QATAR UNIVERSITY

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# GOODNESS OF FIT TESTING FOR THE LOG-LOGISTIC DISTRIBUTION BASED ON TYPE I CENSORED DATA

BY

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# ABSTRACT

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Title: Goodness of Fit Testing for the Log-Logistic distribution Based on Type I Censored Data

Supervisor of Thesis: Prof. Ayman, S, Bakleezi.

The main aim of this thesis is to investigate the problem of the goodness of fit test for Log-Logistic distribution based on empirical distribution function under Type I censored data. The maximum likelihood estimation method is used to estimate the unknown parameters of Log-Logistic distribution. A Monte Carol power studies are conducted to evaluate and compare the performance of the proposed method which is an extension to the test procedure by Pakyari and Balakrishnan (2017) with the existing classical method for several alternative distributions. The proposed method exhibits higher power compared to classical method. Additionally, applications on Type I censored real datasets for the proposed and classical methods are considered for illustrative purposes. As result from the real data it was found that the Log-Logistic model has good fit for the data.

# DEDICATION

*This thesis is dedicated to my parents for their endless love, support, and encouragement.*

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to my supervisor Professor Ayman Bakleezi, who has guided and encouraged me throughout the thesis. I would also like to thank him for supporting me when I needed it the most. Without his persistent help, the goal of this thesis would not have been achieved. In addition, I would like to express my deep gratitude to Dr. Reza Pakyari for his patience, guidance, enthusiastic encouragement, and useful critiques of this research work. Finally, I would like to thank all my family and friends for their love and support during these two years.

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# CHAPTER 1: INTRODUCTION

## 1.1 Overview

Survival time is well known as event time, lifetime, and failure time. It is denoted by the variable $T$ which is a non-negative continuous random variable. Its actual time represent the waiting time until the occurrence of a well-defined event, that is, the time from starting point to the endpoint of a concerning event for a subject (Klein and Moeschberger, 2006). For example, the time to event can be the duration of stay in a job, interval until recurrences of the symptoms, and unemployment time length. The time can be in form of years, months, days, hours, or even fractions of a second.

Survival analysis is the field of statistics that study and analyze the survival time data, which, mainly focuses on describing the distribution of survival time $T$. Furthermore, there are three methods to model the survival time $T$, which are parametric, non-parametric and semi-parametric. Survival models in which a specific probability distribution is assumed for the $T$ are known as a parametric model. Klein and Moeschberger (2006) mentioned that two quantitative terms are usually measured in any survival analysis i.e. the survival function and the hazard function. The survival function is a non-increasing function and known as the probability of a subject surviving longer than some specified time, as shown below,

$$S(t) = P(T > t) = 1 - F(t), \tag{1}$$

where $F(t) = P(T \leq t)$ is the cumulative distribution function of $T$.

The hazard function (instantaneous failure rate) is defined as the probability of an individual failing in a very short interval, given that the subject has survived to the time $t$ (Lee and Wang, 2003).

$$h(t) = \lim_{\delta t \to 0} \left\{ \frac{p(t \leq T < t + \delta t | T \geq t)}{\delta t} \right\}$$

$$= \lim_{\delta t \to 0} \left\{ \frac{\frac{p(t \leq T < t + \delta t)}{P(T \geq t)}}{\delta t} \right\}$$

$$= \lim_{\delta t \to 0} \left\{ \frac{\frac{F(t + \delta t) - F(t)}{S(t)}}{\delta t} \right\}$$

$$= \lim_{\delta t \to 0} \left\{ \frac{F(t + \delta t) - F(t)}{\delta t} \right\} \frac{1}{\delta(t)} = \frac{f(t)}{s(t)}. \quad (2)$$

In survival analysis, many parametric distributions are used to model the outcome variable $T$ such as Exponential, Weibull, Gamma and Gompertz distributions. The main important feature that differentiates the survival analysis from any other statistical analysis is the presence of censorship in its data. Censoring occurs when having partial information about an individual survival time (Kleinbaum and Klein, 2010).

### 1.2 Censoring

Censoring can occur due to different reasons, it can be unexpected as when a person is lost to follow up in a prospective study because they move away from the area where the study takes place, or it can be predetermined as when a decision is made to terminate a life test before all items have failed (Lawless, 2011). There are many kinds of censoring, such as interval censoring, left censoring and right censoring (Klein and Moeschberger, 2006).

Interval Censoring: the survival analysis can be interval censored when the survival time of a subject is true but unknown within a specific time interval. In this case, the true survival time of individual occures after time $t_1$ and before time $t_2$. Hence, it is interval censored in time interval lie between$(t_1, t_2)$. Right censoring and left censoring are special cases of interval censoring (Klein and Moeschberger, 2006).

Left Censoring: the survival time is censored to the left when the actual survival time is less than or equal to the observed survival time (Klein and Moeschberger, 2006).

Right Censoring: is the most common type of censoring and it is occurs when the survival time is incomplete at the right side of the follow up period. For example, when a person leaves, the study before the event of interest has occurred or the study end before any event of interest has occurred. This type of censoring is very popular in real life application (Klein and Moeschberger, 2006). In addition, the most well-known kinds of right censoring are Type I, Type II and random censoring (Lawless, 2011).

In Type I censoring, the censoring time $c$ is assumed to be fixed, this type of censoring occurs when a study end and no event have happened. The event is observed only if it is occurred before pre-specified time (Lee and Wang, 2003). While, in Type II censoring, when a fixed number of events between the subject has occurred the study ends. Usually this type of censoring (Type II) is used in experiments that involved in the testing equipment of lifetime. Let $r$ represent the pre-determined integer of the failure of individuals, therefore, the study continues until the first $r$ failure of individuals have occurred $(r < n)$. All equipment are put at the same time for the test and it is terminated when $r$ items out of $n$ have failed. These experiments are useful in saving time and money since it takes a long time in order to wait for all items to fail (Klein and Moeschberger, 2006).

Random censoring: is a more general scheme, each unit is associated with a potential censoring time, say $c_i$ and a potential lifetime say $T_i$. These are assumed to be independent random variables (Lawless, 2011).

### 1.3 The Likelihood function under Type I censoring

The likelihood function is the most significant concept in statistics that plays a key role in almost all areas of statistics. The inferential procedures derived from it are known to have optimal properties asymptotically under very general regularity conditions (Lehmann and Casella, 2006).

Assume that $t_i, (i = 1, \dots, n)$ is a random sample from a parametric model with probability density function given by $f\left(t_i, \underline{\theta}\right)$ and survival function is given by $S\left(t_i, \underline{\theta}\right)$, where $\underline{\theta} = (\theta_1, \dots, \theta_k)'$is a parameter vector, $c$ is a censoring constant. Based on Lawless (2011) the observed Type I censored sample is given in the form $(y_i, \delta_i), i = 1, \dots, n$, where

$$y_i = \min(t_i, \mathrm{c}) = \begin{cases} t_i, & t_i \leq \mathrm{c},\ \text{subject is complete} \\ \mathrm{c}, & t_i > \mathrm{c}\ ,\ \text{subject is right censored} \end{cases} \tag{3}$$

$$\delta_i = \begin{cases} 1, & t_i \leq \mathrm{c}, \quad \text{subject is complete} \\ 0, & t_i > \mathrm{c}, \quad \text{subject is right censored} \end{cases} \tag{4}$$

Moreover, $\delta_i$ is the event indicator.

The likelihood function under Type I censored data is given by (Lawless, 2011):

$$L\left(y_i, \underline{\theta}\right) = \prod_{i=1}^{n} f\left(y_i, \underline{\theta}\right)^{\delta_i} S\left(y_i, \underline{\theta}\right)^{1-\delta_i}. \tag{5}$$

### 1.4 The Log-Logistic distribution

The Log-Logistic distribution is an important survival parametric model that is used the field of science, actuarial, hydrology, survival analysis, reliability, and economics (Al-Shomrani, Shawky et al. 2016).

Suppose that $T$ is a Log-Logistic random variable with the cumulative probability function (cdf) given by (Lawless, 2011) as:

$$F(t,\alpha,\beta) = \frac{\left(t/\alpha\right)^{\beta}}{1+\left(t/\alpha\right)^{\beta}}, \quad t > 0, \alpha > 0, \beta > 0. \tag{6}$$

Differentiating equation (6) with respect to $t$ to obtain the probability density function (pdf) of the Log- Logistic distribution is obtained and given as:

$$f(t,\alpha,\beta) = \frac{\beta/\alpha \left(t/\alpha\right)^{\beta-1}}{\left(1+\left(t/\alpha\right)^{\beta}\right)^{2}}, t > 0, \tag{7}$$

where, $\alpha > 0$ is the scale parameter, $\beta > 0$ is the shape parameter, which controls the shape of the distribution as illustrated in Figure 1.

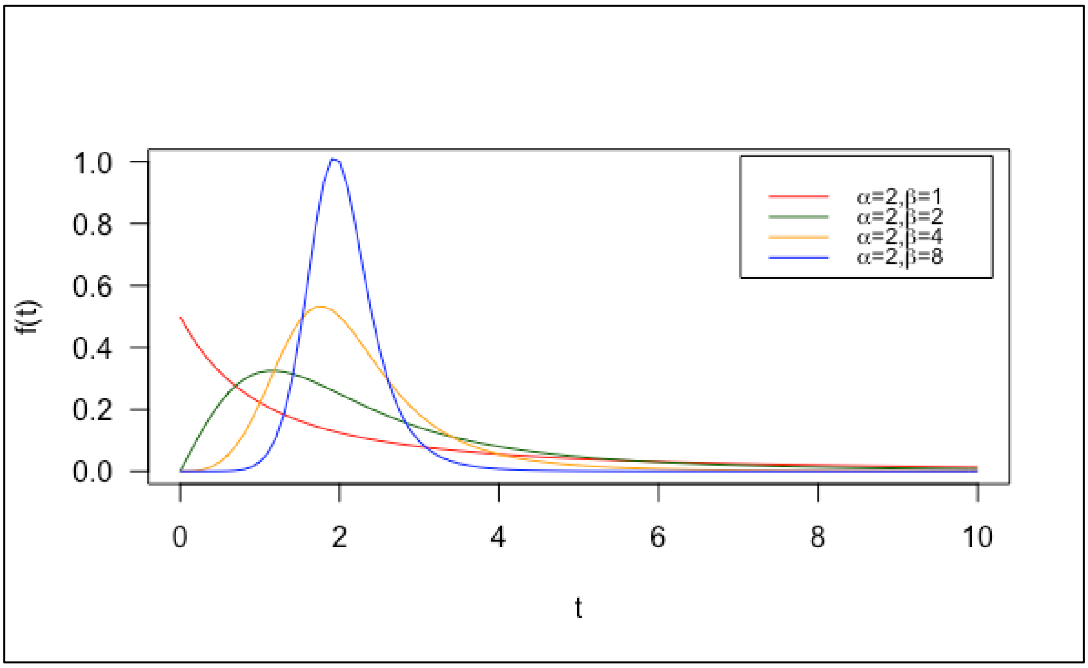


Figure 1. Plot of the pdf of the log-Logistic distribution with $\alpha = 2$ and various values of $\beta$.

Figure 1 shows that the Log-Logistic distribution has different shapes; it can be unimodal, right skewed, or decreasing. It is clear that from Figure 1, as $\beta$ increases the shape of the distribution is closer to be symmetric.

The survival function of Log-Logistic distribution is given by (Lawless, 2011) as:

$$S(t) = \frac{1}{1 + \left({}^{t}/_{\alpha}\right)^{\beta}}, t > 0. \tag{8}$$

Figure 2 illustrates the survival function of Log-Logistic distribution.

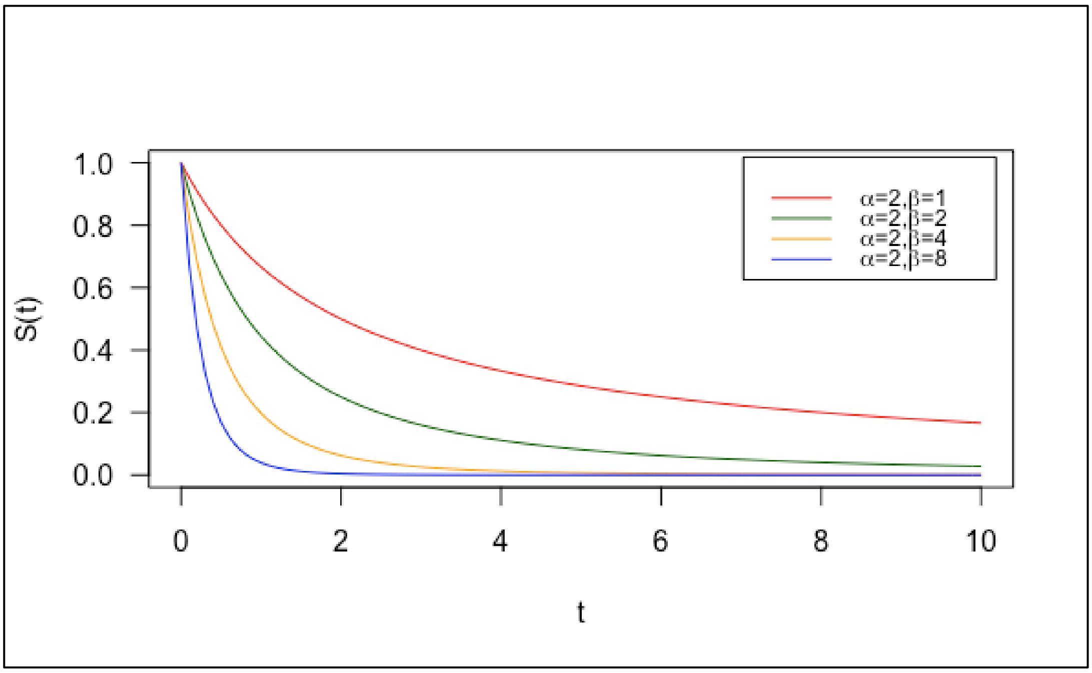


Figure 2. Plot of the survival function of the Log-Logistic distribution with α=2 and various values of β.

The survival function of the Log-Logistic distribution is a non-increasing function, which is shown in Figure 2 the hazard function of Log-Logistic distribution is obtained by taking the ratio of the pdf in equation (7) and the survival function in equation (8).

$$h(t) = \frac{f(t)}{s(t)} = \frac{{}^{\beta}/_{\alpha}\,({}^{t}/_{\alpha})^{\beta-1}}{1 + \left({}^{t}/_{\alpha}\right)^{\beta}} \tag{9}$$

Figure 3 illustrates the hazard function of Log-Logistic distribution.

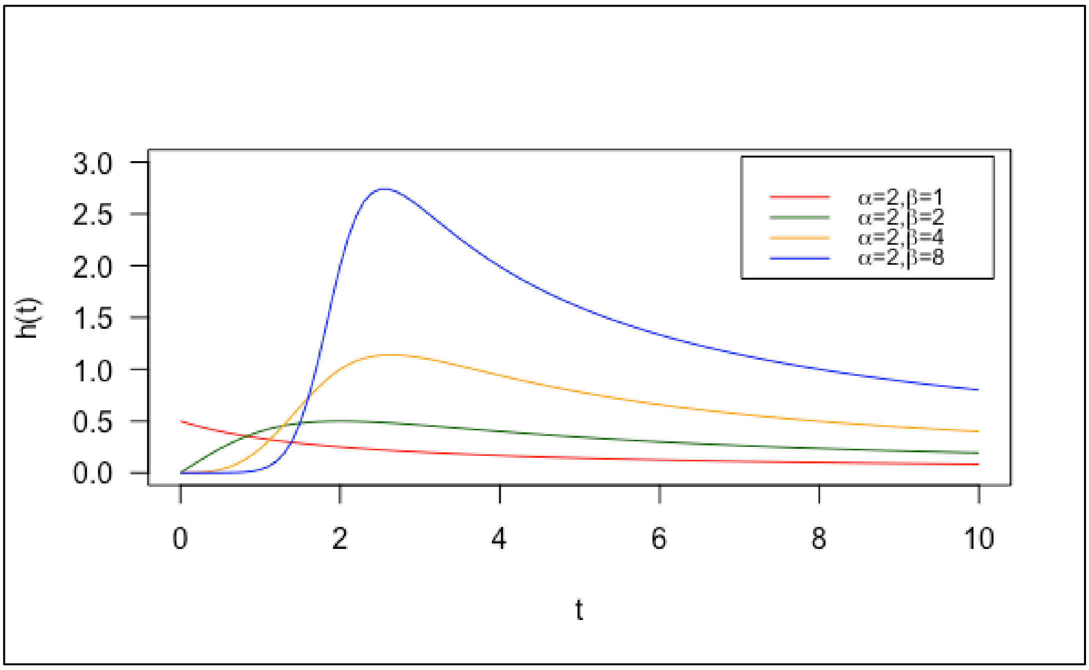


Figure 3. Plot of the hazard function of the Log-Logistic distribution with α=2 and various values of β.

Figure 3 shows that the hazard function of the Log-Logistic distribution can have different shapes for instance, it can be increasing, decreasing, and a hump shape. Thus, the Log-Logistic distribution can be used quite effectively in analyzing lifetime data.

### 1.5 Goodness of fit test based on EDF

The goodness of fit (GOF) test is a formal method to test the significance of difference between an empirical distribution function $F_n(x)$ and theoretical distribution function $F(x)$. Suppose that $F(x)$ is a continues cumulative distribution function, the hypothesis under the test is $H_0: F(x,\theta) = F_0(x,\theta)$ verses $H_1: F(x,\theta) \neq F_0(x,\theta)$. The most commonly used goodness of fit tests based on empirical distribution function statistics is Kolmogorov Smirnov, Cramer von Mises and Anderson Darling tests (D’Agostino and Stephens, 1986).

The empirical distribution function for any value of $x$ is a function of the observations less than or equal to $x$ which gives the probability that an observation is less than $x$. Thus, the empirical distribution function of a sample is defined as follows:

$$F_n(x) = \begin{cases} 0 & x < x_{(1)} \\ \frac{i}{n}; \quad x_{(i)} \le x < x_{(i+1)} & i = 1,2,\dots,n-1, \\ 1 & x \ge x_{(n)} \end{cases} \tag{10}$$

Kolmogorov proposed a test based on the discrepancy:

$$z_n(x) = F_n(x) - F(x), \tag{11}$$

where $F_n(x)$ is the empirical function and $F(x)$ is the theoretical distribution function. The Kolmogorov and Smirnov proposed a test called Kolmogorov-Smirnov test and it is given as follows (Stephens, 1986);

$$D = \max(D^+, D^-), \tag{12}$$

where $D^+ = sup_x\{z_n(x)\}$; $D^- = sup_x\{-z_n(x)\}$.

