QATAR UNIVERSITY

COLLEGE OF EDUCATION

THE IMPACT OF USING LESSON STUDY APPROACH ON STUDENTS'

MATHEMATICAL PROBLEM-SOLVING ABILITY IN PRIMARY SCHOOLS IN

QATAR

BY

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ABSTRACT

ALSHWAL, HANA, A., Masters : January : [2022:], Masters of Arts in Curriculum and Instruction

Title:_The Impact of Using Lesson Study Approach on Students' Mathematical Problem-solving Ability in Primary Schools in Qatar

Supervisor of Thesis: Areej I. Barham.

Problem-solving ability, as a major element of mathematics curriculum, helps learners become competent individuals qualified and prepared to positively impact in their societies. This research study was designed to explore the impact of using lesson study approach with problem-based learning (PBL) on students' mathematical problem solving ability. A Mixed methods research design was utilized to collect data, with a sample of 117 fifth grade students and 10 of their teachers in primary schools in Qatar. Quantitative data were obtained through student's mathematical problem solving test, while semi-structured interviews with teachers and students' work were utilized to collect the qualitative data. The analyses of variance (ANOVA) test was used to analyze quantitative data, while thematic analysis was used to analyze data collected through the semi-structured interviews. Findings revealed that the Lesson Study approach with (PBL) had significantly improved students' ability to solve mathematical problems, and promoted students' skills, including perseverance, students' thinking, reasoning, and using multiple representations.

DEDICATION

To my parents, who have always believed in me and supported me to follow my dreams through long years of education. To my family, who have been the real joy to my life, and to my sisters and brothers who have shown me how much I could influence others' life.

I love you!

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CHAPTER ONE: INTRODUCTION

Due to globalization, students must be equipped with the necessary skills to successfully deal with the changing and continuously evolving world (Stewart, 2012). Besides knowledge, students have to be adept in the much-needed 21st century skills such as creativity, collaboration, reasoning, critical thinking, and problem solving (Stewart, 2012).

Problem-solving ability is considered the most important of all 21st century skills as complex challenges facing the global community are increasing (Barell, 2007). Kyllonen (2012) reported that several studies on Measurement of 21st Century Skills within the Common Core State Standards (CCSS) were correlated with the importance of problem-solving ability. For instance, a study directed by the Boston Advanced Technological Educational Connection, inquired employers' opinions regarding what skills were the most essential in an employee. Results showed that problem-solving proficiency was the first top ten skills recognized and lacked in potential employees. Conley and Darling-Hammond (2013, p. 1) also noted that employees were required to be able to demonstrate higher thinking and communication skills, problem-solving proficiency and create strategies for workplace success.

Mathematics assists us to make aware and understand of the world in which we live as we go about our daily lives. It is a known fact that Mathematics is used in various fields of knowledge and plays an important role in enhancing skills especially developing thinking and the ability to solve problems. The National Research Council Committee on Early Childhood Mathematics (2009) noted that the increasing needs of international competition in the 21st century demand a workforce that is qualified in and comfortable with mathematics. Moreover, the Common Core State Standards (2010) emphasized that students with math proficiency can implement the learned mathematics

skills to solve problems they face in their daily lives.

In addition, Mathematical skills assist in developing logical, conceptual and procedural knowledge, promoting positive attitudes, building self-esteem, and providing opportunities for cooperation in today's world (Zenvenberg et al., 2004). Moreover, Students' math skills can influence one's competitive spirit and participation in technology and thereby, participation in the global economy as these skills are essential for economic growth (Norford, 2012). On this background, further attention to mathematics teaching and learning in schools is essential (Salim & Samman, 2011).

Learning mathematics is the process of accumulating knowledge and skills and applying them in different contexts and real-life situations. Therefore, effective mathematics instruction requires teachers to know what students need to learn, their previous knowledge and experiences, to provide a level of challenge that stimulates students' thinking, and the needed practice that helps students learn well (NCTM, 2000). In addition, teaching mathematics includes a set of complex learning experiences that focuses on numerical procedures and concepts in mathematics. This complexity urges teachers to adopt new practices and differentiate learning experiences based on students' needs (Lujan, 2020). Thus, teachers need to be involved in continuous professional development programs where they learn new skills, teaching strategies, and exchange experiences to be able to improve students' ability to solve problems, and develop their learning and performance in mathematics. Exchanging experiences that are reflected on creating rich educational environment, and developing students' problem solving ability, learning and performance (NCTM, 2000).

The National Council of Teachers of Mathematics (NCTM) and The Common Core State Standards (CCSS) have recommended changes in mathematical teaching to develop mathematics curriculum and instructions (NCTM, 2000; CCSS, 2010). They also highlighted the importance for mathematics teachers to shift from directed instruction to problem-based learning (CCSSI, 2010; Lesh & Zawojewski, 2007; NCTM, 2000). Because students need to change the way they deal with mathematics, they need opportunities to deal with problems in scenarios that can be solved in various ways and may have more than one solution. They also need to apply acquired knowledge and skills to real world situations, while thinking, discussing, collaborating, and building various paths to reach a result. Additionally, students need to experience justifying their logic to others, as well as responding to others' arguments (Pinter, 2017).

Problem based learning (PBL) is an approach to teach and learn mathematics curriculum. It is an effective method if not the best method for students to learn how to use and apply their learning, understanding and skills in new contexts (Larmer, Mergendoller & Boss, 2015). In this approach, students have the opportunities to learn concepts, varied solution strategies, and improve self-learning skills by solving challenging problems collaboratively, reflecting on their experiences, presenting their reasoning, and engaging in self- directed study (Demiroren, Turan & Oztuna, 2016, p.2). Problem based learning approach allows students to be more actively involved and responsible for their own learning, work collaboratively with other students, listen to others' thinking, share and analyze their own ideas, and create a logical answer, all in real-world contexts (Hmelo-Silver, 2004, p. 236).

Problem based learning is a teaching design in which a new concept is taught by giving students opportunities to be self-directed learners and to grapple with purposeful and challenging tasks. It has been demonstrated to generate student learning, increase conceptual understanding of mathematical content, and significantly affect students' problem solving proficiency (Boaler & Staples, 2008; Boaler, 2002; Cai, 2003; D'Ambrosio, 2003; Schoenfeld, 1992). Even though mathematical education is moving towards problem-based learning, there is still a need to implement an educational model that supports both teachers and students while teaching and learning using this approach. One model that improves teaching quality and helps teachers teach using problem based learning simultaneously is the Lesson Study approach.

Lesson study Approach

Lesson Study approach is an approach investigating classroom instruction implemented cooperatively by a group of teachers to enhance teaching and learning quality (Tsui & Law, 2007). This approach is also implemented to improve student's learning through collaborative lesson development and is commonly used by teachers in Japan (Cajkler & Wood, 2013). In Lesson Study process, teachers concentrate on their students' work and thinking (Isoda, 2010; Stigler & Hiebert, 2000). Teachers use effective strategies to capture students' thinking using data collection tools that guide their observations during live lessons (Lewis et al., 2009). Lesson Study also plays a key role in transforming classrooms into student-centered learning environment and is noted to significantly improve teaching and learning (Calhoun, 2018).

Problem solving ability

With access to the internet and online search engines, resources become available, and as a result, the capability to remember and recall information from resources is not as significant as the capability to apply the information in solving authentic problems. Developing mathematical problem solving ability is defined as the process through which a student attempts to accomplish mathematical goals and find solutions for problems that may not have immediate answer, which ultimately leads to increasing conceptual understandings and problem solving strategies (Lester, 2013). In order for students to be effective and efficient problem solvers, they should be able to work through failure, access and analyze information, discover answers, and understand interconnected systems. In addition, they have to effectively justify their thinking to others, orally and in writing. This will entail them to learn and practice how to use and apply acquired knowledge and skills in problem solving competently (Salim & Samman, 2011).

In primary schools, Problem-solving is an essential attribute to be a successful learner in mathematics. It has been cited as an effective approach to construct deeper students' conceptual understandings of mathematical content, (Leong et al., 2013) and support learning that increases long-term knowledge retention, generates motivation in student, transfers learning, and increases positive attitudes towards math (Lambdin, 2003).

Woodward et al. (2012) noted that problem-solving ability is essential to succeed in advanced mathematical levels, and achieving scores at and above grade four as a result of the complex mathematical concepts and the greater than before focus on problem solving that appear on exams. Consequently, teaching concepts through problem solving, and teaching problem solving effectively is an essential element to the learning of mathematics within primary schools (NCTM, 2014). In conclusion, problem solving has become both a primary focus of mathematical learning and an essential methodology for improving mathematical ability (Lester, 2013).

The transition into problem-based learning is taking place in Qatar as well, the context of the current study as it is evident in Qatari curriculum, tests, and expectations of what skills and competencies students have to obtain when they graduate. This has gradually highlighted the importance of problem solving particularly at and above grade four. Thus, teachers are required to have a greater understanding of how to effectively teach via problem solving.

Education in the State of Qatar

The Human Development pillar, stated in Qatar National Vision 2030, aims to empower citizens of Qatar so they can maintain and support the growth of their country. Although Qatar has benefited greatly from oil and gas resources, and continues to expand in knowledge-based economy, Qatar is proactively expanding its educational system to compete and, in some areas exceed the best ones in the world (QNV2030). However, International comparison data reported that Qatari students' performance in math tests is far lower than other countries (TIMSS, 2015). In addition, students' performance in math in national exams is reported to be unsatisfactory (education Report, 2017-2018).

Qatar has always adopted best practices used by countries with the highest students' competencies in mathematics, and in order to improve students' performance in math, it is necessary to implement a global educational approach based on problem solving that develops students' ability to solve mathematical problems. Therefore, the Faculty of Education at Qatar University represented by the National Center for Educational Development (NCED), and International Professional Development of Mathematics Teachers Using Lesson Study (IMPULS) at the University of Tokyo Gakuji have implemented QU-IMPULS project at Qatar primary and middle schools since the beginning of the academic year 2014-2015. In QU-IMPULS project, Collaborative Lesson Research (CLR), which is an expanded and developed format for lesson study (Takahashi and McDougall, 2016) is utilized to enhance the ability of Qatari teachers to teach mathematical concept via problem solving, and the ability of students to get higher achievement in mathematics.

Problem Statement

Education is a process of creating knowledge and experience to apply and use in real-life situations. It is the key to economic success and growth (Stewart, 2012). Qatar

is seeking to shift to knowledge economy, however, international math assessment (TIMSS) data for the years 2007, 2011, 2015 and 2019, showed that Qatar's students scored lower than the required level of the proficiency in mathematics, and their average scores were below the cut-of points for the average of international standards (Provasnik et al., 2016; Mullis et al., 2020). In addition to these results, the national exams in Qatar demonstrated that students' average scores in mathematics are still less than 40% (Ministry of Education and Higher Education report, 2018).

These results can be attributed to many reasons, including the use of teaching methods and strategies that are unable to develop students' abilities to solve mathematical problems and thinking skills and thus, students cannot reach higher levels in acquiring mathematical skills. To solve this problem, it is necessary to search for a global educational approach based on problem solving that can improve students' ability to solve mathematical problems and increase academic performance.

The NCED at Qatar University and IMPULS at the University of Tokyo Gakuji have implemented the project of Lesson Study approach for Qatar primary and middle schools since the academic year 2014-2015.

Although there are wide ranges of researches on lesson study approach around the word, rare studies were implemented in Qatari context. To the best of the researcher's knowledge, only one study was conducted on Lesson Study as a model for professional development measuring the impact of this development on students' achievement in mathematics in the academic year 2011-2012 (before QU-IMPLUS project). Another research done by a faculty member at Qatar University, in cooperation with (IMPULS) team, was on the project's impact on teachers' professional development in 2019. Further studies are needed to investigate the influence of the Lesson Study approach as problem based learning on students' performance. This study aims to investigate the impact of the QU-IMPLUS project by applying Lesson Study approach on students' performance. More specifically, the study explores the impact of using Lesson Study approach on students' mathematical problem solving ability in primary schools in Qatar.

Research Question

This study focuses on the impact of Lesson Study approach on students' mathematical problem solving ability in primary schools. The main question of the study is:

What is the impact of using lesson study approach on students' mathematical problem solving ability in primary schools in Qatar?

To address the main question, the study attempts to investigate the following secondary research questions:

 Does mathematical problem solving ability differ between students who learn via lesson study approach compared with students learning in conventional classes?
 To answer this question, the study examines the following statistical question

• Are there any statistically significant differences (α = 0.05) between students' mean scores of problem solving test among the three groups participated in the study?

2) How do students demonstrate their problem solving ability?

Objective of the study

The study seeks to investigate the impact of Lesson Study approach as problem based learning approach utilized as a teaching tool to enhance students' ability to solve mathematical problems.

Relevance and significance of the study

• The current study became more important because it seeks to achieve Qatar National Vision 2030, which is building an educational system that conforms to

modern scientific standards and is parallel to the best educational systems in the world.

- This study provides an opportunity to examine Lesson Study approach in government primary schools, and assess its influence on teachers' ability to improve students' overall performance in mathematics as they develop skills necessary to accurately and effectively solve problems and explain their thinking to others.
- This study provides a better understanding of the Lesson Study approach impact on students' mathematical problem solving ability.
- One rationale behind the study is review of literature showing scarcity of data regarding the impact of this approach on student learning. Because most of researchers around the world look at Lesson Study as professional development for teachers. Therefore, we hope this study will further enrich educational literature with results obtained.
- The study meets researchers' recommendations such as English and Gainsburg (2016) who asserted that there is a need for studies to provide insights on how to use and implement problem solving in classrooms and to decide if teaching concepts through problem solving leads to enhanced students' proficiency.

Definitions of terms

Several terms are used throughout the study. To prevent any misunderstanding, these terms are defined below:

Problem solving: a process by which students work to achieve a mathematical outcome that they are able to acquire, but to which, the methods of solution is not immediately apparent, and that leads to construct a deep understanding of mathematical

concepts, strategies, or connections (Hiebert et al., 1997; Lester, 2013).

Problem-Based Learning (PBL): a methodology to teaching and learning in which students face and experience real problems, and work collaboratively to study how to solve problems. These problems have many entry points and more than one way to solve (Larmer, Mergendoller, & Boss, 2015).

Problem -Based Learning (PBL): a student-centered educational approach that combines problem-solving abilities with real-life situations (Roopashree, 2014; Savery, 2006) to promote higher order thinking skills, self-directed learning skills, cooperation, and knowledge-seeking skills (Yeo &Tan, 2014).

Students' mathematical problem solving ability: students' ability to analyze problems, prepare strategies, implement selected strategies, and re-examine problem solving in order to develop alternative solutions or improve problem solving skills (Kuzle, 2013; OECD, 2004; Polya, 1973; Szetela & Nicol, 1992).

Lesson study approach: an educational approach adopts learning based on problem solving; provides opportunities for mathematical thinking, mathematical application and deepening of knowledge. This approach presents mathematical problems through scenarios that have more than one solution method, then discusses a variety of methods of solution, and comparing between them. It also requires cooperation between students to reach conclusion and generalization of the concept or the new mathematical knowledge that students must learn.

Operational definitions

Problem: a task with higher order thinking that cannot immediately be achieved and causes a cognitive disequilibrium as a student attempts to find a solution (Lambdin, 2003). Items of the problems related to the three strands "algebra", "geometry", and

"data handling" in the problem solving test represents the operational definition of problem in this study.

Students' mathematical problem solving ability: In this study, it refers to students' ability to use related concepts, generalizations, procedures and skills to solve "algebra", "geometry", and "data handling" problems. The total score from the problem-solving test will be used to determine ability level at which students are producing solutions for mathematical problems.

Organization of the thesis

For this study, chapter one provides a brief overview of the research, including the background of the study; the problem statement, significance of the study, as well as the research questions that guided the study. Chapter two, presents a review of the literature related to the topic, definition of key terms, and the theoretical framework. Chapter three details the research design and methodology used to answer the research questions and describes data collection methods, the target population and sample, the procedures that guided data collection, instruments used for data collection, in addition to a review of the ethical considerations related to this study. Then Chapter four presents the data analysis, the results of that analysis and reports the findings. Finally, Chapter five discusses the study findings, and provides an interpretation of the results. In addition, the chapter offers recommendations for further research that relate to these results. The list of the references is presented at the end of this study and the appendices used within this research follow it.

CHAPTER TWO: LITERATURE REVIEW

This study seeks to investigate the impact of Lesson Study approach on students' problem solving ability in primary schools at mathematics classrooms. Review of related literature focuses on mathematical education and problem solving, problem solving ability, and problem based learning. The review also focuses on description of the Lesson Study approach, its components, and techniques educators can apply in their classrooms. In addition, this chapter reviews relevant researches and studies on Lesson Study approach and its influence on students' ability of critical thinking, creative thinking, reasoning, metacognitive and problem solving.

Mathematics Education and Problem Solving

Mathematics is a crucial tool for success in life. It is considered a significant and essential part of the world (Danisman & Erginer, 2017). Thus, building next generation of researchers, scientists, and educators depends on whether education from Kindergarten to Grade 12 enhances mathematical abilities (NCTM, 2009; Kilpatrick et al., 2001; Stein, Remillard & Smith, 2007).

Mathematics education is a vital, ongoing, and dynamic process, where mathematical learning activities assist learners to enhance their logical, methodical, critical, and comprehensive thinking and reasoning skills, as well as take an objective and open approach while addressing problems (Sumarmo, 2004).

Mathematicians' existence revolve around solving problems (Paul Halmos, 1980, p. 519), which is considered mathematical education's most effective feature (Otten, 2010). Therefore, Students who are proficient in mathematics can apply what they have learned to solve problems that arise in their daily lives (CCSS, 2010).

As mathematics is considered the most challenging field of study in school subjects (Bishara, 2016), focus is placed on constructing students' mathematical

abilities in today's classroom. To address concerns regarding mathematics education, most researchers, psychologists and educators have attempted to define how students can further achieve excellence in mathematics through multiple studies on gender, socioeconomic status, talented students, and students with learning disabilities (Pinter, 2017).

Mathematical rules and procedures have been taught in mathematics schools for far too long without any clarification of "why and who cares" (Germain-McCarthy, 2001). As Mathematics is a scientific field concerned with number relationships and problem solving, Mathematical education became more than just a computational process that produces correct or wrong response. Mathematical power comes in its capacity to combine a number of scenarios and thereby to implement problem-solving approach in diverse cases (National Research Council, 2009, p. 43).

In the history of mathematics education, most of teachers emphasized that their students are well skilled with various mathematical concepts and models; nevertheless, they failed upon different assessments to attain the application of these skills in different contexts. In addition, most of students are keen on learning rules and procedures for solving direct mathematical questions, however, most of them lack the understanding of applying these rules and procedures in different context (Pinter, 2017). Moreover, Students' lack of mathematical abilities exhibited ineffective mathematical performances, for instance, trying to solve problems without understanding the context of the problems (CCSSO, 2010), and being unlikely to utilize their prior understanding of mathematical knowledge while solving problems (CCSSO, 2010).

Multiple studies have demonstrated that, practicing rules and procedures does not provide students with opportunities to learn how mathematics could be applied in different situations outside school (Boaler, 2016; Van de Walle, Karp, & Bay-Williams,

2015). Therefore, they continue struggling in solving mathematical problems (Pinter, 2017).

