On the Propagation of Elastic-plastic Wave in the Half Space

by

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ABSTRACT

The problem of the propagation of the elastic-plastic wave in the half-space occupied by and elastic-plastic medium has been studied. Treatments were carried out under the Assumption of perpendicular load, on the boundary, which propagates with a constant speed D.

The assumption further involved that displacements were in the direction of the load, whereas the lateral displacements were neglected.

The method of quadratic error was used throughout the solution of the wave equation. the theoretical calculations of space-dependent stress were displayed with respect to the distance from the half-space boundary.

انتشار الموجات المرنة اللدنة في نصف الفراغ

الدكتور/فوزي شعبان الدويك
قسم الرياضيات ـ كلية العلوم
جامعة قطر

ملخص

في هذا البحث عولجت مسألة انتشار الموجات المرنة اللدنة في نصف الفراغ المرن اللدن بفرض ان الحمل يؤثر عمودياً على حدود نصف الفراغ وانه ينتشر بسرعة ثابتة . علاوة على ذلك اننا افترضنا ان الازاحات في اتجاه الحمل بينما اهملنا الازاحات المستعرضة .

وخلال حل معادلة الموجة استعملنا طريقة الخطأ التربيعي .

*key words – (propagation, elestic-plastic, wave.)

Introduction

Among various authors, Rakmatulin [1], Shapiro [2] and Sokolovskii [3] have studied the problem of elastic-plastic wave propagation in the half-space.

The three-dimensional problem of elastic wave propagation has been solved by Fawze El-Dewik [4]. The problem has been studied when an instantaneous constant load acts on the boundary of the elastic half-space. The load is taken to act perpendicular to the boundary and the lateral displacements were neglected. A similar study of this problem was carried in the case where the load is time-dependent [5].

The two-dimensional problem was treated under the assumption that the propagation load acts perpendicular to the boundary [6], and the solution was obtained by taking into account the verticle displacements, whereas the lateral displacements were neglected.

Nevertheless, this problem was recently solved [7] taking into consideration the lateral displacements as well as the verticle ones.

In the present work, the two-dimensional problem of elastic-plastic wave propagation has been studied, whereas the boundary load moves with a constant velocity.

1. Basic Equation and its Solution

Let the semi elastic-plastic material occupy the domain $Z \geqslant 0, -\infty < x, y < \infty$ in the cartesian coordinates.

Assume that a load exists perpendicular on the boundary of the half-space and we assume that this load propagates with a constant speed D. if we neglect the lateral displacement then, the vertical displacement W satisfies the wave equation:

$$\frac{\partial^2 W}{\partial t^2} = a^2(e)\left[\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right], \tag{1.1}$$

where a is the velocity of the longitudinal wave which is a function of intensity deformation e.

The initial condition is

$$W(x, y, o) = \frac{\partial}{\partial t} W(x, y, o) = 0 \tag{1.2}$$

And the boundary conditions are

$$\left.\begin{aligned} \left(\frac{\partial W}{\partial y}\right)_{y=0} &= \xi(x, t) && 0 \leqslant |x| \leqslant Dt \\ \left(\frac{\partial W}{\partial y}\right)_{y=0} &= 0 && |x| > Dt \end{aligned}\right\} \tag{1.3}$$

The Propagation of Elastic Wave in the Half-space

The solution of the problem in this case may be written in the form

$$W = \iint_{\sigma} \frac{C(\xi,\tau)\,d\xi\,d\tau}{\sqrt{a^2(t-\tau)^2 - (x-\xi)^2 - y^2}}\,, \tag{2.1}$$

where δ is the region bounded by

$$\tau = t - \frac{1}{a}\sqrt{(x-\tau)^2 + y^2}\,, \tag{2.2}$$

$$\tau = \pm D\tau \tag{2.3}$$

Consequently, one can prove that the expression (2.1) satisfies the wave equation (1.1) and then we get

$$\left(\frac{\partial W}{\partial y}\right)_{y=0} = -\Pi C(x,t) \tag{2.4}$$

Therefore, the expression (2.1) takes the form

$$W = -\frac{1}{\Pi}\iint_{\sigma} \frac{\xi(\xi,\tau)\,d\xi\,d\tau}{\sqrt{a^2(t-\tau)^2 - (x-\tau)^2 - y^2}} \tag{2.5}$$

