Engineering Journal of the University of Qatar, Vol. 12, 1999, PP. 151-164

MULTICRITERION OPTIMIZATION APPROACH FOR THE DESIGN OF INDUCTION MOTORS

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ABSTRACT

In most engineering design problems, trade offs are to be made among competing factors. This is obvious in the design of electric motors, when more than viewpoint is to be considered. In this paper the problem of induction motor design is treated as a multicriterion optimization problem with three objective functions. The preferred solutions are investigated by implementation of weighting min-max approach.

INTRODUCTION

In the design of induction motors, as in all design problems, the engineer is faced with making a decision in the face of competing objectives. The design problem is considered as an optimization process by which the designer seeks the best (optimum) design. In certain approaches, the optimization is achieved by applying the principles of nonlinear programming. In stating a nonlinear programming problem, the basic variables are to be defined and an objective function is to be minimized (or maximized) subjected to non violated constraints. The objective function is a scalar when the designer is interested in achieving one goal.

These are many papers [1-14] that represent the design problem of an induction motor (IM) as a scalar nonlinear programming problem. In the application of nonlinear programming techniques to optimize the design of IM the objective function may be the annual cost, the material cost, the power factor, or the weight as in airplanes, or a certain performance parameter. In such cases the designer deals with the objective function in a scalar form.

In fact, the designer of an induction motor, as in all design problems, is interested in achieving more than one goal. In such case the objective function is defined as a vector function consisting elements each one representing a goal. The goals may be given equal or different weights according to the design requirements. The functions are non-commensurable and usually some of them are in conflict with others. In general there is no single solution which gives the best values for all functions. But the designer will concern on a set of solutions known as non-dominant (non-inferior, Pareto) set. Where he can select the preferred one which achieve his goal to large extent and this design is considered as the optimal solution for the problem. In this case the problem is known as a multicriterion optimization problem.

The only available paper to the authors , which represents the design problem of a three-phase IM as a multicriterion optimization problem, is that due to Jazdzynski [15]. In this paper the idea of multicriterion optimization approach is applied to the design of induction motor. The multicriterion optimization problem is defined as a multiple criteria decision making (MCDM) problem. In this paper the design of a three-phase induction motor is represented as a multicriterion optimization problem with three objective functions.

MULTICRITERION OPTIMIZATION [6-19]

For a set of n design parameters $[x_1,x_2.....,x_n]$ which are known as basic variables. Let the n-vector $\underline{X}$ be defined as:

$$\underline{X}=[x_1,x_2.....,x_n]$$

So $\underline{X}$ may be viewed as a point in the n-dimensional basic variable space. The k-design objectives f_j , j=1,2,...k will be denoted by the k-vector $\underline{F}$

$$\underline{F}(\underline{X})= [f_1(\underline{X}),f_2(\underline{X}),......,f_k(\underline{X})]$$

So $\underline{F}(\underline{X})$ may be viewed as a point in the k-dimensional objective function space.

The objective function is to be optimized subject to unequal constraint $\underline{G}(\underline{X})$ and equal constraints $\underline{H}(\underline{X})$

Which can be expressed as:

$\underline{G}(\underline{X}) < 0$
$\underline{H}(\underline{X}) = 0$

So, the multicriterion optimization problem can be stated as:

Find ($\underline{X}$)

which :Minimize (Maximize) $\underline{F}(\underline{X})$
Subject to:

$\underline{G}(\underline{X}) < 0$
$\underline{H}(\underline{X}) = 0$

That is we wish to simultaneously minimize the individual components of $\underline{F}$ subject to the given constraints. Usually some of the components of $\underline{F}$ are in competitive. So, there will be no optimal solution to the multicriterion optimization problems. The concept of non-inferiority (non-dominated) is used to characterize the solution to the problem. The non-dominated set of solutions includes all feasible points which, are characterized by the impossibility of transforming from one point to another without decreasing one objective function. Generally there are large number of non-inferior points for a given multicriterion optimization problem. The non-inferior set is the collection of the non-inferior points. The image of the non-inferior set by $\underline{F}$ is called the non-inferior solution set. So the procedure for solving MCDM problems will generate local non-inferior points.

It is not possible to discuss all the techniques used to find the preferred solution from the non-inferior set. However the weighting min-max method is discussed here.