The Cramer-von Mises considered a family of tests statistics which is based on the integral of $z_n(x)$. The Cramer-von Mises family of statistics is given as:

$$C = n\int_{-\infty}^{\infty}\{z_n(x)\}^2\psi(x)\, dF(x), \tag{13}$$

where $\psi(x)$ is a weight function, when $\psi(x) = 1$ the Cramer-von Mises statistics is obtained, and when $\psi(x) = \{F(x)[1 - F(x)]\}^{-1}$, the Anderson-Darling statistics is provided.

To simplify the computation, the three tests can be re-written as (Stephens, 1986):

Kolmogorov-Smirnov (KS);

$$D = \max(D^{+}, D^{-}), \tag{14}$$

where $D^{+} = \max\left[ z_{(i)} - \frac{(i-1)}{n}\right]$ and $D^{-} = \max\left[ z_{(i)} - \frac{(i-1)}{n}\right]$

Cramer-von Mises (W);

$$W^{2} = \frac{1}{12n} + \sum_{i} \{ z_{(i)} - \frac{2i-1}{2n} \}^{2}, \tag{15}$$

Anderson-Darling (AD);

$$A^{2} = -n - \frac{1}{n} c z_{(i)} + \log(1 - z_{(n+1-i)})], \tag{16}$$

,where $z_{(i)} = F_{0}\left(x_{(i)}, \theta\right)$ is the theoretical cdf of the interest model.

## 1.6 Literature reviews

The purpose of the literature review is to gain an understanding of the current research relevant to a certain topic or study. Over the years, many researchers have considered the goodness of fit test problem. The main goal of these tests is to check whether a certain sample comes from a specific distribution by using different techniques.

### 1.6.1 Goodness of fit test based on empirical distribution function

Davis and Stephens (1989) applied the EDF tests on normal and Exponential distribution; they have obtained an approximate significance level for Cramer-von Mises, Anderson Darling test statistics, and Watson statistics. The Anderson–Darling statistics test was more appropriate and powerful in testing goodness of fit for the Normal and Exponential distribution. Some authors have worked on developing the

EDF efficiency, Al-Subh et al. (2009) improved the efficiency of EDF tests using ranked set sampling (RSS), which gives more information about the population of interest than simple random sampling (SRS). These tests were applied to the Logistic distribution and the results showed that the EDF tests were more efficient under the RSS compared to the SRS technique. Likewise, Ibrahim et al. (2009) studied the power of a set of modified empirical distribution function tests under SRS and extreme ranked set sampling (ERSS), which is a modification of RSS. They have shown that the power of set modified EDF tests was improved if the sample was collected via ERSS compared to SRS.

### 1.6.2 Goodness of fit test for Logistic distribution

It is well known that the Log-Logistic and Logistic are equivalent statistical models, any statistical technique developed for one distribution can be applied for the other distribution. Based on the available literature, there is no study considered the problem of goodness of fit test for Log-Logistic distribution. However, there are several studies focused on the goodness of fit tests (GOF) for the Logistic distribution using different methods.

Meintanis (2004) investigated the GOF test for the Logistic distribution based on weighted integrals involving two methods of empirical transformations, the first method was utilized by the empirical characteristic function (ECF), while the second utilized by the empirical moment generating function (EMGF). Gulati and Shapiro (2009) proposed a new GOF test for Logistic distribution, the proposed test was based on higher order spacing (m-step spacings) and it was a modification of the Greenwood statistics. In order to compute the test statistic, the parameters of the Logistics distribution were estimated by the maximum likelihood estimation and the method of moments. Hence, a power comparison for two methods of estimation was investigated. The results of the Monte Carlo simulation showed that the method of moments established higher power than the maximum likelihood estimation. Al-Subh et al. (2011) developed the GOF test for Logistic distribution based on Kullback-Leibler information, the Logistic parameters were estimated using several methods such as maximum likelihood estimation, order statistic, method of moments, L-moments and LQ-moments. In addition, the performance of the Kullback-Leibler information under a SRS was investigated. The test statistic based on estimators found by the method of moment and LQ-moment established the highest power in most of the cases. Alizadeh Noughabi (2015) worked on the GOF test for the Logistic model based on the empirical likelihood ratio (ELR), the location and scale parameters of the Logistics distribution were estimated using the maximum likelihood estimation method. A comparison of the power for the proposed and the classical tests based on EDF was carried out.The results of Monte Carol simulation revealed that the proposed test based on ELR outperformed in most of the cases. Alizadeh Noughabi (2017) studied the GOF for the Logistic distribution based on the Gini Index estimator, the approximate maximum likelihood estimator (AMLEs) was used to estimate the parameters of interest model. From the simulation results, the proposed test was more powerful compared to the EDF tests.

### 1.6.3 Goodness of fit test under for Type I censored sample

In addition, some authors investigated the problem of the GOF under Type I censoring. Bispo et al. (2012) studied the GOF test based on EDF under Type I right censored samples for various null and alternative lifetime models such as Weibull, Log-Normal, Exponential, and Log-Logistic distributions. The statistical power of Kolmogorov Simonov, Anderson Darling, and Cramer- von Mises statistics were evaluated using different sizes of a sample, significance level and various censoring proportion, whereas Pakyari and Balakrishnan (2013) developed the GOF test using Exponential model under Type I censored sample. The proposed method was based on considering the Type I censored sample as order statistics from a complete sample of size $d$ from Exponential distribution with right truncation, and then the classical GOF test for the complete sample was obtained. A comparison between the classical test and the proposed test were performed. The analysis of the power for several alternative distributions such as Weibull, Log-Normal, Lomax, and Gamma distribution was obtained. The authors concluded that the proposed test was more powerful than the classical test under Type I censoring. Further, Pakyari and Nia (2017) extended the method of Pakyari and Balakrishnan (2013) from the simple Exponential distribution to Log-Normal and Weibull distributions, they have concluded that this method is powerful for shape scale and location-scale models. Pakgohar et al. (2019) have studied the GOF for Normal and Exponential models under Type I censoring using two methods, the Lin-Wong divergences (LW) and the EDF. They have compared the performance of the two methods and concluded that when LW divergence measure was used to evaluate the distance, the results exhibited more powerful than the EDF under Type I censoring in measuring the difference between two distributions pattern.

Overall, most of the available literature reviews have studied the GOF test either for Logistics distribution with different techniques or various distributions under Type I censoring. Additionally, it was noticed that the problem of the GOF test for Log-Logistics based on EDF under type I censoring was insufficiently explored.

## 1.7 Problem statement

The Log-Logistic model is a continuous probability model with a non-negative random variable. In the survival analysis, it is used as a parametric distribution for modeling time to event to occur. The hazard function of the Log-Logistic model can have different shapes for instance; it can be increasing, decreasing, and a hump shape. Thus, Log-Logistic distribution can be used effectively in analyzing lifetime data. Type I censoring is common in survival analysis, the event is observed only if it is occurred before pre-specified time. This kind of censoring is found in many areas, such as medicine, biomedical sciences, and engineering.

The goodness of fit test procedure is important in the statistical analysis of lifetime data. It is used to verify the assumed distribution, which adequately fits the data. Previous studies have investigated the problem of the goodness of fit using the empirical distribution function for Type I censoring and for different lifetime models for example Weibull, Exponential, Log-Normal etc. Bispo et al. (2012) suggested to use of the empirical distribution function statistics in presence of Type-I censoring for some lifetime distribution including the Log-Logistic model. Pakyari and Balakrishnan (2013) developed a GOF test using the Exponential model under Type I censored sample. The proposed method was based on considering the type I sample as order statistics from Exponential distribution with the right truncation and by treating this sample as a complete sample, then obtaining the classical tests for the complete sample. Pakyari and Nia (2017) extended the method of Pakyari and Balakrishnan (2013) from simple Exponential distribution to Log-Normal and Weibull distribution.

However, there is no prior study investigates the problem of the goodness of fit tests for log-Logistic distributions under Type I censored sample. Thus, due to work limitation on the goodness of fit test for the Log-Logistic distribution, this study aims to consider the problem of the goodness of fit test based on the empirical distribution function for the Log-Logistic distribution when the available data is in the form of Type I censoring. In this study, A test procedure proposed by Pakyari and Nia (2017) will be applied and a Monte Carlo power study will be conducted to assess the performance of the proposed test with the aim to exhibit higher power as compared to the existing classical method. For this purpose, several alternative models such as, Log-normal, Weibull, Exponential, Gamma, and Lomax distributions will be considered. Over the past few years, the Log-Logistic distribution has been widely used in analyzing lifetime data, owing to its flexibility. Type I censoring is very common in survival analysis. According to Bispo et al. (2012) suggested the use of EDF statistics in presence of Type-I censoring for some lifetime distribution including Log-Logistic model. Pakyari and Balakrishnan (2013) and Pakyari and Nia (2017) used a new methodology for conducting goodness of fit test under Type-I censoring scheme. Therefore, due literatures reviews and work limitation on goodness of fit test, this study aims to investigate the problem of the goodness of fit test of Log-Logistic distribution under Type I censored sample. A test procedure proposed by Pakyari and Balakrishnan (2013) will be used to compare the power with the existing classical method, as well as, the power analysis of the proposed test will be evaluated for different alternative models using Monte carlo studies and real data application.

## 1.8 Research objective and Significant of the study

The main objective is to derive the maximum likelihood estimator for the scale and shape parameters of the Log-Logistic model under Type I censoring, which will be used to compute the proposed and classical goodness of fit tests. Furthermore, a power comparison for the proposed and classical tests with several alternative models such as Gompertz, Weibull, BurrX, and Exponential will be considered. The results of the power analysis for the Log-Logistic model under Type I censored data based on empirical distribution function will illustrate the flexibility and significant of the Log-Logistic distribution in the field of survival analysis.

### 1.8.1 Research Specific Objectives

This study aims to investigate the following specific objectives:

1. Obtain the maximum likelihood estimator for the unknown parameters for the Log-Logistics distribution under Type I censored data.
2. Compute the proposed goodness of tests (GOF) based on the empirical distribution function (EDF) for the Log-Logistic distribution.
3. Compute the classical goodness of tests (GOF) based on the empirical distribution function (EDF) for the Log-Logistic distribution.
4. Calculate the critical points for the proposed and classical tests.
5. Conduct a Monte Carlo power studies to compare the performance of the classical and proposed tests for the Log-Logistic distribution.

## 1.9 Scope of Study

As mentioned above, the work in this thesis is involved with the goodness of fit technique for testing Log-Logistic model under Type I censored sample based on the EDF. Some fundamental concepts in the survival analysis field, such as survival function, hazard function, censoring schemes, and background of the Log-Logistic parametric model are introduced in Chapter 1. As well as, overview of the goodness of fit test based on an empirical distribution function and reviews of the literature related to GOF tests also delivered in Chapter 1. While in Chapter 2 computations of the goodness of fit test and the maximum likelihood estimator for Log-Logistic model under Type I censored sample will be considered. Furthermore, the proposed test and the classical test based on the EDF will be introduced in Chapter 2. A Monte Carlo power study and discussion of the results will be investigated in Chapter 3. While in Chapter 4 real data applications under Type I censoring will be explored. Lastly, Summary, conclusion, and suggestion for further studies will be given in Chapter 5.

# CHAPTER 2: GOODNESS OF FIT TEST TECHNIQUE

The goodness of fit test technique is important in the statistical analysis of lifetime data. It is used to verify the assumed distribution, which adequately fits the data. The most common method is the empirical distribution function (EDF) test statistics, which compare the theoretical continuous distribution function with the empirical distribution function (Huber-Carol, Balakrishnan et al. 2012).

## 2.1 Goodness of fit test for testing the Log Logistic distribution under Type I censored sample.

This section discusses the procedure of goodness of fit test for testing the Log-Logistic distribution under Type I censored sample based on the empirical distribution function. It is known that Log-Logistic and Logistic are equivalent statistical models. For example, any statistical technique developed for one distribution can be easily applied for the other distribution. However, since the Logistic model is the location-scale distribution, the distribution of the empirical distribution function statistics will not depend on the true values of the unknown parameters, while the Log-Logistic model is the scale-shape distribution and hence does not have this useful property. If the random variable $X$ follow Log-Logistic distribution, then $Y = log(X)$ is the Logistic random variable with pdf given as;

$$f(y,\mu,\sigma) = \frac{exp\left(\frac{y-\mu}{\sigma}\right)}{\sigma\left[1+ exp\left(\frac{y-\mu}{\sigma}\right)\right]^2}, \qquad -\infty < y < \infty, -\infty < \mu < \infty, \sigma > 0, \qquad (17)$$

where $\mu = log(\alpha)$ and $\sigma = \frac{1}{\beta}$ (Lawless, 2011).

Integrating equation (17) with respect to variable y then, the cumulative probability function (cdf) of the Logistic distribution is obtained and given as follows:

$$\mathrm{F}(y,\alpha,\beta) = \frac{1}{1+\ exp\left(-\frac{y-\mu}{\sigma}\right)}, \quad -\infty < \ y < \infty\,. \tag{18}$$

Suppose $X_1, \ldots, X_n$ is a random sample of Type-I censored from a probability distribution function F and let $c > 0$ is a pre-fixed censored time for the life testing experiment. In this study, Type I censored sample is treated as order statistics from a complete sample of size $d$ and right truncated at c . In other words, only subjects with observed time less than or equal the censoring time will be considered, which is the complete failure subjects of Type I censored samples, $X_1, \ldots, X_d$ of size $d \leq n$.

Therefore, we are interested in testing that $X_1, \ldots, X_d$ follow a Log-Logistic distribution or equivalently we are interested in testing that the log transformed data $Y_i = log(X_i), i = (1, \ldots, d)$ follow Logistic model with mean $\mu = log(\alpha)$ and standard deviation $\sigma = \frac{1}{\beta}$ .

Thus, we are interested in testing the goodness of fit test hypothesis that:

$$H_0: F(y,\mu,\sigma) = \frac{1}{1+exp\left(-\frac{y-\mu}{\sigma}\right)}$$

$$H_1: F(y,\mu,\sigma) \neq \frac{1}{1+exp\left(-\frac{y-\mu}{\sigma}\right)}, \quad -\infty \leq \ y \leq \infty\,, -\infty \leq \ \mu \leq \infty.\ \ \sigma \geq 0. \tag{19}$$

To test the null hypothesis in (19) the MLE's of the Logistic distribution is required.

### 2.2 Maximum Likelihood Estimator for Log-Logistic and Logistic distributions

Suppose $Y_i, (i = 1, \ldots, n)$ is a Type I random sample from Logistic model. Then $X_i = exp(Y_i), (i = 1, \ldots, n)$ is a Type I censored lifetime from Log-Logistic distribution. Based on Lawless (2011) the likelihood function for Log-Logistic distribution under Type I censored lifetime is given as,

$$L(x_i, \alpha, \beta) = \prod_{i=1}^{n} f(x_i, \alpha, \beta)^{\delta_i} \, S(x_i, \alpha, \beta)^{1-\delta_i}. \tag{20}$$

and, the log-likelihood function of Log-Logistic distribution under Type I censored lifetime is given as,

$$Log(L) = \sum_{i=1}^{n} \delta_i \; log\left({}^{\beta}/_{\alpha}\right) + (\beta - 1) \sum_{i=1}^{n} \delta_i \; log\left(\frac{x_i}{\alpha}\right) - 2 \sum_{i=1}^{n} \delta_i \; log\left(1 + \left(\frac{x_i}{\alpha}\right)^{\beta}\right)$$

$$- \sum_{i=1}^{n} (1 - \delta_i) \log(1 + \left(\frac{x_i}{\alpha}\right)^{\beta}). \tag{21}$$

The first partial derivative of the log-likelihood function with respect to the parameters $(\alpha, \beta)$ in equation (21) and by equating the derivative to zero, obtain the following:

$$\frac{\partial \, log(L)}{\partial \alpha} = \sum_{i\,=\,1}^{n} (1 - \delta_i) \left[ \frac{\beta x_i \left(\frac{x_i}{\alpha}\right)^{\beta - 1}}{\alpha^2 \left( 1 + \left(\frac{x_i}{\alpha}\right)^{\beta} \right)} \right] - \sum_{i\,=\,1}^{n} \frac{\delta_i}{\alpha} - (\beta - 1) \sum_{i\,=\,1}^{n} \frac{\delta_i}{\alpha}$$

$$+ 2 \sum_{i\,=\,1}^{n} \delta_i \left[ \frac{\beta x_i \left(\frac{x_i}{\alpha}\right)^{\beta - 1}}{\alpha^2 \left( 1 + \left(\frac{x_i}{\alpha}\right)^{\beta} \right)} \right] = 0\,. \tag{22}$$

$$\frac{\partial \, log(L)}{\partial \beta} = - \sum_{i\,=\,1}^{n} (1 - \delta_i) \left[ \frac{log\left(\frac{x_i}{\alpha}\right) \left(\frac{x_i}{\alpha}\right)^{\beta}}{1 + \left(\frac{x_i}{\alpha}\right)^{\beta}} \right] + \sum_{i\,=\,1}^{n} \frac{\delta_i}{\beta} + \sum_{i\,=\,1}^{n} \delta_i \, log\left(\frac{x_i}{\alpha}\right)$$

$$-2 \sum_{i\,=\,1}^{n} \delta_i \left[ \frac{log\left(\frac{x_i}{\alpha}\right) \left(\frac{x_i}{\alpha}\right)^{\beta}}{1 + \left(\frac{x_i}{\alpha}\right)^{\beta}} \right] = 0\,. \tag{23}$$

The root of these equations (22 and 23) is maximum likelihood estimator ($\hat{\alpha}$ and $\hat{\beta}$) of the Log-Logistic distribution. These equations will be solved simultaneously using Newton-Raphson iterative method, as these equations cannot be solved analytically.

Using the invariant property of the maximum likelihood estimator the MLE's $(\hat{\mu}, \hat{\sigma})$ of the Logistic distribution is obtained Lawless (2011) and given as the following;

$$\hat{\mu} = log(\hat{\alpha}) \text{ and } \hat{\sigma} = \frac{1}{\hat{\beta}}. \tag{24}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the MLE's of Log-Logistic parameters.

## 2.3 Compute GOF tests based on EDF statistics

In this section, the GOF tests based on EDF statistics will be presented for proposed and the classical methods. The Kolmogorov-Smirnov (KS), Cramer-von Mises (W) and Anderson-Darling (AD) tests are the most commonly used EDF statistics of GOF tests. These tests compare the theoretical continuous distribution function to the empirical distribution function of the samples (Stephens, 1974).

### 2.3.1 Proposed method

Transform the Type I sample of the Logistic model $Y_1, \ldots, Y_d$ to uniformity order statistic from the uniform distribution by using the transformation,

$$u_i = \frac{1}{1 + exp\left(-\frac{y_i - \hat{\mu}}{\hat{\sigma}}\right)} \Bigg/ \frac{1}{1 + exp\left(-\frac{c - \hat{\mu}}{\hat{\sigma}}\right)} \quad , i = (1, \ldots, d) \tag{25}$$

$u_i$ is calculated based on the ratio of cdf of the Logistic distribution evaluated at $y_i$ and censoring time c where $(\hat{\mu}, \hat{\sigma})$ are the MLE's of the Logistic distribution obtained in (2.2).