Using (2.2) and (2.3), then the bounds of the integration in (2.5) will be the following

$$W = -\frac{1}{\Pi}\int_0^{\tau_c} d\tau \int_{-D\tau}^{D\tau} \frac{C(\xi,\tau)\,d\xi}{\sqrt{a^2(t-\tau)^2 - (x-\xi)^2 - y^2}} -$$

$$-\frac{1}{\Pi}\int_{\tau_c}^{\tau_1} d\tau \int_{\xi_1}^{D\tau} \frac{C(\xi,\tau)\,d\xi}{\sqrt{a^2(t-\tau)^2 - (x-\xi)^2 - y^2}} -$$

$$-\frac{1}{\Pi}\int_{\tau_1}^{\tau_m} d\tau \int_{\xi_1}^{\xi_2} \frac{C(\xi,\tau)\,d\xi}{\sqrt{a^2(t-\tau)^2 - (x-\xi)^2 - y^2}} -$$

where $\tau_c, \tau_1, \tau_m, \xi_1$, and ξ_2 are defined from the equations

$$a^2(t-\tau_c)^2 - (x-\xi)^2 = y^2,\ \xi = -D\tau_c,$$

$$a^2(t-\tau_1)^2 - (x-\xi)^2 = y^2,\ \xi = -D\tau_1,$$

$$a^2(t-\tau_m) = y^2,$$

$$\xi_1 = x - \sqrt{a^2(t-\tau)^2 - y^2} \quad \text{and}$$

$$\xi_2 = x + \sqrt{a^2(t-\tau)^2 - y^2}.$$

We shall discuss the problem when

$$\xi(\zeta) = b_o + b_1\zeta + b_2\zeta^2$$

Then we have

$$W = -\frac{1}{\Pi}(b_0 - b_1 x + b_2 x^2)\iint_\sigma \frac{d\zeta\, d\tau}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} -$$

$$-\frac{1}{\Pi}(\tfrac{1}{2}b_1 - b_2 x)\iint_\sigma \frac{2(x-\zeta)d\zeta\, d\tau}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} -$$

$$-\frac{b_2}{\Pi}\iint_\sigma \frac{(x-\zeta)^2 d\zeta\, d\tau}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} \quad . \tag{2.6}$$

i.e.

$$W = -\frac{1}{\Pi}(b_0 - b_1 x + b_2 x^2)\left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{d\zeta}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} - \right.$$

$$\left. - \int_{\tau_0}^{\tau_1} d\tau \int_{x-\sqrt{a^2(t-\tau)^2 + y^2}}^{D\tau} \frac{d\zeta}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}}\right] -$$

$$= -\frac{1}{\Pi}(\tfrac{1}{2}b_1 - b_2 x)\left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{2(x-\zeta)d\xi}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} - \right.$$

$$\left. - \int_{\tau_0}^{\tau_1} d\tau \int_{x-\sqrt{a^2(t-\tau)^2 + y^2}}^{D\tau} \frac{2(x-\zeta)^2 d\zeta}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}}\right]$$

$$-\frac{b_2}{\Pi}\left[\int_0^{\tau_0} d\tau \int_0^{D\tau} \frac{(x-\zeta)^2 d\zeta}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}} - \right.$$

$$\left. - \int_{\tau}^{\tau_1} d\tau \int_{x-\sqrt{a^2(t-\tau)^2 + y^2}}^{D\tau} \frac{(x-\zeta)^2 d\zeta}{\sqrt{a^2(t-\tau)^2 - (x-\zeta)^2 - y^2}}\right] \quad . \tag{2.7}$$

Where τ_0 may be obtained by solving (2.2) and the straight line $\xi = 0$

Thus $\tau_0 = at - \sqrt{x^2 + y^2}$

Also τ_1 - may be obtained by solving (2.2) and the two straight lines $\xi = \pm D\tau$

Thus $\tau_1 = \frac{1}{(1-D^2)}\left[-(Dx-at) \pm \sqrt{(Dx-at)^2 - (1-D)^2(a^2t^2 - x^2 - y^2)}\right]$

See figure (1).