Boaler (2015) noted that a prominent reason why many students have anxiety and struggle in mathematics is the focus on rote memorization of mathematical facts through repetition of the multiplication table. Because of memorization difficulties, which hampers their efforts to achieve fluency, students experience a disadvantage with mathematical problem solving ability. Students' ability to utilize numbers flexibly or in a variety of mathematical contexts is limited by memorizing without acquiring number sense.

Miller (2015) documented that, as students moved from one grade to the next, mathematical problems taught would become more difficult to solve. This is partially due to lack of mathematical fluency in concepts, knowledge or skills that students did not acquire until the time they leave the primary level of learning. Therefore, Students must attain and grasp effectively mathematical skills at early stage in order to show progress as they move up through the grades (Stacy, Cartwright, Arwood, Canfield, & Kloos, 2017).

John Dewey (1938) highlighted that children learn by working on real world problems. Therefore, the essential requirements of students learning mathematics includes not only the grasp of mathematical information, but also critical reasoning, mathematical communication, and challenging problem solving (OECD, 2018).

Numerous mechanisms such as general mathematical processes of thinking, modeling, communicating, and problem solving are mandatory needs to be encounter by students. Students acquire these mechanism will have the ability to step in either directions between the abstract mathematics and real life situations (Pinter, 2017). Therefore, students must be given multiple opportunities to examine and solve tasks

that require considerable thinking time and effort. While solving problems, students should be allowed amble amount of time to rethink and reflect on their own problem solving techniques and strategies. By solving problems in mathematics, students tend to develop methods of thinking, habits of curiosity, tenacity, and confidence in unexpected circumstances, which will help them well in everyday life (NCTM, 2000).

The NCTM advocates problem solving as a significant aspect in assisting children to achieve mathematical competency (NCTM, 2000). Moreover, The CCSS emphasizes the significance of problem solving by highlighting phrases such as understanding problem, persistence in solution, and modeling and thinking abstractly (CCSS, 2010). In addition, multiple studies confirmed that, teachers of mathematics at all grade levels must promote mathematical thinking in their classes through solving mathematical problems (e.g Bruner, 1995; CBMS, 2012; Fennell, 2008; Leung, 2013; Rigelman, 2007; Schoenfeld, 2013; Tsao, 2012) as problem solving is a crucial mental process not only in education but also in students' lives (Jonassen, 2003).

Solving problems is an inherent aspect of mathematics, and an important component of the mathematical learning context (Lott, 2003), not an isolated part of the mathematics curriculum. In addition, improving problem solving disposition and problem solving ability is crucial to be a proficient mathematics' learner (Leong et al., 2013). Problem solving helps students' mathematics understanding of both procedures and content. It also reflects the meaning of doing mathematics (Otten, 2010, p. 17), leads to the joy of discovery and pleasures of doing mathematics (Schoenfeld, 2013), provides students with metacognitive awareness and resilience, and assists them to better prepare for many aspects of their daily life outside the classroom (Otten, 2010, p. 17).

Problem solving is a complex activity that demands critical and higher order

thinking as the problem solver is required to find the right strategy, or alternative strategies, in order to solve the problem (NCTM, 2000).

Problem solving includes analyzing, modifying, and integrating information in order to reach the best solution and obtain goals even though the method's solution is unknown in advance (Bransford & Stein, 1993; NCTM, 2014; Newall & Simon, 1972).

The NCTM described problem solving as enhancing deep learning of mathematical content by working on challenging tasks in which mathematical learning is involved (NCTM, 2000, p. 270). The Organization for Economic Co-operation and Development (OECD) defined problem solving proficiency as student's ability to participate in mental process to learn and solve problem situation where solution method is unknown in advance. It involves the willingness to participate in such settings to identify individual's potential as a productive critical thinker (OECD, 2013, p. 6).

McLeod (1985) defined mathematical problem solving as student's performance on mathematical activities in which a method of solution is not apparent in advance, and there is no clear algorithm for students to apply (p. 267). Problem solving is not only what you understand; it is also how, where, and when you apply your understanding (Schoenfeld, 1992, p. 60). In general, problem solving is what you are doing when you do not know what you have to do (Wheatley, 1984, p. 2).

Problem solving has been an area of emphasis in mathematical education since the 1980s. Initially, in 1980, the NCTM declared that, the main goal of mathematical education is problem solving (NCTM, 1980). In the 1990s, the Council asserted that problem solving is the first priority in mathematical education. Because of this continued emphasis on problem solving, studies on problem solving increased. Some of these studies focused on educational approaches that enhance problem-solving proficiency such as studies conducted by (Kroll, Masingila, & Mau, 1992; Maher & Martino, 1992; Silverman, Winograd, & Strohauer, 1992). Other researchers concentrated on using problem solving in teaching such as studies conducted by (Carey, 1991; Duren & Cherrington, 1992; Kloosterman, 1992; Proudfit, 1992).

This trend toward prioritizing problem solving in mathematical teaching and learning has persisted in the following decades. In 2000s, The National Council of Supervisors of Mathematics (NCSM) declared that, learning methods of problem solving is the primary reason for learning mathematics (NCSM, 2000, p.1). Furthermore, the document entitled "Principles and Standards for School Mathematics" specified problem solving as one of the main six criteria of learning mathematics from kindergarten to grade 12 (NCTM, 2000). In 2010, the Common Core State Standards for Mathematics (CCSSO) has advocated prioritizing mathematical problem solving in classroom education by placing it on the top of their criteria for students' mathematical activities (NGA Center & CCSSO, 2010).

In response to these recommendations to prioritize problem solving in education, educators asserted that problem solving should be considered an integral part of everyday lessons, as it is essential and crucial to learning and applying mathematics (Ball, Ferrini-Mundy, Kilpatrick, Milgram, Schmid, & Schaar, 2005; Kilpatrick, Swafford & Findell, 2001; Lester, 1994). In addition, a lot of researches and articles have emphasized the relevance of problem solving in today's curriculum (NCTM (2000); Stonewater (2005); Ronis (2008); Almaleki (2010)). Moreover, Senk and Thompson (2003) reported that the tendency for mathematics curriculum to teach students how to successfully solve problems.

Students' Mathematical Problem Solving Ability

The focus of education is to improve students' ability to learn, reflect, think, and use their cognitive abilities in order to become qualified problem solvers (Gagné, 1980). Many educators and psychologists discussed the concept of problem solving as more than just an educational norm, but also as an educational product that assists students to improve critical lifelong skills such as flexibility, communication, and creativity (Jonassen, 2000).

In today's society, the ability to solve problems has been identified as an essential skill (Greiff, Holt& Funke, 2013), thus; educators' priority raised to be empowering students to develop their ability to solve problems (Barak 2013, p. 657). While problem solving is more than just completing exercises that can be addressed using remembered techniques (Schoenfeld, 2011), problem solving extends to include more than regular reasoning and thinking skills students use when trying to solve exercise (Polya 1945, 2004; Verschaffel, Greer, & De Corte, 2000). Problem solving means to think deeply about problems, potential solutions techniques, relevant situation, their representations, and problem modeling (English & Halford, 1995; Mayer, 1992; Mayer & Wittrock, 2006; Verschaffel et al., 2000).

Problem solving is a highly valued skill in which students must demonstrate various levels of ability to solve routine and non-routine problems (Stanic & Kilpatrick, 1988). Solving none routine problems is a challenging task that might demand evaluation, planning, using several strategies, and apply of previously relevant experience or understanding of concepts (Huba & Freed, 2000).

Nitko et al. (2011) defined students' problem solving ability as the ability of students to apply several high-level cognitive process to find solutions to problems they encounter. While Polya, (1973); Sajadi et al., (2013); Saragih and Habeahan, (2014); and Batubara et al., (2017) defined students' mathematical problem solving ability as their ability to understand problems, design strategies to solve them, implement selected solution's strategies, and re-examine these solutions to provide responses in systematic ways with appropriate representations.

Mathematical thinking is a mathematical process that includes five aspects; one of them is to solve mathematical problems (Heleni et al., 2018). Mathematical problem-solving ability is one of the basic abilities that should be mastered by students; this is because problem-solving ability is considered the core of mathematics (Putra et al., 2018).

Developing student's ability to solve problems is a vital product of K-12 mathematics (English & Gainsburg, 2016; Lester, 2013; NCTM, 2000, 2014). In 2012, The NCTM established standards that outline goals schools must adopt in order for all students to succeed in mathematics. These goals involved learning to respect mathematics, being confident in one's capacity to perform mathematics, learning to think and communicate mathematically, and becoming mathematical problem solver (Foegan & Deno, 2001). In addition, the NCTM (2000) called for the inclusion of mathematical problem solving skills to help students learn to observe, evaluate, and improve their knowledge in mathematical subject as well as their ability to solve mathematical problems.

The importance of developing problem solving ability

Researchers discussed various reasons to justify the need for students to improve their ability to solve problems. A major reason refers to enabling learners to think critically and creatively by applying advanced and challenging cognitive process (Radzali, 2007). In addition, skills obtained while lerning problem solving, such as metacognitive and flexibility, prepare students for real life situations (Otten, 2010). Moreover, with problem solving ability, students can enhance their cognitive abilities, carry out procedures, and obtain a deeper grasp of concepts (Ranjan & Gunendra, 2013). Jobs of the 21st century rely heavily on students graduating from schools prepared to analyze, assess, and justify reasons (Pinter, 2017). With the growth in STEM occupations over the last 10 years and jobs requiring strong problem solving abilities for non-routine tasks (OECD, 2013), students are required to display advanced communication skills, critical thinking, design strategies, and problem solving ability in workplace in order to accomplish non-routine jobs (Conley & Darling-Hammond, 2013, p. 1). Furthermore, students in the 21st century are expected to become independent learners and thinkers, as well as inventive problem solver with the ability to generate solutions to problems emerge from a changing society (Trilling &Fadel, 2009).

Based on previous studies' discussion, students have to improve their essential skills to investigate and explain multiple problematical situations, and be able to locate, analyze, synthesize, frame, apply information in new settings, address non-routine problems and provide solutions.

How to develop problem solving ability?

Constructivist educators believe that children acquire their abilities via experimentation, thinking growth, and adaption of one's own learning via communication with teachers and peers (Cobb & Bauersfeld, 1995).

Polya (1957) called for a more heuristic approach as opposed to the procedural approaches usually utilized in mathematics instruction. His techniques suggest that, while problem solving, students should consider a relevant problem they could solve and assess whether this knowledge can assist them solve the current problem. He also recommended that students should look for several strategies to solve a problem, and think how approaches they are adopting can be used in real world. Throughout this process, students should engage in internal conversation to clarify what the problem includes and enhance planning, estimating, monitoring and supporting problem solving process.

According to (TIMSS) 2011, the core of problem solving is applying knowledge. They explain that the term "applying" refers to students' ability to use their understanding and knowledge in problem contexts, such as how well students work on regular tasks. While, students' ability to answer non-routine or unexpected tasks is identified as "reasoning" (Mullis et al. 2012). As a result, knowledge demands to be transformed before it can be applied to a specific situation. Transfer occurs in new learning when past related information and experiences are implemented in new situations (Bransford et al. 1999).

"Knowing" and "applying knowledge" in routine and non-routine problems assist leaners in improving their problem solving ability. In order to increase students' problem-solving abilities, mathematics' education should emphasize on the three cognitive domains: "knowing," "applying," and "reasoning."

Different approaches utilized in teaching design field to meet the need to develop students' ability to solve problems. One learning model that can develop students' ability to solve mathematical problems is problem based learning approach (Vikriyah, 2015).

Problem based learning approach (PBL)

One educational approach that enhances students' problem solving ability is teaching through problem solving or problem based learning approach. It is defined as a student centered educational approach that is created to involve learners in authentic situations (Savery, 2006). This teaching and learning approach is based on constructivist principles where teachers present a problem, and students are encouraged to shape their own learning through the process of applying understanding and skills in order to reach practical solutions in collaboration with others (Savery, 2006; Bruning, Schraw, & Norby, 2011; Nariman & Chrispeels, 2015; Wilder, 2015).

Problem based learning (PBL) approach originated in medical schools as a

response to the need of medical students to develop problem-solving abilities and learn how to apply understanding to solve problems in their daily lives (Mustaffa, Ismail, Tasir, & Said, 2016). It eventually expanded into various majors at colleges, vocational schools, secondary schools, preparatory schools, and primary schools (Savery, 2006).

Using problem solving as a main methodology in mathematical education has increased dramatically during the last two decades (NCTM, 2000; Hung et al., 2008; Westwood 2011). NCTM released Curriculum and Evaluation for Math School in 1989 emphasizing the process of doing mathematics and placing significance on problem solving with its mathematical pedagogy, standards, as well as assessment recommendations. In 2000, the NCTM recognized mathematical problem solving as the top of the five required process standards that student must apply to develop deep understanding of mathematical subject (NCTM, 2000). The NCTM council assured the need for teaching mathematical problem solving as an integral component of mathematics curriculum (Cai & Lester, 2010). In 2010, the CCSS described mathematically proficient students' practices students through the eight Criteria for Mathematical Practice, the first one was "Making sense of problems and persevere in solving these problems".

Importance of problem based learning approach

Slavin, Lake and Groff (2010) in a comprehensive review of educational approaches in mathematics concluded that problem based learning is clearly superior to traditional forms of education. Furthermore, new cognitive researches suggest that the optimum learning occurs when students are fully involved in the PBL process (Ronis, 2008). Schoenfeld (1989) in his research indicated that Problem solving as an instructional approach improved problem-solving success. Delisle (1997) asserted that PBL is a structured method that teachers use to improve students' critical thinking and their ability to solve problems while learning essential subject knowledge.

PBL is a learning model which, argues for problem solving to be the curriculum's intellectual focus (Barrows, 1986; Barrows, 1996; Jonassen & Hung, 2008), uses problems as the core to develop problem solving ability (Eggen et al., 2012). PBL allows students to practice problem-solving processes and gain knowledge. During the process of PBL, students apply knowledge learned to generate solutions to problems in hand (Hmelo-Silver, 2004; Savery, 2015; Yeo & Tan, 2014). Because of that, multiple studies showed that integrating or supplementing daily mathematics education with problem-solving activities improves students' ability to solve problems (e.g., Charles & Lester, 1984; Sigurdson, Olson, & Mason, 1994; Verschaffel et al., 1999).

PBL is vital in mathematical classrooms because it could serve three purposes: The first is to achieve the goal of education to learn procedures and basic math facts. The second is to acquire new concepts via discovery and exploration, in addition to the creation and development of algorithms to solve problems (Wilson, Fernandez, & Hadaway, 1993). The third is to enhance student achievement and improve 21st century skills by concentrating on higher order thinking skills, cooperation, and ability to solve problems (Dağyar & Demirel, 2015; Savery, 2015; Tee & Lee, 2013; Yeo & Tan, 2014). In addition, using PBL in classrooms helps teachers to present challenging mathematical tasks and to evaluate students' ability in applying mathematical learning at levels that do not involve recreating algorithms or procedures practiced in current lessons, such as common or relative errors, or discussion mathematical hypotheses, which helps them move to different levels of mathematics (Bishara, 2016).

PBL also has various advantages, including increasing motivation for learning, information retention, and the capacity to apply learning across academic disciplines and life circumstances (Larrier, et al., 2016). In addition, PBL benefits students by

having them reformulate hypotheses, choose how to solve problems based on numerous sources and points of view, and defend their point of view (Aufdenspring, 2003).

Teacher's role in problem based learning approach

Teachers are pivotal to provide opportunities for students in PBL approach (Lambros, 2004) and play a very active role during PBL process. While students attempt their way to solve a problem, teachers act as facilitators by monitoring and guiding students, allowing them to achieve the intended learning goals, and reassuring or redirecting them as needed. They also provide scaffolding through the process to support students as they build their own learning interrelated to the problem rather than strictly provide content and knowledge (Roopashree, 2014; Chang et al., 2012; Jo & Ku, 2011). With this support from teachers, students can improve deep understandings of mathematics subject (Salinitri et al., 2015).

How to solve mathematical problem

John Dewey believes that once students are curious, they will solve problems and achieve more. Hence, the environment must be nurtured with problem-based learning activities to stimulate socially collaborative and motivating classroom environment that promotes active learning (Tracey & Morrow, 2012).

To solve a problem, a student could follow strategies or phases formulated by Polya (1973) in his book "how to solve", where student should first understand the problem; realize obviously what is the problem required to find, what kind of information is available, and what the situation is. On the second phase, students should recognize how things relate, how the required is related to information provided in order to obtain ideas and plan solutions. Third, students have to carry out the plan. Fourth, student should examine and check results by paying attention to solutions achieved, reviewing and discussing them again.

The process of solving a mathematical problem requires utilizing previously

learned skills and relevant experiences, and progressing via increased comprehension and reformulation and constructing options to solve the problem (Kilpatrick, 2016; Lester & Cai, 2016; Pólya, 1945).

Students who are proficient in mathematics demonstrate problem-solving behaviors, such as carefully reading and understanding problems, developing models, and making guesses regarding solutions and their strategies (Kilpatrick et al., 2001). While, students who are mathematically incompetent exhibit unproductive solution methods such as solving problems out of context without applying proper understanding of mathematical concepts while solving problems (CCSSO, 2010).

Problems in problem based learning approach

TIMSS provided information and data on teaching approaches and curriculum. It indicated that most highest-achieving countries had a consistent mathematical curriculum. Teachers in these countries applied most mathematical tasks as problems with higher order thinking (Stigler & Hiebert, 2004).

Mathematical tasks presented as problems requiring higher order thinking to solve is challenging. However, it helps students demonstrate curiosity, creativity, create connections with previous knowledge, and collaborate with each other (Boaler, 2016). It also enhances students' motivation in solving mathematical problems (Zambo, 1996, p. 82) and their reasoning, which allows for multiple entry points, diverse solution techniques, and using various levels of the revised version of Bloom's Taxonomy (Principles to Action, 2014). Literature highlighted that students' previous understanding influences their perseverance and motivation with challenging tasks as they transfer through mathematical learning (Boaler, 2016; Pasquale, 2015; Principles to Action, 2014).

The NCTM defined a problem, which is the subject of the problem-solving
process, as a task for which the answer is not immediately accessible (NCTM, 2000, p. 52). A problem is a task that cannot be achieved through direct effort and will demand some creative thinking in order to be accomplished (Liljedahl 2008; Mason et al. 1982; Pólya 1965). Liljedahl (2008) noted that mathematics educators, experts and teachers agree with the definition given by Resnick and Glaser (1976), which states that a problem is something that you do not have the experience to solve.

George Pólya (1945, 1957) defines exercise as "a routine problem", a task that can be solved either by replacing particular information into a previously solved general problem, or by following step by step process and some well-worn procedure, without any hint of creativity. While Kantowski (1980) mentioned that a situation is said to be a problem when students have to apply their new knowledge in a new context in order to solve the problem. As a result, a certain degree of innovation is required to solve mathematical problems (Pólya, 1945, 1957). Since the student is attempting to find an unknown answer or to do something without knowing a direct method (Schoenfeld, 2013). If the students can immediately recognize the procedures demanded, the situation is a routine task or exercise.

In conclusion, researchers particularly emphasize that solving a challenging task should involve a demand for mathematical understanding and skills, as well as the creation of a level of challenge to the task and its solution so that it does not have an immediate solution.