The order statistic $u_1, u_2, \ldots, u_d$ will be treated as a complete sample of size $d$ and any usual goodness of fit test technique available for complete data can be applied from (Pakyari and Balakrishnan, 2013). The proposed EDF tests calculated as follows:

Kolmogorov-Smirnov statistics;

$$D_d = max_{1\le i\le d}\left[max\left\{\frac{i}{d} - u_i, u_i - \frac{i-1}{d}\right\}\right], \tag{26}$$

Cramer-von Mises statistics;

$$W_d^2 = \sum_{i=1}^{d}(u_i - \frac{2i-1}{2d})^2 + \frac{1}{12d}, \tag{27}$$

Anderson-Darling statistics;

$$A_d^2 = -d - \frac{1}{d}\sum_{i=1}^{d}(2i-1)\{log(u_i) + log(1 - u_{d+1-i})\}, \tag{28}$$

EDF tests in equations (26, 27, and 28) are calculated based on $u_i$ that obtained in equation (25).

### 2.3.2. Classical method

For the Type I sample of Logistic model $Y_1, \dots, Y_d$ $u_i$ is calculated as,

$$u_i = \frac{1}{1+exp\left(-\frac{y_i-\hat{\mu}}{\hat{\sigma}}\right)}, i = (1, \dots, d). \tag{29}$$

$u_i$ is calculated based on the cdf of the Logistic distribution evaluated at $y_i$ where $(\hat{\mu}, \hat{\sigma})$ are the MLE's of the Logistic distribution obtained in (2.2).

Considering the order statistics, $u_1, u_2, \dots, u_d$ the classical EDF tests are calculated as follows:

Kolmogorov-Smirnov statistics is proposed by D'Agostino and Stephens (1986);

$$D_{n,p=}max_{1\le i\le d}\left[max\left\{\frac{i}{n} - u_i, u_i - \frac{i-1}{n}\right\}\right]. \tag{30}$$

Cramer-von Mises statistics is proposed by Pettitt and Stephens (1976);

$$W_{n,p}^2 = \sum_{i=1}^{d}(u_i - \frac{2i-1}{2n})^2 - \frac{d(4d-1)}{12n^2} + n\,u_d\left(\frac{d^2}{n^2} - u_d\frac{d}{n} + \frac{1}{3}{u_d}^2\right) \tag{31}$$

Anderson–Darling statistics is proposed by Pettitt and Stephens (1976);

$$A_{n,p}^{2}=\sum_{i=1}^{d}\left(\frac{2i-1}{n}\right)[log(1-u_i)-log(u_i)]-2\sum_{i=1}^{d}log(1-u_i)$$
$$+n\left[\frac{2d}{n}-\left(\frac{d}{n}\right)^{2}-1\right]log(1-u_d)+\frac{d^2}{n}log(u_d)-n\,u_d \tag{32}$$

where $n$ is the size of Type I censored sample, while $d$ is size of complete failure subjects of Type I censored and $u_d$ is the value of the cdf of Logistic distribution evaluated at $c$ and $u_i$ is the value of the cdf of Logistic distribution evaluated at $y_i$ which is found it in equation (29).

## 2.4 Critical points and Empirical significant levels

### 2.4.1 Critical points

The critical values are required for testing the Log-Logistic model which statistically equivalent for testing the hypothesis in equation (19). The steps of finding the critical values for the proposed and the classical methods will be as follows:

**Step1**: Choose c from a standard Logistic distribution with different proportions of failure, $F(c\ )=\frac{1}{1+exp\left(-\frac{c-\mu}{\sigma}\right)}$ which is based on the cdf of Logistic distribution evaluated at c.

**Step 2**: Generate a Type I sample, $Y_1,\dots,Y_n$ with a chosen sample size n from a standard Logistic distribution $(i.e.\mu\ =\ 0\ and\ \sigma\ =1)$. For more details, see appendix A.

**Step 3**: Calculate $X_i=exp(Y_i\ )\ ,i\ =(1,\dots,n)$, the distribution of $x_i$*'s* will be Log-Logistic distribution with parameters $\alpha\ =exp(\mu)$ and $\beta\ =\frac{1}{\sigma}$, $x_i,\alpha,\beta\geq0$.

**Step 4**: From the data that are obtained in step 3, calculate the MLE's $(\hat{\alpha}, \hat{\beta})$ for the parameters of Log-Logistic distribution then, by applying the invariant property of the MLE's $(\hat{\mu}, \hat{\sigma})$ of the Logistic distribution are obtained as follows $\hat{\mu} = log(\hat{\alpha})$ and $\hat{\sigma} = \frac{1}{\hat{\beta}}$.

**Step 5**: From the data that are obtained in step 2, $Y_1, \ldots, Y_n$ only the complete failure subjects with observed time less than or equal the censoring time will be considered, $Y_1, \ldots, Y_d$ and the sample will be of size $d$.

**Step 6**: Using the data obtained in step 5, the values of $u_i$ is calculated as follows: For the proposed method using equation (25), which is based on the ratio of the cdf of the Logistic distribution evaluated at the values $y_i$ and c. For the Classical method $u_i$ is calculated as equation (29), which is based on the cdf of the Logistic distribution evaluated at $y_i$.

**Step 7**: Using the order statistics of $u_i$ and calculate the EDF statistics for the proposed method use equations (26, 27, and 28) and for the classical method use equations (30, 31, and 32).

**Step 8**: Repeat Steps 1–7 many times and calculate the $(1 - \alpha)^{th}$ quantile of the proposed and the classical EDF tests statistic as the required critical values with the significant levels $\alpha = 5\%\ and\ 10\%$.

### 2.4.2 Empirical significant level

To check the validity of the critical points in 2.4.1 the empirical significant level is needed. The significant level is defined as the probability of rejecting the distribution under the null given that the null hypothesis is assumed to be true.

The steps of calculating the empirical significant level will be as follows:

1. Calculate the same steps 1-7 of the critical points in section 2.4.1.
2. Repeat Steps 1–7 many times and calculate the empirical significant level of the proposed and classical EDF tests as the proportion of replications in which the test statistic exceeded its corresponding critical value obtained in section 2.4.1.
3. The empirical significant levels are obtained at two nominal levels $\alpha = 5\%$ and $10\%$.

## 2.5 Power Analysis

Power of the tests is the probability of rejecting the null hypothesis when the alternative hypothesis is true and can be obtained as follows:

$$Power = p(reject\ H_0 \mid H_1 true).$$

Power analysis is conducted to evaluate the behavior of the proposed tests and classical EDF statistics by comparing their power, several alternative models such as Gompertz, Weibull, BurrX, and Exponential distributions are considered. The probability density function for the alternative models are as follows:

The Gompertz distribution (Wu et al., 2003);

$$f(t,p,d) = d\ e^{pt} e^{\left[\frac{d}{p}\left(1-e^{pt}\right)\right]},\quad t > 0\,, p > 0, d > 0. \tag{33}$$

The Weibull distribution (Collett, 2015);

$$f(t,b,a) = abt^{b-1}\, e^{-at^{b}},\quad t > 0\,, b > 0\ a > 0. \tag{34}$$

The Burr X distribution (Al-Nasser and Baklizi, 2004);

$$f(t,\nu,\theta) = \frac{2\nu t}{\theta^2}\left( e^{-\left(\frac{t}{\theta}\right)^2}\right)\left(1 - e^{-\left(\frac{t}{\theta}\right)^2}\right)^{\nu-1}, \quad t > 0\,, \nu, \theta > 0. \tag{35}$$

The Exponential distribution (Choi et al., 2004);

$$f(t,\lambda) = \lambda\, e^{-\lambda t}, \quad t > 0\,, \lambda > 0. \tag{36}$$

### 2.5.1 Empirical power of the tests

This section provides an explanation about conducting the power analysis and calculating the Empirical power for testing the log-Logistic distribution. The steps of calculating the power of EDF tests for the proposed and classical methods as the following:

**Step 1**: Choose c from the alternative distributions with different proportions of failure, $F(c)$ that based on the cdf of alternative distributions evaluated at c.

**Step 2**: Generate a Type I censored sample, $Y_1, \dots, Y_n$with a chosen sample size $n$ from the alternative distributions.

**Step 3**: Using the data obtained in step 2, calculate the MLE's $(\hat{\alpha}, \hat{\beta})$ for the parameters of Log-Logistic distribution then, by applying the invariant property of the MLE's $(\hat{\mu}, \hat{\sigma})$ of the Logistic distribution are obtained as follows $\hat{\mu} = log(\hat{\alpha})$ and $\hat{\sigma} = \frac{1}{\hat{\beta}}$.

**Step 4**: From the data that are obtained in step 2, $Y_1, \dots, Y_n$ only the complete failure subjects with observed time less than or equal the censoring time will be considered, $Y_1, \dots, Y_d$ and the sample will be of size $d$.

**Step 5**: Using the data that are obtained in step 4, the values of $u_i$ is calculated as follows:

For the proposed method using equation (25), which is based on the ratio of the cdf of the Logistic distribution evaluated at $log(c)$ and $log(y_i)$ where c and $y_i$found in step (1 and 2). For the classical method $u_i$ is calculated as equation (29), which is based on the cdf of the Logistic distribution evaluated at $log(y_i)$ where $y_i$ is found in step 2.

**Step 6**: Using the order statistics of $u_i$ and calculate the EDF statistics for the proposed method use equations (26, 27, and 28) and for the classical method use equations (30, 31, and 32).

**Step 7:** Repeat Steps 1–6 many times and calculate the power of the proposed and classical EDF tests as the proportion of replications in which the test statistic exceeded its corresponding critical value obtained in section 2.4.1

# CHAPTER 3: FORMATTING OR MODIFYING HEADINGS IN A MANUSCRIPT

The importance of simulation studies that they are used in obtaining empirical results to evaluate the efficiency of statistical methods in certain scenarios, since it can understand the behavior of statistical methods. This assists the researchers to consider several properties of statistical methods. In addition, Monte carlo studies are statistical procedures that obtain numerical results depending on repeated random sampling (Morris et al., 2019).

## 3.1 Monte Carlo simulation studies

In this section, Monte Carlo simulation studies are considered for testing the Log-Logistic model under Type I censored data. Power analysis is carried out to assess the performance of the Kolmogorov Smirnov (KS), Cramer von Mises (W), the Anderson Darling (AD) tests for the proposed and classical methods. Several alternative distributions with different shape parameter values are considered for power analysis such as Gompertz, Weibull, Burr X, and Exponential distributions. Moreover, the critical points for both methods are calculated to obtain empirical power while the empirical significance level ($\alpha$) is calculated to check the validity of the critical points. Hence, the critical points and the empirical significance level are computed at two significant levels ($\alpha$=0.10 and 0.05). 10,000 iterations with a chosen sample size ($n$=35, 60, 90) and observed proportions of failures F(c) =0.40 (small), F(c) =0.60 (moderate), F(c) =0.80 (heavy) are used to obtain the critical points, the empirical significance levels, and the power analysis. All the simulation studies are implemented using the R program. Figures 4 - 6 display the plots of the densities for the Log-Logistic verse the alternative distributions with different parameters values.

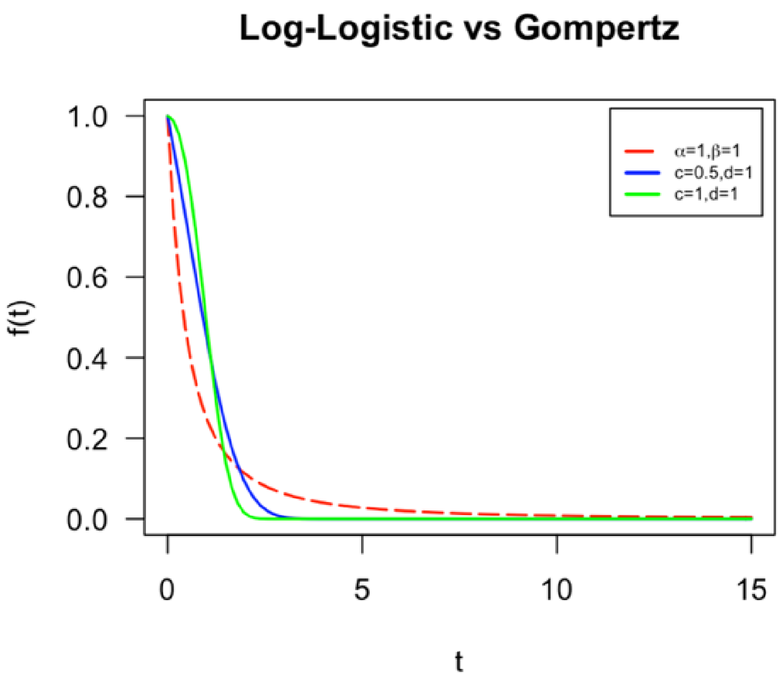


Figure 4. The plots of densities for the Log-Logistics (dashed lines) verse Gompertz (solid line).

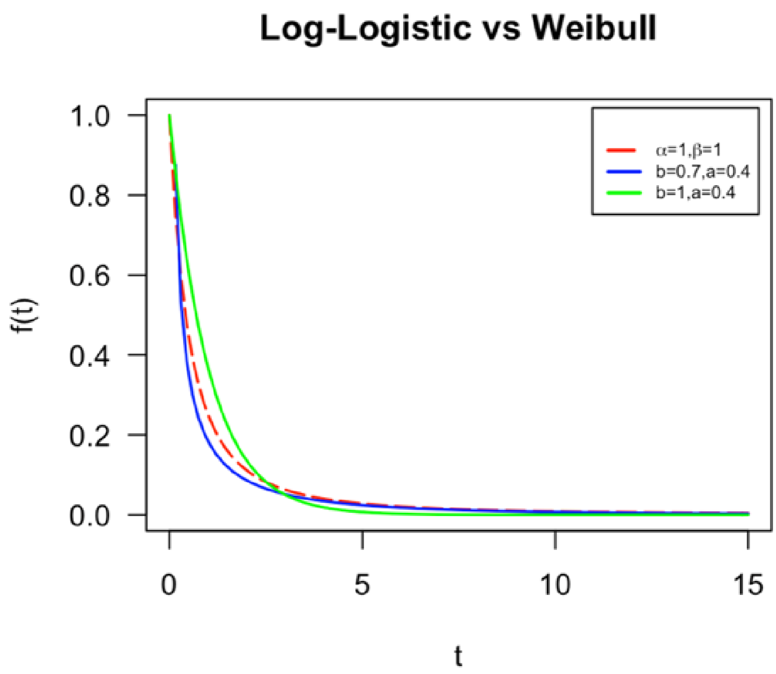

Figure 5. The plots of densities for the Log-Logistics (dashed lines) verse Weibull (solid line).

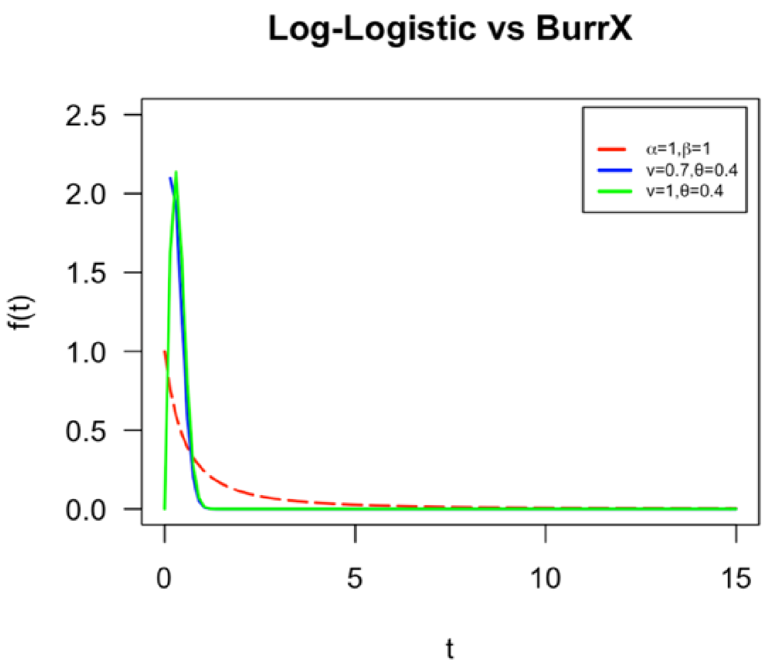


Figure 6. The plots of densities for the Log-Logistics (dashed lines) verse BurrX (solid line).

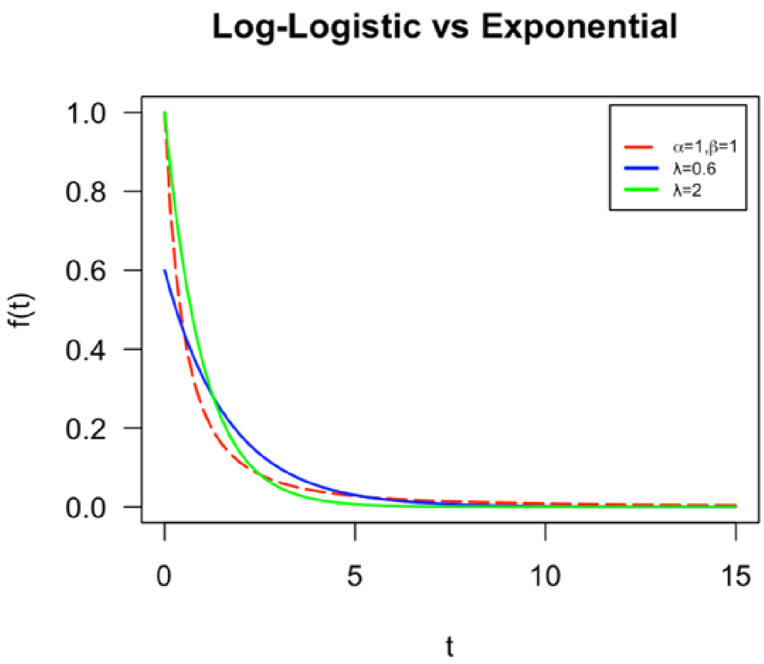


Figure 7. The plots of densities for the Log-Logistics (dashed lines) verse Exponential (solid line).

### 3.2 Results and Comparison

Table 1 presents the critical points for EDF tests at nominal levels α = 0.10 and 0.05 for different choices of $n$ and observed proportions of failures F(c). The following proportions of failures $\mathrm{F(c)} = 0.40$, $\mathrm{F(c)} = 0.60$ and $\mathrm{F(c)} = 0.80$ are calculated by substituting the following values of censoring time $\mathrm{c} = -0.40$, $\mathrm{c} = 0.41$ and $\mathrm{c} = 1.41$ respectively in the cdf of Logistic distribution with $\mu = 0$ and $\sigma = 1$. For more details about the critical points, see section 2.4.1.