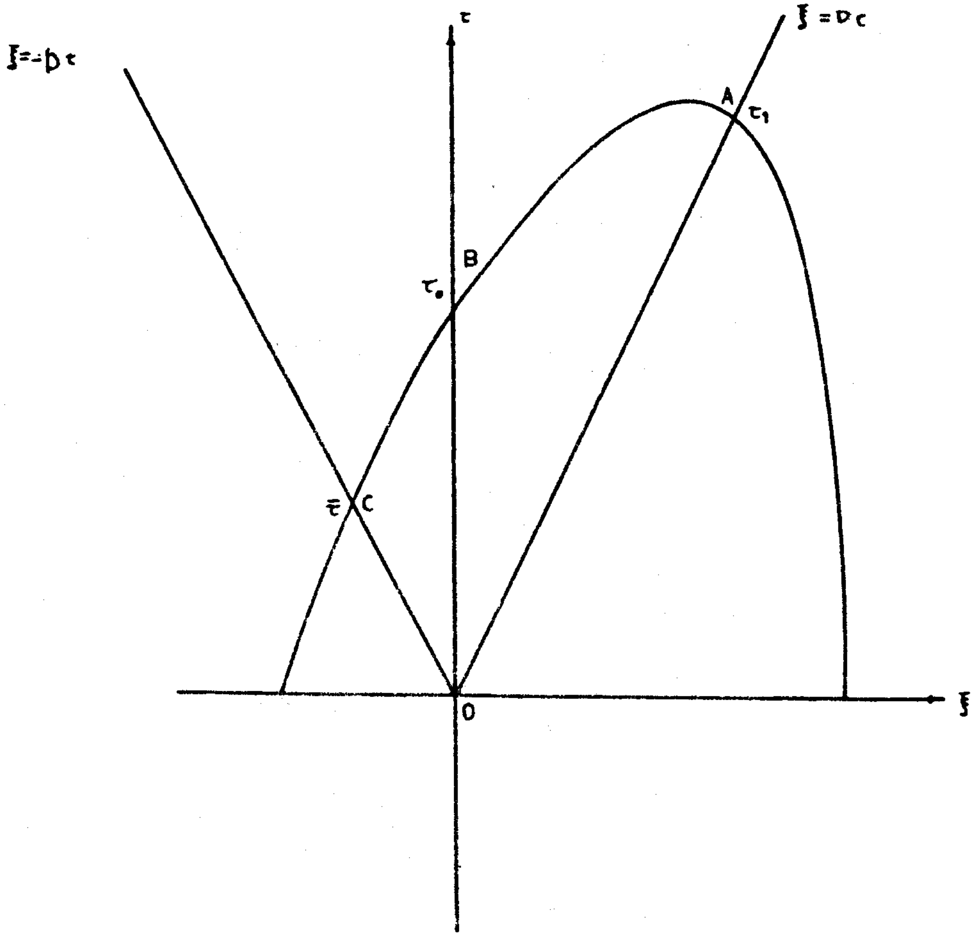

Fig. 1.

Differentiating (2.7) with respect to x and y respectively we get

$$\frac{\partial W}{\partial x} = -\frac{1}{\Pi}-(b_0-b_1x+b_2x^2)\left\{\left[\cos^{-1}\frac{(x-D\tau_1)}{\sqrt{a^2(t-\tau_1)^2-y^2}}\,\frac{\partial\tau_1}{\partial x}-\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_0)^2-y^2}}\,\frac{\partial\tau_0}{\partial x}\right]\right.$$

$$+\frac{1}{\sqrt{D^2-a^2}}\left[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Dt)}{\sqrt{a^2(x-t)^2+(a^2-D^2)y^2}}-\sin^{-1}\frac{(a^2-D^2)t-D(x-Dt)}{\sqrt{a^2(x-Dt)^2+(a^2-D^2)y^2}}\right]-$$

$$\left.-\frac{1}{a}\left[1\text{ an}\left|(t-\tau_0)+\sqrt{(t-\tau_0)^2-\frac{x^2+y^2}{a^2}}-\right|-\log\left|t+\sqrt{t^2-\frac{x^2+y^2}{a^2}}\right|\right]\right\}-\frac{1}{\Pi}(½b_1-b_2x)\ X$$