Isoda (2010, p. 17) stated that problem solving approach originated in Japan more than a century ago using lesson study approach. It is regarded as a teaching theory for the topic of mathematics, involving the instillation of self-learning in Japanese schoolchildren, which embraces learning how to learn. This approach lead Japanese students to attain higher educational achievement when compared to similar student groups in the United States (Stigler & Hiebert, 1999; TIMSS, 1999).

Lesson Study Approach

Lesson study, also called the Japanese method of teaching, is defined as an educational approach based on problem solving, careful observation of how students learn, and the focus on their thinking. In this approach teachers work collaboratively to improve a set of lessons that are known as "research lessons." Work on these lessons includes planning, teaching, observing, analyzing and reviewing lessons in an ongoing cycle (Fernandez and Yoshida, 2004).

Lesson study approach revolutionized classroom teaching because it holds social constructivism theory at its core. During the class, students analyze new concepts according to their previous knowledge, and build new knowledge in a social environment, starting with the assumption that learning is an integrated and structural activity that affects the social environment and the contexts in which learning happens (Wright, 2009).

In Japan, teachers have been using lesson study approach for more than a century. The international education community first paid attention to it in 1999, after Stigler and Hiebert (1999) published a book called "The Teaching Gap: The Best Ideas of Teachers to Improve Education in the Classroom". The book described results drawn from lessons depicted in several countries while working in (TIMSS video study). Authors singled out a chapter on education in Japan named "Japan's approach to improving teaching in the classroom", in which, the Japanese model structure of mathematics education was shown in classrooms. Stigler and Hebert referred to this teaching approach as "systematic problem solving". However, majority of international educators' interest was in utilizing lesson study as an effective professional development model, and the focus was less on applying the Japanese technique in

teaching mathematics, which is based on problem-based learning (Fujii, 2018).

Lesson study and structured problem solving

Japanese lesson study in mathematics is conducted with a base in structured problem solving (Hino, 2007) and utilize open- ended approach (Miyakawa & Winslow, 2009, p. 200). Fujii (2018) described lesson study along with teaching mathematics via solving problems like two wheels of the same cart: one cannot walk without the other. In this approach, the structured problem-solving lessons are used to introduce new mathematical concepts and engage students in evaluating alternative solutions to select and apply the best strategies and reach the most effective solutions (Inoue, 2010).

Structured problem-solving lessons in lesson study approach, also called "research lessons", are characterized by their own distinct and clear style, which often fascinates educators all over the world (Fujii, 2018).

Research Lesson

The research lesson depends on the structure of problem-based learning, in which the lesson is taught through presenting only one task (mathematical problem) during the lesson. As students complete the task, they achieve all the lesson goals, which makes selecting the suitable and appropriate task essential. Japanese mathematics teachers recognized this type of lesson structure, with challenging task and time to explore and examine solution techniques, as the best approach to provide students with learning opportunities (Shimizu, 1999, pp. 109- 111). Some educators may be skeptical about giving students one problem only in a math class. However, Japanese educators confirm through research the excellence and success of this approach.

Following, we will review the four stages of mathematics teaching through the

lesson study, the structured problem solving approach. These four stages are the framework of the research lesson. The length of each stage is 5 to 20 minutes, consisting of a 45 to 50 minutes lesson.

Presentation of the problem (5-10 minutes)

The term "problem presentation" in a lesson study approach means getting students to understand and interpret the context of a particular task and the corresponding mathematical conditions utilized to solve it. It is critical to select the task wisely and carefully. If applied well, it allows learning new mathematical concepts through mathematical ideas that appear in the comparison and class discussion stage (Fujii, 2018). Fujii (2016) believes that the ideal task is one that is appropriate and of sporting value in terms of lesson objectives, arise students' motivation, and of appropriate level of challenge and difficulty, that can be solved in several ways.

In Japan, "problem presentation" is not intended to clarify and explain procedures for solving the problem. It means that the teacher presents a problem to students to solve without providing any support or assistance for how to solve it, which is one of the advantages of learning based on problem solving in lesson study approach. Japanese teachers also distinguish between (teaching how to solve a problem) and (learning mathematics through problem solving). However, in the United States, teachers clarify procedures to solve the problem before assigning it to students when learning based on problem solving.

Students' work on the problem (10-20 minutes)

If the problem interests students, they will start working on solving it on their own without any assistance from their teacher. As students attempt to solve the problem, the teacher passes between students, and practices what is called the intended monitoring that determines how each student decided to solve the problem. As identifying the expected solutions, including the wrong solutions is part of the lesson plan, the teacher takes notes of how majority of students solve the problem, provides hints to struggling students if necessary, and asks early finishers to solve the problem in other ways. The teacher also plans what to do during the comparison and discussion phase (Fujii, 2018). It is significant for the teacher to look beyond the answers and acknowledge how all students in a class solve the task and how the mathematical concept develop while students reach the solution. In this stage, the teacher decides which solution methods will be presented first, and how to lead the whole-class discussion (Fujii, 2018).

Comparison and discussion (10-20 minutes)

The third stage of the research lesson, comparison and discussion, is considered a crucial stage of the structured problem solving approach. Hiebert, Stigler and Manaster, 1999, Stigler and Hiebert, 1999 noted that one goal of structured problem solving is to hold a whole-class discussion about different solution strategies that students offer.

As students come up with different strategies and methods to solve the problem, the teacher achieves the stage as comparisons and discussion of these methods take place. At this point, student solutions are written on the board in order from "simple" to "advanced" (Fujii, 2018). All solutions are written on the board to compare thinking, results and organization. The comparison and discussion stage gives teachers an opportunity to note and measure their students' mathematical thinking. Furthermore, it is an opportunity to focus on the new mathematical concept. Japanese mathematical teachers teach mathematics via problem solving using method described earlier believe that it stimulates students to think about mathematics and assists them to become selfdirected learners, which is one of the ultimate educational goals (Fujii, 2018).

Lesson Summarization (5 minutes)

Fujii, Kumagai, Shimizu and Sugiyama (1998) consider this stage as one of the main differences between problem-based learning in the United States and Japan. While teachers in the USA do not summarize the lesson at the end of the lesson, Japan's teachers see that without this stage, the lesson ends with students only comfortable with their work. Nevertheless, by including summaries at the end of the class, each student can also feel satisfied with mathematical concepts explored. The "lesson summary" stage is brief but very important. It helps to relate mathematical concepts and knowledge and build on them. Students listen to the teacher as the lesson concludes with a simple review of mathematical procedure, which students benefit from. If teachers summarize the lesson by reviewing how students build their understanding, it is as if they are sending a message to their students and teaching them how to think. In other words, throughout this stage, the teacher reviews the content and the thinking processes (Fujii, 2018).

As stated previously, the importance of giving students the skill to solve mathematical problems is clear as students continue to learn steps to solve problems, find keywords, deal with important information, report on chosen appropriate process for the problem, and verify those that ensure accuracy and reasonableness.

Researchers argue that these steps may provide students with methods to reach the answer, but they do not provide the opportunity to analyze, find relationships, think about the connections between mathematics strands, and provide various alternatives and different ways to solve problems as in lesson study approach (Pinter, 2017). Another argument of the current study presented when the researcher focuses on using the educational approach (Lesson Study), which depends on problem-based learning in mathematical learning, and its effectiveness in developing students' mathematical problem solving ability in order to obtain higher scores in math tests.

Theoretical framework for lesson study approach

Lesson Study approach adopted constructivist beliefs as both identified areas of incompetence in students' content knowledge. When gaps are identified, teachers work toward improving the environment where new learning can be devolved via scaffolding and collaboration with the teacher and other students and through research lessons constructed by teachers concentrating on real-world applications, students will build a better understanding of mathematical concepts (Wright, 2009).

Theoretical framework for problem based learning

The NCTM (2000, 2014) called for the improvement of mathematics learning via problem solving. In response to this advocacy, researches recommended that a large part of learning must be construct in a social environment (Bandura, 2005; LaMorte, 2016). The literature supports a clear sequence between constructivism learning theory and the problem based learning (PBL) educational model (Savery & Duffy, 2001). Ravitz (2008) asserted that PBL is a thoughtful constructive learning approach that involves students in learning and promotes a deeper conceptual understanding through relevance, accuracy, and connections. Tandogan and Akinoglu (2007); Larrier, Hall, Linton, Bakerson, Larrier, and Shirley (2016) reported that problem based learning is anchored in a theoretical framework of social construction where students engage in open-ended problems and work collaboratively. PBL includes an active learning process that allows students to solve authentic problems utilizing their critical thinking and analysis skills making students the driving force in the learning process and teachers act as facilitators of learning (Han et al., 2015). Moreover, in PBL, students utilize their own previous problem solving knowledge and skills they become actively involved in mathematical problem solving, resulting in gaining new learning (Bruner, 1960; Dewey, 1916; Flores et al., 2015; Hughes et al.,

2015; Kennedy, 2012; Rosli et al., 2015).

Related studies

This section presents a review of the studies related to the impact of Lesson Study approach on improving students' problem solving ability. However, examining the previous literature revealed a lack of in the field of study. Therefore, the researcher included studies related to the impact of Lesson Study on other abilities such as reasoning, creative thinking, critical thinking and metacognitive, which are considered essential for developing students problem solving ability.

First: Studies that addressed the effect of lesson study approach on students' mathematical learning

In a recent study, Kusumawati's (2020) study investigated the impact of lesson study on enhancing students' ability in mathematical reasoning. Twenty-four students in grade nine participated in this study. Instruments used included learning achievement test, observation, student worksheets, and interviews. Results revealed that lesson study approach has developed students' ability in mathematical reasoning.

Moreover, Lundbäck and Egerhag's (2020) study described how teachers utilize Lesson Study to promote students' mathematical learning in two settings (the first years of school, and school-age educare). The study adopted the case study design in which 4 teachers and 37 grade one to three students from an elementary school in Sweden took part. Qualitative data were collected through teachers' recorded conversations with the researchers, while quantitative data were collected using students' questionnaire. The study results showed that mapping students' understanding before and after the teaching sessions, assisted teachers to know how to design teaching situations that would benefit their students. Therefore, the researchers advocated for the use of Lesson Study approach throughout the school. Another study conducted by Risnanosanti, Susyla and Syofiana (2019), aimed to describe the impact of lesson study approach on students' critical thinking skills. This research applied descriptive qualitative method, with a sample of 33 eighth grade students in Indonesia. The data were collected from students' critical thinking tests and classroom observations. Results showed that there are significant effects of utilizing lesson study approach on developing students' critical thinking ability in mathematics classroom based on their SOLO thinking ability.

Indrawanti, Hobri, Hadi, and Fauziah's (2019) quasi-experimental mixed method study intended to investigate the influence of applying Lesson Study approach and Discovery Learning model on students' critical thinking ability. The Data were collected using achievement tests and questionnaires from grade seven Indonesian students (one class as control group and one class as experiment group). Findings revealed that there is a significant difference in critical thinking ability between the two groups.

Similarly, Fauziah, Hobri, Yuliati, and Indrawanti's (2019) a quasiexperimental mixed method study was undertaken to explore the differences in terms of creative thinking abilities between two groups of grade 7 students. The experimental group was taught using Lesson Study approach with structured problem solving, and the control group was taught in the conventional method. Data collection included lesson plans, students' worksheets, and creative thinking tests. Findings showed that the group who apply Lesson Study approach with (PBL) have better creative thinking abilities than those that do not. It also showed that lesson study with structured problem solving could enhance students understanding of mathematical lessons.

Moreover, Saadah, Hobri and Irvan's (2019) study aimed to examine the impact of applying Lesson Study approach with (PBL) on students' creative thinking ability.

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The study utilized the Four D Model, and the sample included 28 grade 8 students. The data were obtained from lesson plans, students' work sheets, and tests. Findings revealed that the implementation of Lesson Study with (PBL) has significantly affected students' creative thinking ability. Furthermore, Students' creative thinking process increased after using lesson study approach with (PBL) model.

Nuha, Waluya, and Junaedi's (2018) carried out a study to find out the impact of utilizing Problem Posing Model with Lesson Study on students' mathematical creative thinking ability in Digital Classes. This research utilized a quasi-experimental design mixed method to collect data using creative thinking tests and classroom observations. The sample comprised two classes of eighth grade. The results showed that, in Digital Class, Problem Posing Model with Lesson Study approach promote creative thinking skills.

In a similar vein, Hobri, Romlah, Prihandoko, Safitri and Nazareth's (2018) conducted a quasi- experimental design mixed method study to analyse students' metacognitive skills through the implementation of Lesson Study with structured problem solving. The sample included 72 grade 7 students at Junior High School in Indonesia (36 students in the experimental class and 36 students in the control class). Data were collected using test, students' questionnaires and interviews. Study results indicated that the implementation of structured problem solving based on lesson study led to increase student activity, and students' problem solving ability that resulted in increasing of students' metacognitive ability.

Salim and Samaan's (2011) experimental study aimed to explore lesson study impact on teachers' teaching ability and their effect on their students' achievement. The sample consisted of 6 male teachers and 150 grade 7 students from a school in Qatar. Data were collected using achievement tests in mathematics. Findings showed that the

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lesson study approach contributed significantly to the professional development of the teachers, and enhanced students' learning. Based on this result, the researchers recommended that it is necessary to further train teachers on lesson study approach.

Review of previous study

This section focused on reviewing previous studies relevant to the scope of this study. The review showed the following:

All studies and the current study investigated Lesson Study as an effective approach used to enhance student learning, except the study of Salim and Samman (2011), which investigated lesson study as a professional development model for teachers, then measured its impact on students' learning. In addition, Studies of Fauziah et al. (2019), Saadah et al. (2019), and Nuha et al. (2018) assessed the impact of lesson study approach on students' creative thinking ability. Studies of Risnanosanti et al. (2019), and Indrawanti et al. (2019) examined the impact of lesson study approach on improving students' critical thinking ability. Studies of Salim and Samaan (2011) investigated the effect of using lesson study approach on students' achievement. Lundbäck and Egerhag's (2020) study investigated the impact of lesson study approach as a tool to improve students' mathematical learning in general. Kusumawati's (2020) study aimed to examine the effect of lesson study approach to enhance students' mathematical reasoning skills. Hobri et al. (2018) investigated the effect of using lesson study approach on students' metacognitive skill.

Moreover, the current study, and most studies (Salim and Samaan (2011); Hobri et al. (2018); Nuha et al. (2018); Fauziah et al. (2019); and Indrawanti et al. (2019)) applied mixed method approach as a research methodology. While studies of Risnanosanti et al. (2019). On the other hand, Kusumawati (2020) utilized action research, Risnanosanti et al. (2019), Lundbäck and Egerhag (2020) applied case study design, and Saadah et al. (2019) utilized Four D Model.

Furthermore, most studies targeted the students only to collect data about the impact of lesson study except for Lundbäck and Egerhag (2020), Salem and Samman (2011), and the current study who included students and teachers in their sample. The majority of the studies (Kusumawati (2020), Indrawanti et al. (2019), Risnanosanti et al. (2019), Fauziah et al. (2019), Saadah et al. (2019), Nuha, et al. (2018), Hobri et al. (2018), and Salim and Samaan (2011) were conducted in preparatory schools. While only the current study, and Lundbäck and Egerhag's (2020) study were conducted in primary schools.

Salim and Samaan's (2011) study recommended using lesson study approach in teachers' professional development to enhance their abilities to teach mathematics. Lundbäck and Egerhag's (2020) called for applying Lesson Study approach throughout the school.

Second: Studies that addressed the impact of lesson study on students' problem solving ability

To address the impact of lesson study on students' ability to solve problems, Bintoro, Zaenuri, and Wardono (2021) conducted a recent study aimed to investigate the impact of implementing technology based Lesson Study on students' mathematical problem solving ability. The study applied a quantitative method with a quasiexperimental (one group's pre-test post-test) design. To collect data, undergraduate students from the Mathematics Education study program at UMK University in Indonesia were recruited and data were obtained from mathematical problem-solving tests. Findings indicated that learning mathematics using technology based lesson study is effective in developing students' mathematical problem-solving ability.

Furthermore, Khalid, Saad, Abdul Hamid, Abdullah, Ibrahim and Shahrill

(2020) conducted a quasi-experimental mixed method study to examine changes in students learning of mathematics concerning their creativity and problem-solving ability by utilizing lesson study approach. The study sample consisted of 172 grade 7 students divided into treatment and comparison groups. Data collection included Torrance Test of Creative Thinking, mathematical problem solving tests, students' interviews and observation checklists. Findings showed statistically a significant increase in students' creativity and problem solving tests.

Purwandi, Susanto and Hobri's (2020) conducted an experimental mixed method study aimed to explore the impact of applied remedial mathematical learning tools based on Lesson Study approach on students' problem solving ability. The sample included 68 grade seven students from Indonesia (34 as control class and 34 as experimental class). Qualitative data were obtained from teachers' lesson plans and students' worksheets, while quantitative data were collected using students' questionnaires, and students' achievement tests. Results indicated that students' problem solving ability in the experimental class has significantly increased. Therefore, the researcher concluded that the implementation of remedial tools based on lesson study approach has a significant effect on students' problem solving ability.

Loc, Uyen, Tong, and Ngoi's (2020) study examined the impact of implementing lesson study approach on developing students' problem solving ability to solve two mathematical problems about straight-line equations. The research applied a case study and quasi- experimental design. The study included 74 high school students from Vietnam distributed into experimental and control groups. Data collected from students' worksheets revealed that students in experimental class answered teachers' questions and were able to solve the posed problems. Moreover, the quality of the lesson was significantly improved and led to a positive change in students' problem solving ability.

Gholami, Yunus, Ayub, and Kamarudin's (2019) study investigated the impact of implementing the lesson study approach on students' achievement in mathematical problem solving. The quasi-experimental study was conducted on 8 lecturers and 95 students (45 in control group and 50 in experimental group) at a Foundation Centre in Malaysia. Data were obtained from five tests on mathematics function topic and analyzed by an independent t-test, and one-way ANOVA test. Findings indicated that the application of lesson study approach on the experimental group increased their ability in problem solving significantly, while no significant differences were established in control group.

The focus of Ningrum, Hobri, Susanto, Dafik, Lutvita, and Lestari's (2019) study was to examine the use of STEM learning based lesson study on students' problem solving abilities in mathematics. This study applied multiphase mixed method, as well as a quasi-experimental design. Ninety six students in grade 11 in Indonesia participated in this study and were divided into three groups each class consisted of 32 students (control group, first experimental group and the second experimental group. Data were gathered through lesson plans, students' worksheets, students' interviews and learning achievement tests. Findings revealed that the implementation of STEM learning based lesson study had a significant impact on students' problems solving ability.

Bradshaw and Hazell's (2017) study aimed to investigate differences in students' approaches to problem solving through Lesson Study. The researcher used case study design and the sample included two classes of 32 grade seven students from UK. Instruments used were students' interviews and teachers' observations. Data were analyzed using a thematic analysis. Study findings indicated that students' ability to

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solve problems appeared to differ for a variety of reasons such as ability, gender, motivation, and confidence. In addition, findings showed that all students were motivated to solve the problem even though some were more competitive than others. This appeared to have an impact on their problem-solving ability because they were too obsessed with finding the answers and did not give time to examine the problem, and thus overlooked important aspects needed to solve the problem set correctly.

