OPTIMAL PRICING AND ORDER QUANTITY STRATEGIES FOR A FIRM OFFERING MULTIPLE PRODUCTS FACING CUSTOMERS CANNIBALIZATION AND RANDOM MARKET DEMAND

HAMOUDEH SALEH AL-ZAINI
HSBC Bank Middle East Limited
E-mail: hamoudehelzaini@hsbc.com

SYED ASIF RAZA
College of Business and Economics
Qatar University
E-mail: syedar@qu.edu.qa

ABSTRACT

This paper studies a firm's profitability problem offering its products into different market segments at differentiated prices. In order to improve the firms' profitability the firm needs to decide the prices and order quantities allocations for each market segment. In perfect market segmentation, it is assumed that the customers do not cannibalize between market segments. Whereas, in the case of imperfect market segmentation, the customers are assumed to cannibalize from a high price market segment to a lower price segment. Models to determine the optimal strategies for pricing and order quantity for the perfect as well as for the imperfect market segmentations are proposed with both the deterministic and stochastic Customers' demand. The study has shown that the perfect market segmentation always yields higher revenues compared to no segmentation for a firm facing both the deterministic and stochastic demand situations. In addition to this, the study has also shown that when cannibalization exists, a firm is still able to yield higher revenues compared to the case of no market segmentation facing both the deterministic and stochastic demands, however, greater the extent of cannibalization could result substantial losses in the profitability.

Keywords: Revenue Management, Pricing, Market Segmentation, Cannibalization, Inventory control

1. INTRODUCTION

Revenue Management (RM) has been well recognized as an essential practice in many businesses. RM is loosely defined as the set of strategies adopted by a business to improve its profitability (Philips, 2005). According to a detailed literature review presented in McGill and Van Ryzin (1999) it is notified that among many businesses, the airline industry is perhaps the major user of RM tools, however, most RM tools find applications in many other industries, such as, Retail stores, Hotels, Car rentals, etc. An airline RM practices are categorized into four: forecasting; overbooking; inventory control (booking control); and pricing. All of these categories are well researched, however, in experts' opinion the integration of pricing and inventory control is expected

Studies in Business and Economics Vol. 15 No. 2

to improve firms' revenues significantly (Cote et al., 2003).

In addition to this in marketing and strategy, the cannibalization refers to a reduction in the sales volume, sales revenue, or market share of one product as a result of the introduction of a new product by the same producer. For example, if Pepsi, were to introduce a similar product such as Diet Pepsi, then this new product could take some of the sales away from the original Pepsi. Cannibalization is a key consideration in product portfolio analysis and product line design. A second commonly noticed example of cannibalization is when companies, particularly retail companies, open outlets too close to each other. Much of the customers for the new outlet could have come from the old outlet. Thus, the potential for cannibalization is often discussed when considering companies with many outlets in an area, such as Pizza Hut, Carrefour. Another example of cannibalization is when a firm creates a promotion like 20% discount for one item (for example Pepsi). The tendency of consumers is to buy the discounted item (Pepsi) rather than the other items with a higher price. However when the promotion event is over, the regular drinker of Coke will resume buying Coke. By this behavior, there is a temporary cannibalization happening due to a promotion event.

Although business cannibalization may seem inherently negative, it can be a positive thing. It sometimes involves a carefully planned strategy, and it also forces a company to think outside the box in order to evolve with the changing needs of both the marketplace and the consumer. In the world of e-commerce for example, some companies intentionally cannibalize their retail sales through lower prices on their online product offerings. Seeing these discounts on items, more number of consumers than usual would buy these items. Even though their in-store sales might decline, the company may see overall positive gains. In project evaluation, the earnings on the lost sales must reduce the estimated profit generated from the new product.

The paper is organized in the following sections. In Section (2), we outline a brief literature review about RM and pricing problem. In Sections (3), we discuss the background of the problem of achieving Optimal pricing in order to maximize the generated revenue. In section (4), we define the problem and explain our approach in solving the problem in details. In addition, we illustrate the optimal pricing approach by using numerical example. In section (5), we describe and analyze our results. Finally, in section (6), we state our conclusion and our recommendation for future research.