$$X\left\{2\sqrt{a^2(t-\tau_1)^2-(x-D\tau_1)^2-y^2}\,.\,\frac{\partial\tau_1}{\partial x}-2\sqrt{a^2(t-\tau)^2-x^2-y^2}\,.\,\frac{\partial\tau_0}{\partial x}-\right.$$

$$-\frac{-2D}{(a^2-D^2)}\left[\sqrt{(a^2-D^2)(t-\tau_1)^2-2D(x-Dt)(t-\tau_1)-(x-Dt)^2-y^2}\right.-$$

$$\left.-\sqrt{(a^2-D^2)t^2-2D(x-Dt)-(x-Dt)^2-y^2}\right]-\frac{2x}{a}\left[\log\left|(t-\tau_0)+\sqrt{(t-\tau_0)^2-\frac{x^2+y^2}{a^2}}\right.\right.-$$

$$\left.-\log\left|t+\sqrt{t^2-\frac{x^2+y^2}{a^2}}\right|\right]-\frac{2a^2(x-Dt)}{(a^2-D^2)^{3/2}}\left[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Dt)}{\sqrt{a^2(x-Dt)^2+(a^2-D^2)y^2}}\right.-$$

$$\left.\left.-\sin^{-1}\frac{(a2-D^2)t-D(x-Dt)}{\sqrt{a^2(x-Dt)^2+(a^2-D^2)y^2}}\right]\right\}-\frac{b_2}{\Pi}\left\{\left[½(x-D\tau_0)\sqrt{a^2(t-\tau_0)^2-y^2(x-D\tau_0)^2}\right.\right.-$$

$$\left.-x\sqrt{a^2(t-\tau_0)^2-y^2-x^2}\right]\frac{\partial\tau_0}{\partial x}-(a^2(t-\tau_0)^2-y^2)\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_0)^2-y^2}}\cdot\frac{\partial\tau_0}{\partial x}-$$

$$-\frac{x}{2}\sqrt{a^2(t-\tau_1)^2-y^2-x^2}\,.\,\frac{\partial\tau}{\partial x}-½\left[a^2(t-\tau)^2-y^2\right]\cos^{-1}\frac{x-D\tau_0}{\sqrt{a^2(t-\tau_0)^2-y^2}}\cdot\frac{\partial\tau_0}{\partial x}$$

$$-½(t-\tau_0)\sqrt{(t-\tau_0)^2-x^2-y^2}-t\sqrt{t^2-y^2-x^2}+½\frac{a^2}{\sqrt{x^2+y^2}}\left(\sin^{-1}\frac{(t-\tau_0)\sqrt{x^2+y^2}}{a}\right.-$$

$$-\sin^{-1}\frac{t\sqrt{x^2+y^2}}{a}+\frac{(x^2+y^2)}{a}\left[\sin^{-1}\frac{a(t-\tau_0)}{\sqrt{x^2+y^2}}-\sin^{-1}\frac{at}{\sqrt{x^2+y^2}}\right]+$$

$$+\left[\frac{a^2(t-\tau_1)^2-3D(x-Dt)}{2(a^2-D^2)}\right]\left[\sqrt{(a^2-D^2)(t-\tau_1)^2-2D(x-DT)(t-\tau_1)-(x-Dt)^2-y^2}\right]-$$

$$-\left[\sqrt{(a^2-D^2)t^2-2D(x-Dt)t-(x-Dt)^2-y^2}\right]\left(\frac{a^2t-3D(x-Dt)}{2(a^2-D^2)}\right)-\frac{1}{\sqrt{a^2-D^2}}\left[y^2-\frac{3D(x-Dt)}{(a^2-D^2)}\right.$$

$$\left.\left.-\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)+D(x-Dt)}{\sqrt{a^2(x-Dt)^2+y^2(a^2-D^2)}}-\sin^{-1}\frac{(a^2-D^2)t+D(x-Dt)}{\sqrt{a^2(x-Dt)^2+y^2(a^2-D^2)}}\right]\right\} \tag{2.8}$$