WEIGHTING MIN-MAX METHOD [16]

This method of optimization utilizes two concepts, the min-max concept and the weighting concept. The min-max optimum compares the relative deviations from the separately attainable minima (the ideal solution-f_i^0). Consider the ith objective function for which the relative deviation can be calculated from:

$$z_i'(\underline{x}) = \frac{\left|f_i(\underline{x}) - f_i^0\right|}{\left|f_i^0\right|} \tag{1}$$

or from

$$z_i''(\underline{x}) = \frac{\left|f_i(\underline{x}) - f_i^0\right|}{\left|f_i(\underline{x})\right|} \tag{2}$$

Let $\underline{z}(\underline{x}) = [z_1(\underline{x}),\ldots.., z_i(\underline{x}),\ldots, z_k(\underline{x})]$ to be vector of the relative increments which are defined in E^k. The components of the vector $\underline{z}(x)$ will be evaluated from the formula:

$$\bigwedge_{i\in I}\left[z_i(x) = \max\left\{z_i'(x), z_i''(x)\right\}\right] \tag{3}$$

where $i=[1,2,\ldots.,k]$ is used to denote the set of indices for all the objective functions.

The min-max optimum is defined as follows:

A point $\underline{x}^* \in X$ (X- the feasible set) is min-max optimal, if for every $\underline{x} \in X$ the following recurrence formula is satisfied:

Step 1:

$$v_1\left(\underline{x}^*\right) = \min_{x\in X}\ \max_{i\in I}\left\{z_i(\underline{x})\right\} \tag{4}$$

and then $I_1=\{i_1\}$, where i_1 is the index for which the value of $z_i(x)$ is maximal .

If there is a set of solutions $X_1 \subset X$ which satisfies step 1, then

Step 2:

$$v_2\left(\underline{x}^*\right) = \min_{x\in X_1}\ \max_{\substack{i\in I\\ i\notin I_1}}\left\{z_i(\underline{x})\right\} \tag{5}$$

and then $I_1=\{i_1, i_2\}$, where i_2 is the index for which the value of $z_i(x)$ in this step is maximal

If there is a set of solutions $X_{r-1} \subset X$ which satisfies step r- 1 then

Step r:

$$v_r\left(\underline{x}^*\right)= \min_{x\in X_{r-1}} \max_{\substack{i\in I \\ i\notin I_{r-1}}} \{z_i(\underline{x})\} \tag{6}$$

and then $I_1=\{I_{r-1}, i_r\}$, where i_r is the index for which the value of $z_i(x)$ in the rth step is maximal

If there is a set of solutions $X_{k-1} \subset X$ which satisfies step k- 1, then

Step k:

$$v_k\left(\underline{x}^*\right)= \min_{x\in X_{k-1}} \{z_i(\underline{x})\} \qquad \text{for } i \in I \text{ and } i \notin I_{k-1} \tag{7}$$

where $v_1(\underline{x}^*),\ldots.., v_k(\underline{x}^*)$ is the set of optimal values of fractional deviations ordered non-increasingly.

The optimum can be described as follows: Knowing the extremes of the objective functions, which can be obtained by solving the optimization problem for each criterion separately, the desirable solution is that one which gives the smallest values for the relative increments of all the objective functions. In most optimization models the min-max optimum is determined in the first step eq.(1). The min-max approach gives a solution, which treats all the criteria on terms of equal importance.

The weighting concept may be used together with min-max approach to get the weighting min-max approach. A set of weights is used to represent the criteria importance. In this case a weighting coefficients are assigned to the relative deviations and eq. (3) become:

$$\bigwedge_{i\in I}\left[z_i(\underline{x}) = \max\left\{w_i z_i'(\underline{x})\ , w_i z_i''(\underline{x})\right\}\right] \tag{8}$$

Since in eq.(8) the weighting coefficients refer to relative deviations which are non-dimensional, the assumed values of w_i reflect exactly the priority of the criteria. The weighting min-max can be used interactively by suggesting a set of weights, which represent the importance of criteria to the DM. If the solution obtained by this approach is not satisfactory to the DM, then he may suggest another set of weights in the neighborhood of the region of interest and all of the solutions are obtained from the non dominated set.

IMPLEMENTATION OF MCDM TO THE DESIGN OF A THREE PHASE IM

The application of the multicriterion optimization to the design of an induction motor is achieved by two steps; in the first step the design problem is formulated as a multicriterion optimization problem. In the second step, the problem is solved using the scalar and vector optimization techniques.

Problem Formulation

The problem is formulated by defining the basic variables, constraints and objective functions in terms of motor parameters.