Table 1. Critical points for EDF tests at nominal level $\alpha$ = 0.05 and 0.10 for different sample size $n$ and different proportion of failures F(c).

| | | | | $\mathrm{F(c)} = \frac{1}{1+\exp\left(-\frac{c-\mu}{\sigma}\right)}$ | | |
|---|---|---|---|---|---|---|
| $\alpha$ | $n$ | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
| 0.05 | 35 | Proposed | KS | 0.28990 | 0.23291 | 0.19113 |
| | | | W | 0.21999 | 0.21537 | 0.18675 |
| | | | AD | 1.28454 | 1.28056 | 1.14355 |
| | | Classical | KS | 0.10113 | 0.11629 | 1.14355 |
| | | | W | 0.06529 | 0.09213 | 0.05538 |
| | | | AD | 0.22037 | 0.33514 | 0.46526 |
| | 60 | Proposed | KS | 0.22139 | 0.17817 | 0.14736 |
| | | | W | 0.21930 | 0.21103 | 0.18699 |
| | | | AD | 1.31015 | 1.25761 | 1.15640 |
| | | Classical | KS | 0.07867 | 0.08960 | 0.09703 |
| | | | W | 0.11482 | 0.15377 | 0.08725 |
| | | | AD | 0.21723 | 0.32549 | 0.45741 |
| | 90 | Proposed | KS | 0.18160 | 0.14548 | 0.12170 |
| | | | W | 0.22247 | 0.21390 | 0.19153 |
| | | | AD | 1.32615 | 1.27203 | 1.16713 |
| | | Classical | KS | 0.06404 | 0.07363 | 0.07980 |
| | | | W | 0.17127 | 0.23742 | 0.12450 |
| | | | AD | 0.21509 | 0.32834 | 0.46155 |
| 0.10 | 35 | Proposed | KS | 0.26351 | 0.21062 | 0.17366 |
| | | | W | 0.17617 | 0.17084 | 0.14931 |
| | | | AD | 1.04411 | 1.03227 | 0.93222 |
| | | Classical | KS | 0.09202 | 0.10675 | 0.11585 |
| | | | W | 0.09202 | 0.06569 | 0.03486 |
| | | | AD | 0.18085 | 0.27574 | 0.39087 |
| | 60 | Proposed | KS | 0.19980 | 0.16081 | 0.13349 |
| | | | W | 0.17503 | 0.16883 | 0.14851 |
| | | | AD | 1.05352 | 1.02113 | 0.93303 |
| | | Classical | KS | 0.07158 | 0.08264 | 0.08971 |
| | | | W | 0.08131 | 0.11165 | 0.05589 |
| | | | AD | 0.17831 | 0.27314 | 0.38264 |
| | 90 | Proposed | KS | 0.16292 | 0.13110 | 0.10985 |
| | | | W | 0.17554 | 0.16857 | 0.15021 |
| | | | AD | 1.04992 | 1.02576 | 0.94012 |
| | | Classical | KS | 0.05830 | 0.06783 | 0.07366 |
| | | | W | 0.12018 | 0.16806 | 0.08595 |
| | | | AD | 0.17789 | 0.27428 | 0.38694 |

Table 2. Estimated Empirical level for EDF tests at nominal level α = 0.05 and 0.10 for different sample size $n$ and different proportion of failures F(c).

| α | n | Methods | Test statistics | $F(c) = \frac{1}{1+\exp\left(-\frac{c-\mu}{\sigma}\right)}$ 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| 0.05 | 35 | Proposed | KS | 0.054 | 0.051 | 0.050 |
| | | | W | 0.049 | 0.045 | 0.052 |
| | | | AD | 0.047 | 0.046 | 0.050 |
| | | Classical | KS | 0.047 | 0.045 | 0.046 |
| | | | W | 0.047 | 0.047 | 0.048 |
| | | | AD | 0.048 | 0.045 | 0.050 |
| | 60 | Proposed | KS | 0.047 | 0.046 | 0.050 |
| | | | W | 0.048 | 0.050 | 0.052 |
| | | | AD | 0.046 | 0.053 | 0.049 |
| | | Classical | KS | 0.045 | 0.045 | 0.047 |
| | | | W | 0.046 | 0.054 | 0.051 |
| | | | AD | 0.046 | 0.048 | 0.051 |
| | 90 | Proposed | KS | 0.050 | 0.048 | 0.048 |
| | | | W | 0.052 | 0.050 | 0.048 |
| | | | AD | 0.049 | 0.052 | 0.050 |
| | | Classical | KS | 0.050 | 0.051 | 0.050 |
| | | | W | 0.046 | 0.054 | 0.048 |
| | | | AD | 0.051 | 0.049 | 0.052 |
| 0.10 | 35 | Proposed | KS | 0.098 | 0.097 | 0.101 |
| | | | W | 0.096 | 0.093 | 0.099 |
| | | | AD | 0.097 | 0.091 | 0.102 |
| | | Classical | KS | 0.096 | 0.095 | 0.097 |
| | | | W | 0.093 | 0.093 | 0.101 |
| | | | AD | 0.097 | 0.093 | 0.099 |
| | 60 | Proposed | KS | 0.095 | 0.095 | 0.099 |
| | | | W | 0.096 | 0.098 | 0.103 |
| | | | AD | 0.093 | 0.103 | 0.102 |
| | | Classical | KS | 0.094 | 0.093 | 0.094 |
| | | | W | 0.089 | 0.099 | 0.103 |
| | | | AD | 0.098 | 0.095 | 0.104 |
| | 90 | Proposed | KS | 0.102 | 0.103 | 0.098 |
| | | | W | 0.096 | 0.100 | 0.099 |
| | | | AD | 0.097 | 0.101 | 0.103 |
| | | Classical | KS | 0.099 | 0.100 | 0.101 |
| | | | W | 0.098 | 0.105 | 0.092 |
| | | | AD | 0.100 | 0.095 | 0.098 |

Tables 2 shows the estimated empirical levels for EDF tests at nominal levels α = 0.05 and 0.10 for different choices of n and observed proportion of failures F(c). From Tables 2 it is observed that the estimated empirical levels are close to the nominal levels α = 0.05 and 0.10 for both proposed and classical methods, this gives a good indication about the validity of the critical points in Tables 1. In addition, from Table 2 it observed that the estimated significance levels of the proposed method are closer to the nominal level than the classical method in most of the cases. Furthermore, Tables 3-6 present the estimated power for testing the Log-Logistic against several alternatives at nominal level α = 0.05 and Tables 7-10 present the estimated power for testing the Log-Logistic against several alternatives at nominal level α = 0.10.

Table 3. Estimated power for Gompertz distribution with different sizes and proportion of failures F(c) at nominal level 0.05.

| | | | | $F(c,p,d) = 1 - e^{\left[\frac{d}{p}(1-e^{pc})\right]}$ | | |
|---|---|---|---|---|---|---|
| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
| 35 | Gompertz (0.5, 1) | Proposed | KS | 0.056 | 0.090 | 0.235 |
| | | | W | 0.063 | 0.112 | 0.291 |
| | | | AD | 0.076 | 0.132 | 0.312 |
| | | Classical | KS | 0.059 | 0.083 | 0.144 |
| | | | W | 0.066 | 0.094 | 0.134 |
| | | | AD | 0.067 | 0.097 | 0.186 |
| 60 | | Proposed | KS | 0.068 | 0.133 | 0.372 |
| | | | W | 0.079 | 0.182 | 0.462 |
| | | | AD | 0.089 | 0.201 | 0.467 |
| | | Classical | KS | 0.062 | 0.114 | 0.220 |
| | | | W | 0.079 | 0.177 | 0.341 |
| | | | AD | 0.078 | 0.146 | 0.301 |
| 90 | | Proposed | KS | 0.075 | 0.195 | 0.518 |
| | | | W | 0.090 | 0.240 | 0.615 |
| | | | AD | 0.098 | 0.258 | 0.628 |
| | | Classical | KS | 0.078 | 0.151 | 0.322 |
| | | | W | 0.094 | 0.242 | 0.587 |

| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| | | | AD | 0.088 | 0.185 | 0.427 |
| 35 | Gompertz (1, 1) | Proposed | KS | 0.059 | 0.112 | 0.301 |
| | | | W | 0.072 | 0.140 | 0.368 |
| | | | AD | 0.087 | 0.164 | 0.394 |
| | | Classical | KS | 0.068 | 0.097 | 0.180 |
| | | | W | 0.077 | 0.123 | 0.197 |
| | | | AD | 0.077 | 0.117 | 0.230 |
| 60 | | Proposed | KS | 0.076 | 0.178 | 0.474 |
| | | | W | 0.097 | 0.235 | 0.572 |
| | | | AD | 0.109 | 0.254 | 0.577 |
| | | Classical | KS | 0.073 | 0.141 | 0.285 |
| | | | W | 0.099 | 0.233 | 0.475 |
| | | | AD | 0.094 | 0.185 | 0.384 |
| 90 | | Proposed | KS | 0.089 | 0.260 | 0.645 |
| | | | W | 0.116 | 0.319 | 0.740 |
| | | | AD | 0.127 | 0.339 | 0.745 |
| | | Classical | KS | 0.096 | 0.192 | 0.420 |
| | | | W | 0.124 | 0.329 | 0.738 |
| | | | AD | 0.113 | 0.242 | 0.534 |

Table 3 presents the power for testing the Log-Logistic when the Gompertz distribution is an alternative at a nominal level of 0.05. For n=35, with Gompertz (0.5, 1) and Gompertz (1, 1) it is observed that at high proportions of failure (F(c)=0.60 and 0.80) the proposed tests (KS, W, AD) perform better than the classical in terms of power, while at F(c)=0.40 for Gompertz (0.5, 1) the classical KS and W tests outperform the proposed tests, where for Gompertz (1, 1) only the classical W test is better. In n=60, with Gompertz (0.5, 1) the proposed tests outperform the classical tests at all F(c). However, for Gompertz (1, 1) the proposed KS and AD tests provide a higher power than the classical at small F(c) =0.40, while at F(c) =0.60 and 0.80 the proposed tests are better than the classical. At n=90, for both Gompertz (0.5, 1) and Gompertz (1, 1) at F(c) =0.40, the classical KS and W tests perform better than the proposed tests while at F(c) =0.60 the proposed KS and AD tests are better than the classical tests but at F(c) =0.80 all proposed tests perform better than the classical.

Table 4. Estimated power for Weibull distribution with different sizes and proportion of failures F(c) at nominal level 0.05.

| n | Alternative Model | Methods | Test statistics | $F(c,b,a) = 1 - e^{-ac^b}$ 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| 35 | Weibull (1, 0.5) | Proposed | KS | 0.051 | 0.063 | 0.141 |
| | | | W | 0.055 | 0.077 | 0.174 |
| | | | AD | 0.063 | 0.092 | 0.192 |
| | | Classical | KS | 0.055 | 0.069 | 0.095 |
| | | | W | 0.059 | 0.063 | 0.068 |
| | | | AD | 0.060 | 0.077 | 0.117 |
| 60 | | Proposed | KS | 0.058 | 0.087 | 0.208 |
| | | | W | 0.064 | 0.114 | 0.271 |
| | | | AD | 0.070 | 0.127 | 0.276 |
| | | Classical | KS | 0.055 | 0.084 | 0.130 |
| | | | W | 0.065 | 0.106 | 0.169 |
| | | | AD | 0.065 | 0.103 | 0.175 |
| 90 | | Proposed | KS | 0.058 | 0.110 | 0.290 |
| | | | W | 0.071 | 0.144 | 0.365 |
| | | | AD | 0.077 | 0.155 | 0.375 |
| | | Classical | KS | 0.065 | 0.102 | 0.179 |
| | | | W | 0.076 | 0.141 | 0.312 |
| | | | AD | 0.071 | 0.125 | 0.242 |
| 35 | Weibull (1, 1) | Proposed | KS | 0.051 | 0.064 | 0.140 |
| | | | W | 0.055 | 0.076 | 0.174 |
| | | | AD | 0.063 | 0.090 | 0.191 |
| | | Classical | KS | 0.055 | 0.067 | 0.096 |
| | | | W | 0.058 | 0.063 | 0.064 |
| | | | AD | 0.060 | 0.072 | 0.121 |
| 60 | | Proposed | KS | 0.057 | 0.089 | 0.205 |
| | | | W | 0.064 | 0.112 | 0.272 |
| | | | AD | 0.070 | 0.127 | 0.277 |
| | | Classical | KS | 0.056 | 0.082 | 0.132 |
| | | | W | 0.066 | 0.106 | 0.165 |
| | | | AD | 0.065 | 0.097 | 0.180 |
| 90 | | Proposed | KS | 0.058 | 0.111 | 0.289 |
| | | | W | 0.072 | 0.141 | 0.367 |
| | | | AD | 0.078 | 0.154 | 0.379 |
| | | Classical | KS | 0.066 | 0.096 | 0.182 |
| | | | W | 0.076 | 0.142 | 0.306 |
| | | | AD | 0.071 | 0.117 | 0.248 |

When Weibull distribution is considered as an alternative as an alternative. The results in Table 4 show that at n=35, with F(c) =0.40 for Weibull (0.5, 1) only the classical W presents slightly higher power than the corresponding proposed W test, however for Weibull (1,1) two classical tests KS and W present higher power than the proposed tests. While, for Weibull (0.5, 1) and Weibull (1, 1) at failure 0.60, the proposed W and AD perform better power and at F(c) = 0.80 all proposed tests show higher power than the classical. Similarly, at n=60 for both Weibull (0.5, 1) and Weibull (1, 1) it is observed that the proposed tests perform better than the classical at higher proportions of failure (F(c) =0.60 and 0.80). Whereas, at F(c) =0.40 the proposed KS and AD tests outperform the classical tests. At the largest sample size n=90, for Weibull (0.5, 1) all proposed tests perform better than the classical at higher proportions of failure but at F(c) =0.40 the proposed W and AD tests are better than the classical. For Weibull (1, 1) at F(c) =0.40 the classical KS and W tests are better, while at F(c) =0.60 the proposed KS and AD tests are better but at F(c) =0.80 all the proposed test outperforms the classical.

Table 5. Estimated power for Burr X distribution with different sizes and proportion of failures F(c) at nominal level 0.05.

| | | | | $F(c,\nu,\theta) = \left(1 - e^{-\left(\frac{c}{\theta}\right)^2}\right)^{\nu}$ | | |
|---|---|---|---|---|---|---|
| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
| 35 | Burr (0.7, 0.4) | Proposed | KS | 0.062 | 0.113 | 0.310 |
| | | | W | 0.073 | 0.154 | 0.386 |
| | | | AD | 0.090 | 0.177 | 0.404 |
| | | Classical | KS | 0.064 | 0.106 | 0.193 |
| | | | W | 0.077 | 0.132 | 0.186 |

| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| | | | AD | 0.077 | 0.129 | 0.249 |
| 60 | | Proposed | KS | 0.080 | 0.185 | 0.494 |
| | | | W | 0.096 | 0.256 | 0.599 |
| | | | AD | 0.108 | 0.279 | 0.604 |
| | | Classical | KS | 0.070 | 0.152 | 0.311 |
| | | | W | 0.094 | 0.254 | 0.472 |
| | | | AD | 0.090 | 0.206 | 0.420 |
| 90 | | Proposed | KS | 0.091 | 0.274 | 0.666 |
| | | | W | 0.116 | 0.349 | 0.764 |
| | | | AD | 0.130 | 0.370 | 0.770 |
| | | Classical | KS | 0.093 | 0.211 | 0.770 |
| | | | W | 0.120 | 0.362 | 0.745 |
| | | | AD | 0.110 | 0.270 | 0.578 |
| 35 | Burr (1, 0.4) | Proposed | KS | 0.061 | 0.114 | 0.309 |
| | | | W | 0.074 | 0.150 | 0.381 |
| | | | AD | 0.090 | 0.174 | 0.408 |
| | | Classical | KS | 0.065 | 0.104 | 0.189 |
| | | | W | 0.078 | 0.129 | 0.198 |
| | | | AD | 0.078 | 0.124 | 0.241 |
| 60 | | Proposed | KS | 0.079 | 0.186 | 0.490 |
| | | | W | 0.098 | 0.252 | 0.588 |
| | | | AD | 0.112 | 0.273 | 0.593 |
| | | Classical | KS | 0.072 | 0.149 | 0.300 |
| | | | W | 0.097 | 0.251 | 0.482 |
| | | | AD | 0.093 | 0.200 | 0.406 |
| 90 | | Proposed | KS | 0.092 | 0.274 | 0.662 |
| | | | W | 0.116 | 0.342 | 0.756 |
| | | | AD | 0.130 | 0.361 | 0.761 |
| | | Classical | KS | 0.094 | 0.206 | 0.441 |
| | | | W | 0.123 | 0.349 | 0.749 |
| | | | AD | 0.112 | 0.263 | 0.560 |

When Burr X distribution is considered as an alternative, the results in Table 5 indicate that at n=35, for both Burr (0.7, 0.4) and Burr (1,0.4) at F(c) =0.40 the classical KS and W tests establish higher power the proposed tests but at the high proportion of failure it is observed that the proposed tests are more powerful than classical tests. Moreover, when the sample size is increase from n=35 to n=60 the proposed tests are more powerful compared to the classical tests for Burr (0.7,0.4) and Burr (1,0.4). However, at n=90, for Burr (0.7,0.4) the classical KS and W tests perform better than the proposed tests at F(c)=0.40 and at F(c)=0.60 all the proposed tests perform better than the classical but at F(c)=0.80 the proposed W and AD tests are better. For Burr (1,0.4) the classical KS test outperforms at F(c)=0.40 while at F(c)=0.60 and 0.80 the proposed tests display higher power the classical tests.