In another study conducted by Marwiang, Klaharn, Saree, and Junpeng (2017), the researchers adopted the applied a case study mixed methods design to assess students' problem solving ability by utilizing Lesson Study in mathematical classrooms. The study used students test to collect data from 7 fifth grade students in Thailand. Findings indicated that students need to develop their ability to understand basic knowledge, use rules and formulas in solving simple to complex problems, analyze problems to eliminate misunderstanding, provide logical reasoning, check solutions, calculate accurately, and utilize numbers, and symbols to represent the problem.

Furthermore, Khotimah and Masduki's (2016) study explored how contextual teaching and learning model based on lesson study approach could develop students' ability to solve problems in differential equations course. Using the action research, data were obtained through observations, students' test, field records, and interviews. The sample of this research was 34 students of mathematics education in Muhammadiyah Surakarta University in Indonesia. Findings showed that learning with lesson study approach improved students' problem solving ability in differential equations learning.

In another study, Khalid and Ali (2016) investigated the impact of lesson study with structured problem solving on students' ability to solve problems and their

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knowledge of the "rate" topic. The sample composed of three classes, consisting of 58 grade 5 students. This research applied a mixed method, and data were obtained from student's pre- and post-test, observation checklists, researcher's field notes, teachers' interviews and reflections. Research findings highlighted that students understanding of the problem context was enhanced as well as their ability to utilize understanding to create the best solution to the problem. Consequently, lesson study was found to assist teachers to produce effective lessons that incorporated student-centered real -life lessons as well as students' creativity in problem solving.

Review of previous studies

All previous studies reviewed used Lesson Study as an independent variable to measure its impact on students' problem solving ability, and utilized Lesson Study as an approach to enhance learning. The current study and studies of (Khalid et al. (2020); Purwandi et al. (2020); and Ningrum et al. (2019) applied a mixed method approach as a research methodology, while studies of (Loc et al. (2020); Marwiang et al (2017), and Bradshaw and Hazell (2017) applied case study design.

In their studies, (Bintoro et al. 2021); (Khalid et al. 2020); (Purwandi et al. (2020); (Gholami et al. (2019), (Ningrum et al. (2019), Khotimah and Masduki (2016), and Khalid and Ali (2016) used students' test as a tool to obtain data, while Loc et al. (2020) used students' worksheet, and Bradshaw and Hazell's (2017) used students' interview and teachers' observation. To obtain data, for the current study, the researcher selected participants from grade 5 students, similar to Khalid and Ali's (2016) study. The current study also used students' test as an instrument to obtain data similar to other studies of (Bintoro et al. (2021), Khalid et al. (2020), Purwandi et al. (2020), Gholami et al. (2019), Ningrum et al. (2019), Marwiang et al. (2017), Khotimah and Masduki (2016), and Khalid and Ali , (2016).

The current study differs from previous studies in that:

The current study focuses on students' ability to solve mathematical problems in three strands (algebra, geometry, and data handling). It differs from Loc et al. (2020) study, which focused on a specific unit only. Their study aimed to examine the impact of implementing lesson study approach on students' problem solving ability to solve two mathematical problems about straight-line equations. It also differs from the study conducted by Khotimah and Masduki (2016) in its aim as the lateral aimed to describe the effect of lesson study approach on students' ability to solve problems in differential equations course. Furthermore, the current study differed from the specific topic objective targeted in Khalid and Ali's (2016) study, which aimed to investigate the impact of lesson study in students' problem solving ability and their knowledge of the topic "rate".

Based on the above background, this study is conducted to investigate the effectiveness of lesson study approach on students' ability in solving mathematical problems in primary schools in the State of Qatar.

CHAPTER THREE: METHODOLOGY

The aim of the study is to investigate the impact of the lesson study approach on student's problem solving ability in primary public Qatari schools. This chapter describes and justifies the research design, population, subject and instruments. These are followed by a detailed description of data collection method and data coding and analyses for major themes, as well as ethical considerations applied while conducting this study.

Research design

This study employed descriptive research design, which is a research method used to provide detailed description of existing phenomena and their features accurately and systematically (Dulock, 1993; Nassaji, 2015; Atmowardoyo, 2018). Thus, descriptive research is an appropriate method selected to investigate the influence of lesson study approach on students' ability to solve mathematical problems.

The current study applied the mixed-method approach. This approach is a combination of qualitative and quantitative techniques. It is used when both techniques are needed to offer a better understanding of the study questions (Creswell, 2015, p. 15). A mixed-method approach incorporates the strengths and advantages of both the qualitative and quantitative research. Creswell (2009) noted that while the quantitative research offers closed-ended responses and facilitates generalization of the findings from the sample to the targeted population, the qualitative research provides open-ended data, meanings, and interprets, and allows the researchers to understand participants' views and experiences.

To study primary students' mathematical problem solving ability and its relationship to lesson study approach, the study adopted explanatory sequential mixed methods design as shown in Figure (1).



Figure 1. Explanatory sequential mixed method design

In this design, there were two phases. On the first phase, quantitative data were obtained via mathematical problem solving test to examine whether lesson study approach influenced students' problem solving ability. Findings were then analyzed to inform the collection of the qualitative data.

On the second phase, data were gathered using semi-structured interview. The purpose of using qualitative technique in phase 2 is to provide in depth description and interpretation of quantitative results and strengthen conclusions (Creswell, 2015). In addition, utilizing multiple sources to gather data ensures reliability and credibility of the study (Mills, 2003).

Research Population and Sample

Population

The study population consisted of the teachers and students in six primary public schools in the State of Qatar for the academic year 2020-2021, which participating in the lesson study project at least for one year.

Sample (Subject)

To fulfill the purpose of the study, purposeful sampling was most appropriate to use. Creswell and Plano Clark (2011) reported that identifying groups with experience in the phenomenon desired a study is recognized through purposeful sampling. According to Creswell (2014), purposive sampling allows for an in-depth investigation of the specific characteristics identified in the sample in order to obtain insight or understanding of the core phenomena of interest. Therefore, a purposive sample was used to identify and select 117 female fifth grade students from two participated schools, and the 10 mathematical teachers who taught the sample students in the third, fourth and fifth grades.

Students participating in the study were divided into three groups. The first group included 38 fifth grade students who were taught using lesson study approach for 2 years. The second group included 39 fifth grade students who were taught using lesson study approach for 1 year. The third group included 40 fifth grade students who were taught using traditional instructions. In other words, Group 1 and 2 have gone through at least one year of learning mathematics using the lesson study approach, while students' in group 3 have not.

In terms of teacher, participants in this study were female, and had different years of experience with lesson study and varied teaching experience. They were selected based on their teaching of the sample of students over the past three years. Teachers' demographic characteristics include educational level, teaching experience, and Prior Lesson Study experience. The following table (1) provides a brief description of the teacher participants.

Teacher	Years of	Current	Highest	Prior Lesson Study
	Teaching	Teaching	Degree	Experience
	Experience	Position	attained and	
			Study Area	
T1	6 years,	Grade 5	B.A.	Lesson Study team member
	taught grade	mathematics	Mathematics	for three years, Teacher for
	J		Education	several research lessons in
	mathematics			and out her school
T2	5 years.	Grade 4	B.A.	Lesson Study team member
	taught grade	Mathematics	Mathematics	for three years; Teacher for
	4		Education	research lessons, Observed
	mathematics			several research lessons in
				and out her school.
T3	5 years,	Grade 3	B.A.	Lesson Study team member
	taught grade	Mathematics	Mathematics	for three years, Teacher for
	3 mathematics		Education	research lessons, Observed
	mathematics			and out her school
T4	14 years.	Grade 5	B.A of	Lesson Study team member
	taught	Mathematics	Science in	for two years, Teacher for
	mathematics		Mathematics	research lessons, Observed
	for students		Education	several research lessons in
	in primary			and out her school.
m-	stage			
15	3 years,	Grade 5	B.A Environmental	Lesson Study team member
	s	manematics	engineering	research lessons. Observed
	mathematics		engineering	several research lessons in
				her school.
T6	9 years	Grade 4	B.A.	Lesson Study team member
	taught grade	Mathematics	Mathematics	for two years, Teacher for
	3		and science	research lessons, Observed
	mathematics		Education	several research lessons in
Т7	11 voore	Grade 1	ВΛ	her school. Lesson Study team member
1/	taught	Mathematics	Mathematics	for two years Teacher for
	mathematics	1.1utileinuties	Education	research lessons, Observed
	for students			several research lessons in
	in primary			her school.
	stage			
T8	5 years taught	Grade 5	B.A.	Attended the Professional
	grade5	Mathematics	Education	Study Project Decement
	manicillatics		Euucation	teach using lesson study
				approach. Observed 3
				research lessons in her
				school.

Table 1: Participating teachers' demographics and Lesson Study experience.

Teacher	Years of	Current	Highest	Prior Lesson Study
	Teaching	Teaching	Degree	Experience
	Experience	Position	attained and	
			Study Area	
Т9	9 years	Grade 5	B.A.	Attended the Professional
	taught grade5	Mathematics	Mathematics	Development for Lesson
	mathematics		and science	Study Project, Does not
			Education	teach using lesson study
				approach, Observed 2
				research lessons in her
				school.
T10	6 years	Grade 4	B.A.	Attended the Professional
	taught grade4	Mathematics	Mathematics	Development for Lesson
	mathematics		and science	Study Project, Does not
			Education	teach using Lesson Study
				approach, Observed 2
				research lessons in her
				school

Table (1) above shows that the total of participant teachers is 10. The majority of the teachers hold a bachelor's degree in mathematics education, and one teacher hold a bachelor's degree in environmental engineering. In terms of teaching experience, around half of the participants have more than 5 years of experience. Four teachers have 5 years as an experience in teaching mathematics, two teachers have 6 years as an experience, two teachers have 9 years of experience, one teacher has 3 years of experience. Moreover, all teachers have professional development related to the lesson study approach, but they have varied experience in teaching through problem solving. Three teachers have three years of experience of teaching through problem solving, and three teachers do not have any experience of teaching through problem solving.

Research Variables

Instructional methods were considered as the independent variable between the

three groups in this study in order to examine how each instructional approach affects students' problem-solving ability. Student's ability to solve problems, the dependent variable, was assessed using mathematical problem-solving test and teachers' responses through the semi-structured interview.

Research Instruments

Researchers often assess the effect of Lesson Study approach on students' learning in the following ways: (1) providing tests; (2) obtaining feedback from students and teachers about development; and (3) in some cases tracking those changes over time (Dudley, Vermunt & Lang, 2019). Therefore, the study utilized two main instruments: problem solving mathematical test, and teacher's interview.

The rationale behind using two instruments is that relying on a single source for data collection methodology exposes the study to weaknesses (Wolcott, 1988). In addition, applying a variety of data collection methods helps in exploring complex issues over an extended period of time (Pelto & Pelto, 1978).

Mathematical problem solving test was used to assess students' problem solving ability in mathematics, while the interview design held the possibility for fruitful discussion that lead to richer and more productive conversation. The interview form included a list of direct and open questions to collect data related to the study. Upon participants' responses, questions were modified to investigate new ideas (Merriam & Tisdell, 2015).

The Mathematical Problem-Solving Test

The study utilized mathematical problem-solving test to measure students' ability in mathematical problem solving. The researcher developed the test based on Qatar's mathematics curriculum and TIMSS items announced in 2011 and 2015 (see Appendix A). This test instrument combined TIMSS items from 'knowing', 'applying' and 'reasoning' domains in the three mathematical strands "algebra", "geometry", and

"data handling". The criterion for selecting items was the average correct responses of students in the State of Qatar to these items was less than 35%.

The test adapted multiple-choice items format, however, it required students to demonstrate their work and thinking as multiple-choice assessment is not enough to assess the PBL outcomes (Roopashree, 2014). The test in its last version consisted of 20 items. Completing this test needed 45 minutes approximately were students solved the problems and showed their work as well. A higher score on the mathematical assessment indicated higher mathematics performance and the ability to solve problem, while a lower score represented lower mathematics performance.

For the purposes of the study, the mathematical problem-solving test was translated into Arabic.

Rubric

The researcher created a rubric (see Appendix B) to eliminate any potential bias while grading. To understand the participants' thinking process, the researcher used different grading systems for some quotations other than the original TIMSS's rubric. For instance, the TIMSS rubric accepted the right choices or accurate answer as full credits, while developed rubric assessed responses and justifications to calculate full, partial, or no scores. The rubric assesses the understanding of the problem, planning, strategies, calculations, and solution for each question.

The rubric is considered crucial to ensure the consistency of the scoring system and to avoid any subjectivity in evaluation. In addition, all possible solutions to solve open-ended questions were considered. Moreover, to ensure that marking and scoring were done correctly, the researcher scored a random sample of students' solutions from each class.

Validity of the Mathematical problem - solving test:

To ensure the validity of the problem-solving test, a panel of 10 experts in mathematics education including professors from Qatar University, educational supervisors from the Ministry of Education and Higher Education, professional development specialists in Mathematics, English and Arabic at the National Center for Educational Development reviewed the test items. Suggestions were adopted to enhance the test validity.

Reliability of the Mathematical problem - solving test:

To evaluate the test's reliability, a pilot study was implemented on 10 subjects taken from the sample. Fifteen days later, this test was re-administered to the same subjects. The outcomes were correlated utilizing coefficient Cronbach's Alpha (see table 2), Cronbach's Alpha reflects that good reliability of questions as all reliability coefficients are greater than 0.7. Researchers suggest that a level of 0.67 or higher is acceptable (Cohen et al., 2018).

Table 2:	Reliability	of mathematic	al problem	solving	test in	each	category	by	using
Cronbac	h's Alpha c	oefficient.							

Item	Reliability measure
First group: students who studied using	0.926
lesson study approach from 2 years	
Second group : students who studied	0.781
using lesson study approach from 1 year	
Third group : students who studied using	0.730
conventional instructions	
Total	0.872

Teachers' Semi-structured Interviews:

Hatch (2002) and Creswell's (2007) indicated that interviews, as a method to

collect data, contribute to understand the phenomenon in depth, and are significant to utilize data to lead the study.

In this study, the researcher interviewed each teacher individually, following pre-set semi-structured interviews and their protocol (see Appendix C) to explore teachers' mathematical instruction, pedagogical strategies to teach and assess problem solving, and knowledge related to student's problem solving ability. The interview protocol was e-mailed to each teacher before the interview sessions. This helped teachers to review questions, clarify any misunderstandings about them, and confirm that all teachers had the same understanding of the questions.

The researcher interviewed each teacher for 30-40 minutes regarding using the lesson study approach with structured problem solving, its effect on her instructional strategies, and student's problem solving ability in the grade level.

After completing the interviews, the data were transcribed, reviewed several times to coded, and then identify initial themes. Data were translated from Arabic to English afterwards and were reorganized based on research questions.

Validity and Reliability

Methods of trustworthiness in the qualitative research work are used as an attempt to ensure results' accuracy. For this study, the researcher aimed to ensure creating consistent and trustworthy information using qualitative method in an ethical manner, thus that the results of the study can be trusted (Merriam & Tisdell, 2015).

Initially, before the interview session, the researcher requested permission to audio record the interview to ensure the accuracy of data collection and the validity of the study (Benatar, McKibbin, & Stewart, 2012).

According to Anfara, Brown, and Mangione (2002), using a detailed description can enhance external validity of results, which are also supported by connecting them

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to the quantitative data collected in this mixed methods study.

The researcher conducted interviews in private to encourage participants to be responsive and honest with their feelings and thoughts about their students' ability to solve mathematical problems. Participants were informed that all data would be kept confidential to provide honest and accurate responses during the interviews. Moreover, all participants were aware that they could refuse or withdraw their participation at any time of the study without providing explanation to the researcher to ensure that data collected would only involve participants who would voluntary provide it (May & Luth, 2013).

Lastly, all participants reviewed transcripts and confirmed that they reflect their responses accurately.

Research Procedures

The study was conducted after receiving the approval from Qatar University Institutional Review Board (QU-IRB) on the second semester of the academic year 2020-2021.

First, possible schools, teachers and students who meet criteria were identified. In each school, the researcher met with the principle, explained the nature and the purpose of the study, the reason for choosing the school and participants, and requested approval from the principle to meet with the mathematical department in the school.

After that, the researcher met with the teachers to explain the nature and the purpose of the study, invited them to participate in the study, discussed time and method of data collection, and provided consent form for teachers welling to participate in order to sign.

Next, the researcher spent about 10 minutes during the first period at each class to explain the study to grade five students, distributed consent forms to be signed by their parents, and asked them to hand them over to the math teacher. Three days later, the researcher visited the school and collected the forms.

After this, the test was conducted during the first two periods for the three groups over a period of three days on the third week of February 2021. The researcher collected exam papers, and spent two months to grade and analyze them.

Finally, the researcher contacted the teachers to arrange individual interviews during the months of May, June and July 2021. Interview appointments were set based on participants convenient.

Data collection and analysis

To explore the effectiveness of using structured problem solving based lesson study approach on students' mathematical problem-solving proficiency compared to classrooms that do not use this approach, data were collected from 10 teachers and 117 students in 2 public schools in Qatar. The study assessed students' problem-solving ability through mathematical problem solving test, then teachers' interviews were conducted to provide further insight of students' problem solving ability. The following table demonstrates how each research question was answered using which research tools.

Research questions	Data Sources
Q1	Main Source: scores of mathematical problem solving test. Supporting Source: teachers' interview.
Q2	Main Source: teachers' interview Supporting Source: students' work

Table 3: Data Sources: Research instruments based on the research questions

An overall goal of this analysis was to examine students' problem solving ability in relation to lesson study approach. This allowed the researcher to explore the impact of lesson study approach on students' problem solving ability.

In this mix method study, the researcher analyzed the quantitative data to provide background information about the difference in ability of problem solving between the three groups of students, then collected and analyzed the qualitative data to clarify and explain statistical outcomes in depth.

Quantitative data were analyzed using descriptive statistics as well as analysis of variance (ANOVA). The researcher calculated the Mathematical problem-solving test scores for the three groups. Tables with mean scores and standard deviations for this test scores were generated to compare the grades of each of algebra, geometry, data handling, and the total grade between the three groups. Then ANOVA test and Post hoc tests were used to determine if there was a significant difference in the means of the three groups. The researcher also highlighted the differences among these groups by giving excerpts from their responses in the test.

Interviews and written responses to interview questions provided data that led to a deeper understanding of the phenomenon and are best used when the researcher wants to gain detailed insights from individual participants (Bloomberg & Volpe, 2015). Therefore, Separate interview were conducted with each participant and audio recorded for professional transcription. Each participant answered the same main questions, probing questions were asked based on their responses for more details and clarifications. Each interview took about 35 minutes. The researcher analyzed the data to examine how do students demonstrate their ability to solve problems. The researcher used transcriptions of the audio recordings to create a coding scheme, then organized and conceptualized these codes into categories and overarching themes. The researcher used interview transcripts to support the depth of analysis and provide further evidence of students' problem solving ability. All interview data were transcribed into "word document' format to integrate participants' responses. These data were gathered and analyzed to determine common themes mentioned in their responses. Following Creswell (2014) recommendation, the first step in data analysis involved getting an overview of all the information. Once all data were transcribed, the researcher reviewed all the collected information several times to get an understanding of the overall data. Moreover, the researcher documented results in reflective notes as a first sorting out process. Following that, a coding system was developed to reduce the data, identify and highlight particular themes, and sort the text into categories. Data reduction was followed by the creation of a graphic table displaying the information. As a result, themes were reduced into seven categories.