II. LITERATURE REVIEW

A single period newsvendor problem is a building block in stochastic inventory control. It incorporates fundamental techniques of stochastic decision-making and can be applied to a much broader scope. The problem is well researched that its history traces back to Edgeworth (1888), where it first appeared in the banking context. During the 1950's, war effects enabled the expansion of research in this area, leading to the formulation of this problem as the inventory control problem. Arrow et al. (1951) showed that it is critical to have optimal buffer stocks in an inventory control system. Porteus (1990), and Lee and Nahmias (1990) presented although review of the newsvendor problem using a stochastic demand. In most studies, the pricing is considered as a fixed parameter rather than a decision variable. Whitin (1955) was the first to discuss the pricing issues in the inventory control theory. Mills (1959) extended Whitin's work by modeling the uncertainty of the price sensitive demand. He suggested an additive form for the study and assumed that the stochastic demand was a summation of the price-dependent risk-less demand and of the random factor. The risk-less demand is considered a deterministic function of the price. The most evident benefit of such modeling is that the random behavior of the demand is captured using standard distributions independent of the pricing. Karlin and Carr (1962) presented a multiplicative form of the demand. In this model, the random demand is considered as the product of the riskless demand function and of the random factor. Both the additive and multiplicative models are fundamental to the pricing problem. Some subsequent contributions to the additive model are due to Ernst (1970), Young (1978), Lau and Lau (1996) and Petruzzi and Dada (1999). The contributions to the multiplicative model include Nevins (1966), Zabel (1970), Young (1978) and Petruzzi and Dada (1999). Mieghem and Dada (1999) studied the quantity and pricing of the price versus the production postponement in the competitive market. A coordination of the dynamic joint pricing and production in a supply chain is studied in Zhao and Wang (2002) using a leader/follower game. Optimal control policies are identified for the channel coordination. Bish and Wang (2004) studied the optimal resource investment decision on a two-product, price-setting firm that operates in a monopolistic market and that employs a postponed pricing scheme. The principles on the firm's optimal resource investment decision are provided. Gupta et al. (2006) developed a pricing model and heuristic solution procedures for clearing end-of-season inventory. Yao et al. (2006) revisited the standard newsvendor problem and its extension with pricing. The work generalizes the problem under both the additive and multiplicative modeling approaches and shows quasi-concavity of the revenue function of the problem under various stochastic demand distributions. Chen et al. (2006) addressed dynamic adjustment of the production rate and of the sale price to maximize the long run discounted profit. An algorithm is proposed to compute the base stock level and price switch threshold. Extension to the multiple price choices is also presented. Bell and Zhang (2006) examined the different decisions surrounding the implementation of an aggressive RM pricing in a firm facing a single period stochastic pricing and a stocking problem. They also identified decisions that have large financial effects. Bhargava et al. (2006) studied the optimal stock out compensation in the electronic retail industry using price as the decision tool.

Differentiated pricing is among essential RM techniques to improve the profitability of a firm. Firms offer their products to their customers in different markets who may have different willingness to pay. There are several papers discussing the price discrimination. Narasimhan (1984) discussed the price discrimination theory of coupon. Smith et al. (1991) studied the issue of fairness in consumer pricing. Philips (2005) presented a riskless model for revenue optimization with differentiated pricing of a single product offered into two market segments, full (high) and discounted (low) price markets. The market segmentation was assumed imperfect and thus the customers who belong to the group with high willingness to pay could opt to purchase the product from lower price market segment. This issue of imperfect market segmentation is known in marketing literature as *Cannibalization*.

In the next section, we discuss the pricing differentiation's background; define the problem of pricing differentiation for situations of riskless and stochastic customers' demand in a segmented market.

III. BACKGROUND AND PROBLEM DEFINITION

Many firms, such as airlines, car rentals, hotels, restaurants, etc., usually face difficulties in forecasting the customers' future demand while planning for their pricing strategy since the historical information represents only the sales and not the customers demand. This concept is explained in Figure 1, where the total quantity demand for a product is the area (0AC), but the revenue generated at the selling price (p_1) and demand (q_1) is actually the area under (0p1Bq1). Thus, the difference between the two areas is surplus demand, or the lost sale opportunity.

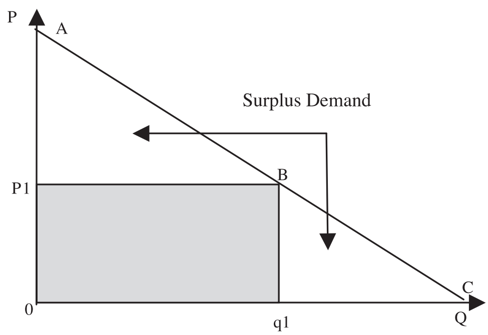
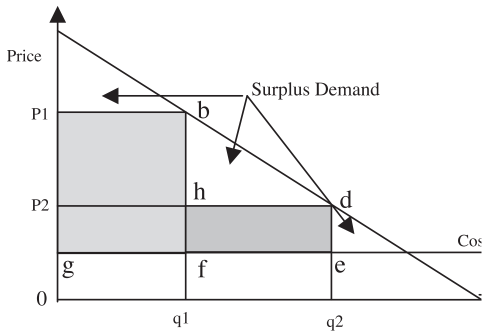


Figure 1: Product Demand vs. Actual Sales

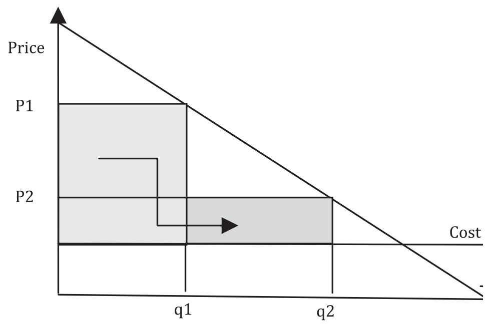
Since firms usually try to maximize their generated revenues from their products, they tend to group their customer into different segments based on the customers' price willingness-to-pay attitude, and by offering each segment tailored products with different prices. Figure 2 shows an example of two-customer segments were the high-price segment is offered a product with selling price p_1 and the relevant demand is q_1, and the low-price segment product price is p_2 and the related customers' demand is q_2. Because of the market segmentation, the total revenue is increased and it is now equal to both areas (p1bfg) and (hdef). Although the customers' surplus demand is now reduced, yet more revenue is captured. The unsatisfied demand level still can be noticed, and more revenue could be generated.