$$\frac{\partial W}{\partial y}=-\frac{1}{\Pi}(b_0-b_1+b_2x^2)\left\{\cos^{-1}\frac{x-D\tau_1}{\sqrt{a^2(t-\tau_1)^2-y^2}}\cdot\frac{\partial\tau_1}{\partial y}-\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_0)^2-y^2}}\cdot\frac{\partial\tau_0}{\partial y}+\right.$$

$$+\frac{1}{2a}\left[\mathrm{Sin}^{-1}\frac{[a(t-\tau_1)-y]\,[y(1-D^2)-D(x-Dt)]-[Dy-[Dy+(x-Dt)]^2}{[a(t-\tau_1)-y]\sqrt{y^2(1-D^2)+(x-Dt)^2}}-\right.$$

$$\left.-\sin^{-1}\frac{(at-y)\,[y(1-D^2)-D(x-Dt)]-[Dy+(x-Dt)]^2}{(at-y)\sqrt{y^2(1-D^2)-(x-Dt)^2}}\right]+$$

$$+\frac{1}{2a}\left[\sin^{-1}\frac{[a(t-\tau_1)+y]\,[y(1-D^2)+D(x-Dt)]+[Dy-(x-Dt)]^2}{[a(t-\tau_1)+y]\sqrt{y^2(1-D^2)+(x-Dt)^2}}-\right.$$

$$\left.-\sin^{-1}\frac{(at+y)\,[y(1-D^2)+D(x-Dt)]+[Dy-(x-Dt)]^2}{(at+y)\sqrt{y^2(1-D^2)+(x-Dt)^2}}\right]+\left[\sin^{-1}\frac{a(t-\tau_1)y-y^2-x^2}{[a(t-\tau_1)-y]\sqrt{x^2+y^2}}-\right.$$

$$\left.-\sin^{-1}\frac{aty-y^2-x^2}{(at-y)\sqrt{x^2+y^2}}\right]-\left[\sin^{-1}\frac{a(t-\tau_1)+y^2-x^2}{[a(t-\tau_1)+y]\sqrt{x^2+y^2}}-\right.$$

$$\left.\left.-\sin^{-1}\frac{aty+y^2-x^2}{(at+y)\sqrt{x^2+y^2}}\right]\right\}-\frac{1}{\Pi}(\tfrac{1}{2}b_1-b_2x)\left\{2\sqrt{a^2(t-\tau_1)^2-(x-D\tau_1)^2-y^2}\cdot\frac{\partial\tau_1}{\partial y}-\right.$$

$$-2\sqrt{a^2(t-\tau_0)^2-x^2-y^2}\cdot\frac{\partial\tau_0}{\partial y}+\frac{2y}{\sqrt{D^2-a^2}}\left[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Mt)}{\sqrt{a^2(x-Dt)^2+(x-Dt)^2}}-\right.$$

$$- \sin^{-1} \frac{(a^2-D^2)t-D(x-Dt)}{\sqrt{a^2(x-Dt)^2+(a^2-D^2)y^2}}\Bigg] - \frac{2y}{a}\Bigg[\log\Bigg|(t-\tau_0)+\sqrt{(t-\tau_0)^2-\frac{x^2+y^2}{a^2}}\Bigg|$$

$$-\log\ t+\sqrt{t^2-\frac{x^2+y^2}{a^2}}\Bigg]\Bigg\}-\frac{b_2}{\Pi}\Bigg\{-\tfrac{1}{2}(x-D\tau_0)\sqrt{a^2(t-\tau_0)^2-y^2(x-D\tau_0)^2}\,.\,\frac{\partial\tau_0}{\partial y}$$

$$+X\sqrt{a^2(t-\tau_0)^2-y^2-x^2}\,.\,\frac{\partial\tau_0}{\partial y}-\frac{7}{2}\,[a^2(t-\tau_0)^2-y^2]\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_0)^2-y^2}}\,.\,\frac{\partial\tau_0}{\partial y}+$$

$$+\frac{3}{2}\,(a^2(t-\tau_0)^2-y^2)\cos^{-1}\frac{x-D\tau_0}{\sqrt{a^2(t-\tau_0)^2-y^2}}\,.\,\frac{\partial\tau_0}{\partial y}+$$