Table 6. Estimated power for Exponential distribution with different sizes and proportion of failures F(c) at nominal level 0.05

| n | Alternative Model | Methods | Test statistics | $F(c,\lambda)=1-e^{-\lambda c}$ | | |
|---|---|---|---|---|---|---|
| | | | | 0.40 | 0.60 | 0.80 |
| 35 | Exponential (0.6) | Proposed | KS | 0.052 | 0.064 | 0.141 |
| | | | W | 0.055 | 0.076 | 0.175 |
| | | | AD | 0.065 | 0.092 | 0.192 |
| | | Classical | KS | 0.054 | 0.068 | 0.095 |
| | | | W | 0.058 | 0.062 | 0.066 |
| | | | AD | 0.060 | 0.076 | 0.119 |
| 60 | | Proposed | KS | 0.057 | 0.087 | 0.207 |
| | | | W | 0.064 | 0.114 | 0.275 |
| | | | AD | 0.070 | 0.128 | 0.282 |
| | | Classical | KS | 0.053 | 0.086 | 0.135 |
| | | | W | 0.063 | 0.106 | 0.158 |
| | | | AD | 0.063 | 0.104 | 0.186 |
| 90 | | Proposed | KS | 0.060 | 0.110 | 0.290 |
| | | | W | 0.069 | 0.146 | 0.374 |
| | | | AD | 0.074 | 0.156 | 0.385 |
| | | Classical | KS | 0.064 | 0.102 | 0.187 |
| | | | W | 0.072 | 0.141 | 0.296 |
| | | | AD | 0.069 | 0.125 | 0.256 |
| 35 | Exponential (2) | Proposed | KS | 0.051 | 0.064 | 0.141 |
| | | | W | 0.055 | 0.076 | 0.175 |
| | | | AD | 0.063 | 0.090 | 0.192 |
| | | Classical | KS | 0.055 | 0.067 | 0.095 |

| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| | | | W | 0.058 | 0.063 | 0.066 |
| | | | AD | 0.060 | 0.072 | 0.119 |
| 60 | | Proposed | KS | 0.057 | 0.089 | 0.206 |
| | | | W | 0.064 | 0.112 | 0.271 |
| | | | AD | 0.070 | 0.127 | 0.278 |
| | | Classical | KS | 0.056 | 0.082 | 0.130 |
| | | | W | 0.066 | 0.106 | 0.167 |
| | | | AD | 0.065 | 0.097 | 0.177 |
| 90 | | Proposed | KS | 0.058 | 0.111 | 0.290 |
| | | | W | 0.072 | 0.141 | 0.366 |
| | | | AD | 0.078 | 0.154 | 0.376 |
| | | Classical | KS | 0.066 | 0.096 | 0.180 |
| | | | W | 0.076 | 0.142 | 0.309 |
| | | | AD | 0.071 | 0.117 | 0.244 |

Table 6 presents the Exponential model when it is considered as an alternative, the results show that at n=35, for Exponential (0.6) and Exponential (2) the classical KS and W are better than the proposed with F(c)=0.40 but at F(c)=0.60 the proposed W and AD tests establish higher power than the classical and at failure 0.80 all the proposed tests outperform the classical tests. For n=60, with Exponential (0.60) the proposed tests seem to exhibit higher power at all F(c) this is also noticeable with Exponential (2) but at higher proportions of failure while at F(c)=0.40 the proposed Kolmogorov and Anderson Darling tests are better than the classical test. Moreover, for n=90, both Exponential (0.6) and Exponential (2) the classical Kolmogorov Kolmogorov–Smirnov and Cramer-von tests perform well than the proposed tests at F(c)=0.40. While for Exponential(0.6) all the proposed tests possess better power than the classical tests at higher proportions of failure but for Exponential(2) the proposed KS and AD tests are better than the classical tests at F(c)=0.60 and at F(c)=0.80 all proposed tests perform better than the classical.

Table 7. Estimated power for Gompertz distribution with different sizes and proportion of failures F(c) at nominal level 0.10

| n | Alternative Model | Methods | Test statistics | $F(c,p,d) = 1 - e^{\left[\frac{d}{p}(1-e^{pc})\right]}$ | | |
|---|---|---|---|---|---|---|
| | | | | 0.40 | 0.60 | 0.80 |
| 35 | Gompertz (0.5, 1) | Proposed | KS | 0.103 | 0.156 | 0.342 |
| | | | W | 0.113 | 0.180 | 0.393 |
| | | | AD | 0.127 | 0.203 | 0.418 |
| | | Classical | KS | 0.115 | 0.149 | 0.234 |
| | | | W | 0.119 | 0.165 | 0.272 |
| | | | AD | 0.119 | 0.166 | 0.275 |
| 60 | | Proposed | KS | 0.121 | 0.222 | 0.500 |
| | | | W | 0.134 | 0.262 | 0.572 |
| | | | AD | 0.146 | 0.281 | 0.579 |
| | | Classical | KS | 0.118 | 0.183 | 0.326 |
| | | | W | 0.133 | 0.259 | 0.531 |
| | | | AD | 0.140 | 0.224 | 0.419 |
| 90 | | Proposed | KS | 0.137 | 0.297 | 0.653 |
| | | | W | 0.150 | 0.336 | 0.722 |
| | | | AD | 0.167 | 0.355 | 0.727 |
| | | Classical | KS | 0.138 | 0.232 | 0.450 |
| | | | W | 0.159 | 0.348 | 0.727 |
| | | | AD | 0.153 | 0.275 | 0.544 |
| 35 | Gompertz (1, 1) | Proposed | KS | 0.106 | 0.187 | 0.418 |
| | | | W | 0.123 | 0.218 | 0.475 |
| | | | AD | 0.142 | 0.243 | 0.494 |
| | | Classical | KS | 0.123 | 0.168 | 0.278 |
| | | | W | 0.131 | 0.202 | 0.356 |
| | | | AD | 0.130 | 0.191 | 0.328 |
| 60 | | Proposed | KS | 0.133 | 0.280 | 0.609 |
| | | | W | 0.156 | 0.322 | 0.676 |
| | | | AD | 0.172 | 0.345 | 0.679 |
| | | Classical | KS | 0.134 | 0.219 | 0.407 |
| | | | W | 0.154 | 0.328 | 0.660 |
| | | | AD | 0.160 | 0.271 | 0.506 |
| 90 | | Proposed | KS | 0.157 | 0.374 | 0.762 |
| | | | W | 0.182 | 0.425 | 0.818 |
| | | | AD | 0.201 | 0.444 | 0.824 |
| | | Classical | KS | 0.162 | 0.282 | 0.549 |
| | | | W | 0.200 | 0.439 | 0.842 |
| | | | AD | 0.187 | 0.344 | 0.648 |

Table 7 presents the estimated power when the Gompertz distribution is an alternative at a nominal level of 0.10. The results show that at a small sample size n=35, with F(c) =0.40 (a small failure which is heavy censoring) for Gompertz (0.5, 1) the classical KS and W tests performs better than the proposed tests while for Gompertz (1, 1) only the classical KS test is better. But, when increasing the proportions of failure (F(c) =0.60 and 0.80) with Gompertz (0.5, 1) and Gompertz (1, 1) the proposed tests (KS, W, AD) possesses better power than the classical tests. Furthermore, at n=60 Gompertz (0.5, 1) is shown that the proposed tests are powerful as compared to classical tests at all proportions of failure while with Gompertz (1, 1) at higher proportions of failure (F(c) =0.60 and 0.80) the proposed tests perform better than the classical but at F(c) =0.40 the classical KS test slightly better than the proposed KS test. Whereas, at n=90 for Gompertz (0.5, 1) and Gompertz (1, 1) alternatives, at F(c) =0.40 the classical KS and W tests possesses a better power than the proposed tests, while at higher proportions of failure (F(c) =0.60 and 0.80) it is observed the proposed KS and AD tests reveal higher power than the classical tests.

Table 8. Estimated power for Weibull distribution with different sizes and proportion of failures F(c) at nominal level 0.10

| n | Alternative Model | Methods | Test statistics | $F(c,b,a) = 1 - e^{-ac^b}$ | | |
|---|---|---|---|---|---|---|
| | | | | 0.40 | 0.60 | 0.80 |
| 35 | Weibull (1, 0.5) | Proposed | KS | 0.094 | 0.121 | 0.228 |
| | | | W | 0.100 | 0.134 | 0.258 |
| | | | AD | 0.113 | 0.152 | 0.275 |
| | | Classical | KS | 0.108 | 0.127 | 0.168 |
| | | | W | 0.108 | 0.122 | 0.162 |
| | | | AD | 0.110 | 0.139 | 0.191 |

| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| 60 | | Proposed | KS | 0.107 | 0.157 | 0.317 |
| | | | W | 0.116 | 0.181 | 0.370 |
| | | | AD | 0.125 | 0.196 | 0.383 |
| | | Classical | KS | 0.107 | 0.146 | 0.216 |
| | | | W | 0.114 | 0.180 | 0.316 |
| | | | AD | 0.122 | 0.169 | 0.268 |
| 90 | | Proposed | KS | 0.114 | 0.201 | 0.423 |
| | | | W | 0.125 | 0.224 | 0.483 |
| | | | AD | 0.136 | 0.236 | 0.488 |
| | | Classical | KS | 0.123 | 0.175 | 0.279 |
| | | | W | 0.130 | 0.228 | 0.461 |
| | | | AD | 0.127 | 0.196 | 0.351 |
| 35 | Weibull (1, 1) | Proposed | KS | 0.094 | 0.125 | 0.227 |
| | | | W | 0.101 | 0.133 | 0.260 |
| | | | AD | 0.114 | 0.150 | 0.275 |
| | | Classical | KS | 0.109 | 0.123 | 0.170 |
| | | | W | 0.108 | 0.122 | 0.157 |
| | | | AD | 0.110 | 0.133 | 0.196 |
| 60 | | Proposed | KS | 0.107 | 0.160 | 0.318 |
| | | | W | 0.116 | 0.179 | 0.372 |
| | | | AD | 0.127 | 0.194 | 0.384 |
| | | Classical | KS | 0.107 | 0.141 | 0.218 |
| | | | W | 0.115 | 0.175 | 0.307 |
| | | | AD | 0.123 | 0.160 | 0.275 |
| 90 | | Proposed | KS | 0.114 | 0.202 | 0.426 |
| | | | W | 0.125 | 0.220 | 0.487 |
| | | | AD | 0.136 | 0.234 | 0.492 |
| | | Classical | KS | 0.124 | 0.168 | 0.284 |
| | | | W | 0.130 | 0.224 | 0.453 |
| | | | AD | 0.128 | 0.187 | 0.359 |

By considering Weibull distribution as an alternative, the results in Table 8 show that at small sample size n=35, and F(c) =0.40 for Weibull(0.5, 1) and Weibull(1, 1) the classical KS and W tests are better than the proposed tests but when increasing the proportions of failure to F(c) =0.60 and =0.80 the proposed tests perform better than the classical tests. And, for n=60 with Weibull (0.5, 1) and Weibull

(1, 1) the proposed tests outperform the classical at all proportions of failure. Additionally, at large sample size n=90 and F(c)=0.40 for Weibull (0.5, 1) and Weibull (1, 1) the classical KS and W tests display higher power than the proposed tests, while at F(c)=0.60 the proposed KS and AD tests are more powerful than the classical and at F(c)=0.80 all the proposed tests outperform the classical tests.

Table 9. Estimated power for Burr X distribution with different sizes and proportion of failures F(c) at nominal level 0.10

| | | | | $F(c,\nu,\theta) = \left(1 - e^{-\left(\frac{c}{\theta}\right)^2}\right)^{\nu}$ | | |
|---|---|---|---|---|---|---|
| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
| 35 | Burr (0.7, 0.4) | Proposed | KS | 0.111 | 0.194 | 0.429 |
| | | | W | 0.124 | 0.231 | 0.492 |
| | | | AD | 0.124 | 0.257 | 0.516 |
| | | Classical | KS | 0.121 | 0.180 | 0.296 |
| | | | W | 0.129 | 0.213 | 0.349 |
| | | | AD | 0.128 | 0.208 | 0.355 |
| 60 | | Proposed | KS | 0.139 | 0.290 | 0.626 |
| | | | W | 0.155 | 0.346 | 0.696 |
| | | | AD | 0.168 | 0.369 | 0.703 |
| | | Classical | KS | 0.133 | 0.237 | 0.434 |
| | | | W | 0.152 | 0.346 | 0.663 |
| | | | AD | 0.155 | 0.295 | 0.543 |
| 90 | | Proposed | KS | 0.166 | 0.392 | 0.781 |
| | | | W | 0.185 | 0.458 | 0.842 |
| | | | AD | 0.206 | 0.474 | 0.845 |
| | | Classical | KS | 0.161 | 0.308 | 0.582 |
| | | | W | 0.196 | 0.473 | 0.850 |
| | | | AD | 0.184 | 0.377 | 0.688 |
| 35 | Burr (1, 0.4) | Proposed | KS | 0.110 | 0.191 | 0.428 |
| | | | W | 0.125 | 0.228 | 0.490 |
| | | | AD | 0.142 | 0.254 | 0.508 |
| | | Classical | KS | 0.122 | 0.176 | 0.290 |
| | | | W | 0.131 | 0.214 | 0.362 |
| | | | AD | 0.129 | 0.202 | 0.345 |

| n | Alternative Model | Methods | Test statistics | 0.40 | 0.60 | 0.80 |
|---|---|---|---|---|---|---|
| 60 | | Proposed | KS | 0.139 | 0.290 | 0.623 |
| | | | W | 0.158 | 0.339 | 0.691 |
| | | | AD | 0.173 | 0.362 | 0.693 |
| | | Classical | KS | 0.135 | 0.233 | 0.423 |
| | | | W | 0.156 | 0.340 | 0.669 |
| | | | AD | 0.157 | 0.287 | 0.529 |
| 90 | | Proposed | KS | 0.165 | 0.392 | 0.776 |
| | | | W | 0.185 | 0.451 | 0.835 |
| | | | AD | 0.208 | 0.470 | 0.840 |
| | | Classical | KS | 0.164 | 0.299 | 0.569 |
| | | | W | 0.200 | 0.469 | 0.853 |
| | | | AD | 0.186 | 0.368 | 0.672 |

When Burr X distribution is considered as an alternative, the results in Table 9 display that at n=35 and small failure proportion F(c)=0.40 for Burr(0.7,0.4) all the classical tests perform better than the proposed tests in teams of power, the same result is obtained for Burr (1,0.4) except that the proposed AD test is better than the corresponding classical test. But, at higher proportions of failure, the proposed tests outperform the classical tests for the two alternatives Burr (0.7,0.4) and Burr (1,0.4). Moreover, when increasing the sample size from n=35 to n=60 with Burr (0.7, 0.4) the performance of the proposed tests is better than the classical tests in terms of power at all proportion of failure. But, for Burr (1,0.4) the performance of the proposed tests is better than the classical at proportions of failure F(c)=0.40 and 0.80, while at F(c)=0.60 the proposed KS and AD tests are better. Furthermore, at n=90 for both Burr (0.7,0.4) and Burr (1,0.4), the results show that the classical W test possesses better power at all F(c). Although, it seems that the proposed KS and AD tests display higher power all proportions of failure.

Table 10. Estimated power for Exponential distribution with different sizes and proportion of failures F(c) at nominal level 0.10

| n | Alternative Model | Methods | Test statistics | $F(c,\lambda)=1-e^{-\lambda c}$ | | |
|---|---|---|---|---|---|---|
| | | | | 0.40 | 0.60 | 0.80 |
| 35 | Exponential (0.6) | Proposed | KS | 0.097 | 0.121 | 0.227 |
| | | | W | 0.101 | 0.133 | 0.257 |
| | | | AD | 0.114 | 0.153 | 0.275 |
| | | Classical | KS | 0.107 | 0.127 | 0.170 |
| | | | W | 0.106 | 0.121 | 0.160 |
| | | | AD | 0.108 | 0.138 | 0.193 |
| 60 | | Proposed | KS | 0.109 | 0.157 | 0.318 |
| | | | W | 0.114 | 0.180 | 0.377 |
| | | | AD | 0.123 | 0.196 | 0.387 |
| | | Classical | KS | 0.105 | 0.147 | 0.221 |
| | | | W | 0.110 | 0.178 | 0.298 |
| | | | AD | 0.119 | 0.170 | 0.282 |
| 90 | | Proposed | KS | 0.117 | 0.201 | 0.426 |
| | | | W | 0.123 | 0.224 | 0.494 |
| | | | AD | 0.138 | 0.238 | 0.503 |
| | | Classical | KS | 0.119 | 0.176 | 0.290 |
| | | | W | 0.125 | 0.176 | 0.453 |
| | | | AD | 0.123 | 0.197 | 0.368 |
| 35 | Exponential (2) | Proposed | KS | 0.094 | 0.125 | 0.227 |
| | | | W | 0.101 | 0.133 | 0.257 |
| | | | AD | 0.114 | 0.150 | 0.275 |
| | | Classical | KS | 0.109 | 0.123 | 0.170 |
| | | | W | 0.108 | 0.122 | 0.160 |
| | | | AD | 0.110 | 0.133 | 0.193 |
| 60 | | Proposed | KS | 0.107 | 0.160 | 0.318 |
| | | | W | 0.116 | 0.179 | 0.372 |
| | | | AD | 0.127 | 0.194 | 0.385 |
| | | Classical | KS | 0.107 | 0.141 | 0.217 |
| | | | W | 0.115 | 0.175 | 0.313 |
| | | | AD | 0.123 | 0.160 | 0.272 |
| 90 | | Proposed | KS | 0.114 | 0.202 | 0.424 |
| | | | W | 0.125 | 0.220 | 0.485 |
| | | | AD | 0.136 | 0.234 | 0.491 |
| | | Classical | KS | 0.124 | 0.168 | 0.281 |
| | | | W | 0.130 | 0.224 | 0.460 |
| | | | AD | 0.128 | 0.187 | 0.355 |

Table 10 presents the Exponential distribution when it is considered as an alternative, the results show that at n=35, and F(c)=0.40 both Exponential (0.6) and Exponential (2) the classical KS and W tests perform better than the proposed tests. While, for Exponential (0.6) at F(c)=0.60 the proposed W and AD tests exhibit higher power than the classical tests where at F(c)=0.80 the proposed tests perform better power than classical tests. But, for Exponential (2) with high failure proportions all the proposed tests possess better power than the classical tests. However, at n=60 with Exponential (0.6) and Exponential (2) the proposed tests perform better the classical tests at all F(c). Moreover, at n=90 with Exponential (0.6) and Exponential (2) the classical KS and W tests seems to perform better than the proposed tests at F(c)=0.40. But, with Exponential (0.6) at F(c)=0.60 and 0.80 all the proposed tests show higher power than the classical tests, while with Exponential(2) the proposed KS and AD tests are better at F(c)=0.60, and at F(c)=0.80 all proposed tests display higher power than the classical tests.

Overall, from Tables 3-10 the results show that the power values of the proposed and classical tests increase when the sample size and the proportion of failure increases. It is observed that in most of the cases the proposed tests outperform the classical tests. However, in some cases the classical KS and W tests have shown slightly higher power than the corresponding proposed tests at small and moderate proportions of failure. This suggests that the proposed KS and W test appears to lose in power due to the transformation performed in the samples. Moreover, by increasing the significance level to 0.10 the results display that, the power levels increase in all the proposed and classical tests for all the alternative models and sample sizes compared to the 0.05 significance level. In addition, the results show that under different censoring conditions, the Anderson–Darling and Cramer–von Mises statistics for both proposed and classical methods show higher power levels than the Kolmogorov–Smirnov test. Thus, it seems advisable to use these two statistics when working with Type-I right-censored data.

# CHAPTER 4: REAL DATA ANALYSIS

In this chapter, real data applications under Type I censored sample are considered. In order to see whether a given sample follows a Log-Logistic distribution by applying the proposed and classical methods. Therefore, two data sets are analyzed, the first data set have been taken from (Nelson, 2003, Table 1.1, page 105) and represents the times of breakdown of insulation fluid samples (in minutes) tested at 32 kV. While, the second data set is found originally by Schmee and Nelson (1977) and it has been analyzed by (Dube et al., 2011). This data set shows the number of thousand miles at different locomotive controls failed in a life test involving 96 controls.