Ethical considerations in the Study

In all research studies, it is critical to safeguard human participants by following appropriate ethical guidelines. Therefore, ethical principles were reviewed and followed to ensure the well-being of all participants.

As required, permissions and approvals to conduct this research were obtained from Ministry of Education and Higher Education, Qatar University Institutional Review Board QU-IRB and the schools. All guidelines established by the QU-IRB were followed to ensure safe and ethical handling of participation in this study.

This study does not pose any risk on students. Students' safety and well-being has been at the forefront of the study at all times. Purpose and overview of the study were explained to the students, and were communicated to their parents. Throughout the research, participating students engaged in their regular mathematical curricula and school evaluations. Students and schools names were not included in the findings.

Students were required to provide their assent to be part of the study. Since these students are young, it was critical to obtain assent form their parents as well. Students were informed that they have the freedom to withdraw from this study for any reason without providing explanation to the researcher.

Building an ongoing professional connections between the researcher and the participants, as well as providing clear explanation of the study goal, methods, and findings, was an essential element in maintaining clear communication regarding the study's purpose, procedures, and outcomes.

During the study, the researcher considered objectivity, ensured the analysis accuracy, and provided feedback for emerging themes.

This study paid close attention to ethical concerns such as protecting subjects' privacy and confidentiality. All materials used in the study such as participants' written consent and assents, audio recordings, transcripts, and interview notes are stored and kept in a locked cabinet at home that is accessible only by the researcher. Moreover, all electronic material is stored on the researcher personal laptop and were kept secured by a password that is known only to the researcher. Five years later, all participant data will be destroyed, thus safeguarding participants' privacy.

CHAPTER FOUR: RESULTS

This study aims to explore the impact of lesson study approach on mathematical problem solving ability of students in Qatari government schools. To serve this purpose, the following guiding questions and sub questions were answered through the study.

1) Does mathematical problem solving ability differ between students who learn via lesson study approach compared to students learning in traditional classes?

• Is there any statistically significant difference (α = 0.05) between students' mean score of problem solving test among the three groups?

2) How do students demonstrate their problem solving ability?

This chapter provides the mathematical problem-solving test results given during the academic school year 2020-2021 at two primary schools in Qatar. Data gathered from participating teachers' interviews are also provided. Descriptive statistics and analysis of variance (ANOVA) were utilized to analyze quantitative data collected from the test. While the qualitative data collected during teachers' interviews were analyzed using thematic analysis. These results are presented in response to each of the above-mentioned questions.

Research Question 1: Does mathematical problem solving ability differ between students who learn via lesson study approach compared to students learning in traditional classes?

Quantitative Results (mathematical problem solving test)

Quantitative data used to answer the first sub-research question, "Are there any significant statistical difference (α = 0.05) between students' mean score of problem solving test among the three groups?" Data were analyzed to show if there is any difference in the ability to solve mathematical problems among students who received learning via lesson study approach for two years, students who received learning via

lesson study approach for a year, and students who followed the conventional learning.

Mathematical problem solving test is used to assess students' mathematical problem solving ability in algebra, geometry and data handling. The following strands were tested: (a) Algebra: 60 % of the total mark of the exam. (b) Geometry: 30 % of the total mark of the exam. (c) Data handling: 20 % of the total mark of the exam. All grades were adjusted to be out of 100 and the total to be out of 300.

Mathematical problem-solving test scores for all groups were calculated and represented in tables including mean scores and standard deviations to compare scores in algebra, geometry, and data handling, and the total scores between the three groups. In addition, the Analysis of variance (ANOVA) test and Post hoc tests were used to determine if there is a significant difference in the means scores of the three groups.

Testing normality

In order to further prove and determine if there was a significant difference in the means scores between the three groups, the ANOVA test and Post hoc tests were used. Before conducting the ANOVA test analysis, testing normality was conducted to ensure data could be analyzed using the ANOVA test.

	group			Shapiro-Wilk
	group	Statistic	df	Sig.
	1	0.977	38	0.595
Algebra	2	0.97	39	0.377
	3	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		
	1	0.962	38	0.219
Geometry	2	0.975	39	0.526
	3	0.952	40	0.092
	1	0.925	38	0.014
Data handling	2	0.926	39	0.013
	3	0.874	40	0.00
	1	0.987	38	0.93
Total	2	0.96	39	0.173
	3	0.938	40	0.129

	Table ((4):	Tests	of Nor	rmality
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Grades of mathematical problem solving test of (algebra, geometry, and the total) for the 3 groups are normally distributed as p-values for Shapiro-Wilk test and are greater than 0.05. Therefore, the researcher can use ANOVA to test significance of difference between the three groups in all grades, even data handling grades, which is not normally distributed. This is because with large enough sample sizes (greater than 30) the violation of the normality assumption does not cause problems (Pallant, 2007); even if the data is not normally distributed, we can run parametric methods (Elliott, Woodward, 2007).

Descriptive statistics

				Data	
group		Algebra	Geometry	handling	Total
First group:	Mean	72.06	71.84	74.47	218.38
applied	Ν	38	38	38	38
lesson study	Std.	9 400	0.029	12 012	27.044
approach for	Deviation	8.422	9.028	12.015	27.844
2 years	Maximum	88	90	100	278
	Minimum	55	47	40	142
	Skewness	.070	375	166	177
	Kurtosis	781	.775	.974	.605
Second group	Mean	61.92	59.15	60.00	181.07
: applied	Ν	39	39	39	39
lesson study	Std.	0, (00)	12 106	10.010	26.024
approach for	Deviation	9.609	13.106	18.918	36.034
1 year	Maximum	83	87	100	238
	Minimum	45	33	30	112
	Skewness	.309	.101	197	186
	Kurtosis	543	523	615	-1.015
students who	Mean	54.33	48.92	50.75	154.00
studied using	Ν	40	40	40	40
traditional	Std.		10.051	20 155	a - - a a
instructions	Deviation	9.920	12.251	20.177	35.728
	Maximum	82	77	100	258
	Minimum	37	23	30	103
	Skewness	.589	.518	.543	.845
	Kurtosis	.316	128	697	.541

Table (5) shows descriptive statistics, of the average grades in (Algebra, Geometry, Data handling, and the Total). The first group's mean scores are (72.06, 7.84, 74.47and 218.3, respectively), which are greater than the second group's scores with average (61.92, 59.15, 60.00, and 181.07). Both groups' scores are greater than the third group with average grade (54.33, 54.33, 50.75, and 154). Figure 2 and Figure 3 represent results described above graphically.



Figure 2: The mean of grades across groups



Figure 3: the mean of total grades across groups

Analysis of variance (ANOVA)

An ANOVA test was used to analyze mean differences of problem solving mathematical test results between groups. The independent variable was the instructional approach and the dependent variable was students' ability to solve mathematical problems. The null hypothesis in ANOVA is always that there is no difference in means. The alternative hypothesis of the ANOVA test is that at least one grades' average of one group is different from others. Therefore, if we accept the alternative hypothesis, then we need to run the post hoc tests to know which groups are different specifically. Table 6 shows results of the ANOVA tests for each group.

Algebra

Table (6): ANOVA test for Algebra mean scores

		Sum of Squares	df	Mean Square	F	Sig.
Algebra	Between Groups	6153.020	2	3076.510	35.174	.000
	Within Groups 9970	9970.959	114	87.465		
	Total	16123.979	116			

As shown from the above table, there was statistically a significant difference at less than 0.05 level related to students' ability to solve algebra problems for all groups: F(2, 114) = 35.174, p = 0.00, which means that at least one group is different from others, so we ran the Post-hoc test to know which groups are significantly different.
						95% Co	nfidence
			Mean			Inte	rval
			Difference	Std.		Lower	Upper
Dependent Variable		(I-J)	Error	Sig.	Bound	Bound	
Algebra	1	2	10.138^{*}	2.132	.000	5.08	15.20
		3	17.728^*	2.119	.000	12.70	22.76
	2	3	7.590^*	2.105	.001	2.59	12.59

Table (7): Tukey HSD for Algebra scores

A post hoc test (Tukey HSD) was used to identify differences in means, the test showed that the first group's mean score (M = 10.138, SD = 2.132) was significantly different than the second group, and third group (M = 17.728, SD = 2.119). In addition, the mean score for the second group (M = 7.590, SD = 2.105) was significantly different than the third group.



Figure 4: the Mean of Algebra grades across groups

From the above tables and graph, results show that the average of algebra grades for the first group is 72.06, which is significantly greater than the second group and the third group with average of 61.92, and 54.33, respectively. This means the average of the algebra for the first group is significantly greater than the second group by 10 grades and third group by 17 grades, this with confident 95%, as the p-value is less than 0.05. Moreover, the average of algebra grades for the second group is 61.92, which is significantly greater than the third group' average = 54.33. This means that the average of algebra scores of the second group is significantly greater than the third group by eight grades with confident of 95%, as the p-value is less than 0.05.

Geometry

		Sum of Squares	df	Mean Square	F	Sig.
Geometry	Between Groups	10264.971	2	5132.485	38.005	.000
	Within Groups	15395.618	114	135.049		
	Total	25660.589	116			

Table (8): ANOVA test for Geometry mean scores

The above table shows that there was statistically a significant difference at less than 0.05 level related to students' ability to solve Geometry problems for all groups: F(2, 114) = 38.005, p = 0.00, which means that at least one group is different from others, so we ran the Post-hoc test to identify which groups differed significantly.

Tabla	(0)	Tukov	цср	for	Geometry	scores
Table	(9):	тикеу	пэр	101	Geometry	scores

						95% Co	nfidence
			Mean			Inte	rval
			Difference	Std.		Lower	Upper
Dependent	Varia	able	(I-J)	Error	Sig.	Bound	Bound
Geometry	1	2	12.697^{*}	2.649	.000	6.41	18.99
		3	22.925^{*}	2.633	.000	16.67	29.18
	2	3	10.229^{*}	2.615	.000	4.02	16.44

Running a post hoc test (Tukey HSD) to identify exactly where the means differed, results showed that the first group's mean score (M = 12.697, SD = 2.649) was significantly different than the second group, and third group (M = 22.925, SD = 2.633). In addition, the mean score for the second group (M = 10.229, SD = 2.615) was significantly different than the third group.



Figure 5: the Mean of Geometry grades across groups

From the above tables and graph, we can conclude that the average of geometry grades for first group is 71.84, which is significantly greater than the second and third groups with average of 59.15, and 48.92 respectively. This means that the average of the geometry for the first group is significantly greater than the second group by 13 grades and the third group by 23 grades and with confident of 95%, as the p-value is less than 0.05. In addition, the average of geometry grades for the second group is 59.15, which is significantly greater than the third group is average, 48.92. This means that the average of geometry results of the second group is significantly greater than the third group by 10 grades and with confident of 95%, as the p-value is less than 0.05.

Data handling

		Sum of		Mean		
		Squares	df	Square	F	Sig.
Data handling	Between Groups	11106.103	2	5553.052	18.182	.000
	Within Groups	34816.974	114	305.412		
	Total	45923.077	116			

Table (10): ANOVA test for Data handling mean scores

From the above table we conclude that there was statistically a significant difference at p-value (sig) less than 0.05 level related to students' ability to solve Data handling problems for all groups: F(2, 114) = 18.182, p = 0.00, which means that at least one group is different from others, so we will use the results of Post-hoc test to know which groups are significantly different.

						95% Coi	nfidence
			Mean			Inte	rval
			Difference	Std.		Lower	Upper
Dependent Variable		riable	(I-J)	Error	Sig.	Bound	Bound
Data	1	2	14.474*	3.983	.001	5.01	23.93
handling		3	23.724^{*}	3.959	.000	14.32	33.12
	2	3	9.250*	3.933	.043	.09	18.59

Table (11): Tukey HSD for Data handling scores

Using a post hoc test (Tukey HSD) to identify differences in means, the test showed that the first group's mean score (M = 14.474, SD = 3.983) was significantly different than the second group, and third group (M = 23.724, SD = 3.959). In addition, the mean score for the second group (M = 9.250, SD = 3.933) was significantly different than the

third group.



Figure 6: The Mean of Data handling grades across groups

From the above tables and graph, we can conclude that the average of data handling grades for the first group is 74.47, which is significantly greater than the second and third groups' average of 60 and 50.75, respectively. This means that the average of data handling for the first group is significantly greater than the second group by 14.47 grades and the third group by 24 grades and with confident of 95%, as the p-value is less than 0.05. In addition, the average of data handling grades for the second group is 60 which is significantly greater than the third group with average = 50.75. This means that the average of data handling for the second group is significantly greater than the third group by nine grades with confident 95%, as the p-value is less than 0.05.

Total grades

		Sum of		Mean		
		Squares	df	Square	F	Sig.
Total	Between Groups	81242.708	2	40621.354	36.232	.000
	Within Groups	127809.9	114	1121.139		
	Total	209052.6	116			

Table (12): ANOVA test for the total mean scores

From the above table, we can conclude that there was statistically a significant difference at p-value (sig) is less than 0.05 level related to students' problem solving ability for all groups: F(2, 114) = 36.232, p = 0.00, which means that the null hypothesis is rejected and at least one group is different from others. Results of Post-hoc test are presented to identify which groups are significantly different.

						95% Co	nfidence
			Mean			Inte	rval
			Difference	Std.		Lower	Upper
Depe	endent	Variable	(I-J)	Error	Sig.	Bound	Bound
Total	l 1	2	37.309^{*}	7.632	.000	19.18	55.43
		3	64.377^{*}	7.585	.000	46.36	82.39
	2	3	27.068^*	7.535	.001	9.17	44.96

Table (13): Tukey HSD for total scores

A post hoc test (Tukey HSD) was used to identify where the means differed, results showed that the first group's mean score (M = 37.309, SD = 7.632) was significantly different than the second group, and third group (M = 64.377, SD = 7.585). In addition, the mean score for the second group (M = 27.068, SD = 7.535) was significantly different than the third group.



Figure 7: The Mean of total grades across groups

From the above table and graph, we can conclude that the average of total grades for the first group is 218.38, which is significantly greater than the second and the third groups' average of 181.07 and 154, respectively. This means that the average of data handling results for the first group is significantly greater than the second group by 37 grades and the third group by 64 grades with confident 95%, as the p-value is less than 0.05. In addition, the average of total grades for the second group is 181.07, which is significantly greater than the third group's average of 154, which means the average of data handling for the second group is significantly greater than the third group by 27 grades and with confident of 95%, as the p-value is less than 0.05.

Summary of the quantitative results.

There is a significant impact of Lesson Study project as problem based learning approach used as a teaching tool to enhance students' mathematical problem solving ability. Results show that students who learned using lesson study approach for 2 years have greater ability than students who studied using lesson study approach for 1 year. Results also show that both groups have higher ability than students who learned via traditional instructions, with confident 95%.

Does mathematical problem solving ability differ between students who learn via lesson study approach compared to students learning in traditional classes?

Qualitative Results (semi-structured teacher's interview)

For this study, the researcher conducted semi structured interviews in order to further investigate how lesson study approach affects students' mathematical problem solving ability in comparison to conventional teaching methods. Interviews' transcripts analysis yielded seven interesting themes related to students' ability to solve problems. Four of these themes related to the first research question. These themes are; the effect of the instructional approach on students' ability to solve mathematical problems, evaluation of students' problem solving ability, factors that facilitate or hinder students' ability to solve problems, and development of students' ability to solve problems during the last 2 years. Although these are classified as different themes, there is a considerable overlap between them.

Theme 1: The effect of the instructional approach on students' mathematical problem solving ability.

One of the most obvious themes that emerged was the effect of instructional approach on students' problem solving ability. Teachers' responses varied regarding the effect of the instructional approach on students' ability to solve mathematical problems.

Teachers of the first Group

All participants reported that the Lesson Study with (PBL) as teaching method affected positively students' problem solving ability. Teachers' noticed that students' scores mostly increased in quizzes and exams after applying lesson study approach for a good period. Moreover, other improvements were observed including explaining and justifying answers clearly; using appropriate strategies ; using multiple representations; speaking mathematically; making connections between words; using, acquiring and applying accurately symbols and numbers; and having number sense. For examples one of the teachers' said"

"I noticed that this approach had a clear effect on learning mathematics. There was an improvement and development for all students. All students developed and progressed in their ability to solve problems... they used multiple representations, ... they made connections between words, symbols and numbers, provided reasoning and justification for their answers."

Another teacher added: "Lesson study allowed students to construct more conceptual understanding of mathematics. It was noticeable, without any doubt that teaching through problem solving promotes student's mathematical knowledge, their problem solving ability and learn a set of strategies ...their reasoning skills and how to express their thinking".

Moreover, a third teacher added: "Well this approach have students realize the importance of mathematics in their life, and it improved students mathematics learning and their ability to solve problems. For example, students improved in making sense of the problem, planning, speaking mathematically, as well as showing acceptable level of reviewing solutions, utilizing multiple representations, and using diverse methods to solve the same problem".

Teachers of the second Group

Teachers of the second group credited the instructional approach of lesson study they were utilizing as the main contributor to the improvement of their students' ability to solve problems, as it helped them understand mathematical concepts. In addition, teachers stated that students came to be more willing to express their thoughts, justify their answers, and listen to others' thinking. Teachers talked in details about their students' progress in problem solving ability. Examples of their responses are written below.

"I have noticed an increase in problem solving ability over time. I think that in the last 2 years the problem solving ability has been improving all the time. I contribute this to the lesson study and problem-solving environment." T4

"I think the lesson study allows for a deeper conceptual understanding of mathematics and improve problem solving ability of students. With the problembased learning, challenging problems, and the ways that students used to solve problems are very beneficial. It helps to develop students' problem solving ability."

T5

"Lesson study approach as being positively accepted by the students, helps them understand the mathematical concepts and improve problem solving ability. Encouraged students to utilize different problem solving strategies." T6

Teachers of the third group

Teachers of the third group reported that instructions implemented did not have noticeable impact on students' problem solving ability. The interviews with teachers also looked into students' weaknesses and factors could influence their problem solving ability. These included reliance on memory, lacking perseverance, lacking of strategies to apply, difficulty understanding the meaning of the problem.

"I think that the students' ability to solve problems has not developed significantly, many students still suffer in mathematical problems and prefer direct questions in which they can use direct formulae to solve them because they depended on their memory, ... most students lack perseverance and did not spend more than three minutes to solve the problem." T8

"I think that the method of teaching has nothing to do with the students' problem solving ability. Most students lack strategies and find it difficult to understand the problem. I think that the curriculum is the important factor in the development of students' ability to solve problems, as well as the students themselves, and the attention of their parents and follow-up in solving school assignments." T10

Theme 2: Development of students' problem-solving ability during the last 2 years.

Teachers varied in what they thought about students' level of development in problem-solving ability. Teachers of the first and second group noticed students' progress in mathematical problem solving ability, whereas teachers of the third group did not see any changes in student's abilities.

Teachers who used lesson study as instructional approach reported development and gave credit to the structured problem solving method. All teachers in the first group stated that there was improvement in students' problem solving ability over time. All asserted that they noticed improvement on students' problem solving ability compared to students from previous years. Examples of their responses are prescribed below.