$$+\tfrac{1}{2}(a^2(t-\tau_1)^2-y^2)\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_1)^2-y^2}}\,.\,\frac{\partial\tau_1}{\partial y}+$$

$$+2y(\tau_1\cos^{-1}\frac{x}{\sqrt{a^2(t-\tau_1)^2-y^2}}-\tau_0\cos^{-1}\frac{x-D\tau_0}{\sqrt{a^2(t-\tau_0)^2-y^2}})-$$

$$-\frac{(x-Dt)-y^2}{\sqrt{a^2-D^2}}\Bigg[\sin^{-1}\frac{(a^2-D^2)(t-\tau_1)-D(x-Dt)}{\sqrt{a^2(x-Dt)^2+y^2(a^2-D^2)}}-$$

$$-\sin^{-1}\frac{(a^2-D^2)t+D(x-Dt)}{\sqrt{a^2(x-Dt)+y^2(a^2-D^2)}}\Bigg]-\frac{x}{a}\Bigg[\sin^{-1}\frac{a(t-\tau_0)}{\sqrt{x^2+y^2}}-\sin^{-1}\frac{at}{\sqrt{x^2+y^2}}\Bigg]\Bigg\}$$

The numerical values for $\frac{\partial W}{\partial y}$ and $\frac{\partial W}{\partial x}$ were calculated for the different values of x. The plots of $\frac{\partial W}{\partial y}$ and $\frac{\partial W}{\partial x}$ are represented in figures (2) & (3).

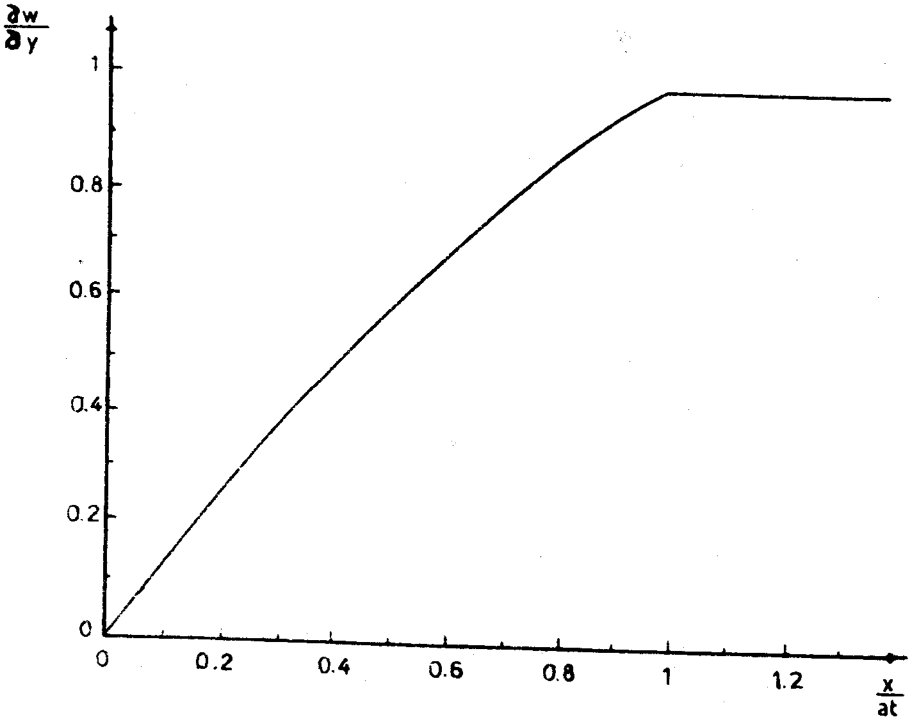

Fig. (2)