The following parameters are chosen as the basic variables:

1. Stack length (L)
2. Stator bore diameter (D)
3. Mean stator slot height (h_1)
4. Mean stator slot width (w_1)
5. Mean rotor slot height (h_2)
6. Mean rotor slot width (w_2)
7. Wire diameter (W_d)
8. Maximum air gap flux density (B_g)
9. End ring height (R_h)
10. 10.End ring width (R_w)
11. 11.Stator back iron or yoke depth (Y_d).

These variables (all in cm) represent the motor volume (L & D),the motor laminations (h_1,h_2,w_1,w_2,Y_d), winding dimensions (W_d,B_g) and end ring dimensions(R_h,R_w).

The most common constraints for three-phase motor are:

1) P.u. starting current (g_1).
2) P.u. starting torque (g_2).
3) P.u. pull out torque (g_3).
4) Full load slip (g_4).
5) Full load current density (g_5).
6) Slot fullness (g_6).
7) Saturation factor (g_7).

These constraints are chosen to ensure acceptable motor performance(g_1,g_2,g_3,g_4) ,limited temperature rise(g_5)[20], reliable motor winding (g_6). Also it can be secured that flux density at any magnetic path will not exceed the maximum possible value by limiting the maximum allowable value of the air gap flux density and the saturation factor.

In this problem the vector objective function chosen is of three elements, efficiency (f_1), active material cost (f_2) and power factor (f_3).The active material cost is normalized by dividing it by a base value to make it comparable with other objective function elements.

These elements represent non-commensurable functions. Each function represents the desired goal from one viewpoint. The efficiency represents the goal for the consumer, material cost represents the manufacturer goal while the power factor is the electric utility goal.

Problem Solution

The problem solution is achieved by suggesting a design, which takes in consideration the three goals mentioned in the previous section. The problem is solved by building a computer package for this task. The package is written in FORTRAN 77 language and with 1475 main statements.

The approach of MCDM is applied to optimize the design of a three- phase IM, which is already in production by Sate Electrical Industries Company (SEICO). The motor is 380 V, 50 Hz, 11 kW, 4-poles, Y-connected, squirrel cage rotor. The original motor dimensions are given in table (1). These values represent the starting (initial) values of basic variables in the optimization procedure.

The table gives also the original values of the elements of objective function vector. The design procedure is achieved by representing the motor by its equivalent circuit based on the revolving magnetic field theory. The motor performance functions are predicted from this circuit. The equivalent circuit approach will give results, which are close to the test results if the necessary modifications are taken into account. These modifications are due to skew, temperature, saturation and skin effects. The iron, friction and windage losses are taken into consideration [21]. The validity of performance prediction program is verified by comparing the program results with the manufacturer test results.

The optimization problem is solved using weighting min-max algorithm. In this algorithm, a set of weights is suggested by the Decision Maker (DM) and the algorithm will find the optimal solution according to these weights. The algorithm will search for the optimal solution using a scalar optimization technique as a tool. The scalar optimization technique used in this package is the flexible tolerance method [22]. In this method a polyhedron is constructed in variable space and an iterative procedure is adopted to improve the function values at the vertices of this polyhedron. The search is terminated when the polyhedron is converged practically to a point.

Table 1: Initial values of basic variables and objective function vectors

(All dimensions are in cm)

L	D	h_1	w_1	h_2	W_2	W_d	B_g	R_h	R_w	Y_d
15.0	15.20	1.837	1.01	2.2	0.4	0.23	0.894	3.2	1.6	2.435

f_1	f_2	f_3
-0.8646	0.9807	-0.8830

The first step in the weighting min-max is to find the ideal solution. The ideal solutions of this problem are shown in tables (2-4). It is seen from these tables, that the maximum efficiency is 91.82% with very high cost (77) p.u. This design represents a motor with large dimensions and low flux density in order to reduce the motor losses. The large value of cost indicates that efficiency has limited upper value even for a very large cost and it could never reach 100% in contrary to "modal low"[1,2,5]. From table (3) it is seen that maximum power factor is 92.63% which was obtained for motor with large D & L but slot dimensions are limited and end rings are reduced in order to increase the ratio R/X.