Figure 2: Two-Product Perfect Market Segmentation



Thus, it is in the best interest of a firm to segment the market demand. If the market segments are completely independent and the firm faces no capacity constraints, then the determination of the differentiated prices is quite simple. However, when the market segmentation is incompletely independent then there exists a phenomenon called cannibalization. Unfortunately, in most market situations perfect market price differentiation is usually impossible. Cannibalization is likely to happen whenever the customers cannot be perfectly segmented based on their willingness to pay and rationalities. Arbitrage is likely whenever a third party can purchase the product at a low price and resale it at a high price. Regional pricing is subject to arbitrage whenever a product can be purchased in a low-price region and transported cheaply to be resold at a higher price elsewhere. Figure 3 shows the phenomenon of high price market segment customers cannibalized to low price market segment.

Figure 3: High-Price Customers Cannibalize to low-Price product



IV. THE MATHEMATICAL MODELS

In this section, we first discuss the riskless model. The model is presented in Phillips (2005) which considers the effect of cannibalization under price dependent riskless demand. Later in this study, the models presented in Phillips (2005) are extend to the case in which the market demand is stochastic and price dependent, and thus the model enables consideration of market demand risk.

1. The riskless model:

To analyze differentiated pricing with cannibalization. First, we briefly discuss the problem of riskless pricing of the product that is offered to two market segments. We also assume that cannibalization exists, and customer cannibalize from high priced market segment to a lower priced segment. Under cannibalization, we consider a product offered by a firm to two marketing segments. Customers' segments are assumed to have cannibalization due to imperfect market segments. The product has a cost c, and it is offered to two markets segments with two different prices p_1 and p_2. Market segment 1 is restricted to the customers who are willing to pay high price p_1, and market segment 2 is priced at lower price p_2. Since the first market segment has customers with more willingness to pay compared to the second segment, thus. $p_1 > p_2 > c$.

The customers' riskless price dependent demand, $y_1(p_1)$ related to the high price market segment follows increasing price elasticity (IPE) property. Similarly, the customers' riskless price dependent demand related to the lower prices market segment is, $y_2(p_2)$ and it follows the IPE property. Both the demands are assumed linear, thus $y_1(p_1) = [\alpha_1 - \beta_1 p_1]^+$ where $[x]^+ = max\{x, 0\}$. Likewise, for the lower price segment, the riskless demand is assumed to be $y_2(p_2) = [\alpha_2 - \beta_2 p_2]^+$. Thus, the maximum price which a firm can set for market segment2 1 and 2 are and $\overline{p_1} = \frac{\alpha_1}{\beta_1}$ and $\overline{p_2} = \frac{\alpha_2}{\beta_2}$ respectively. Given that β_1 and β_2 are equal, α_1- α_2 is the maximum demand that attributes to the high price customer segment. Assuming there is market cannibalization represented by a factor θ, the adjusted demand for each market segment would be:

$$d_1(p_1,\theta) = (1-\theta)\ y_1(p_1) \qquad (1)$$

$$d_2(p_2,\theta) = y_2(p_2) + (\alpha_1 - \alpha_2)\ \theta \qquad (2)$$

With the conditions

$\overline{p_2} < p_1 \leq \overline{p_1},\ c < p_2 \leq \overline{p_2}$, and $0 \leq \theta \leq 1$

Thus, the pricing optimization problem would be:

$$\pi(p_1,p_2) = \max_{p_1,p_2}(p_1 - c)\, d_1(p_1,\theta) + (p_2 - c)\, d_2(p_2,\theta) \qquad (3)$$

Subject to:

$\overline{p_2} < p_1 \leq \overline{p_1},\ c < p_2 \leq \overline{p_2}$, and $0 \leq \theta \leq 1$

2. The risk based model

Now, we consider the problem in which the firm is facing stochastic price dependent demand in both market segments, and with the affect of cannibalization. It is assumed that RD_1 is the stochastic price dependent demand experienced in full price market segment 1, and likewise RD_2 is the stochastic price dependent demand for the discounted price market segment 2. Additive approach is used to model both RD_1 and RD_2.

RD_1 has an additive random factor ξ_1 such that $\xi_1 \in [\underline{\xi 1}, \overline{\xi 1}]$, and similarly, RD_2 has an additive random factor ξ_2 such that $\xi_2 \in [\underline{\xi 2}, \overline{\xi 2}]$. For simplification in the analysis, the expected for both the ξ_1 and ξ_2 is zero.