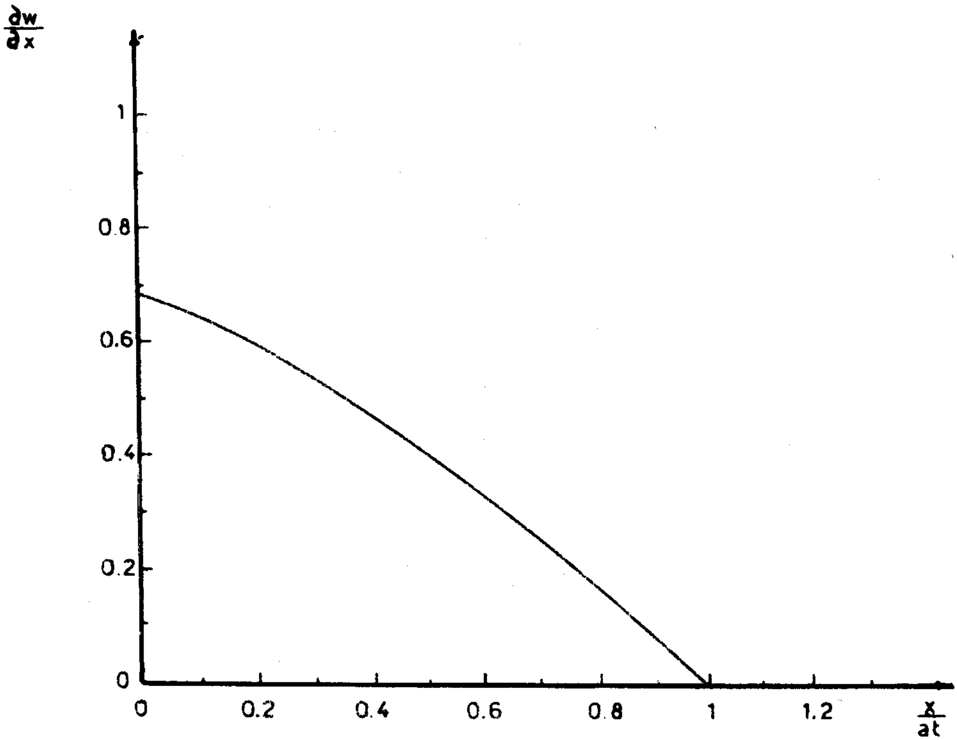

Fig. (3)

The Propagation of Elastic-plastic Wave in the Half-space:

In this case if we take into consideration Prantel's diagram, then the elastic-plastic wave propagation may be illustrated as shown in figure (4). The region W_1 is the elastic medium and the surface A B C B A is the discontinuous deformation interface. If the lateral displacements were neglected, then the vertical displacement W in the plastic medium satisfies the wave equation (1.1) with the boundary conditions (1.3) where

$$W = W_1 + W_2 \tag{3.1}$$

and W_1 & W_2 represent the vertical displacements in the elastic and plastic regions respectively. (see figure (4)).

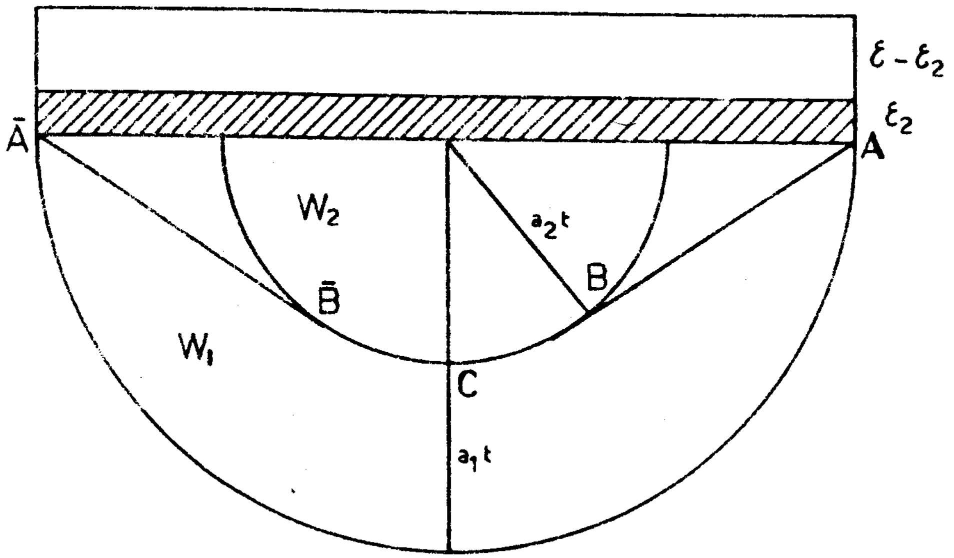

Fig. (4)