Table 2: Ideal (absolute) solution of efficiency objective function and the corresponding basic variables

(All dimensions are in cm)

L	D	h_1	W_1	h_2	W_2	W_d	B_g	R_h	R_w	Y_d
35.10	20.82	35.67	4.93	2.03	0.437	2.34	0.528	1.2	10.51	11.97

f_1	f_2	f_3
-0.9182	77.195	-0.8406

Table 3: Ideal (absolute) solution of power factor objective function and the corresponding basic variables

(All dimensions are in cm)

L	D	h_1	W_1	h_2	W_2	W_d	B_g	R_h	R_w	Y_d
34.66	24.44	1.837	0.101	0.101	0.84	0.203	0.388	0.176	0.105	21.47

f_1	f_2	f_3
-0.8397	13.623	-0.9263

Table 4: Ideal (absolute) solution of cost objective function and the corresponding basic variables

(All dimensions are in cm)

L	D	h_1	W_1	h_2	W_2	W_d	B_g	R_h	R_w	Y_d
11.85	13.09	1.711	1.037	0.561	0.881	0.209	0.999	5.60	0.707	1.981

f_1	f_2	f_3
-0.8391	0.6696	-0.8834

The above two ideal solution designs could not be of practical importance due to their large dimensions and cost but they indicate the ability of the algorithm to cover wide space of variables. The ideal solutions could be practical designs if explicit constraints are added on the variables.

The third ideal solution is shown in table (4) for minimum cost. The minimum cost obtained is about 0.67 p.u. with the other two objective functions are not much deviated from the their values of the original design. Hence the minimum cost design may be accepted from manufacturer viewpoint.

The DM will suggest weights and the algorithm will select the design from Pareto set according to these weights. To give an idea about the application of this algorithm in this problem, the following set of weights are given (1/3,1/3,1/3), (0.1,0.8,0.1), (0.8,0.1,0.1),(0.1,0.1,0.8). The sets represent different degrees of importance of objective functions. For example the set (1/3,1/3,1/3) will give a design which takes the three objective functions with equal importance. The other sets, as shown, concern on one objective function while other two functions are taken with lower importance. The results are shown in tables (5,6). As seen from these tables, the suggested design give values of objective functions according to their weights. For example, when the cost of prime importance (0.8) the objective function value is 0.687 while efficiency and power factor are less then the initial values. For (0.8,0.1,0.1) the efficiency is 0.87 and the cost is less than the initial value but the power factor is less than the initial value. For (0.1,0.1,0.8) it is seen that an improvements are achieved in the power factor and cost while a reduction is occurred in the efficiency. From these results it is observed that it could not improve all the objective functions simultaneously because the original design is belonging to the Pareto set. This is achieved by long experience of the manufacturer. But the suggested designs may be superior to the original design from different points of view.

The designs in tables (5,6) are obtained with realistic dimensions, hence the designs are of practical importance unlike the ideal solutions for efficiency and power factor.

From these results it is observed that a sensible reduction in the active material cost is achieved with the variations in the efficiency and power factor are limited.

The DM may continue giving weights and check the designs suggested by the algorithm. The procedure may continue until reaching the 'preferred solution' which verify the implicit requirements of the DM.

Table 5: Multiobjective optimization results obtained by weighting min-max method with equal function weights

(All dimensions are in cm)

L	D	h_1	W_1	h_2	W_2	W_d	B_g	R_h	R_w	Y_d
14.45	13.31	1.402	1.021	1.09	0.771	0.208	0.995	3.261	1.171	2.03

f_1	f_2	f_3
-0.8576	0.7235	-0.8633

Table 6: Multiobjective optimization results obtained by weighting min-max method with different function weights

(All dimensions are in cm)

L	D	h_1	W_1	H_2	W_2	W_d	B_g	R_h	R_w	Y_d
13.20	12.84	1.601	0.99	0.845	0.887	0.209	1.00	3.688	0.846	1.939

f_1	f_2	f_3	w_1	w_2	w_3
-0.8482	0.6872	-0.8642	0.1	0.8	0.1

L	D	h_1	W_1	H_2	W_2	W_d	B_g	R_h	R_w	Y_d
14.70	13.24	2.041	1.051	0.907	0.860	0.254	0.998	4.94	1.50	2.122

f_1	f_2	f_3	w_1	w_2	w_3
-0.8789	0.9156	-0.8552	0.8	0.1	0.1

L	D	h_1	W_1	H_2	W_2	W_d	B_g	R_h	R_w	Y_d
9.812	16.45	1.48	1.20	0.386	0.953	0.208	1.00	4.68	0.803	3.452

f_1	f_2	f_3	w_1	w_2	w_3
-0.8476	0.8091	-0.9062	0.1	0.1	0.8

CONCLUSIONS

The design problem of a three phase induction motor is represented as a multicriterion decision making problem. The weighting min-max algorithm is used to solve this problem. The method is applied to optimize the design of a motor which is produced locally by SEICO. The results obtained indicate the capability of this approach to suggest different solutions according to the design requirements.

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