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S.G. CONNECTED SPACES

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ABSTRACT

The purpose of this paper is to introduce and study a strongr type of connectedness called s.g connectedness in topological spaces.

1. PRELIMINARIES

Definition 1.1: A subset A of a space X is said to be semiopen [5] if there exists of open set G such that G⊂A⊂∈1 (G). A subset B of X is said to be semiclosed [5] if its complement is semiopen.

Remark 1.1: Every open set is semiopen but the converse may not be true. [5]

Definition 1.2: The intersection of all semiclosed sets which contains a subset A of a space X is called the semi closure of A. It is denoted by Scl (A) [4].

Definition 1.3: A subset A of space X is said to be g-closed [6] (resp. sg.-closed [1]) if cl(A) ⊂ O (resp. scl(A) ⊂ 0) and O is open (resp. semiopen), complement of a g-closed (resp. s.g. closed) set is called g-open (resp. sg-open).

Remark 1.2: Every closed set is semiclosed (resp. g-closed) and every semiclosed (resp. g-closed) set is sg.-closed by the separate converse may not be true. [1,6].

Definition 1.4: A space X is said to be s-connected [7] if it is not the union of two non empty disjoint semiopen sets.

Remark 1.3: Every s-connected space is connected but the converse may be false [7].

Definion 1.5: A space (x, τ) is called semi T1/s [8] if every s.g.-closed set is semi-closed.

Definition 1.6: A mapping f: X – Y is said to be s.g. continuous [8] (resp. s.g.-irresolute [8] if the inverse image of each closed (resp. s.g.-closed) set of Y is sg.-closed in X.

2. S.G. CONNECTEDNESS

Definition 2.1: A topological space X is said to be s.g. conneced if X cannot be written as a disjoint union of two non empty s.g. open sets.

Theorem 2.1: Every sg connected space is s-connected.

Proof: Suppose that a space X is sg-connected but not s-connected. Then X is the union of two non empty disjoint semiopen sets. Since every semiopne set is sg.-open. It follows that X can not be s.g.-connected, a contradiction.

Remark 2.1: The converse of theorem 2.1 may not be true for,

Example 2.1: Let X = (a,b,c) and J = (∅, (a), X) be a topology on X. Then (x, τ) is s-connected but it is not sg-connected. However.

Theorem 2.2: If a space X is semi T 1/2 then X is s.g-connected if and only if it is s-connected.

Proof: Follows on utilizing def. 1.5, Def. 2.1 and theorem 2.1.

Theorem 2.3: A space X is sg.-connected if and only if the only subsets of X which are both s.g.-open and s.g.-closed are the empty set Ø and X.

Proof Necessity: Let U be a s.g. open and s.g. closed subset of X. then X-U is both s.g. open and s.g. closed. Since X is the disjoint union of the s.g. open set U and X-U, one of these must be empty, that is U= Ø or U=X.

Sufficiency: Suppose that X=A U B where A and B are disjoint non empty s.g. open subsets of X. Then A is both s.g. open and s.g. closed. By assumption, A = Ø or X.

Therefore X is s.g. connected.

Theorem 2.4: A space X is s.g.-connected if an only if each s.g.-continuous mapping of X into a discrete space Y with atleast two points is constant.

Proof Necessity: Let f : X → Y be a s.g. continuous mapping, then X is covered by s.g. open and s.g. closed covering (f^1 (Y) : y ∈ Y). By theorem 2.3 f^1 (Y) = Ø or X for each y ∈ Y. If f^1 (y) = Ø for all ∈ Y then f fails to be a map. Then, these exists only one point y ∈ Y such that f^1 (y) = Ø and hence f^1 (y) = x which shows that f is a constant map.

Sufficiency: Let U be both s.g. open and s.g. closed in X. Suppose ≠ Ø. Let f:X → Y be a s.g. continuous map defined by f(U) = {y} and f (X-U) = {w} for some distinct points y and w in Y. By assumption, f is constant. Therefore we have U=X.

Theorem 2.5: If f:X → Y is a s.g. continuous surjection and X is s.g. connected then Y is connected.

Proof: Suppose that Y is not connected. Let Y=A UB where A and B and disjoint non empty open sets in Y. Since f is s.g. cotinuous and onto, X=f^1 (A) U f^1 (B) where f^1 (A) and f^1 (B) are disjoint non empty and s.g. open in X. It contradicts the fact that X is s.g. connected. Hence Y is connected.

Theorem 2.6: If f:X → Y is a s.g.-irresolute surjection and X is s.g.-connected then Y is s.g. connected.

Proof: Analogous to the proof of theorem 2.5.

Lemma 2.1: If (X, τ) = X (X_α, T_α): α ∈ ▲) and if A_α is s.g. closed in X_α for each A in ▲ then X (A_α : a c ∈ ▲) is s.g. closed in X [9].

Theorem 2.7: If the product space of two non empty spaces is s.g. connected, then each factor space is s.g. connected.

Proof : Let X x Y be the product space of non empty spaces X and Y. By using lemma 2.1, the projection P:X x Y → X from X x Y onto X, is s.g. irresolute. By theorem 2.6, the s.g. irresolute image P(X x Y) (X) of s.g. connected space X x Y, is s.g. connected. The proof for a space Y is similar to the case of X.

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