## 4.1 Times of breakdown of insulation fluid

Table 11 presents the data of the times to breakdown of an insulating fluid (in minutes) tested at 32 kV that contains 15 observations. Suppose a decision is made to terminate life testing after 27 min. Then, sex observations are censored, an asterisk is used to mark the censored observations. This data has n=15, d=9 (complete failure observations) with 0.60 proportion of failure and censoring time or termination time c=27.

Table 11. Times to breakdown in minutes of an insulating fluid at 32 kV voltage level

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0.27 | 0.40 | 0.69 | 0.79 | 2.75 | 3.91 | 9.88 | 13.95 | 15.93 | 27.80* |
| 53.24* | 82.85* | 89.29* | 100.58* | 215.10* | | | | | |

As mentioned above, we are interested to test whether the times to breakdown follow Log-Logistic distribution. To carry through, the parameters of the Log-Logistic are estimated using the MLE ($\hat{\alpha} = 11.957943$, $\hat{\beta} = 0.642404$), then the MLE's of Logistic distribution parameters are obtained ($\hat{\mu} = 2.481396$, $\hat{\sigma}$ =1.556653) as given in equation (24), these MLE's ($\hat{\mu}, \hat{\sigma}$)are used to calculate the value $u_{i:d}$ for the proposed as equation (25) and for the classical as in equation (29) which are used to calculate the EDF statistics (KS,W, AD) for the proposed method follow equations (26,27, and 28) and for the classical method follow equations (30,3, and 31).After finding the observed tests statistics for the sample, the p-value of the test is calculated by generating 10,000 samples from the distribution under the null hypothesis and then obtaining the EDF statistics for these samples. Thus, the proportion of samples that exceed the corresponding observed EDF tests is the p-value and the calculation of the critical points follows the procedure that is explained in section 2.5. Table 12 presents the EDF statistics with corresponding p-values and critical points for times to breakdown of an insulating fluid.

Table 12. EDF statistics with corresponding p-values and critical points for times to breakdown of an insulating fluid.

| | **Proposed method** | | | **Classical method** | | |
|---|---|---|---|---|---|---|
| | KS | W | AD | KS | W | AD |
| Test Statistics | 0.20775 | 0.05822 | 0.38320 | 0.11805 | 0.01045 | 0.14295 |
| Critical points | 0.35577 | 0.20946 | 1.24055 | 0.17115 | 0.03885 | 0.34339 |
| P-value | 0.5628 | 0.6387 | 0.6479 | 0.4601 | 0.3875 | 0.4733 |

From Table 12 by comparing the proposed and classical tests statistics with the corresponding critical points, it is appeared that all values of the proposed and classical tests statistics less than the corresponding critical points. As well as, all the proposed and classical p-values are greater than the significant level $\alpha = 0.05$. This implies that, the Log-Logistic distribution have a good fit for the data and hence the sample follows the Log-Logistic model.

### 4.2 Locomotive controls

Table 13 presents the number of thousand miles which different locomotive controls failed in life test. The data contains 96 observations and the test was terminated after 135,000 miles. Hence, 59 observations are censored. This data has $n = 96$, $d = 37$ (complete failure observations) with 0.40 proportion of failure and censoring time or termination time $c = 135$. Table 3 display only the 37 failed units while the other 59 censored units all are equal to 135.

Table 13. The failed units of locomotive controls

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 22.5 | 37.5 | 46.0 | 38.5 | 51.5 | 53.0 | 57.5 | 66.5 | 68.0 | 69.5 |
| 76.5 | 77.0 | 78.5 | 80.0 | 81.5 | 83.0 | 84.0 | 91.5 | 93.5 | 102.5 |
| 107.0 | 108.5 | 112.5 | 113.5 | 116.0 | 117.0 | 118.5 | 119.0 | 129.0 | 120.0 |
| 122.5 | 123.0 | 127.5 | 131.0 | 132.5 | 132.5 | 134.0 | | | |

To tests weather the locomotive controls failed data follow Log-Logistic distribution. The parameters of the Log-Logistic given the observed Type I censored data set are estimated using the MLE ($\hat{\alpha} = 161.2482$, $\hat{\beta} = 2.60637$), then the MLE's of Logistic distribution parameters are obtained ($\hat{\mu} = 5.082945$ $\hat{\sigma} = 0.3836754$). Thus, the EDF statistics (KS, W, AD) for the proposed and classical methods which are obtained followed equations (25, 26,27,30,31 and 32) respectively. Table 14 presents the EDF statistics with corresponding p-values and critical points for the locomotive controls failed in life test.

Table 14. EDF statistics with corresponding p-values and critical points for the failed units of locomotive controls

| | **Proposed method** | | | **Classical method** | | |
|---|---|---|---|---|---|---|
| | KS | W | AD | KS | W | AD |
| Test Statistics | 0.11350 | 0.05577 | 0.29511 | 0.04341 | 0.00095 | 0.066113 |
| Critical points | 0.17503 | 0.22122 | 1.32546 | 0.06170 | 0.17967 | 0.21770 |
| P-value | 0.4576 | 0.6678 | 0.8310 | 0.4059 | 0.9071 | 0.7090 |

Table 14 shows that all the proposed and classical tests statistics are less than the critical points and all the p-values are more the significant level $\alpha = 0.05$. This indicates that the proposed and classical tests (KS, W, AD) are supporting the null hypothesis and conclude that the locomotive controls failed data set follows Log-Logistic distribution.

# CHAPTER 5: SUMMARY, CONCLUSION AND SUGGESTIONS FOR FURTHER STUDY

## 5.1 Summary

This thesis focuses on survival analysis application for Log-Logistic distribution. The hazard function of Log-Logistic distribution can have different forms such as increasing, decreasing and hump shape. Hence, this model can be used quite effectively in analyzing real lifetime data. Mainly, this thesis investigated the problem of the goodness of fit test for Log-Logistic distribution under Type I censored sample based on the EDF statistics for the proposed and classical methods. The maximum likelihood estimation method was used to estimate the unknown parameters of Log-Logistic distribution. Therefore, Newton-Raphson was used to obtain an approximate solution since the MLE cannot be obtained in close form. The MLE's were used in calculating the proposed and the classical tests as explained in chapter 2. In addition, the proposed method used in this thesis was developed by Pakyari and Balakrishnan (2017) while the classical method was developed by Pettitt and Stephens (1976).

On other hand, in order to compare the performance of the EDF statistics; the Kolmogorov Smirnov (KS), Cramer von Mises (W) and the Anderson Darling (AD) tests for the proposed and classical methods, Monte Carlo power studies were carried out for 10000 replications with various values of sample sizes n and different observed proportions of failures F(c). Also, several alternative distributions with different shape parameter values were considered for power analysis such as Gompertz, Weibull, Burr X, and Exponential distributions. Additionally, a simulation study was also conducted to calculate the critical points and the empirical significance level ($\alpha$=0.05 and 0.10) for both methods. Whereas, the selection of these significance levels were based on these two literatures Bispo et al. (2012) and Pakyari and Nia (2017). The critical points helped to derive conclusions about the EDF tests while; the empirical power was easily calculated for the proposed and classical EDF tests, as the proportion of replications in which the test statistic exceeded its corresponding critical value. While the empirical significance level (α) assisted to check the validity of the critical points. However, the critical points and the empirical significance level were both computed at two significant levels ($\alpha$=0.10 and 0.05). Finally, applications on some real data sets (Type I right censored) were applied to illustrate the testing procedure of Log-Logistic under Type I censored data for the classical and the proposed methods.

## 5.2 Conclusion

From Monte Carlo power studies, the results revealed that the proposed method outperforms the classical method in most of the cases also the power values of the two methods increased when the sample size and the proportion of failure increased. Moreover, under various censoring conditions the AD and W statistics for both proposed and classical methods displayed higher power than the KS test. Hence, AD and W statistics are recommended when working with Type I censored data.

Furthermore, from the two real datasets with Type I censored and different proportion of failures the results of the critical points and p-values for the proposed and classical methods showed that the two data sets followed the Log-Logistic distribution.

## 5.3 Suggestions for Further Study

The work in this thesis involved the procedure of goodness of fit test for testing the Log-Logistic distribution under Type I censored sample based on the empirical distribution function statistics the Kolmogorov Smirnov (KS), Cramer von Mises (W) and the Anderson Darling (AD) for the classical and the proposed methods. Future researchers could use the same test procedures applied in this thesis and can be applied on other lifetime distributions such as Gamma and Gumbel distributions. They could also use other tests statistics of the EDF such as the Kuiper Statistics and Watson Statistics. Future work can also study the procedure of goodness of fit test under other censoring schemes such as Type II censored samples and progressive Type II censoring samples. Moreover, future researchers can also consider other classical methods of parameter estimation with censored data, such as the least square and the method of moment.

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## APPENDIX A:THE INVERSE TRANSFORMATION TECHNIQUE

For generating a random sample from log-logistic distribution under Type I censored data. Let $m$ be a uniform (0, 1) random variable. To obtain random numbers such $(t)$ from Logistic distribution we need to solve this equation $m = F(t)$. Where $F(t)$ the cumulative distribution function of Logistic distribution followed equation (18).

$$m = \frac{1}{1 + \exp\left(-\frac{t-\mu}{\sigma}\right)}$$

$$1 + \exp\left(-\frac{t-\mu}{\sigma}\right) = \frac{1}{m}$$

$$\exp\left(-\frac{t-\mu}{\sigma}\right) = \frac{1}{m} - 1$$

By taking the Log for both sides,

$$\log\left(\exp\left(-\frac{t-\mu}{\sigma}\right)\right) = \log\left(\frac{1}{m} - 1\right)$$

$$-\frac{t-\mu}{\sigma} = \log\left(\frac{1}{m} - 1\right)$$

$$t = -\sigma \log\left(\frac{1}{m} - 1\right) + \mu$$

Then, this formula $(t)$ is used to generate data from logistic model.

Let c is censoring constant chosen from logistic. The Type I data from logistic distribution is obtained as follows:

$$Y_i = \min(t_i, \mathrm{c})$$

Then $T_i = \exp(Y_i)$ is a Type I censored sample from Log-Logistic distribution.