"Students' problem-solving ability has improved ...They have clearly improved from the first year .. Well, I think that because of the process this approach have, students realize the importance of mathematics in their lives, and because students learned mathematics through problem-solving ...they think about the mathematical concepts in depth rather than memorizing." T2

"I think my students perform better than before. It seems they get a little bit better and more improved in mathematics each year. ..You know this year is our

fourth year implementing lesson study, I think that the way we focused a lot on mathematical problem-solving in the last few years has helped our students along with the lesson study process." T1

"My students' problem-solving ability had improved.. I noticed a big change in problem-solving ability compared to students from previous years; they do not seem to be as afraid to address problems. ... They want to be called out to express their opinions... They are also speaking more effectively about their strategies. I credit that improvement in students' problem-solving ability to the process of lesson study approach and problem-based learning." T3

Teachers of the second group

Teachers of the second group had similar thoughts about students' progress in solving mathematical problems. However, as for the level of progress, they stressed students' needs for more efforts to achieve more progress.

"Prior to use problem-solving approach, I had done a lot of examples and exercises; it was practically made students memorize. However, with problem-based learning, I am taking a step back and letting students decide their own strategies to solve the problem and their own methods to check the answers.as a result, students showed more confidence selecting problem-solving strategies, and attempting to apply these strategies during the first year."T4

"I noticed improvement in students' ability to solve problems ...They do not give up as quickly as prior students did. Students benefit from PBL by choosing how to solve problems through multiple methods, and by explaining their methods. They perseverance improved ...they tend to keep trying different ways to solve a problem until they find the solution, try new approaches, and are more confident because they can use another way to solve the same problem and check the answer by themselves. I believe they have improved and are becoming better, but there is still need to improve more." T5

"Well, they have a confidence boost as they can find solutions and can solve problems. So, I think that it helped them persevere during problem solving. While prior to this year, students could only persevere for less than three minutes before giving up, becoming frustrated, or raising their hand asking for support and assistance." T6

"At the beginning of that year, students struggled to select strategies, use it to find solutions regarding problem solving. However, during school year we applied lesson study; students started to ask more relevant questions and developed alternative methods to solve the problems."T7

Teachers of the third group

Teachers of the third group did not observe problem-solving ability changes much from previous years. They cited that students' abilities are the same as previous years in solving problems. They have problems with problem comprehension, selecting operations, using strategies and reaching the correct answers. They also noted that their perseverance also did not change much either.

"Students' ability to solve problems has not changed much compare to students in previous years. The only thing that I would ask for would be more time to practice. Sometimes we will go through a lesson and I just want them to practice operations/ skills a little more." T8

"Problem solving can be a hard skill for students and it actually is as we see in classes. They still find difficulty understanding the problem, choosing correct operations and applying suitable strategies. Their perseverance in solving problems did not change much." T9

"I think students' problem solving ability is about the same. We will always have that group of students who strive on trying to solve problems and then another group who find difficulty solving problems. They are good at arithmetic, but they are not able to apply it. I did not notice an improvement over time, ...I think providing real-life problems to classrooms is important."T10

Theme 3: Factors that facilitate or hinder students' ability to solve problems.

When talking about what factors affect problem solving ability, Teachers of groups one and two reported some factors they perceived as having a major role affecting and improving students' problem solving ability. The process of the Lesson Study with PBL is the main factors includes in problem solving, the nature of tasks presented to students, and students' prior knowledge.

"Students seem to enjoy mathematics problems overall... The nature of problems given to students contribute to improving their ability to solve problems. Students' problem solving ability increased when they were asked to solve challenging problems. Problems should be adequate that they are able to solve them and challenging enough to motivate them. .. The nature of the problem that it can be solved in more than one-way and access prior knowledge is very important. Therefore, when a solution strategy fails, they search for another way. In addition, high expectations of students in the classroom, holding students to be responsible, and allowing them to take ownership of their learning which helped with improving problem solving ability."T1

"Students need problem based learning process to develop their ability to solve mathematical problems. Doing, thinking, justifying, sharing, making mistakes, trying again and evaluating answer, help students to learn mathematics and improve their ability to solve problems" T2 "The lesson study and problem-based learning process played an important role in students' problem solve ability development... the level of problems, ways of making sure students have prior knowledge and know how to use it to solve problem also changed. I just think that applying what they know and being able to address the problem in order to make sense of it helps... Also the modeling in mathematics is important. Therefore, a lot of modeling to try to get them to figure out how to work through the problems helps." T6

"The impact of lesson study and process it presents, the type of problems and that it can be solved in many ways ...the level of challenge presented to students; these all were important factors to be considered when looking at students' improved problem solving ability. ... Other factors also considered such as students' understanding of content, previous experience, and discussions, all helped because students get to share their arguments".T5

"The type of problems given contributed to the students' progress of problem solving ability. These problems can reach the solution in more than one way. Even for students who are struggling, these problems enable students to work at their own pace, find a way to get the answer, select appropriate methods and apply knowledge to create their solutions."T3

"Learning through problem solving has made a big difference in students learning than trying to start off the lesson easy with an example. Students are acquiring mathematical knowledge. They are learning when and how to apply it. I think the nature of the task played a role... Good tasks helped students use their prior knowledge and build on it. Various solutions' methods also allowed students to work at their own pace."T7

"I have gradually seen students have a better understanding of mathematics

and improve problem solving ability over the past year. this is contributed directly to the problem based learning process.... allowing students to control their learning, giving them time to solve, and the opportunities to solve using methods they prefer, discussing methods with class, and having those methods appreciated by the teacher in class, helped with learning mathematics and problem solving." T4

Theme 4: Evaluation of students' problem solving ability.

Teachers were asked to rate their students' ability to solve mathematical problems. Teachers responded by evaluating the ability of their students to solve problems as shown in table (14). Teachers who taught students in-group 1 reported that students' level of ability to solve problems ranged between 80% and 85%. While teachers of the second group stated that, their students' level in solving problems was around 70%. Teachers of the third group rated their students' ability to solve problem with less than 50%. Examples of teachers' responses are written below.

"Most students did well when they address mathematical problems,... I'm proud of them, I give them 80%." T2

"Um... I can give those students between 80 to 85%." T3

"They are good, but they still need more time to be excellent. I gave them 70%,.. I think if they continue learning using this approach, they will be better." T4 "Students' ability to solve problems was improved. I can give them around 70%." T7

"I give most students 40 to 45%. They need hard work to rise their ability to solve problems." T9

"There are good students in my class, but I can rate the ability of the majority of students as less than 50%." T10

Teachers	by	Frequency	Percent	Quotation Examples
groups				
teachers of	the	2	80 %	"Most students did well when they
first group				address mathematical problems, I
(3 teachers)		1	85 %	am proud of them. I give them 80%. "
				T 2
				"Um I can give those students
				between 80 to 85%." T3
Teachers of	the			"They are good but they still need
second group		4	70%	more time to be excellent. I gave
(4 teachers)				them 70%, I think if they continue
				learning using the same approach,
				they will be better than now. "T4
				"Students' ability to solve problems
				was improved. I can give them around
				70%." T7
			T	
Teachers of	the	2	Less	and hard work to improve their
third group		5	50%	ability to solve problems" TO
(3 teachers)			30%	"There are good students in my class
				but I can rate the ability of the majority
				of students as less than 50% "T10
				of students as ress than $50/0$. 110

Table (14): Teachers rate of students' problem solving ability

Research question 2: How do students demonstrate their ability to solve problems?

In order to address the second research question, asking how do students demonstrate their problem solving ability, teachers were asked to explain how students demonstrate their problem solving ability. An analysis of the qualitative data yielded to two themes. The fifth and sixth theme is related to the second research question, which aimed to explore how students demonstrate their problem solving ability. The fifth them is: Assessment methods of students' ability to solve problems. While the sixth theme is about how students demonstrate their problem solving ability.

Theme 5: Assessment methods of students' ability to solve problems

Another prominent theme detected throughout data collected focused on how

teachers assess their students' ability to solve problem. Most teachers confirmed that assessing students' ability to solve problems had several methods, including test, observing students while working on solving problems, and presenting and discussing their solutions. As for the evidence teachers found indicating the development of students' ability to solve problems, they were as follows: increase in students' scores in quizzes; using multiple representations; explaining and justifying their answers clearly; modeling the problem; using one or more strategies to solve problems, and finding the correct answer. Examples responses are written below.

"To assess students' problem solving ability improvement, we need to assess them during and after problem solving lessons. However, possible methods to assess problem solving ability require knowing both the procedures and the last answer demonstrated by students, which reflect not only on how they apply understanding conceptions to solve none routine problems, but also their explanation, reasoning and justification, for the solution.... In terms of evidence indicating the ability to solve problems ... increasing scores in the tests; using multiple representations; explaining answers clearly; using appropriate strategies, and finding the correct answer" T1

"To evaluate students' ability to solve problems, Teachers need to focus on asking specific open-ended problems that allow students to present their own ways of solving problems through selecting some strategies and explaining in their own words how these strategies would work. Evidence indicated students' ability to solve problem are: increasing scores in unit tests; using different representations; explaining answers with reasoning; using more than one strategy to solve the same problem." T2

"Students are accountable to justify alternative strategies to solutions, using prior knowledge as a background resource for informing decisions and selecting strategies and provide explanations and reasoning." T3

"Students' ability to solve problems can be assessed by examining the process followed to solve a problem rather than the final answer found only. Students are also required to explain their reasoning when giving the answer." T4

"Students' problem solving ability can be measured by seeing how they apply their learning in different activities, exercises and tests... our focus should be on methods students use to solve problems, rather than the correct answer" T5

Theme 6: How students demonstrated their ability to solve problem

In the first and second group, most students attempted to understand the problems and come up with plans to solve them. Most students persevered to solve a problem before moving to the next one; tried new methods; were comfortable using more than one strategy and most of them were able to present their arguments accurately. However, compiling, applying strategies and evaluating the results were less accurate. Students generally showed acceptable level about explanation, using multiple representations, and attempting various methods to solve the same problem. This was stated clearly in teachers' responses as follows:

"Students provide explanations, and use diagrams to show their solutions to problems. Most of them are able to distinguish between relevant and irrelevant information when solving problems. "

"Students' problem solving ability depend on their ability to use specific strategies, make sense of a problem, identify appropriate methods, sequence solving problem, construct a solution, and examine problem solving result." T1

"Students' demonstrated their ability to solve problems in three ways: identifying and defining problem, finding solution by applying their understanding to a problem, giving reasoning to support this solution, and providing argument related to mathematics concepts clearly."T2

"Understanding the problem and identifying information given and what they need to do; introducing alternative methods to find solution and evaluating results demonstrated students' ability to solve problems." T3

Theme 7: Challenges of applying a structured problem solving approach.

Based on the interviews, teachers highlighted two challenges that hinder applying lesson study with (PBL) approach in their classrooms as shown in table (15). Among these challenges, the time factor, which was the most prominent as all participants emphasized that the application of such an approach needs more time. Another challenge emerged based on teachers' point of view related to the application of lesson study approach is students' prior knowledge. (70%) of the teachers think that students' lack of prior knowledge is the challenge faced when applying structured problem solving in their classrooms, examples of teachers' responses are listed below:

"Students' lack of prior knowledge affects their academic performance. Solving current problems, students need to use and build on their prior learning in order to gain new knowledge." (T7)

"We need to address students' prior learning before beginning new learning using problem-solving approach." (T4).

Challenges	Frequency	Percent	Quotation Examples
Time conquine	10	1000/	Daily school schedule does not
1 me-consuming	10	100%	opportunities for students to grapple with challenging problems.
			The time factor is a challenge 45 minutes is not enough. Students need time to think about the problem and present their argument and then discuss solutions, come to generalizations or conclusions, and

Table (15): List of problem based learning approach Challenges (Qualitative)

Challenges	Frequency	Percent	Quotation Examples
			then another question to make sure students understand. I think PBL requires more time than other approaches.
lack of students' prior knowledge	7	70%	Prior learning experiences do not prepare students well for problem- based learning.
			"I noticed that some students have problems because of the gap in their previous knowledge or lack of certain skills. This hindered them from cooperating and collaborating in the class." "Students' lack of prior knowledge affects their academic performance. Solving current problems, students need to use and build on prior learning in order to gain new
			learning.

Students' work

To answer research questions regarding the difference between groups' problem-solving ability, and how they demonstrate their ability, excerpts from students' responses to problems in the test given were highlighted. For each question in the test, the researcher looked for evidence of students' learning and how do students demonstrate their mathematical problem solving ability in each group.

The following figures are an example of students' work. In the picture below, students were asked to solve the following problem: "*Fatima wants to send letters to 12 of her friends*. *Half of the letters will need one page each, and the other half will need two pages each. How many pages will be needed altogether?*"

Group (1)

In the first group, most students were able to interpret the problem by modeling it appropriately, plan to solve problems, and explain the problem. Furthermore, students were also able to apply more than one appropriate strategy to find the answer, and present problems' solutions accurately. Only four students struggled to use various strategies.



Figure (8): Sample of students' work from group 1

The student was able to interpret the problem using drawing. Other strategies used to solve the problem included counting, and dividing 12 into two groups, one has six and the other has 12 in the last step, the student used addition to find the answer.

Group (2)

In the second group, most students generally demonstrated significant levels of understanding problems, tried different methods to solve problems, and used one or more strategies to solve problems in the test. However, some students' methods were usually right but quite often their answers were incorrect. Only nine students struggled to use various strategies in some problems. Moreover, students began to apply some mathematical explanation as they were solving problems.

Figure (9): Sample of students' work from group 2

The picture above displays the answer of a student in the second group. The student was able to interpret the problem by dividing the 12 pages into two groups of six, then multiply one six by 1 to get 6, and multiply the other six by 2 to get 12. The last step was adding the results of the two multiplication sentences to find the final answer.

Group (3)

Students in the third group generally received lower scores in mathematical problem solving ability tests. While some students write the final answer without any explanation, most Students wrote answers without understanding what problems actually required. They also had difficulties in devising plans to solve problems and select appropriate strategies to find the answer. Students focused on the procedural calculations in solving problems only and did not understand the meaning behind the solution. Furthermore, the analysis of the mathematical problem-solving test identified some common misunderstandings of specific mathematical concepts as well as common methods used among students, such as doing random calculations without clear plan and utilizing incorrect formulae.

$$52$$

تم يد فاطمة إرسال رسائل إلى (1) صديقة من صديقاتها. تحتاج نصف هذه الرسائل إلى صفحة واحد
المك رسالة، ويحتاج النصف الآخر إلى صفحة من كال رسالة. ما العدد الكلي للصفحات المطلوبة
وضح إجابتك.
 $12 + 2 = 14$

Figure (10): Sample of students' work from group3

In the third group, most students did not find the correct answer as they took the numbers and added them altogether. They did not actually analyze what is being asked in the problem and thus, the student could not find the correct answer because she failed to analyze what the problem requires.

CHAPTER FIVE: DISCUSSION AND CONCLUSION

Mathematics assists us to make aware and understand of the world in which we live as we go about our daily lives. This mixed-method study aims to describe the impact of using lesson study approach with structured problem solving on students' mathematical problem solving ability. This chapter includes a discussion of results; interpretation of these results; comparison of the findings with previous literature, conclusions based on results, as well as recommendations for further research to be conducted.

The first question investigated the difference in students' mathematical problem-solving ability between students who apply the lesson study approach and those in conventional classes. The mixed-method, quantitative and qualitative data were collected, analyzed and interpreted to provide the opportunity to examine thoroughly a deeper and comprehensive understanding of the addressed research questions.

Quantitative data were gathered and analyzed from the mathematical problemsolving test. Its scores, the mean of these scores, and the standard deviations for all groups of students were calculated. Data then statistically compared to see if there is a significant difference in the mean scores between the two groups who learned using the lesson study approach and those who studied using conventional methods.

Results show that the mean scores of the total grades for the first group is significantly greater than the second group and the third group. In addition, the mean scores of the total grades for the second group is significantly greater than the third group. These results agree with the findings of Bintoro et al. (2021); Khalid et al. (2020); Purwandi et al. (2020); Gholami et al. (2019); Ningrum et al. (2019); Khotimah and Masduki (2016); and Khalid and Ali (2016) finding who highlighted that the application of lesson study approach with (PBL) increased students' ability in

problem solving significantly.

The analyses also revealed that the mean scores of algebra grades for the first group is significantly greater than the second group and the third group. Moreover, the mean scores of algebra grades for the second group is significantly greater than the third group. This indicate that the Lesson Study approach with (PBL) positively affects students' ability to solve problems in algebra strand. This study supports conclusions found in the study written by Loc et al. (2020) which have reported that the lesson study approach has led to a positive change in the ability of students to solve problems in straight-line equations, a part of algebra. The current study also supports conclusions found in the study written by Khotimah and Masduki's (2016) which have concluded that Lesson Study approach improved students' problem solving ability in differential equations learning that is part of algebra as well.

Furthermore, the mean scores of geometry and data-handling grades for the first group is significantly greater than the second group and the third group. While, the mean scores of geometry and data handling grades for the second group is significantly greater than the third group. This indicate that the Lesson Study approach with (PBL) positively affects students' ability to solve problems in all strand and enhance their learning and achievement. This result was consistent with prior researches written by Salim and Samaan (2011) which have reported that the lesson study approach contributed significantly to enhance students' learning and their achievement.

In addition to data obtained from the quantitative analysis, the qualitative analysis for the interviews with teachers revealed that lesson study has positive impact on students' problem-solving ability. Teachers who taught the first and the second groups confirmed that there was improvement in their students' ability to solve mathematical problems, while the third group teachers responded that there was no

significant improvement in students' level or their ability since the beginning of the year. Moreover, teachers asserted that lesson study with (PBL) is more positive than conventional methods as they found it effective in developing problem-solving ability, learning mathematics as well as improving perseverance and as the reasoning skill. Ranjan and Gunendra (2013) stated that with the development of students' problem solving ability; students can enhance their cognitive abilities, carry out procedures, and obtain a deeper grasp of concepts. These results are consistent with prior researches written by Fauziah et al. (2019) which have reported that lesson study with structured problem solving could enhance students understanding of mathematical lessons, and Kusumawati's (2020) which have asserted that lesson study approach has developed students' ability in mathematical reasoning.

Teachers who participated in this study attributed this development to the process of lesson study with (PBL) includes in problem solving, the nature of tasks presented to students, and students' prior knowledge. Evidence from the literature indicated that mathematical tasks presented as problems requiring higher order thinking help students demonstrate curiosity, creativity, create connections with previous knowledge, and collaborate with each other. It also enhances students' motivation in solving mathematical problems and their reasoning skills, which allows for multiple entry points and diverse solution techniques (Boaler, 2016; Principles to Action, 2014; Zambo, 1996, p. 82). That aligned with the Literature, which highlighted that students' previous understanding influences their perseverance and motivation with challenging tasks as they transfer through mathematical learning (e.g., Boaler, 2016; Principles to Action, 2014).

To answer the second research question investigating how students demonstrate their ability to solve mathematical problems, students' mathematical problem solving

test results were analyzed. For each question in the test, the researcher looked for evidence of students' learning and how do students demonstrate their problem solving ability in each group.