Thus, RD_1 and RD_2 are given by:

$$RD_1 = d_1(p_1,\theta) + \xi_1 \qquad (4)$$

$$RD_2 = d_2(p_2,\theta) + \xi_2 \qquad (5)$$

The total revenue for the two products is written as:

$$\pi(\boldsymbol{p},\boldsymbol{q}) = \max_{\boldsymbol{p},\boldsymbol{q}}(p_1 - c)\ min\{q_1, RD_1\} + (p_2 - c)\ min\{q_2, RD_2\} \qquad (6)$$

Subject to:

$p_2 < p_1 \leq p_1,\ c < p_2 \leq p_2$, and $0 \leq \theta \leq 1$

Where: : $\boldsymbol{p} = (p_1, p_2)$ and $\boldsymbol{q} = (q_1, q_2)$

Assuming that each of the two random factors ξ_1 and ξ_2 follow a continuous Probability Distribution Functions (PDFs) $f_1(.)$, and $f_2(.)$ respectively. Whereas, their cumulative Probability Distribution Functions (CFDs) are $F_1(.)$, and $F_2(.)$ respectively.

Upon simplification, the following expression is derived for the revenue:

$$\pi(\boldsymbol{p},\boldsymbol{q}) = p_1q_1 + p_2q_2 - p_1\int_{\underline{\xi_1}}^{q_1 - d_1(p_1)} F_1(\xi_1)\,d\xi_1 - p_2\int_{\xi_2}^{q_2 - d_2(p_2)} F_2(\xi_2)\,d\xi_2 - c(q_1 + q_2) \qquad (7)$$

Subject to:

$p_2 < p_1 \leq p_1,\ c < p_2 \leq p_2,$

and $0 \leq \theta \leq 1$

The revenue function presented in Equation 7 is the expected revenue. The objective is the find the prices and order quantities $\boldsymbol{p},\boldsymbol{q}$ that maximize the total revenue $\pi(\boldsymbol{p},\boldsymbol{q})$. This is a non-linear optimization problem, in order to solve the problem the Karush-Kuhn Tuker (KKT) optimality conditions could be explored (Bertsekas, 1999). The other possibilities are to use the commercially available software such as MATHEMATICA or MATLAB in order to solve the problem. This study uses a MATHEMATICA based built-in numerical optimization procedure, NMaximize to achieve this task. The Nelder & Mead method (http://mathworld.wolfram.com/Nelder-MeadMethod.html) is selected as the search algorithm for the NMaximize procedure of the MATHEMATICA.

V. RESULTS ANALYSIS

We provide below a numerical example to illustrate our model and explain the results. The example is adapted from Phillips (2005).

1. Riskless Analysis

When the firm is trying to find out the optimal price for a product offered to a single market segment. Assuming linear riskless price dependent demand function, $y(p) = (\alpha - \beta p)^+$, where $\alpha = 9000$, and $\beta = 800$. The cost of product per unit c is \$5. In this case, the maximum selling price is $\frac{\alpha}{\beta}$. The optimal price p* which yields the maximum revenue must satisfy the first order optimality condition

$$y(p^*) = -y'(p^*)(p^* - c)$$

Where, $y' = \frac{dy}{dp} = -\beta$, resulting $p^* = \$8.13$

Thus, when the firm sets an optimal selling price $p^* = \$8.13$, the total unit sales would be equal to 2,500 units, and the total revenue is equal to \$7,812.50. Now, when the firm applies market segmentation and offers the same product into two market segments. When the firm applies two-market segments, it is assumed that

$c = 5$; $\alpha_1 = 9000$; $\alpha_2 = 5600$; $\beta_1 = \beta_2 = 800$;

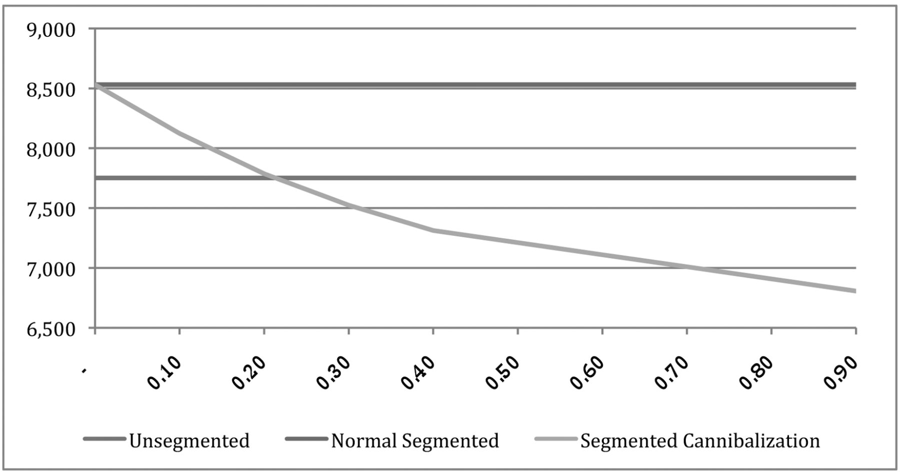
The prices that maximize the revenue in this example would be:

$$p_1^* = \$8.13$$

$$p_2^* = \min(6 + 2.12\theta, 7)$$

As a result, if $\theta = 0$, then $p_2^* = \$6$, and the total revenue generated increases by 10% and it is equal to \$8,612.50. As θ increases, the cannibalization increases and p_2^* increases as well. When θ=0.22, the optimal low price is increased to 6.47, and the total revenue is equal to \$7,816.60, which is still competitive to the revenue generated from no-market segmentation. However, if the cannibalization rate exceeds 22%, then it is better for the firm to sell its product without market segmentation as segmentation is weak in this case. Figure 4 illustrates the impact of cannibalization under no risk assumption. If the firm does not apply market segmentation and offers one price to all customers, the maximum revenue would be equal to \$7,751.99 when the quantity demand is normally distributed.

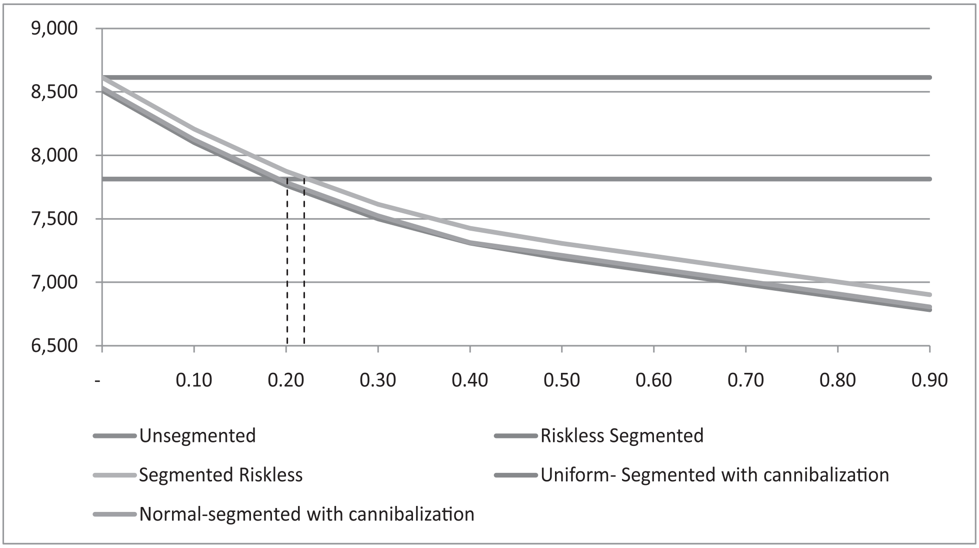
Figure 4: Effect of Cannibalization under no Risk



2. Risk based Analysis

The analysis is extended for the situation in which the price dependent customer demand in each of the two market segments is stochastic. The random demand factor, ξ_1 and ξ_2 follow uniform and normal distributions such that ξ_1 is bounded in [-40, 40], and ξ_2 is bounded in [-30, 30]. Figure 5 graphically shows the impact of cannibalization when the risk is considered. If the firm offers one price for all customers while assuming no risk associated with the quantity demand, the maximum revenue will be 7,812.50. Now, we consider a case in which the customer demand is price dependent stochastic and the market segmentation as long as the cannibalization factor $\theta \leq 0.22$. When cannibalization factor increases further, it will have negative impact on the total revenue. In such a situation, it is better for the firm to apply non-segmented market strategy and offer the product with the same price for all customers in order to generate higher revenues. Following this analysis, we consider the price dependent stochastic demand is uniformly distributed. The finding are consistent and comparable to normal distribution, the firm will still generate higher revenue as long as cannibalization factor $\theta \leq 0.21$.

Figure 5: Effect of cannibalization under risk



follows normal distribution. If the firm divides the customers into two segments based on their willingness-to-pay, the total revenue would increase by 10% and reaches \$8,530.96 compared to un-segmented revenue in case price dependent stochastic demand is normally distributed. In addition from Figure 5, when cannibalization exists between the two-market segments, the firm could generate higher revenue with

3. A sensitivity analysis

In this section, a sensitivity analysis is presented which shows the impact of cannibalization on the firm's profitability, and its strategies for pricing, and order quantity among the products it offers. From Table 1 we see that the optimal total revenue generated from the un-segmented market without any risk is \$7,812.50, whereas under perfect market segmentation (no cannibalization) the revenue increases by 10% to reach \$8,612.50. However, when cannibalization exists the revenue is still higher compared to un-segmented case for the cannibalization factor up to $\theta \leq 0.20$. When the market demand is considered stochastic price dependent, i.e., the random demand factors, ξ_1 and ξ_2 are uniformly distributed, the revenue generated under perfect market segmentation without cannibalization is higher by 10% compared to no market segmentation case, and is \$8,510.79. When cannibalization reaches at $\theta = 0.20$, i.e., about 20% of the high price market segment customers move to low price product the revenue is still competitive to the un-segmented case. But, any further cannibalization rate leads to a situation in which market segmentation can guarantee any revenue improve, but rather it decreases. Similar to case of uniform distribution, when random demand factors, ξ_1 and ξ_2 are normally distributed, the optimal total revenue generated with perfect market segmentation is superior to the un-segmented case by 10%. Similar to the case of uniform distribution, revenue generated with cannibalized market segmentation is superior to the un-segmented case for $\theta \leq 0.20$.