APPENDIX B: MONTE CARLO SIMULATION STUDIES

```
#Proportion Logistic distribution
#at C=-0.40  F(C)=0.40
#at C=0.41   F(C)=0.60
#at C=1.40   F(C)=0.80
library("nlme")
library(matrixStats)
set.seed(2020)
N=10000;n=90;C=1.40;C1=1.27;
#g=0.5;l=1;              #Gompoz
#th=0.70;v1=0.40;       #Burr
#a1=1;b1=2;              #Weibull(a1,b1)
#la=0.5;                 #EXP(la)
#---------------------------------------------------------------
mu=0;sigma=1;# True Parameters Values of Logistic distribution
a=exp(mu);B=(1/sigma)    #Log-Logistic distribution
yi<-vector("double")
dq<-vector("double")
y1<-list()      # Data from Logistic distribution
y2<-list()
T1<-list()      # Data from Log-Logistic distribution
ind<-list()     #indicator
ind1<-list()
#MLE's
a.h<-c()
```

```
B.h<-c()

mu.h<-c()

sigma.h<-c()

# EDF of Proposed

L<-list()     #length of data of size d

Ewf=list()    #EDF

Ewf.1=list()

Ewf.2=list()

Ewf.3=list()

Cum=list()#  CDF at y1

Cum.C=c() #  CDF at C

#Transformed sample for the Proposed

wi=list()    #CDF(y1)/CDF(C)

wd=list()    #wi(d+1-i)/revs wi

## Proposed Tests

# ks test

D.pw<-list() #Dn.plus

D.mw<-list() #D.minus

D.Bind<-list() # Combined (D.pulus,D.minus)

Max1<-list()   # max of row for (D.plus,D.minus)

# Proposed Tests

test.d<-c()

test.w<-c()

test.ad<-c()

#Classical Tests
```

```
# EDf of  classical
EWf=list()
EWf.1=list()
EWf.2=list()
EWf.3=list()
EWf.4=list()
EWf.5=list()
# CDF
Wi=list()       #CDF(y1)
# CDF for logistic distribution  at  C
Wd=c()       #CDF(C)
D1.W<-list() # Dn.plus
D2.W<-list() # Dn.minus
D.bind<-list()  # Combined (D1,D2)
max1<-list()   # max of row(D1,D2)
# Classical Tests
test.D<-c()
test.W<-c()
test.AD<-c()
#--------------------------- critical Points----------------------------
# Frist loop for critical Points
for (i in 1:N){
  q=sort(runif(n,0,1))
  qi=(-sigma*log((1/(q))-1))+mu
  yi=pmin(qi,C)
```

```
 dq=as.numeric(qi<=C)
 y2[[i]]<-yi
 ind1[[i]]=dq
 y1[[i]]<-yi[yi<C]
 ind[[i]]<-dq[dq==1]
 L[[i]]<-length(y1[[i]])   #length of y1 data /d
 T1[[i]]<-exp(y2[[i]])
 L1<-function(p){-(sum((ind1[[i]])*log(p[2]/p[1]))+(p[2]-
1)*sum((ind1[[i]])*log((T1[[i]])/p[1]))-
2*sum((ind1[[i]])*log(1+((T1[[i]])/p[1])^p[2]))-sum((1-
(ind1[[i]]))*log(1+((T1[[i]])/p[1])^p[2])))}
 result1=nlm(L1,p<-c(a,B),hessian = T)
 a.h[i]=result1$estimate[1]
 B.h[i]=result1$estimate[2]
 #MLEs for Logistic
 mu.h[i]=log(a.h[i])
 sigma.h[i]=(1/(B.h[i]))
 #Proposed method
 Cum[[i]]=(1/(1+exp(-(y1[[i]]-mu.h[i])/sigma.h[i])))
 Cum.C[i]=(1/(1+exp(-(C-mu.h[i])/sigma.h[i])))
 # Transformed data
 wi[[i]]=(Cum[[i]]/Cum.C[i]) #Proposed method
 wd[[i]]= rev(wi[[i]]) #reverse order of wi
 # EDF for porposed
 Ewf[[i]]=(1:L[[i]])/L[[i]]#(i/d)
```

```
Ewf.1[[i]]=((1:L[[i]])-1)/L[[i]]#(i-1/d)

Ewf.2[[i]]=(2*(1:L[[i]])-1)/(2*L[[i]])#(2*i-1/2d)

Ewf.3[[i]]=(2*(1:L[[i]])-1)/(L[[i]]) #(2*i-1/d)

D.pw[[i]]=(Ewf[[i]]-wi[[i]])

D.mw[[i]]=(wi[[i]]-Ewf.1[[i]])

D.Bind[[i]]=cbind(D.pw[[i]],D.mw[[i]])

Max1[[i]]=rowMaxs(D.Bind[[i]])

#Proposed Tests

#test.D

test.d[i]=(max(Max1[[i]]))

# test.Wst

test.w[i]=sum((wi[[i]]-Ewf.2[[i]])^2)+(1/(12*L[[i]]))

#test.ad

test.ad[i]=(-L[[i]])-sum((Ewf.3[[i]])*(log(wi[[i]])+log(1-wd[[i]])))

#classical Tests

# CDF for logistic distrbution  at y1  & C

Wi[[i]]=((1/(1+exp(-(y1[[i]]-mu.h[i])/sigma.h[i])))) #

Wd[i]=(1/(1+exp(-(C-mu.h[i])/sigma.h[i])))

# EDF classical

EWf[[i]]=(1:L[[i]])/(n) #(i/n)

EWf.1[[i]]=((1:L[[i]])-1)/(n) #(i-1/n)

EWf.2[[i]]=(2*(1:L[[i]])-1)/(n) #(2*i-1/n)

EWf.3[[i]]=((1:L[[i]])-0.5)/(n)#(*i-0.5)/n

EWf.4[[i]]=((L[[i]])*(4*L[[i]]^2-1))/(12*(n^2))

EWf.5[[i]]=(2*(1:L[[i]])-1)/(2*n) #(2*i-1/2n)
```

```
  D1.W[[i]]=(EWf[[i]]-Wi[[i]])
  D2.W[[i]]=(Wi[[i]]-EWf.1[[i]])
  D.bind[[i]]=cbind(D1.W[[i]],D2.W[[i]])
  max1[[i]]=rowMaxs(D.bind[[i]])
  #classical Tests
  test.D[i]=(max(max1[[i]]))
  test.W[i]=sum(Wi[[i]]-EWf.5[[i]])^2-EWf.4[[i]]+(n*Wd[i])*((L[[i]]^2/n^2)-
(Wd[i]*((L[[i]]/n)))+((1/3)*(Wd[i])^2))# steveen w
  test.AD[i]=sum(EWf.2[[i]]*(log(1-Wi[[i]])-log(Wi[[i]])))-2*sum(log(1-
Wi[[i]]))+n*((2*L[[i]]/n)-((L[[i]]/n)^2)-1)*(log(1-Wd[i]))+((L[[i]])^2/n)*log(Wd[i])-
(n*Wd[i])#          ad          test.AD[i]=sum((EWf.2[[i]])*(log.1.Wi[[i]]-log.Wi[[i]]))-
(2*sum(log.1.Wi[[i]]))+n*((2*L[[i]]/n)-((L[[i]]/n)^2)-
1)*(log.1.Wd[i])+((L[[i]])^2/n)*(log.Wd[i])-(n* Wd[i])# ad
}
#Proposed Method
(C.V1=quantile((test.d),.95,na.rm=TRUE))
(C.V2=quantile((test.w),.95,na.rm=TRUE))
(C.V3=quantile((test.ad),.95,na.rm=TRUE))
#Classical Method
(C.V4=quantile((test.D),.95,na.rm=TRUE))
(C.V5=quantile((test.W),.95,na.rm=TRUE))
(C.V6=quantile((test.AD),.95,na.rm=TRUE))
R.1=round(C.V1,5)
R.2=round(C.V2,5)
R.3=round(C.V3,5)
```

```
R.4=round(C.V4,5)
R.5=round(C.V5,5)
R.6=round(C.V6,5)
Result.CV=matrix(c(R.1,R.2,R.3,R.4,R.5,R.6), nrow = 3, ncol = 2)
colnames(Result.CV) <- c("10% Crtical Proposed ","10% Crtical Classical ")
rownames(Result.CV)<-c("CV1.KS","CV2.W","CV3.AD")
#---------------------------- Emprical alpha ------------------------------
mu.=0;sigma.=1;
a.=exp(mu.);B.=(1/sigma.)
yj<-vector("double")
dqq<-vector("double")
y11<-list()
y22<-list()
T11<-list()
Ind<-list()    #Indicator d
Ind1<-list()
a.hh<-c()
B.hh<-c()
mu.hh<-c()
sigma.hh<-c()
# EDF- porposed
Lq<-list()   #d
EwF=list()
EwF.1=list()
EwF.2=list()
```

```
EwF.3=list()

CUM=list()

CUM.C=c()

wii=list()  # proposed method

log.wii=list()

wdd=list()#w(d+1-i)

log.1.wdd=list() #log(1-w(d+1-i))

D.wp<-list() #Dn.plus

D.wm<-list() #D.minus

D11.Bind<-list()

Max11<-list()

# Proposed test

Test.d<-c()

Test.w<-c()

Test.ad<-c()

# EDF classical

EWF=list()

EWF.1=list()

EWF.2=list()

EWF.3=list()

EWF.4=list()

EWF.5=list()

# CDF for logistic distrbution  at y11 & C

Wii=list() #CDF(y11)

# CDF for logistic distrbution  at  C
```

```
Wdd=c()#W(d+1)

D11<-list() # Dn.plus

D22<-list() # Dn.minus

D11.bind<-list()

Max.11<-list()

# Classical Tests

Test.D<-c()

Test.W<-c()

Test.AD<-c()

# Second loop for emprical alpha

for (j in 1:N){

  qq=sort(runif(n,0,1))

  qj=(-sigma.*log((1/(qq))-1))+mu.

  yj=pmin(qj,C)

  dqq=as.numeric(qj<=C)

  y22[[j]]<-yj

  y11[[j]]<-yj[yj<C]

  Ind1[[j]]<-dqq

  Ind[[j]]<-dqq[dqq==1]

  Lq[[j]]<-length(y11[[j]])   # length of y11 data/d

  T11[[j]]<-exp(y22[[j]])

  L2<-function(P){-(sum((Ind1[[j]])*log(P[2]/P[1]))+(P[2]-

1)*sum((Ind1[[j]])*log((T11[[j]])/P[1]))-

2*sum((Ind1[[j]])*log(1+((T11[[j]])/P[1])^P[2]))-sum((1-

(Ind1[[j]]))*log(1+((T11[[j]])/P[1])^P[2])))}
```

```
result2=nlm(L2,P<-c(a.,B.),hessian = T)

a.hh[j]=result2$estimate[1]

B.hh[j]=result2$estimate[2]

#MLEs for Logistic

mu.hh[j]=log(a.hh[j])

sigma.hh[j]=(1/(B.hh[j]))

#Proposed method

CUM[[j]]=(1/(1+exp(-(y11[[j]]-mu.hh[j])/sigma.hh[j])))

CUM.C[j]=(1/(1+exp(-(C-mu.hh[j])/sigma.hh[j])))

wii[[j]]=(CUM[[j]]/CUM.C[j]) #Proposed method

wdd[[j]]= rev(wii[[j]]) #reverse order of wii

# EDf - proposed

EwF[[j]]=(1:Lq[[j]])/Lq[[j]]#(i/d)

EwF.1[[j]]=((1:Lq[[j]])-1)/Lq[[j]]#(j-1/d)

EwF.2[[j]]=(2*(1:Lq[[j]])-1)/(2*Lq[[j]])#(2*j-1/d)

EwF.3[[j]]=(2*(1:Lq[[j]])-1)/(Lq[[j]]) #(2*j-1/d)

D.wp[[j]]=(EwF[[j]]-wii[[j]])

D.wm[[j]]=(wii[[j]]-EwF.1[[j]])

D11.bind[[j]]=cbind(D.wp[[j]],D.wm[[j]])

Max.11[[j]]=rowMaxs(D11.bind[[j]])

# Propsoed Tests

Test.d[j]=(max(Max.11[[j]]))

Test.w[j]=sum((wii[[j]]-EwF.2[[j]])^2)+(1/(12*Lq[[j]]))

Test.ad[j]=(-Lq[[j]])-sum((EwF.3[[j]])*(log(wii[[j]])+log(1-wdd[[j]])))

#Classical method
```

```
  # CDF for logistic distrbution  at y11 & C
  Wii[[j]]=((1/(1+exp(-(y11[[j]]-mu.hh[j])/sigma.hh[j]))))
  Wdd[j]=(1/(1+exp(-(C-mu.hh[j])/sigma.hh[j])))
  # EDf classical
  EWF[[j]]=(1:Lq[[j]])/(n) #(i/n)
  EWF.1[[j]]=((1:Lq[[j]])-1)/(n) #(i-1/n)
  EWF.2[[j]]=(2*(1:Lq[[j]])-1)/(n) #(2*i-1/n)
  EWF.3[[j]]=((1:Lq[[j]])-0.5)/(n) #(i-0.5/n)
  EWF.4[[j]]=((Lq[[j]])*(4*Lq[[j]]^2-1))/(12*(n^2))
  EWF.5[[j]]=(2*(1:Lq[[j]])-1)/(2*n) #(2*i-1/2n)
  D11[[j]]=(EWF[[j]]-Wii[[j]])
  D22[[j]]=(Wii[[j]]-EWF.1[[j]])
  D11.Bind[[j]]=cbind(D11[[j]],D22[[j]])
  Max11[[j]]=rowMaxs(D11.Bind[[j]])
  #Classical
  Test.D[j]=(max(D11.Bind[[j]]))
  Test.W[j]=sum(Wii[[j]]-EWF.5[[j]])^2-EWF.4[[j]]+(n*Wdd[j])*((Lq[[j]]^2/n^2)-
(Wdd[j]*((Lq[[j]]/n)))+((1/3)*(Wdd[j])^2))# steveen
  Test.AD[j]=sum(EWF.2[[j]]*(log(1-Wii[[j]])-log(Wii[[j]])))-2*sum(log(1-
Wii[[j]]))+n*((2*Lq[[j]]/n)-((Lq[[j]]/n)^2)-1)*(log(1-
Wdd[j]))+((Lq[[j]])^2/n)*log(Wdd[j])-(n*Wdd[j])
}
a=0;b=0;c=0;d=0;e=0;f=0;
for(j in 1:N){
  # proposed
```

```
  if(Test.d[j]>C.V1) {a=a+1}
  if(Test.w[j]>C.V2){b=b+1}
  if(Test.ad[j]>C.V3) {c=c+1}
  # classical
  if( Test.D[j]>C.V4) {d=d+1}
  if( Test.W[j]>C.V5){e=e+1}
  if( Test.AD[j]>C.V6) {f=f+1}
}
#Proposed
a/N
b/N
c/N
# classical
d/N
e/N
f/N
R.11=round(a/N,3)
R.22=round(b/N,3)
R.33=round(c/N,3)
R.44=round(d/N,3)
R.55=round(e/N,3)
R.66=round(f/N,3)
#Emaprical alpha of classical
Emprical=matrix(c(R.11,R.22,R.33,R.44,R.55,R.66), nrow = 3, ncol = 2)
colnames(Emprical) <- c("10% Emprical Proposed ","10% Emprcial Classical ")
```

```
rownames(Emprical)<-c("Emp.KS","Emp.W","Emp.AD")

#print(Emprical)

print(list("Crtical points"=Result.CV," 10% Emprical alpha"=Emprical)) # for 10%

#---------------------------- Power of tests ------------------------------

muu=0;sigmaa=1;#C1=0.882;   # C1: based on CDF of altenative models

aa=exp(muu);BB=(1/sigmaa)

#True parameters of Alternative models

Yi<-vector("double")

di<-vector("double")

Y1<-list()

Y2<-list()

dd2<-list()

dd1<-list()

ahat<-c()

Bhat<-c()

muhat<-c()

sigmahat<-c()

#Proposed method

# EDF

d<-list()

EDF=list()

EDF.1=list()

EDF.2=list()

EDF.3=list()

# CDF based on logistic distrbution
```

```
CDF=list()

CDF.C=c()

# Transformed data for the Proposed Method CDF

ui=list()

ud=list()#u(d+1-i)

Dn.P<-list() #Dn.plus

Dn.M<-list() #Dn.minus

Dn.Bind<-list()

Max2.D<-list()

# Proposed Tests ui

D.test<-c()

W.test<-c()

AD.test<-c()

#Classical Method

UI=list()

Ud=c()#u(d+1) is CDF(C)

D1<-list() # Dn.plus

D2<-list() # Dn.minus

Dn.bind<-list()

Max2.d<-list()

D.testc<-c()

W.testc<-c()

AD.testc<-c()

EDFc=list()

EDFc.1=list()
```

```
EDFc.2=list()

EDFc.3=list()

EDFc.4=list()

EDFc.5=list()

# Third loop for finding the power

for (r in 1:N){

  m=sort(runif(n,0,1))

  #ti=(1/g)*(log(1-((g/l)*log(1-m))))   #Compozt dist (g,l)

  #ti=(-(th)^2*log(1-(m)^(1/v1)))^0.50  #Burr dist(th,v1)

  #ti=((-1/a1) *log(1-m))^(1/b1)        #Weibull (a1,b1)

  #ti=(-1/la)*log(1-m)                   #EXP(la)

  Yi=pmin(ti,C1)

  di=as.numeric(ti<=C1)

  Y2[[r]]<-Yi

  Y1[[r]]<-Yi[Yi<C1]

  dd2[[r]]<-di

  dd1[[r]]<-di[di==1]

  d[[r]]<-length(Y1[[r]])

  L3<-function(p3){-(sum((dd2[[r]])*log(p3[2]/p3[1]))+(p3[2]-
1)*sum((dd2[[r]])*log((Y2[[r]])/p3[1]))-
2*sum((dd2[[r]])*log(1+((Y2[[r]])/p3[1])^p3[2]))-sum((1-
(dd2[[r]]))*log(1+((Y2[[r]])/p3[1])^p3[2])))}

  result3=nlm(L3,p3<-c(aa,BB),hessian = T)

  ahat[r]=result3$estimate[1]

  Bhat[r]=result3$estimate[2]
```

```
#MLEs for Logistic distribution

muhat[r]=log(ahat[r])

sigmahat[r]=(1/(Bhat[r]))

#Proposed Method

CDF[[r]]=(1/(1+exp(-(log(Y1[[r]])-muhat[r])/sigmahat[r])))

CDF.C[r]=(1/(1+exp(-(log(C1)-muhat[r])/sigmahat[r])))

ui[[r]]=sort(CDF[[r]]/CDF.C[r]) #Proposed method

ud[[r]]= rev(ui[[r]]) #reverse order of ui

# EDF- proposed

EDF[[r]]=(1:d[[r]])/d[[r]]#(r/d)

EDF.1[[r]]=((1:d[[r]])-1)/d[[r]]#(r-1/d)

EDF.2[[r]]=(2*(1:d[[r]])-1)/(2*d[[r]])#(2*r-1/2d)

EDF.3[[r]]=(2*(1:d[[r]])-1)/(d[[r]]) #(2*r-1/d)

Dn.P[[r]]=(EDF[[r]]-ui[[r]])

Dn.M[[r]]=(ui[[r]]-EDF.1[[r]])

Dn.Bind[[r]]=cbind(Dn.P[[r]],Dn.M[[r]])

Max2.D[[r]]=rowMaxs(Dn.Bind[[r]])

# Propsoed tests

#D.test

D.test[r]=(max(Max2.D[[r]]))

# W.test

W.test[r]=sum((ui[[r]]-EDF.2[[r]] )^2)+(1/(12*d[[r]]))

#AD.test

AD.test[r]=(-d[[r]])-(sum((EDF.3[[r]])*(log(ui[[r]])+log(1-ud[[r]]))))

#Classical Method
```

```
  UI[[r]]=((1/(1+exp(-(log(Y1[[r]])-muhat[r])/sigmahat[r]))))
  Ud[r]=(1/(1+exp(-(log(C1)-muhat[r])/sigmahat[r])))
  # EDF -Classical
  EDFc[[r]]=(1:d[[r]])/(n) #(r/n)
  EDFc.1[[r]]=((1:d[[r]])-1)/(n) #(r-1/n)
  EDFc.2[[r]]=(2*(1:d[[r]])-1)/(n) #(2*r-1/n)
  EDFc.3[[r]]=((1:d[[r]])-0.5)/(n) #(r-0.5/n)
  EDFc.4[[r]]=((d[[r]])*(4*d[[r]]^2-1))/(12*(n^2))
  EDFc.5[[r]]=(2*(1:d[[r]])-1)/(2*n) #(2*i-1/2n)
  D1[[r]]=(EDFc[[r]]-UI[[r]])
  D2[[r]]=(UI[[r]]-EDFc.1[[r]])
  Dn.bind[[r]]<-cbind(D1[[r]],D2[[r]])
  Max2.d[[r]]<-rowMaxs(Dn.bind[[r]])
  # Classical tests
  D.testc[r]=(max(Max2.d[[r]]))
  W.testc[r]=sum(UI[[r]]-EDFc.5[[r]])^2-EDFc.4[[r]]+(n*Ud[r])*((d[[r]]^2/n^2)-
(Ud[r]*((d[[r]]/n)))+((1/3)*(Ud[r])^2))# steveen w
  AD.testc[r]=sum((EDFc.2[[r]])*(log(1-UI[[r]])-log(UI[[r]])))-(2*sum(log(1-
UI[[r]])))+n*((2*d[[r]]/n)-((d[[r]]/n)^2)-1)*(log(1-Ud[r]))+((d[[r]])^2/n)*(log(Ud[r]))-
(n*Ud[r])# ad
} #Power of Proposed/Classical
a1=0;b1=0;c1=0;d1=0;e1=0;f1=0;
for(r in 1:N){  # proposed  if(D.test[r]>C.V1) {a1=a1+1}
  if(W.test[r]>C.V2){b1=b1+1}
  if(AD.test[r]>C.V3) {c1=c1+1}
```

```
  # Classical
  if(D.testc[r]>C.V4) {d1=d1+1}
  if(W.testc[r]>C.V5){e1=e1+1}
  if(AD.testc[r]>C.V6) {f1=f1+1}
}# rounding the result of the power for three digit
r.1=round(a1/N,3)
r.2=round(b1/N,3)
r.3=round(c1/N,3)
r.4=round(d1/N,3)
r.5=round(e1/N,3)
r.6=round(f1/N,3)
Power=matrix(c(r.1,r.2,r.3,r.4,r.5,r.6), nrow = 3, ncol = 2)
colnames(Power) <- c("10% Power Proposed Method","10% Power Classical
Method")
rownames(Power)<-c("Power.KS","Power.W","Power.AD")
# Final Results for all
print(list("Crtical points"=Result.CV," 10% Emprical
alpha"=Emprical,"Power"=Power)) # for 10%
```