Referring to the scoring rubric used to correct students' responses to items of the mathematical problem solving ability test for the first and the second groups, students' work generally demonstrated significant level of understanding problems, planning to solve them, using one or more strategies to solve the same problem, attempting different strategies, explaining and justifying their answers clearly, and using different representatives. However, some students' methods were used the correct procedures but the final answers were not accurate. This result is consistent with the study written by Kilpatrick et al. (2001) which asserted that students skilled in mathematics demonstrate problem-solving behaviors, such as carefully reading and understanding problems, developing models, and making guesses about solutions and their strategies.

On the other hand, students in the third group generally received lower scores in mathematical problem solving ability tests. Referring to the scoring rubric used to correct students' responses to items of the mathematical problem solving ability test, most Students in third group wrote answers without understanding what problems actually required. Students' responses to items of the test also demonstrated difficulties in devising plans to solve problems and select appropriate strategies to find the answer. Students' solutions demonstrated that they focused on the procedural calculations in solving problems only and they were not able to understand the meaning behind the solution. As per data collected from teachers' interviews, teachers attributed student's weaknesses to specific factors that affected their problem solving ability such as students' working memory, their lack of perseverance, lack of strategies to implement, difficulty to understand the problem, and poor skills in reasoning and number sense.

As the data acquired from the mathematical problem-solving test showed significant differences between the groups, and described how students' demonstrated their problem solving ability. Data gathered from the interviews described significant development and progress in student mathematical learning, their problem solving ability, and engagement in mathematics lessons with increasing confidence and perseverance. Group 1 and group 2 teachers explained how students demonstrated their ability to solve problems as follow: Students were able to determine the unknown factor in each problem prior to attempting to find the solution. In addition, most students preserved to solve a problem before moving to the next, tried various strategies, were comfortable using more than one strategy, and most of them were able to present problems' solutions accurately. Moreover, teachers noticed that students began to apply some mathematical reasoning as they were solving problems. Teachers also highlighted during the interviews evidence they observed indicating the development of students' ability to solve problems, such as the increase in students' scores in guizzes; students using multiple representations; explaining and justifying their answers clearly; modeling the problem, and finding the correct answer.

Data collected using the students' work and teachers' interviews confirmed that students learned using the lesson study approach were able to understand the problems, create strategies, and use multiple representations. Studies indicate that students who are able to think, learn by themselves, evaluate their outcomes and present them appropriately are more likely to develop their problem-solving ability (Polya, 1973; Sajadi et al., 2013; Saragih & Habeahan, 2014; Batubara et al., 2017). Therefore, and since problem solving ability does not developed naturally (Tan, 2004), it is evident that using the lesson study approach helped in promoting students problem-solving ability in this study. In addition, teachers' interviews asserted that the lesson study approach with (PBL) has positively affected students' problem solving ability and been successful in improving students in making sense, reasoning, and generating multiple strategies. These results are consistent with prior studies, reported that implementing Lesson Study approach lead to improve students' ability to solve problems (Loc et al.,2020), reasoning (Kusumawati, 2020), and generatemultiple strategies (Hmelo-Silver, 2004; Savery, 2015; Yeo & Tan, 2014).

Qualitative data confirms quantitative data findings in terms of the positive impact of Lesson Study approach with (PBL). Results showed that the first group's scores and abilities are better than the second group, and both of them are better than the third group. The quantitative ranking is measured based on the mean scores; while the qualitative ranking is measured based on the number of teachers' responses. Qualitative data also highlighted the time as the most prominent factor as the challenge that hinder applying lesson study with structured problem solving approach in classrooms. All participants emphasized that the application of such an approach needs more time for students to adopt this approach and to improve their ability to solve problems. This clearly explains the differences between group 1 who experienced problem solving for 2 years with better performance compared with group 2 who experienced problem solving for 1 year. The fact that enhancing students' problemsolving ability is usually a time - consuming process that needs consistently integrating problem solving in every day lessons was highlighted in Lester and Cais' (2016) study. Another challenge emerged from the teachers' point of view was students' prior knowledge. In addition, Researches indicate that it takes several months for students to adapt to a new educational style (e.g., Boaler, 2002), which explains the progress of the students in the first group over the students in the second group. Thus, it is suggested that continuing to apply Lesson Study approach over time for students will allow them to fully adjust to this approach, and will lead to greater progress in terms of students' ability to solve problem.

Conclusion

Based on the study results, both the quantitative and qualitative data revealed benefits of applying Lesson Study approach with (PBL) at two primary schools in Qatar. The Analyses of Variance (ANOVA) test revealed that there were a statistically significant differences between the three groups detected on problem-solving ability. The ANOVA test also showed that the number of years where students engaged in lesson study approach affects their ability to solve problems. This study revealed that students who studied using lesson study approach for 2 years have greater ability than students who studied using lesson study approach for 1 year, and both groups have higher ability than students who studied using traditional instructions. These results show that there is a positive statistically significant impact of Lesson Study approach with (PBL) used as a teaching tool on students' mathematical problem solving ability. In addition, data also assured that the implementation of lesson study approach with (PBL) is perceived to positively affect student's problem solving ability as teachers foster and develop students' skills, including improving perseverance, influence students' thinking, reasoning, and using multiple representations.

To conclude, it is evident that the lesson study approach with (PBL) is an effective method to be applied at the primary stage to improve students' ability to solve mathematical problems despite the challenges of implementation stated by the, teachers such as timing and consuming, and students' prior knowledge.

Based on these results, the researcher recommends implementing Lesson Study approach with (PBL) in primary school mathematics classrooms. In addition, applying Lesson Study Approach with (PBL) for a long time is recommended where the results

of the study showed that time plays a significant positive role in enhancing students' problem solving ability.

Limitations of the study

There are two major limitations in this study that could be addressed in future research. Due to the narrow implementation of the lesson study in the schools in Qatar, sampling strategy and size was limited to the cohort of the schools in which Lesson Study project was applied. Therefore, the results in this study may not be subject to generalization to all schools. It is possible that expanding the size of the participants will lead to different results.

Further limitation was related to the gender of the participants. The study focused on exploring the impact of the lesson approach with PBL on female students' mathematical problem solving ability. Qatar adopted a segregation system in its school in compliance with cultural perspectives and Islamic rules; male and female are completely separated in schools. As the researcher is a female, approaching male schools is not feasible. Including both male and female in the research may open opportunities to compare the results based on gender.

Considerations for Future Research

Research considerations may include an experimental study concentrating on the impact of implementing lesson study on student's problem solving ability. In addition, it is recommended to include classroom observations as a data resource to develop better understanding of what typically occurs during mathematics instruction. Moreover, this study did not consider the gender as a factor. Further research may lead to better understanding potential differences in terms of gender.

REFERENCES

- Akinoglu, O., & Tandogan, O. R. (2007). The effects of problem-based active learning in science education on students' academic achievement, attitude and concept learning. *Eurasia Journal of Mathematics, Science & Technology Education*, 3, 71-81.
- Almaleki, A. M. (2010). Effectiveness of the Proposed Training Programs on Skills of Mathematics Teachers, Some Active Learning and its Impact on the Achievements and Attitudes of their Students Towards Mathematics. PhD. Umm Al-Qura University.
- Anfara, V. A., Jr., Brown, K. M., & Mangione, T. L. (2002). Qualitative analysis on stage: Making the research process more public. *Educational Researcher*, 31(7), 28–38. (May & Luth, 2013).
- Aufdenspring, D. (2003). Project- and problem-based learning. In S. O'Hara, National educational technology standards for students curriculum series: Social studies units for grades 9-12 (pp. 15-20). Eugene, OR: ISTE.
- Ball, D. L., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J., Schmid, W., & Schaar, R. (2005, October). Reaching for common ground in K-12 mathematics education. *Notices of the American Mathematical Society*, 52(9), 1055-1058. Retrieved from
- Bandura, A. (2005). The evolution of Social Cognitive Theory. In K. G. Smith & M. A.
 Hill (Eds.), Great minds in management (pp. 9–35).
 https://doi.org/10.5465/amr.2007.23467624
- Barak, M. (2013). Impacts of learning inventive problem-solving principles: Students' transition from systematic searching to heuristic problem solving. *Instructional Science*, 41, 657-679. doi:10.1007/s11251-012-9250-5.
- Barell, J. (2007). *Problem-Based Learning: An Inquiry Approach*, SAGE Publications. https://books.google.com.qa/books?id=Oy0mD7AcznMC
- Barrows, H. S. (1986). A taxonomy of problem-based learning methods. *Medical Education*, 20(6), 481-486. https://doi-org.ezp.scranton.edu/10.1111/j.1365-2923.1986.tb01386.x

- Barrows, H. S. (1996). Problem-based learning in medicine and beyond: A brief overview.
 New Directions for Teaching and Learning, 1996(68), 3-12.
 doi:10.1002/tl.37219966804
- Batubara, N. F., Mukhtar, S. E., & Syahputra, E. (2017). Analysis Of Student Mathematical Problem Solving Ability At Budi Satrya Of Junior High School. *International Journal of Advance Research and Innovative Ideas in Education* (*IJARIIE*), 3(2), ISSN (O) -2395-4396.
- Benatar, J., McKibbin, P., & Stewart, R. (2012). Improving the informed consent process—A booklet on participants' rights in medical research. *New Zealand Medical Journal*, 125(1362), 36-46. https://doi.org/10.1371/journal.pone.0047023
- Bintoro, H. S., Zaenuri, & Wardono, W. (2021). Application of information technology and communication-based lesson study on mathematics problem-solving ability. *Journal of Physics Conference Series*, 1918(4):042105. DOI: 10.1088/1742-6596/1918/4/042105.
- Bishara, S. (2016). Self-regulated math instructions for pupils with learning disabilities.CogentEducation,3(1),doi:http://dx.doi.org.caldwell.idm.oclc.org/10.1080/2331186X.2016.1262306
- Bloomberg, L. D., & Volpe, M. (2015). Completing your qualitative dissertation: A road map from beginning to end (2nd ed.). Thousand Oaks, CA: Sage Publications.
- Boaler, J. (2002). Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student learning. Studies in mathematical thinking and learning. New York, NY: Erlbaum.
- Boaler, J. (2015). What's math got to do with it? How teachers and parents can transform math learning and inspire success. New York: Penguin.
- Boaler, J. (2016). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. San Francisco, VA: Jossey-Bass.
- Boaler, J., & Staples, M. (2008). Creating mathematical futures through an equitable

teaching approach: The case of Railside School. *Teachers College Record*, 110(3), 608–645.

- Bradshaw, Z., & Hazell, A. (2017), "Developing problem-solving skills in mathematics: a lesson study", *International Journal for Lesson and Learning Studies*, 6(1), pp. 32-44. https://doi.org/10.1108/IJLLS-09-2016-0032
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (1999). *How people learn: Brain, mind, experience, and school. National Academy Press.*
- Bransford, J.D. & Stein, B.S. (1993). *The IDEAL problem solver* (2nd ed.), New York: Freeman.
- Bruner, J. S. (1960). *The Process of Education*. Cambridge, MA: Harvard University Press.
- Bruner, J. S. (1995). Meaning and self in cultural perspective. In D. Barkhurst & C. Sypnowich (Eds.), *The social self* (pp. 18-29). London: Sage Publications.
- Bruning, R. H., Schraw, G. J., & Norby, M. M. (2011). Cognitive Psychology and Instruction (5th ed.). Boston, MA: Pearson.
- Cai, J. (2003). What research tells us about teaching mathematics through problem solving. In F. K. Lester & R. I. Charles (Eds.), *Research and issues in teaching mathematics through problem solving* (pp. 241–254). Reston, VA: National Council of Teachers of Mathematics.
- Cai, J., & Lester, F. (2010). Why is teaching with problem solving important to student learning? [Research brief]. Retrieved from National Council of Teachers of Mathematics website: https://www.nctm.org/uploadedFiles/Research_and_Advocacy/research_brief_and_cli ps/Research_brief_14_-_Problem_Solving.pdf
- Cajkler, W. & Wood, P. (2013). The feasibility and effectiveness of using 'lesson study' to investigate classroom pedagogy in initial teacher education: student-teacher perspectives, Conference Paper, Association for Teacher Education in Europe(ATEE) August 2013, Ostfeld University College, Halden, Norway.

- Calhoun, G. T. (2018). Lesson study: The influence of job-embedded professional development and teacher collaboration for student literacy achievement (Order No. 10690251). Available from ProQuest Dissertations & Theses Global. (1994459266). Retrieved from http://o-search.proquest.com.mylibrary.qu.edu.qa/docview/1994459266?accountid=13370
- Căprioară, D. (2015). Problem Solving Purpose And Means Of Learning Mathematics In School. *Social And Behavioral Sciences*, Amsterdam, v. 191, p. 1859 – 1864.
- Carey, D. A. (1991). Number sentences: linking addition and subtraction word problems and symbols. *Journal for Research in Mathematics Education*. 22(4), 266-280.
- Charles, R.I. & Lester, F. K. (1984). An evaluation of a process-oriented mathematical problem-solving instructional program in grades five and seven. *Journal for Research in Mathematics Education*, *15*(1), 15-34.
- Cobb, P., & Bauersfeld, H. (1995). *The Emergence of Mathematical Meaning*. Hillsdale, NJ: Erlbaum.
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge. https://doi.org/10.4324/9781315456539
- Conference Board of the Mathematical Sciences (2012). *The Mathematical Education of Teachers II*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.
- Conley, D.T., & Darling-Hammond, L. (2013). *Creating systems of assessment for deeper learning*. Stanford, CA: Stanford Center for Opportunity Policy in Education.
- Council of Chief State School Officers (CCSSO). (2010). *Common core state standards initiative*. Washington, DC: Author. Retrieved from http://www.corestandards.org/
- Cresswell J. (2011). Plano Clark VL. *Designing and conducting mixed method research*. 2nd Sage; Thousand Oaks, CA: Sages.
- Creswell, J. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (2nd ed.). Thousand Oaks, CA: Sage
- Creswell, J. (2009). *Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: Sage.
- Creswell, J. W. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th Ed.). Los Angeles: Sage Publications, Inc.
- Creswell, J.W. (2015), A Concise Introduction to Mixed Methods Research, SAGE, Los Angeles.
- D'Ambrosio, B. (2003). Teaching mathematics through problem solving: A historical perspective. In F. K. J. Lester & R. I. Charles (Eds.), *Teaching mathematics through problem solving: Prekindergarten - Grade 6* (pp. 37–50). Reston, VA: The National Council of Teachers of Mathematics.
- Dağyar, M., & Demirel, M. (2015). Effects of problem-based learning on academic achievement: A meta-analysis study. *Eğitim ve Bilim*, 40(181),139-174. doi: 10.15390/EB.2015.4429
- Danisman, S., & Erginer, E. (2017). The predictive power of fifth graders' learning styles on their mathematical reasoning and spatial ability. *Cogent Education*, 4(1). DOI: 10.1080/2331186X.2016.1266830.
- Delisle, R. (1997). *How to use problem-based learning in the classroom*. Alexandria, VA: ASCD.
- Demirören, M., Turan, S., & Öztuna, D. (2016). Medical students' self-efficacy in problem-based learning and its relationship with self-regulated learning. *Medical Education Online*, 211-219. doi: 10.3402/meo.v21.30049
- Dewey, J. (1916). *Democracy and education: An introduction to the philosophy of education*. New York: MacMillan.
- Dewey, J. (1938). Experience and education. New York, NY: Free Press.
- Dudley, P., Xu, H., Vermunt, J.D. & Lang, J. (2019), "Empirical evidence of the impact of lesson study on students' achievement, teachers' professional learning and on institutional and system evolution", *European Journal of Education*, 54, pp. 202-217. https://doi.org/10.1111/ejed.12337

- Duren, P. E., & Cherrington, A. (1992). The Effects of Cooperative Group Work Versus Independent Practice on the Learning of Some Problem-Solving Strategies. *School Science and Mathematics*, 92(2), 80-83. Doi: 10.1111/j.1949-8594.1992.tb12146.x
- Eggen, M.H., Stuge, B., Mowinckel, P., Jensen, K.S., & Hagen, K.B. (2012) Can supervised group exercises including ergonomic advice reduce the prevalence and severity of low back pain and pelvic girdle pain in pregnancy? A randomized controlled trial. *Physical Therapy*, 92(6), 781-790. doi:10.2522/ptj.20110119
- Elliott, A. C., & Woodward, W. A. (2007). *Statistical analysis quick reference guidebook with SPSS examples* (1st ed.). London: Sage Publications.
- English, L. & Halford, G. (1995). *Mathematics education: Models and processes*.Mahwah, NJ: Lawrence Erlbaum.
- English, L. D., & Gainsburg, J. (2016). Problem solving in a 21st-century mathematics curriculum. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (3rd ed.). New York, NY: Routledge.
- Fauziah, E. W., Hobri, H., Yuliati, N., & Indrawanti, D. (2019). Student's Creative Thinking Skills in Mathematical Problem Posing Based on Lesson Study for Learning Community. *Earth and Environmental Science*, 243(1), 012142. DOI: 10.1088/1755-1315/243/1/012142
- Fennell, F. (2008). "What algebra? When?" NCTM News Bulletin, 44(6). Reston, VA: National Council of Teachers of Mathematics.
- Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning.* New York, NY: Routledge.
- Flores, M., Veiga Simão, M., Barros, A., & Pereira, D. (2015). Perceptions of Effectiveness, Fairness and Feedback of Assessment Methods: A Study in Higher Education. *Studies in Higher Education*, 40 (9), 1523-1534.
- Foegen, A., & Deno, S.L. (2001). Identifying growth indicators for low-achieving students in middle school mathematics. *Journal of Special Education*, 35, 4-16.
- Fujii, T. (2016). Designing and adapting tasks in lesson planning: A critical process of

lesson study. ZDM Mathematics Education. https://doi.org/10.1007/s11858-016-0770-3

- Fujii, T. (2018). Lesson study and teaching mathematics through problem solving: The two wheels of a cart. In M. Quaresma, C. Winsløw, S. Clivaz, J. P. da Ponte, A. Ní Shúilleabháin & A. Takahashi (Eds.), *Mathematics lesson study around the world: Theoretical and methodological issues* (pp. 1–21). New York, NY: Springer.
- Fujii, T., Kumagai, K., Shimizu, Y. & Sugiyama, Y. (1998). A cross-cultural study of classroom practices based on a common topic. *Tsukuba Journal of Educational Study in Mathematics*, 17, 185-194.
- Gagné, R. M. (1984). Learning outcomes and their effects: Useful categories of human performance. American Psychologist, 39(4), 377–385. https://doi.org/10.1037/0003-066X.39.4.377
- General Secretariat for Development Planning, Qatar National Vision 2030: Advancing Sustainable Development. Qatar's Second Human Development Report (2009), p. iii; General Secretariat for Development Planning, Qatar National Vision 2030 (2008)
- Germain-McCarthy, Y. (2001). *Bringing the NCTM standards to life*. Poughkeepsie, NY: Eye on Education.
- Gholami, H., Yunus, A.S.M., Ayub, A.F.M., & Kamarudin, N. (2020). Impact of lesson study on motivation and achievement in mathematics of Malaysian foundation programme students. *Journal of Mathematics Education*, 5(1), 39-53. http://doi.org/10.31327/jme.v5i1.1179
- Greiff, S., Holt, D.V. & Funke, J. (2013), Perspectives on problem solving in educational assessment: Analytical, interactive, and collaborative problem solving, *Journal of Problem Solving*, Vol. 5/2, pp. 71-91, http://dx.doi.org/10.7771/1932-6246.1153
- Halmos, P.R. (1980). The heart of mathematics. The American Mathematical Monthly, 87(7), 519-524.
- Han, S., Capraro, R., & Capraro, M. M. (2015). How science, technology, engineering, and mathematics (STEM) project-based learning (PBL) affects high, middle, and low

achievers differently: The impact of student factors on achievement. *International Journal of Science and Mathematics Education*, *13*(5), 1089-1113. https://doi.org/10.1007/s10763-014-9526-0