Table 2 presents the related optimal price p_1, settings for the high-price segment corresponding to the optimal revenues reported earlier in Table 1. A decreasing trend in the price p_1 is observed, for instance, the optimal price p_1 for un-segmented market is \$8.13 under riskless situation. Whereas, when demand is considered price dependent stochastic following uniform distribution, then the un-segmented market price decreases to \$8.12. Now, when cannibalization is considered for the market segmentation, at $\theta = 0.20$, the price p_1 further drops to \$8.11. Now, if the price dependent stochastic demand is normally distributed, then the optimal price to generate higher revenue is again, p_1 = \$8.12 at a cannibalization θ=0.20. This behavior is consistent with the other findings, firstly, the consideration of

Table 1: Effect of θ on optimal total revenue

θ	**No Market Segmentation**			**Market Segmentation**					
				No Cannibalization			**Cannibalization**		
	Riskless	**Risk**		**Riskless**	**Risk**		**Riskless**	**Risk**	
		Uniform	**Normal**		**Uniform**	**Normal**		**Uniform**	**Normal**
0	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	8,612.50	8,510.79	8,530.96
0.10	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	8,207.38	8,101.37	8,122.58
0.20	**7,812.50**	**7,735.65**	**7,751.99**	**8,612.50**	**8,510.79**	**8,530.96**	**7,874.50**	**7,764.50**	**7,786.71**
0.30	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,613.87	7,500.14	7,523.30
0.40	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,425.50	7,308.27	7,312.56
0.50	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,306.25	7,186.61	7,211.33
0.60	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,205.00	7,085.40	7,110.09
0.70	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,103.75	6,984.21	7,008.88
0.80	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	7,002.50	6,883.08	6,907.70
0.90	7,812.50	7,735.65	7,751.99	8,612.50	8,510.79	8,530.96	6,901.25	6,782.20	6,806.65

Table 2: Effect of θ on High-Pricing p_1

θ	No Market Segmentation			Market Segmentation					
				No Cannibalization			Cannibalization		
	Riskless	Risk		Riskless	Risk		Riskless	Risk	
		Uniform	Normal		Uniform	Normal		Uniform	Normal
0	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.12	8.12
0.10	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.11	8.12
0.20	**8.13**	**8.12**	**8.12**	**8.13**	**8.12**	**8.12**	**8.13**	**8.11**	**8.12**
0.30	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.11	8.11
0.40	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.11	8.11
0.50	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.11	8.11
0.60	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.10	8.11
0.70	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.09	8.10
0.80	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.08	8.09
0.90	8.13	8.12	8.12	8.13	8.12	8.12	8.13	8.03	8.05

stochastic demand results in a lower pricing and an increase in the order quantity in order to enable the firm protection against demand uncertainties.

Furthermore, p_2 is \$5.99 when the customer demand in both market segment is stochastic and assumed to be normally distributed under perfect market segmentation, and it increases to \$6.42 when the cannibalization factor θ=0.20. Table 3 shows the relevant optimal low price p_2 that generates the revenues appearing on Table 1 under perfect and imperfect market segmentation strategies. The optimal low price p_2 = \$6, when no risk is considered and under perfect market segmentation assumption. However, the price is increased to \$6.43 when cannibalization θ=0.20 In case of considering the risk, the optimal low price is \$5.99 when the risk is uniformly

Table 3: Effect of θ on Low Pricing p_2

θ	Market Segmentation					
	No Cannibalization			Cannibalization		
	Riskless	Risk		Riskless	Risk	
		Uniform	Normal		Uniform	Normal
0	6.00	5.99	5.99	6.00	5.99	5.99
0.10	6.00	5.99	5.99	6.21	6.20	6.20
0.20	**6.00**	**5.99**	**5.99**	**6.43**	**6.41**	**6.42**
0.30	6.00	5.99	5.99	6.64	6.63	6.63
0.40	6.00	5.99	5.99	6.85	6.84	7.00
0.50	6.00	5.99	5.99	7.00	7.00	7.00
0.60	6.00	5.99	5.99	7.00	7.00	7.00
0.70	6.00	5.99	5.99	7.00	7.00	7.00
0.80	6.00	5.99	5.99	7.00	7.00	7.00
0.90	6.00	5.99	5.99	7.00	7.00	7.00

distributed for perfect market segmentation, and it should be increased to \$6.41 when customers' cannibalization rate is 20%. In addition, p_2 is \$5.99 when the risk is normally distributed under perfect market segmentation, and it increases to \$6.42 when the cannibalization factor θ=0.20.