APPENDIX C: CODE OF THE REAL DATA ANALYSIS

```
rm(list=ls(all=TRUE))

# Real data 4.1

library("nlme")

library(matrixStats)

set.seed(2020)

ti=c(0.27, 0.40, 0.69, 0.79, 2.75,3.91, 9.88,13.95, 15.93, 27.80,53.24,
82.85,89.29,100.58,215.10)

(tii= sort(ti))

(n=length(tii))

ind1=c(1,1,1,1,1,1,1,1,1,0,0,0,0,0,0)

c=27;

(yi=pmin(tii,c))

(Data=cbind(yi,ind1))

a=1;B=1

L1<-function(p){-(sum((ind1)*log(p[2]/p[1]))+(p[2]-1)*sum((ind1)*log((yi)/p[1]))-
2*sum((ind1)*log(1+((yi)/p[1])^p[2]))-sum((1-(ind1))*log(1+((yi)/p[1])^p[2])))}

result1=nlm(L1,p<-c(a,B),hessian = T)

print(result1)

(a.h=result1$estimate[1])

(B.h=result1$estimate[2])

#MLEs for Logistic

(mu.h=log(a.h))

(sigma.h=(1/(B.h)))

#The data only take cases ti<c to start testing
```

```
(y1<-yi[yi<c])
(L<-length(y1))   #length of y1 data /d
#Proposed method
(Cum=(1/(1+exp(-(log(y1)-mu.h)/sigma.h))))
(Cum.C=(1/(1+exp(-(log(c)-mu.h)/sigma.h))))
# transformed data
(wi=(Cum/Cum.C))#Proposed method
(wd= rev(wi)) #reverse order of wi
# EDF for porposed
(Ewf=(1:L)/L)#(i/d)
(Ewf.1=((1:L)-1)/L)#(i-1/d)
(Ewf.2=(2*(1:L)-1)/(2*L))#(2*i-1/2d)
(Ewf.3=(2*(1:L)-1)/(L)) #(2*i-1/d)
(D.pw=(Ewf-wi))
(D.mw=(wi-Ewf.1))
(D.Bind=cbind(D.pw,D.mw))
(Max1=rowMaxs(D.Bind))
#Proposed Tests
#test.D
(test.d=(max(Max1)))
# test.Wst
(test.w=sum((wi-Ewf.2)^2)+(1/(12*L)))
#test.ad
(test.ad=(-L)-sum((Ewf.3)*(log(wi)+log(1-wd))))
#Classical Tests
```

```
# CDF for logistic distrbution  at y1  & C
(Wi=((1/(1+exp(-(log(y1)-mu.h)/sigma.h))))) #
(Wd=(1/(1+exp(-(log(c)-mu.h)/sigma.h))))
# EDF classical
(EWf=(1:L)/(n)) #(i/n)
(EWf.1=((1:L)-1)/(n)) #(i-1/n)
(EWf.2=(2*(1:L)-1)/(n)) #(2*i-1/n)
(EWf.3=((1:L)-0.5)/(n))#(*i-0.5)/n
(EWf.4=((L)*(4*L^2-1))/(12*(n^2)))
(EWf.5=(2*(1:L)-1)/(2*n)) #(2*i-1/2n)
(D1.W=(EWf-Wi))
(D2.W=(Wi-EWf.1))
(D.bind=cbind(D1.W,D2.W))
(max1=rowMaxs(D.bind))
#classical Tests
(test.D=(max(max1)))
(test.W=sum(Wi-EWf.5)^2-EWf.4+(n*Wd)*((L^2/n^2)-
(Wd*((L/n)))+((1/3)*(Wd)^2)))  # steveen w
(test.AD=sum(EWf.2*(log(1-Wi)-log(Wi)))-2*sum(log(1-Wi))+n*((2*L/n)-((L/n)^2)-
1)*(log(1-Wd))+((L)^2/n)*log(Wd)-(n*Wd))
########Test statistics#######
Test.Result=matrix(c(test.d,test.w,test.ad,test.D,test.W,test.AD), nrow = 3, ncol = 2)
colnames(Test.Result) <- c(" Proposed tests"," Classical tests")
rownames(Test.Result)<-c("KS","W","AD")
print(list("Test statistics"=Test.Result))
```

```
################
rm(list=ls(all=TRUE))
# Real data 4.2
library("nlme")
library(matrixStats)
set.seed(2020)
a=1;B=1;c=135;
ti=sort(c(22.5,37.5,46.0,48.5,51.5,53.0,54.5,57.5,66.5,68.0,69.5,76.5,77.0,78.5,80.0,8
1.5,82.0,83.0,84.0,91.5,93.5,102.5,107.0,108.5,112.5,113.5,116.0,117.0,118.5,119.0,1
20.0,122.5, 123.0, 127.5, 131.0, 132.5, 134.0 ,135, 135, 135, 135, 135, 135, 135, 135,
135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135, 135,
135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,
135,135,135,135,135,135,135,135,135,135,135,135,135,135,135,135, 135, 135, 135,
135))
ind=c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0)
cbind(ti,ind)
(yi=pmin(ti,c))
(y1<-ti[ti<c])
L1<-function(p){-(sum((ind)*log(p[2]/p[1]))+(p[2]-1)*sum((ind)*log((yi)/p[1]))-
2*sum((ind)*log(1+((yi)/p[1])^p[2]))-sum((1-(ind))*log(1+((yi)/p[1])^p[2])))}
result1=nlm(L1,p<-c(a,B),hessian = T)
print(result1)
(a.h=result1$estimate[1])
```

```
(B.h=result1$estimate[2])
#MLEs for Logistic
(mu.h=log(a.h))
(sigma.h=(1/(B.h)))
################
n=length(ti);L=length(y1);mu=0;sigma=1;
#Proposed method
(Cum=(1/(1+exp(-(log(y1)-mu.h)/sigma.h))))
(Cum.C=(1/(1+exp(-(log(c)-mu.h)/sigma.h))))
# Transformed data
(wi=(Cum/Cum.C))#Proposed method
(wd= rev(wi)) #reverse order of wi
# EDF for porposed
(Ewf=(1:L)/L)#(i/d)
(Ewf.1=((1:L)-1)/L)#(i-1/d)
(Ewf.2=(2*(1:L)-1)/(2*L))#(2*i-1/2d)
(Ewf.3=(2*(1:L)-1)/(L)) #(2*i-1/d)
(D.pw=(Ewf-wi))
(D.mw=(wi-Ewf.1))


(D.Bind=cbind(D.pw,D.mw))
(Max1=rowMaxs(D.Bind))
#Proposed Tests
#test.D
(test.d=(max(Max1)))
```

```
# test.Wst
(test.w=sum((wi-Ewf.2)^2)+(1/(12*L)))
#test.ad
(test.ad=(-L)-sum((Ewf.3)*(log(wi)+log(1-wd))))
####################Classical test##########
#classical Tests
# CDF for logistic distrbution  at y1  & C
(Wi=((1/(1+exp(-(log(y1)-mu.h)/sigma.h))))) #
(Wd=(1/(1+exp(-(log(c)-mu.h)/sigma.h))))
# EDF classical
(EWf=(1:L)/(n)) #(i/n)
(EWf.1=((1:L)-1)/(n)) #(i-1/n)
(EWf.2=(2*(1:L)-1)/(n)) #(2*i-1/n)
(EWf.3=((1:L)-0.5)/(n))#(*i-0.5)/n
(EWf.4=((L)*(4*L^2-1))/(12*(n^2)))
(EWf.5=(2*(1:L)-1)/(2*n)) #(2*i-1/2n)
(D1.W=(EWf-Wi))
(D2.W=(Wi-EWf.1))
(D.bind=cbind(D1.W,D2.W))
(max1=rowMaxs(D.bind))
#classical Tests
(test.D=(max(max1)))
(test.W=sum(Wi-EWf.5)^2-EWf.4+(n*Wd)*((L^2/n^2)-
(Wd*((L/n)))+((1/3)*(Wd)^2)))  # steveen w
(test.AD=sum(EWf.2*(log(1-Wi)-log(Wi)))-2*sum(log(1-Wi))+n*((2*L/n)-((L/n)^2)-
```

```
1)*(log(1-Wd))+((L)^2/n)*log(Wd)-(n*Wd))
########Test statistics#######
Test.Result=matrix(c(test.d,test.w,test.ad,test.D,test.W,test.AD), nrow = 3, ncol = 2)
colnames(Test.Result)<-c(" Proposed tests"," Classical tests")
rownames(Test.Result)<-c("KS","W","AD")
print(list("Test statistics"=Test.Result))
#mu=0;sigma=1;c=0.849;
(prof.F.Logistic=(1/(1+exp(-(c-mu)/sigma))))
#####################
#P-value and Critical points code
#Proportion Logistic distribution
#at C=-0.40  F(C)=0.40
#at C=0.41   F(C)=0.60
#at C=1.40   F(C)=0.80
library("nlme")
library(matrixStats)
set.seed(2020)
N=10000;n=15;C=0.41;
#-------------------------------------------------------------
mu=0;sigma=1;# True Parameters Values of Logistic distribution
a=exp(mu);B=(1/sigma)    #Log-Logistic distribution
yi<-vector("double")
dq<-vector("double")
y1<-list()      # Data from Logistic distribution
y2<-list()
```

```
T1<-list()      # Data from Log-Logistic distribution

ind<-list()     #indicator

ind1<-list()

#MLE's

a.h<-c()

B.h<-c()

mu.h<-c()

sigma.h<-c()

# EDF of Proposed

L<-list()     #length of data of size d

Ewf=list()    #EDF

Ewf.1=list()

Ewf.2=list()

Ewf.3=list()

Cum=list()#  CDF at y1

Cum.C=c() #  CDF at C

#Transformed sample for the Proposed

wi=list()    #CDF(y1)/CDF(C)

wd=list()    #wi(d+1-i)/revs wi

## Proposed Tests

# ks test

D.pw<-list() #Dn.plus

D.mw<-list() #D.minus

D.Bind<-list() # Combined (D.pulus,D.minus)

Max1<-list()   # max of row for (D.plus,D.minus)
```

```
# Proposed Tests
test.d<-c()
test.w<-c()
test.ad<-c()
#Classical Tests
# EDf of  classical
EWf=list()
EWf.1=list()
EWf.2=list()
EWf.3=list()
EWf.4=list()
EWf.5=list()
# CDF
Wi=list()       #CDF(y1)
# CDF for logistic distrbution  at  C
Wd=c()       #CDF(C)
D1.W<-list() # Dn.plus
D2.W<-list() # Dn.minus
D.bind<-list()  # Combined (D1,D2)
max1<-list()   # max of row (D1,D2)
# Classical Tests
test.D<-c()
test.W<-c()
test.AD<-c()
#--------------------------- Critical Points------------------------------
```

```
# Frist loop for critical Points
for (i in 1:N){
  q=sort(runif(n,0,1))
  qi=(-sigma*log((1/(q))-1))+mu
  yi=pmin(qi,C)
  dq=as.numeric(qi<=C)
  y2[[i]]<-yi
  ind1[[i]]=dq
  y1[[i]]<-yi[yi<C]
  ind[[i]]<-dq[dq==1]
  L[[i]]<-length(y1[[i]])   #length of y1 data /d
  T1[[i]]<-exp(y2[[i]])
  L1<-function(p){-(sum((ind1[[i]])*log(p[2]/p[1]))+(p[2]-
1)*sum((ind1[[i]])*log((T1[[i]])/p[1]))-
2*sum((ind1[[i]])*log(1+((T1[[i]])/p[1])^p[2]))-sum((1-
(ind1[[i]]))*log(1+((T1[[i]])/p[1])^p[2])))}
  result1=nlm(L1,p<-c(a,B),hessian = T)
  a.h[i]=result1$estimate[1]
  B.h[i]=result1$estimate[2]
  #MLEs for Logistic
  mu.h[i]=log(a.h[i])
  sigma.h[i]=(1/(B.h[i]))
  #Proposed method
  Cum[[i]]=(1/(1+exp(-(y1[[i]]-mu.h[i])/sigma.h[i])))
  Cum.C[i]=(1/(1+exp(-(C-mu.h[i])/sigma.h[i])))
```

```
# Transformed data
wi[[i]]=(Cum[[i]]/Cum.C[i]) #Proposed method
wd[[i]]= rev(wi[[i]]) #reverse order of wi
# EDF for porposed
Ewf[[i]]=(1:L[[i]])/L[[i]]#(i/d)
Ewf.1[[i]]=((1:L[[i]])-1)/L[[i]]#(i-1/d)
Ewf.2[[i]]=(2*(1:L[[i]])-1)/(2*L[[i]])#(2*i-1/2d)
Ewf.3[[i]]=(2*(1:L[[i]])-1)/(L[[i]]) #(2*i-1/d)
D.pw[[i]]=(Ewf[[i]]-wi[[i]])
D.mw[[i]]=(wi[[i]]-Ewf.1[[i]])
D.Bind[[i]]=cbind(D.pw[[i]],D.mw[[i]])
Max1[[i]]=rowMaxs(D.Bind[[i]])
#Proposed Tests
#test.D
test.d[i]=(max(Max1[[i]]))
# test.Wst
test.w[i]=sum((wi[[i]]-Ewf.2[[i]])^2)+(1/(12*L[[i]]))
#test.ad
test.ad[i]=(-L[[i]])-sum((Ewf.3[[i]])*(log(wi[[i]])+log(1-wd[[i]])))
#classical Tests
# CDF for logistic distrbution  at y1  & C
Wi[[i]]=((1/(1+exp(-(y1[[i]]-mu.h[i])/sigma.h[i])))) #
Wd[i]=(1/(1+exp(-(C-mu.h[i])/sigma.h[i])))
# EDF classical
EWf[[i]]=(1:L[[i]])/(n) #(i/n)
```

```
  EWf.1[[i]]=((1:L[[i]])-1)/(n) #(i-1/n)
  EWf.2[[i]]=(2*(1:L[[i]])-1)/(n) #(2*i-1/n)
  EWf.3[[i]]=((1:L[[i]])-0.5)/(n)#(*i-0.5)/n
  EWf.4[[i]]=((L[[i]])*(4*L[[i]]^2-1))/(12*(n^2))
  EWf.5[[i]]=(2*(1:L[[i]])-1)/(2*n) #(2*i-1/2n)
  D1.W[[i]]=(EWf[[i]]-Wi[[i]])
  D2.W[[i]]=(Wi[[i]]-EWf.1[[i]])
  D.bind[[i]]=cbind(D1.W[[i]],D2.W[[i]])
  max1[[i]]=rowMaxs(D.bind[[i]])
  #classical Tests
  test.D[i]=(max(max1[[i]]))
  test.W[i]=sum(Wi[[i]]-EWf.5[[i]])^2-EWf.4[[i]]+(n*Wd[i])*((L[[i]]^2/n^2)-
(Wd[i]*((L[[i]]/n)))+((1/3)*(Wd[i])^2))# steveen w
  test.AD[i]=sum(EWf.2[[i]]*(log(1-Wi[[i]])-log(Wi[[i]])))-2*sum(log(1-
Wi[[i]]))+n*((2*L[[i]]/n)-((L[[i]]/n)^2)-1)*(log(1-Wd[i]))+((L[[i]])^2/n)*log(Wd[i])-
(n*Wd[i])#  ad  test.AD[i]=sum((EWf.2[[i]])*(log.1.Wi[[i]]-log.Wi[[i]]))-
(2*sum(log.1.Wi[[i]]))+n*((2*L[[i]]/n)-((L[[i]]/n)^2)-
1)*(log.1.Wd[i])+((L[[i]])^2/n)*(log.Wd[i])-(n* Wd[i])# ad
}
#Proposed Method
(C.V1=quantile((test.d),.95,na.rm=TRUE))
(C.V2=quantile((test.w),.95,na.rm=TRUE))
(C.V3=quantile((test.ad),.95,na.rm=TRUE))
#Classical Method
(C.V4=quantile((test.D),.95,na.rm=TRUE))
```

```
(C.V5=quantile((test.W),.95,na.rm=TRUE))

(C.V6=quantile((test.AD),.95,na.rm=TRUE))

R.1=round(C.V1,5)

R.2=round(C.V2,5)

R.3=round(C.V3,5)

R.4=round(C.V4,5)

R.5=round(C.V5,5)

R.6=round(C.V6,5)

Result.CV=matrix(c(R.1,R.2,R.3,R.4,R.5,R.6), nrow = 3, ncol = 2)

colnames(Result.CV) <- c("5% Crtical Proposed ","5% Crtical Classical ")

rownames(Result.CV)<-c("CV1.KS","CV2.W","CV3.AD")

#---------------------------- p-values ------------------------------

mu.=0;sigma.=1;

a.=exp(mu.);B.=(1/sigma.)

yj<-vector("double")

dqq<-vector("double")

y11<-list()

y22<-list()

T11<-list()

Ind<-list()    #Indicator d

Ind1<-list()

a.hh<-c()

B.hh<-c()

mu.hh<-c()

sigma.hh<-c()
```

```
# EDF- porposed
Lq<-list()   #d
EwF=list()
EwF.1=list()
EwF.2=list()
EwF.3=list()
CUM=list()
CUM.C=c()
wii=list()  # proposed method
log.wii=list()
wdd=list()#w(d+1-i)
log.1.wdd=list() #log(1-w(d+1-i))
D.wp<-list() #Dn.plus
D.wm<-list() #D.minus
D11.Bind<-list()
Max11<-list()
# Proposed test
Test.d<-c()
Test.w<-c()
Test.ad<-c()
# EDF classical
EWF=list()
EWF.1=list()
EWF.2=list()
EWF.3=list()
```

```
EWF.4=list()

EWF.5=list()

# CDF for logistic distrbution  at y11 & C

Wii=list() #CDF(y11)

# CDF for logistic distrbution  at  C

Wdd=c()#W(d+1)

D11<-list() # Dn.plus

D22<-list() # Dn.minus

D11.bind<-list()

Max.11<-list()

# Classical Tests

Test.D<-c()

Test.W<-c()

Test.AD<-c()

# Second loop for p-values

for (j in 1:N){

  qq=sort(runif(n,0,1))

  qj=(-sigma.*log((1/(qq))-1))+mu.

  yj=pmin(qj,C)

  dqq=as.numeric(qj<=C)

  y22[[j]]<-yj

  y11[[j]]<-yj[yj<C]

  Ind1[[j]]<-dqq

  Ind[[j]]<-dqq[dqq==1]

  Lq[[j]]<-length(y11[[j]])   # length of y11 data/d
```

```
T11[[j]]<-exp(y22[[j]])
L2<-function(P){-(sum((Ind1[[j]])*log(P[2]/P[1]))+(P[2]-
1)*sum((Ind1[[j]])*log((T11[[j]])/P[1]))-
2*sum((Ind1[[j]])*log(1+((T11[[j]])/P[1])^P[2]))-sum((1-
(Ind1[[j]]))*log(1+((T11[[j]])/P[1])^P[2])))}
result2=nlm(L2,P<-c(a.,B.),hessian = T)
a.hh[j]=result2$estimate[1]
B.hh[j]=result2$estimate[2]
#MLEs for Logistic
mu.hh[j]=log(a.hh[j])
sigma.hh[j]=(1/(B.hh[j]))
#Proposed method
CUM[[j]]=(1/(1+exp(-(y11[[j]]-mu.hh[j])/sigma.hh[j])))
CUM.C[j]=(1/(1+exp(-(C-mu.hh[j])/sigma.hh[j])))
wii[[j]]=(CUM[[j]]/CUM.C[j]) #Proposed method
wdd[[j]]= rev(wii[[j]]) #reverse order of wii
# EDf - proposed
EwF[[j]]=(1:Lq[[j]])/Lq[[j]]#(i/d)
EwF.1[[j]]=((1:Lq[[j]])-1)/Lq[[j]]#(j-1/d)
EwF.2[[j]]=(2*(1:Lq[[j]])-1)/(2*Lq[[j]])#(2*j-1/d)
EwF.3[[j]]=(2*(1:Lq[[j]])-1)/(Lq[[j]]) #(2*j-1/d)
D.wp[[j]]=(EwF[[j]]-wii[[j]])
D.wm[[j]]=(wii[[j]]-EwF.1[[j]])
D11.bind[[j]]=cbind(D.wp[[j]],D.wm[[j]])
Max.11[[j]]=rowMaxs(D11.bind[[j]])
```

```
# Propsoed Tests
Test.d[j]=(max(Max.11[[j]]))
Test.w[j]=sum((wii[[j]]-EwF.2[[j]])^2)+(1/(12*Lq[[j]]))
Test.ad[j]=(-Lq[[j]])-sum((EwF.3[[j]])*(log(wii[[j]])+log(1-wdd[[j]])))
#Classical method
# CDF for logistic distrbution  at y11 & C
Wii[[j]]=((1/(1+exp(-(y11[[j]]-mu.hh[j])/sigma.hh[j]))))
Wdd[j]=(1/(1+exp(-(C-mu.hh[j])/sigma.hh[j])))
# EDf classical
EWF[[j]]=(1:Lq[[j]])/(n) #(i/n)
EWF.1[[j]]=((1:Lq[[j]])-1)/(n) #(i-1/n)
EWF.2[[j]]=(2*(1:Lq[[j]])-1)/(n) #(2*i-1/n)
EWF.3[[j]]=((1:Lq[[j]])-0.5)/(n) #(i-0.5/n)
EWF.4[[j]]=((Lq[[j]])*(4*Lq[[j]]^2-1))/(12*(n^2))
EWF.5[[j]]=(2*(1:Lq[[j]])-1)/(2*n) #(2*i-1/2n)
D11[[j]]=(EWF[[j]]-Wii[[j]])
D22[[j]]=(Wii[[j]]-EWF.1[[j]])
D11.Bind[[j]]=cbind(D11[[j]],D22[[j]])
Max11[[j]]=rowMaxs(D11.Bind[[j]])
#Classical
Test.D[j]=(max(D11.Bind[[j]]))
Test.W[j]=sum(Wii[[j]]-EWF.5[[j]])^2-EWF.4[[j]]+(n*Wdd[j])*((Lq[[j]]^2/n^2)-
(Wdd[j]*((Lq[[j]]/n)))+((1/3)*(Wdd[j])^2))# steveen w
Test.AD[j]=sum(EWF.2[[j]]*(log(1-Wii[[j]])-log(Wii[[j]])))-2*sum(log(1-
Wii[[j]]))+n*((2*Lq[[j]]/n)-((Lq[[j]]/n)^2)-1)*(log(1-
```

```
Wdd[j]))+((Lq[[j]])^2/n)*log(Wdd[j])-(n*Wdd[j])
}
a=0;b=0;c=0;d=0;e=0;f=0;
for(j in 1:N){
  # proposed
  if(Test.d[j]>0.20775346) {a=a+1}
  if(Test.w[j]>0.05822051){b=b+1}
  if(Test.ad[j]>0.38320388) {c=c+1}
  # classical
  if( Test.D[j]>0.11804940) {d=d+1}
  if( Test.W[j]>0.01044565){e=e+1}
  if( Test.AD[j]>0.14294709) {f=f+1}
}#Proposed
a/N
b/N
c/N# classical
d/N
e/N
f/N
P_value=matrix(c(a/N,b/N,c/N,d/N,e/N,f/N), nrow = 3, ncol = 2)
colnames(P_value) <- c(" P-values Proposed "," P-values Classical ")
rownames(P_value)<-c("P-value.KS","P-value.W","P-value.AD")
print(list("Crtical points"=Result.CV," % P-value "=P_value))
```