775 | 0.661 | 0.897 | 0.683 | 0.661 |
| 2.000 | 0.737 | 0.903 | 0.751 | 0.737 | 0.744 | 0.896 | 0.763 | 0.744 | 0.807 | 0.920 | 0.819 | 0.807 | 0.607 | 0.916 | 0.729 | 0.706 |

*Table 4*

***The Power Function For The Inadmissible Tests $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$, and The Corresponding Dominating Tests.***

***( $\beta=3$ )***

| $\theta$ | $\phi_1$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_2$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_3$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_4$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.200 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.400 | 0.956 | 0.956 | 0.996 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 0.956 | 0.956 | 0.996 | 1.000 | 0.792 | 0.792 | 0.989 | 0.999 |
| 0.600 | 0.656 | 0.700 | 0.811 | 0.882 | 0.910 | 0.996 | 0.952 | 0.975 | 0.683 | 0.722 | 0.832 | 0.914 | 0.489 | 0.534 | 0.736 | 0.874 |
| 0.800 | 0.511 | 0.696 | 0.546 | 0.620 | 0.719 | 0.996 | 0.743 | 0.800 | 0.578 | 0.730 | 0.609 | 0.695 | 0.421 | 0.643 | 0.468 | 0.601 |
| 1.000 | 0.498 | 0.784 | 0.498 | 0.525 | 0.620 | 0.997 | 0.620 | 0.650 | 0.598 | 0.812 | 0.598 | 0.628 | 0.439 | 0.771 | 0.439 | 0.491 |
| 1.200 | 0.570 | 0.856 | 0.577 | 0.575 | 0.627 | 0.998 | 0.636 | 0.635 | 0.671 | 0.875 | 0.677 | 0.677 | 0.523 | 0.853 | 0.535 | 0.535 |
| 1,400 | 0.662 | 0.902 | 0.675 | 0.663 | 0.684 | 0.999 | 0.701 | 0.686 | 0.748 | 0.916 | 0.759 | 0.749 | 0.624 | 0.903 | 0.644 | 0.625 |
| 1.600 | 0.742 | 0.932 | 0.756 | 0.742 | 0.748 | 0.999 | 0.767 | 0.748 | 0.810 | 0.941 | 0.822 | 0.810 | 0.711 | 0.933 | 0.734 | 0.711 |
| 1.800 | 0.803 | 0.951 | 0.816 | 0.803 | 0.803 | 0.999 | 0.820 | 0.803 | 0.857 | 0.958 | 0.867 | 0.857 | 0.779 | 0.952 | 0.800 | 0.779 |
| 2.000 | 0.849 | 0.963 | 0.859 | 0.489 | 0.846 | 1.000 | 0.861 | 0.846 | 0.890 | 0.969 | 0.900 | 0.890 | 0.829 | 0.965 | 0.847 | 0.829 |

*Table 5*

***The Power Function For The Inadmissible Tests $\phi_1$, $\phi_2$, $\phi_3$, $\phi_4$, and The Corresponding Dominating Tests.***

***( $\beta=4$ )***

| $\theta$ | $\phi_1$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_2$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_3$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 | $\phi_4$ | $\theta_0$= 0.4 | $\theta_0$= 1 | $\theta_0$= 1.8 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.200 | 1.000 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.400 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.980 | 0.980 | 1.000 | 1.000 |
| 0.600 | 0.796 | 1.000 | 0.937 | 0.974 | 0.979 | 1.000 | 0.994 | 0.998 | 0.806 | 1.000 | 0.948 | 0.985 | 0.579 | 0.787 | 0.891 | 0.973 |
| 0.800 | 0.538 | 1.000 | 0.603 | 0.694 | 0.769 | 1.000 | 0.806 | 0.866 | 0.592 | 1.000 | 0.651 | 0.759 | 0.433 | 0.862 | 0.521 | 0.688 |
| 1.000 | 0.498 | 1.000 | 0.498 | 0.530 | 0.620 | 1.000 | 0.620 | 0.655 | 0.598 | 1.000 | 0.598 | 0.634 | 0.439 | 0.931 | 0.439 | 0.501 |
| 1.200 | 0.604 | 1.000 | 0.614 | 0.609 | 0.646 | 1.000 | 0.658 | 0.652 | 0.701 | 1.000 | 0.710 | 0.706 | 0.562 | 0.964 | 0.577 | 0.572 |
| 1,400 | 0.729 | 1.000 | 0.743 | 0.730 | 0.738 | 1.000 | 0.756 | 0.739 | 0.801 | 1.000 | 0.813 | 0.801 | 0.698 | 0.980 | 0.720 | 0.699 |
| 1.600 | 0.821 | 1.000 | 0.833 | 0.821 | 0.819 | 1.000 | 0.836 | 0.819 | 0.870 | 1.000 | 0.880 | 0.870 | 0.799 | 0.988 | 0.818 | 0.799 |
| 1.800 | 0.881 | 1.000 | 0.890 | 0.881 | 0.877 | 1.000 | 0.830 | 0.877 | 0.914 | 1.000 | 0.922 | 0.914 | 0.865 | 0.993 | 0.880 | 0.865 |
| 2.000 | 0.919 | 1.000 | 0.925 | 0.919 | 0.915 | 1.000 | 0.924 | 0.915 | 0.942 | 1.000 | 0.947 | 0.942 | 0.908 | 0.995 | 0.919 | 0.908 |

## REFERENCES

[1] Al - Hassan Q and Al-Rawwash H. (1995), On the Construction of Admissible Tests for the Exponential Distribution with Numerical Applications, Pakistan J. of Statistics, Vol. 11 (3), pp 203 - 213.

[2] Al - Rawwash H. and Marden (1988), Dominating Inadmissible Tests in Exponential Family Models, in Statistical Decision Theory and Related Topics IV. Edited by S.S. Gupta et. al., Springer - Verlag, pp 225-237.

[3] Burnbaum A. (1955), Characterization of Complets Classes of Some Multiparametric Hypotheses with Application to Likelihood Ratio Tests, Ann. Math. Statist., Vol 28, pp 21 - 36.

[4] Mathes T. K. and Traux D. R. (1967), Tests of Composite Hypotheses for Multidimensional Exponential Families, Ann Math Statist., Vol. 38, PP 221.

[5] Thoman, D. R., Bain, L. J., Antle, C. E. (1969). Inferences on the Parameter of the Weibull Distribution. Technometrics 11,445 - 60.