Table 4 shows the relevant order quantity q_1 related to the high-price market segment corresponding to the revenues reported in Table 1. The optimal order quantity is 2,500 units when the market demand is riskless under both the no market segmentation and the perfect segmentation scenarios. However, the ordered quantity (inventory) decreases to 2,000 for market segmentation when the cannibalization θ = 0.20. The optimal ordered quantity for the high-price segment increases to 2,507.59 units when the risk is uniformly distributed and there is no market segmentation. If perfect market segmentation strategy is applied, then the ordered quantity decreases to 2,498.30 units. In addition, when cannibalization factor is, the optimal ordered quantity decreases further to 1,998.29 units. Similarly, when the risk is normally distributed, the optimal quantity demanded for the high-price segment decreases to 2,505.63 units with no risk assumption. If market segmentation, the quantity demanded related to perfect-market segmentation decreases to 2,499.42 units. When cannibalization factor is θ=0.20 the quantity demanded related to imperfect market segmentation is equal to 1,999.41 units.

Table 5 shows the relevant quantity demanded q_2 related to the low-price market segment, which generates the revenues appearing on Table 1. The optimal ordered quantity is 800 units when no risk is considered for perfect market segmentation. The quantity demanded increases to 1,140.00 for the cannibalization factor, θ=0.20. When price dependent stochastic demands in both market segments are uniformly distributed, the optimal quantity demanded is 790.35 units for perfect market

Table 4: Effect of θ on High-Price order quantities q_1

θ	No Market Segmentation			Market Segmentation					
				No Cannibalization			Cannibalization		
	Riskless	Risk		Riskless	Risk		Riskless	Risk	
		Uniform	Normal		Uniform	Normal		Uniform	Normal
0	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	2,500	2,498.30	2,499.42
0.10	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	2,250	2,248.30	2,249.41
0.20	**2,500**	**2,507.59**	**2,505.63**	**2,500.00**	**2,498.30**	**2,499.42**	**2,000**	**1,998.29**	**1,999.41**
0.30	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	1,750	1,748.29	1,749.41
0.40	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	1,500	1,498.28	1,499.40
0.50	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	1,250	1,248.26	1,249.40
0.60	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	1,000	998.24	999.39
0.70	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	750	748.21	749.37
0.80	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	500	498.14	499.33
0.90	2,500	2,507.59	2,505.63	2,500.00	2,498.30	2,499.42	250	247.93	249.23

segmentation. It increases to 1,132.34 units when cannibalization factor θ=0.20. Next, if the risk is normally distributed, the optimal ordered quantity is 793.01 units with perfect market segmentation strategy, and when cannibalization exists between the two segments it increases to 1,134.79 at θ=0.20.

Table 5: Effect of θ on Low-Price order quantity q_2

Θ	Market Segmentation					
	No Cannibalization			Cannibalization		
	Riskless	Risk		Riskless	Risk	
		Uniform	Normal		Uniform	Normal
0	800	790.35	793.01	800	790.35	793.01
0.10	800	790.35	793.01	970	961.37	963.95
0.20	**800**	**790.35**	**793.01**	**1,140**	**1,132.34**	**1,134.79**
0.30	800	790.35	793.01	1,310	1,303.27	1,305.56
0.40	800	790.35	793.01	1,480	1,474.16	1,351.09
0.50	800	790.35	793.01	1,700	1,687.15	1,691.10
0.60	800	790.35	793.01	2,040	2,027.15	2,031.09
0.70	800	790.35	793.01	2,380	2,367.16	2,371.10
0.80	800	790.35	793.01	2,720	2,707.16	2,711.11
0.90	800	790.35	793.01	3,060	3,047.16	3,051.10

VI. CONCLUSIONS AND FUTURE RESEARCH SUGGESTIONS

The results generated in this study have inferred that the firm's optimal total revenue generated by applying the perfect market segmentation is superior compared to un-segmented and imperfect market segmentation scenarios. However, it is not uncommon that a firm to build its pricing strategy based on cannibalization of the customers. Even though when the cannibalization exists between market segments, the firm will be still able to generate superior revenues compared to un-segmented market situation under both the riskless and risk based situations, given a limited amount of cannibalization is experienced. However, for the higher order of cannibalizationfactor firm experiences adverse affect on its revenues.

The work can be extended to explore the robust analysis to the risk based problem. The robust analysis transforms the stochastic problem to its deterministic problem, thus a large scale optimization algorithm can be developed for a realist problem. Another avenue of this research would be to developed models for revenue management for airline industry under imperfect market segmentation.

REFERENCES

Arrow, K., Harris T., & Marschak J. (1951). Optimal Inventory policy. *Econometrica*, Vol. 19, pp. 250-272

Bell, P. C. & Zhang M. (2006). Managerial decision-making in the single period optimum pricing problem. *The journal of operational research society*, Vol. 57, pp. 377-388

Bish, E. K. and Wang, Q. (2004), `Optimal investment strategies for flexible resources,considering pricing and correlated demands', *Operations Research*, Vol. 52, 954-964.

Chen, Y. H., Ray, S. and Song, Y. Y. (2006), `Optimal pricing and inventory control policy in periodic-review systems with fixed ordering cost and lost sales', *Naval Research Logistics*, Vol. 53, 117- 136.