Suppose that the boundary condition for plastic region is

$$\left.\frac{\partial W_2}{\partial y}\right|_{y=0} = \epsilon_2 \tag{3.2}$$

Then the boundary condition for the elastic region will be

$$\left.\frac{\partial W_1}{\partial y}\right|_{y=0} = \epsilon - \epsilon_2 \tag{3.3}$$

and the solution of the problem may be written in the form

$$W_2 = \iint\limits_{\sigma_2} \frac{C_2(\xi,\tau)\,d\xi\, d\tau}{\sqrt{a_2^2(t-\tau)^2 - (x-\xi)^2 - y^2}} \quad , \tag{3.4}$$

&

$$W_1 = \iint\limits_{\sigma_1} \frac{C_1(\ ,\tau)\,d\xi\, d\tau}{\sqrt{a_1^2(t-\tau)^2 - (x-\xi)^2 - y^2}} \quad , \tag{3.5}$$

where C_1, C_2 and ξ_2 are unknown functions to be determined from the following relations

$$\left(\frac{\partial W_1}{\partial y} + \frac{\partial W_2}{\partial y}\right)_{y=0} = -\Pi C_1 - \Pi C_2 = \xi, \tag{3.6}$$

$$\left(\frac{\partial W_1}{\partial y}\right)_{y=0} = -\Pi C_1 = \epsilon - \epsilon_2 \ , \tag{3.7}$$

$$\underset{ABCBA}{[\epsilon_i]} = \epsilon_s \ , \tag{3.8}$$

where ξ_s is the limit value of the deformation between the elastic and plastic regions and ξ_i is the insensitive deformation.
In the present case ξ_i is reduced to the following form

$$\epsilon_i = \frac{2}{3} \sqrt{\left(\frac{\partial W_1}{Py}\right)^2 + \frac{3}{4}\left(\frac{\partial W_1}{\partial x}\right)^2} \tag{3.9}$$

Then the relation (3.8) takes the form:

$$\epsilon_i^2 = \frac{4}{9}\left[\left(\frac{\partial W_1}{\partial y}\right)^2 - \frac{3}{4}\left(\frac{\partial W_1}{\partial x}\right)^2\right]_{ABCBA} = \epsilon_s^2 \tag{3.10}$$

The relations (3.6), (3.7) and (3.10) are sufficient to determine C_1, C_2 and ξ_S.
Using Taylor's expansion, ${}_2\epsilon\ (\xi)$ may take the form

$$\xi_2(\xi) = b_0 + b_1\xi + b_2\xi^2 \tag{3.11}$$

Where b_0, b_1 and b_2 are unknown constants to be determined by using the method of quadratic error.

Let ϵ_i differ from ϵ_s by an infinitesimal quantity.

i.e. $$\epsilon_i = \epsilon - e(b_0, b_1, b_2) \tag{3.12}$$

Then $$e^2 = \int_0^t (\epsilon_i - \epsilon_s)^2 \, d\tau \tag{3.13}$$

Using (3.9) we get:

$$e^2 = \int_0^t \left[\frac{2}{3}\sqrt{\left(\frac{\partial W_1}{\partial y}\right)^2 + \frac{3}{4}\left(\frac{\partial W_1}{\partial x}\right)^2} - \epsilon_s\right]^2 d\tau, \tag{3.14}$$

Where $\frac{\partial W_1}{\partial x}$ and $\frac{\partial W_1}{\partial y}$ are obtained in (2.8) and (2.9) by replacing a with a (e). Making this error minimum, its first derivative with respect to b_0, b_1 and b_2 must be equal to zero.

Thus: $$\frac{\partial e}{\partial b_0} = 0, \frac{\partial e}{\partial b_1} = 0, \frac{\partial e}{\partial b_2} = 0 \tag{3.15}$$

solving the above conditional relations (3.15) with respect to b_0, b_1, b_2 we get:

$$b_0 = 1.08 \, \epsilon_s.$$

$$b_1 = +0.29 \frac{\epsilon_s}{at}$$

$$b_2 = 0.52 \frac{\epsilon_s}{(at)^2}$$

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