Cote, J.P., Marcotte, P. and Savard, G. (2003) 'A bi-level modeling approach to pricing and fare optimisation in the airline industry', *Journal of Revenue and Pricing Management*, Vol. 2, pp.23–36.

Ernst R.L. (1970). Linear inventory models for a monopolistic firm. *PhD thesis*. University of California, Berkeley

Gupta, D., Hill, A. and Bouzdine-Chameeva, T. (2006), `A pricing model for clearing end-of-season retail inventory', *European Journal of Operational Research*, Vol. 170, pp. 518-540.

Karlin, S. and Carr, C. R. (1962), Studies in Applied Probability and Management Science, Stanford University Press, Stanford, CA, chapter Prices and Optimal Inventory Policies.

Lau, A. H. & Lau, H. S. (1996), The multi-product multi-constraint newsboy problem: applications, formulations, and solutions. *Journal of operations management*, Vol. 13, pp. 153-162

Lee, H. and Nahmias, S. (1990), Handbooks in Operations Research and Management Science, North-Holland, chapter Single-Product, Single Location Models, pp. 1-58.

McGill, J. I. (1995), `Censored regression analysis of multiclass demand data subject to joint capacity constraints', *Annals of Operations Research*, Vol. 60, pp. 209-240.

McGill, J. I. and Ryzin, G. J. V. (1999), `Revenue Management: Research overview and prospects', *Transportation Science*, Vol. 33, pp. 233-256.

Mieghem, J. A. V. and Dada, M. (1999), `Price versus production postponement: Capacity and competition', *Management Science*, Vol. 45, pp. 1631-1649.

Mills, E. S. (1959), `Uncertainity and price theory', *Quarterly Journal of Economics*, Vol. 73, pp. 116-130.

Mills, E. S. (1962), Price, Output and Inventory Policy, John Wiley, New York.

Nevins, A. J. (1966), `Some effects of uncertainity: Simulation of a model of price', *Quarterly Journal of Economics*, Vol. 80, pp. 73-87.

Petruzzi, N. C. and Dada, M. (1999), `Pricing and the news vendor problem: A review with extensions', *Operations Research*, Vol. 47, pp. 183-194.

Philips R. L. (2005), 'Pricing and revenue optimization', Standford business book, Standford university press

Porteus, E. (1990), Handbooks in Operations Research and Management Science, North-Holland, chapter Stochastic Inventory Theory, pp. 605-652.

Whitin, T. M. (1955), `Inventroy control and price theory', *Management Science*, Vol. 2, pp. 61-68.

Yao, L., Chen, Y. F. and Yan, H. (2006), `The newsvendor problem with pricing: Extension', *International Journal of Management Science and Engineering Management*, Vol. 1, pp. 3-16.

Young, L. (1978), `Price, inventory and the structure of uncertain demand', *New Zealand Operations Research*, Vol. 5, pp. 157-177.

Zabel, E. (1970), `Monopoly and uncertainity', *Review of Economic Studies*, Vol. 37, pp. 205-219.

Zhao, W. and Wang, Y. Z. (2002), `Coordination of joint pricing-production decisions in a supply chain', *IIE Transactions*, Vol. 34, pp. 701-715.

Short Bio of Hammoudeh Al-Zaini and Syed Asif Raza

Hammoudeh Al-Zaini is currently working as an Internal Control Manager at HSBC Bank Middle East-Qatar. He has 13 years of experience in Financing and Banking. He holds an MBA degree from Qatar University and a Bachelor of Computer Science Degree from Arab Academy for Science and Technology (AAST). Prior to joining his current position, he served as a faculty member at AAST since 2001 and worked as an Assistant Professor for four years. He taught several courses at AAST which include, Software Engineering, Software Project Management, Theory of Computation, Structure of Programming Languages, and Numerical Methods. He has served in graduation project committee at AAST and supervised in several graduation projects"

Syed Asif Raza, Ph.D. is currently working as an assistant professor of operations management and quantitative methods at the College of Business and Economics, Qatar University. Prior to joining his current position, he held a post doctoral research fellowship at Centre for Research on Transportation, University of Montreal, Canada. He is graduated with a PhD degree from Concordia University, Canada in 2007. He also holds a Master of Science degree from King Fahd University, Saudi Arabia, and a Bachelor Degree from NED University, Pakistan in 2002 and 1999 respectively. His research interests include Revenue Management, Econometrics, Pricing, Supply Chain Management, Meta-Heuristics, Planning & Scheduling, Product Development, and Applied Statistics. He has published around fifteen articles in international journals. His articles have appeared in Journal of Heuristics, Computers & Industrial Engineering, Design Studies, International Journal of Operations Research, International Journal of Revenue Management, Journal of Quality in Maintenance Engineering, and Lecture Notes in Computer Science. His teaching interests are Microeconomics, Managerial Statistics, Operations Management, Quantitative Methods in Business, Revenue Management, and Supply Chain Management. He has received distinguished paper award in DSI annual conference in 2009 held in New Orleans, USA.