- Hatch, J. (2002). *Doing qualitative research in education settings. Albany:* State University of New York Press.
- Heleni, S., & Zulkarnain. (2018). The Influence of Mathematical Thinking Ability with Modified MOORE Method on Learning Outcomes of Basic Mathematic II Chemical Education Students. *Journal of Educational Sciences*, 2(2), 33-41.
- Hiebert, J., & Stigler, J. W. (2000). A proposal for improving classroom teaching: Lessons from the TIMSS video study. *The Elementary School Journal*, 10(1) 3-20.
- Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Human,P., & Olivier, A. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.
- Hiebert, J., Stigler, J. W., & Manaster, A. B. (1999). Mathematical features of lessons in the TIMSS Video Study. Zentralblatt für Didaktik der Mathematik (International Reviews on Mathematical Education), 31(6), 196-201.
- Hino, K. (2007). Toward the problem-center classroom: Trends in mathematical problem solving in Japan. ZDM Mathematics Education, 39(5-6), 503-514. http://doi.org/10.1007/S11858-007-0052-1
- Hmelo-Silver, C.E. (2004). Problem-based learning: What and how do students learn?EducationalPsychologyReview,16(3),235-266.http://dx.doi.org/10.1023/B:EDPR.0000034022.16470.f3
- Hobri, H., Romlah, S., Prihandoko, A. C., Safitri, J., & Nazareth, E. (2018). Students' metacognitive ability in mathematical problem-solving learning based on lesson study for learning community (LSLC). *Journal of Physics: Conference Series*, 1088(1), 12064. IOP Publishing.
- http://eprints.ums.ac.id/32907/9/NASKAH%20PUBLIKASI.pdf

http://smu.edu/education/teachereducation/graduate/Milgram_rdg_comm-schmid.pdf

- Huba, M. E., & Freed, J. E. (2000). Learner-Centered Assessment on College Campuses: Shifting the Focus from Teaching to Learning. Allyn and Bacon. http://assessment.uconn.edu/what/index.html
- Hughes, G., Brendefur, J., & Carney, M. (2015). Reshaping teachers' mathematical perceptions: Analysis of a professional development task. *Mathematics Teacher Educator*, 3(2), 116-129. Retrieved from https://www.researchgate.net/profile/Jonathan_Brendefur/publication/274074433_Res haping_Teachers'_Mathematical_Perceptions_Analysis_of_a_Professional_Developm ent_Task/links/551491200cf283ee08378e43.pdf
- Hung, W., Jonassen, D. H., & Liu, R. (2008). Problem-based learning. In M. Spector, D. Merrill, J. van Merrienboer, & M. Driscoll (Eds.), *Handbook of research on educational communications and technology* (3rd ed., pp. 485–506). New York, NY, USA: Erlbaum.
- Inoue, Y. (2010). Reflections: Two years after implementing a blended educational research course. In Inoue, Y. (Ed.), Cases on online and blended learning technologies in Higher Education: Concepts and Practices, pp. 145-165.
- Isoda, M. (2010). Lesson study: Problem solving approaches in mathematics education as a Japanese experience. *Procedia-Social and Behavioral Sciences*, 8, 17-27.
- Jo, S., & Ku, J.-O. (2011). Problem based learning using real-time data in science education for the gifted. *Gifted Education International*, 27(3), 263-273. Retrieved from

http://journals.sagepub.com.ezp.scranton.edu/doi/pdf/10.1177/026142941102700304

- Jonassen, D. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development*, 48, 63–85
- Jonassen, D. H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review*, 15 (3), 267–296.
- Jonassen, D. H., & Hung, W. (2008). All Problems Are Not Equal: Implications for Problem-Based Learning. *Interdisciplinary Journal of Problem-Based Learning*, 2, 6-28. https://doi.org/10.7771/1541-5015.1080

- Kantowski, M. G. (1980). Some Thoughts on Teaching for Problem Solving. In S. Krulik & R. E. Reys (Eds.), *Problem Solving in School Mathematics*. NCTM Yearbook 1980. (pp. 195–203). Reston (VA): Council.
- Kay Stein, M., Boaler, J., & Silver, E. A. (2003). Teaching mathematics through problem solving: Research perspectives. In *Teaching mathematics through problem solving: Grades 6-12.* Reston, VA: National Council of Teachers of Mathematics.
- Kennedy, N. S. (2012). Lipman, Dewey, and philosophical inquiry in the mathematics classroom. *Education and Culture*, 28(2), 81-94. doi:10.1353/eac.2012.0012
- Khalid, M., & Ali, D. P. (2016). Inculcating Tsunami Awareness in a Mathematics Lesson: Improving Students' Collabirative Problem Solving via lesson Study. *Southeast Asian Mathematics Education Journal*. 6(1), 19-31.
- Khalid, M., Saad, S., Abdul Hamid, S., Abdullah, M. Ibrahim H. & Shahrill, M. (2020). Enhancing creativity and problem solving skills through creative problem solving in teaching mathematics. *Creativity Studies* 13(2):270-291
- Khotimah, R. P., & Masduki, M. (2016) Improving teaching quality and problem solving ability through contextual teaching and learning in differential equations: A lesson study approach. *Journal Research and Advances in Mathematics Education*, 1(1), 1–13.
- Kilpatrick, J. (2016). Reformulating: approaching mathematical problem solving as inquiry. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), *posing and solving mathematical problems: Advances and new perspectives* (pp. 69-81). New York, NY: Springer.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). Adding it up: Helping children learn mathematics. Washington, DC: National Academy Press.
- Kloosterman, P. (1992). Non-routine word problems: One part of a problem-solving program in the elementary school. *School Science and Mathematics*, *92*(1), 31-37.
- Kroll, D. L., Masingila, J. O., & Mau, S. T. (1992). Cooperative problem solving: What about grading? *Arithmetic Teacher*. 39(6), 17-23.

- Kusumawati, R. (2020). The application of Lesson Study for LearningCommunity (LSLC)- based collaborative learning integrated Realistic Mathematics Education (RME) to improve the students' mathematical reasoning ability class IX D of MTSN 5 Jember on quadratic equation material. *Journal of Physics Conference Series*, 1563:012060. DOI: 10.1088/1742-6596/1563/1/012060
- Kuzle, A. (2013). Patterns of Metacognitive Behavior During Mathematics Problem-Solving in a Dynamic Geometry Environment. *International Electronic Journal of Mathematics Education – IΣJMΣ*, 8(1), 20-40.
- Kyllonen, P. C. (2012). Measurement of 21st Century Skills within the Common Core State Standards. In Invitational Research Symposium on Technology Enhanced Assessments (pp. 7-8).
- Lambdin, D. V. (2003). Benefits of teaching through problem solving. In F. K. Lester (Ed.), *Teaching mathematics through problem solving: Prekindergarten Grade 6*.Reston, VA: National Council of Teachers of Mathematics.
- Lambros, A. (2004). *Problem-based learning in middle and high school classrooms: A teacher's guide to implementation.* Corwin Press, Inc. California, USA.
- LaMorte, W. W. (2016). *The social cognitive theory*. Retrieved from http://sphweb.bumc.bu.edu/otlt/MPH-Modules/SB/BehavioralChangeTheories/BehavioralChangeTheories5.html
- Larmer, J., Mergendoller, J., & Boss, S. (2015). Setting the standard for project based learning. Alexandria, VA: ASCD
- Larrier, Y. I., Hall, K., Linton, J. M., Bakerson, M., Larrier, I. M., & Shirley, T. S. (2016). Problem based learning: A viable school counseling intervention to promote student engagement with at-risk high school students. *National Teacher Education Journal*, 9(2), 11-20.
- Leong, Y. H., Yap, S.F. Quek, K.S., Tay, E. G. Tong, C.L., Ong, Y.T., & ... Noorhazman, N. M. (2013). Encouraging problem-solving disposition in a Singapore classroom. *International Journal of Mathematical Education in Science & Technology*, 44(8), 1257-1267. Doi:10.1080/0020739X.2013.790505.

- Lesh, R., & Zawojewski, J. (2007). Problem Solving and Modeling. In F. K. Lester (Ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp. 763-799). Charlotte: National Council of Teachers of Mathematics and Information Age Publishing.
- Lester, F. K. (1994). Musings about mathematical problem-solving research: The first 25 years in JRME. *Journal for Research in Mathematics Education*, 25, 660-675.
- Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, *10*, 245-278. Retrieved from https://scholarworks.umt.edu/tme
- Lester, F. K., & Cai, J. (2016). Can mathematical problem solving be taught? Preliminary answers from 30 years of research. In P. Felmer, E Pehkonen, & J. Kilpatrick (Eds.), *Posing and solving mathematical problems* (pp. 117–135).
- Leung, S. (2013). Teachers Implementing Mathematical Problem Posing in Classroom: Challenges and Strategies. *Educational Studies in Mathematics* 83(1). DOI: 10.1007/s10649-012-9436-4.
- Lewis, C. C., Perry, R. R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education*, 12(4), 285-304.
- Liljedahl, P. (2008). *The AHA! experience: Mathematical contexts, pedagogical implications*. Saarbrücken, Germany: VDM Verlag.
- Liu, M., Wivagg, J., Geurtz, R., Lee, S., & Chang, H. M. (2012). Examining how middle school science teachers implement a multimedia-enriched problem-based learning environment. *Interdisciplinary Journal of Problem-Based Learning*, 6(2), 46-84.
- Loc, N. P., Uyen, B. P., Tong, D. H., & Ngoi, H. T. (2020), A Teaching Process of Fostering Students' Problem-solving Skills: A Case Study of Teaching the Equation of a Line. Universal Journal of Educational Research, 8(5), 1741-1751.
- Lott, J. W. (2003). Grounding mathematics in quantitative literacy. In B. L. Madison & L. A. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges*

(pp. 175-177). Princeton, NJ: The National Council on Education and the Disciplines.

- Lujan, A. M. A. (2020). Working together: A qualitative case study on the effects of implementing cooperative learning strategies in a 30 RW 1S34 RfeSDcfkexd 09rT3rd1RW1S34RfeSDcfkexd09rT3 grade math class on guam (Order No. 27739759). Available from ProQuest Dissertations & Theses Global. (2359472241). Retrieved from http://o-search.proquest.com.mylibrary.qu.edu.qa/docview/2359472241?accountid=13370
- Lundbäck, B. and Egerhag, H. (2020), "Lesson Study as a bridge between two learning contexts", *International Journal for Lesson and Learning Studies*, *9*(3), pp. 289-299. https://doi.org/10.1108/IJLLS-02-2020-0006
- Maher, C. A. & Martino, A. M. (1992). Teachers building on students' thinking. Arithmetic Teacher, 39(7), 32-37.
- Mason J., Burton L., & Stacey K., (1982): *Thinking Mathematically*, Addison Wesley, London.
- Mayer, R. (1992). *Thinking, problem solving, and cognition* (2nd ed.). New York: Freeman.
- Mayer, R., & Wittrock, M. (2006). Problem solving. In P. Alexander & P. Winne (Eds.), *Handbook of educational psychology* (pp. 287-303). Mahwah, NJ: Erlbaum.
- McLeod, D. B. (1985). Affective issues in research on teaching mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 267–279). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Merriam, S., & Tisdell, E. (2015). *Qualitative research: A Guide to Design and Implementation* (4th ed.). San Francisco, CA: Jossey-Bass.
- Miller, J. (2015). Predictors of Student Persistence in the STEM Pipeline: Activities Outside the Classroom, Parent Aspirations, and Student Self-beliefs Using NELS:88 Data (doctoral dissertation). Notre Dame of Maryland University.
- Mills, G. E. (2003). Action research: A guide for the teacher researcher, Ohio: Pearson

Education

- Miyakawa, T., & Winslow, C. (2009). Didactical designs for students' proportional reasoning: An open approach lesson and a fundamental situation. *Educational Studies in Mathematics*, 72, 199–218. doi: 10.1007/s10649-009-9188-y.
- Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). TIMSS 2011 international results in Mathematics. Boston, MA: TIMSS & PIRLS International Study Center.
- Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D. L., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: https://timssandpirls.bc.edu/timss2019/international-results/
- Mustafa, N., Ismail, Z., Tasir, Z., & Mohamad Said, M. H. (2016). A Meta-Analysis on Effective Strategies for Integrated STEM Education. *Advanced Science Letters*, 12, 4225-4229.
- Nariman, N., & Chrispeels, J. (2015). PBL in the era of reform standards: Challenges and benefits perceived by teachers in one elementary school. *Interdisciplinary Journal of Problem-Based Learning*, *10*(1). Retrieved from https://docs.lib.purdue.edu/ijpbl/vol10/iss1/5/
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making. Reston*, VA: Author.
- National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston VA: Author.
- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Common core state standards (college- and career-readiness standards and K–12 standards in English language arts and math). Washington, DC: Author. http://www.corestandards.org

National Research Council (2009). "Mathematics learning in early childhood: Paths

towards excellence and equity." Washington: National Academy Press.

Newell, A., & Simon, H. A. (1972). Human problem solving. Prentice-Hall.

- Ningrum, F. Y. W., Hobri, H., Susanto, Dafik, Lutvita, D., and Lestari, C. F. (2019), The students' problem solving abilities in science, technology, engineering and mathematics (stem) based on lesson study for learning community (lslc), *Journal of Physics: Conference Series*, *1538* 012088.
- Nitko, A.J., & Brookhart, S.M., (2011). *Educational Assessment of Students* (6th ed.). Boston, MA: Pearson Education, Inc.
- Norford, J. A. (2012). Increasing fourth-grade students' proficiency at solving mathematical word problems (Order No. 3518426). Available from ProQuest Dissertations & Theses Global. (1033343265). Retrieved from http://0search.proquest.com.mylibrary.qu.edu.qa/docview/1033343265?accountid=13370
- Nuha, M., Waluya, B. S., & Junaedi, I. (2018), Mathematical Creative Process Wallas Model in Students Problem Posing with Lesson Study Approach, *International Journal* of Instruction, 11(2):527-538. DOI: 10.12973/iji.2018.11236a
- OECD (2004), Learning for Tomorrow's World: First Results from PISA 2003, OECD, Paris.
- OECD. (2010). PISA 2009 results: What students know and can do—Student performance in reading, mathematics and science (volume I). http://dx.doi.org/10.1787/9789264091450-en
- OECD. (2013). PISA 2012 Assessment and Analytical Framework: Mathematics, Reading, Science, Problem Solving and Financial Literacy. https://www.oecdilibrary.org/docserver/9789264190511en.pdf?expires=1580654526&id=id&accname=guest&checksum=074EB03FD4D7F B122A322C1AB81A10BD
- OECD. (2018). Preparing our Youth for an Inclusive and Sustainable World: The OECD PISA Global Competence Framework. Paris: OECD Library.
- Otten, S. (2010). When the ends justify the meanings: Exploring conclusions of middle

school mathematical tasks. Unpublished research practicum, Michigan State University, East Lansing, MI.

- Pallant, J. (2007). SPSS Survival Manual: A Step by Step Guide to Data Analysis Using SPSS for Windows (3rd Edition). McGraw Hill Open University Press, New York.
- Pasquale, M. (2015). Productive struggle in mathematics. *Interactive STEM*. Retrieved from http://interactivestem.org/wp-content/uploads/2015/08/EDC-RPC-Brief-ProductiveStruggle.pdf
- Pelto, P. J., & Pelto, G. H. (1978). Anthropological research: The structure of inquiry (2nd ed.) Cambridge: Cambridge University Press.
- Pinter, H. A. (2017). The effects of problem-based learning on the math anxiety, selfefficacy and math achievement of elementary students (Order No. 10622838). Available from ProQuest Dissertations & Theses Global. (2076521291). Retrieved from http://0search.proquest.com.mylibrary.qu.edu.qa/docview/2076521291?accountid=13370
- Pólya, G. (1945; 2nd edition, 1957). How to solve it. Princeton: Princeton University Press.
- Pólya, G. (1957). How to solve it (2nd ed.). Princeton, NJ: Princeton University Press.
- Polya, G. (1965). *Mathematical discovery: On understanding, learning and teaching problem solving (vol. 2).* New York: Wiley.
- Polya, G. (1973). *How To Solve It: A New Aspect of Mathematical Method*. New Jersey: Princeton University Press.
- Polya, G. (2004). *How to Solve It*. Princeton, NJ: Princeton University Press. (Original work published in 1945)
- *Principles to action: Ensuring mathematical success for all.* (2014). Reston, VA: National Council of Teachers of Mathematics.
- Proudfit, L. (1992). Questioning in the elementary mathematics classroom. *School Science* and Mathematics, 92(3), 113-136.
- Provasnik, S., Malley, L., Stephens, M., Landeros, K., Perkins, R., & Tang, J.H. (2016),

Highlights from TIMSS and TIMSS Advanced 2015: Mathematics and science achievement of U.S. students in grades 4 and 8 and in advanced courses at the end of high school in an international context (NCES 2017-002), U.S. Department of Education, *National Center for Education Statistics*, Washington, DC.

- Purwandi, D., Susanto, & Hobri, H. (2020). Development of remedial mathematics learning based on lesson study for learning community against students' problem solving analysis capabilities. *Journal of Physics Conference Series*, 1563:012038. DOI: 10.1088/1742-6596/1563/1/012038
- Radzali, R. (2007). Mathematics Beliefs, Metacognition, Problem Representation and Problem Solving in Mathematics amongst students. Ph.D. Dissertation. National University of Malaysia.
- Ranjan, & Gunendra Chandra. 2013. Math Anxiety: The Poor Problem Solving Factor in School Mathematics. *International Journal of Scientific and Research Publications* 4(3): 1-5.
- Ravitz, J. (2008, March 27). Project based learning as a catalyst in reforming high schools. Paper presented at the annual meeting of the American Educational Research Association, New York. Retrieved from http://www.bie.org/files/AERA PBL 2008.pdf
- Resnick, L. B., & Glaser, R. (1976). "Problem solving and intelligence", pp. 205–230 inL. B. Resnick (ed.), *The Nature of Intelligence*. Hillsdale, N.J.: Lawrence Erlbaum Associates.
- Rigelman, N. (2007). Fostering Mathematical Thinking and Problem Solving: The Teacher's Role. *Teaching Children Mathematics*, 13(6),308-314. DOI: 10.5951/TCM.13.6.0308.
- Ronis, D., L. (2008). *Problem-based Learning for Math & Science; Integrating Inquiry and the Internet.* California: Corwin Press.
- Roopashree, B.J. (2014). PBL: Future Challenges for Educational Practice and Research. *Journal on School Educational Technology*, 10(2), 9-16.

Rosli, R., Capraro, M. M., Goldsby, D. (2015). Using Manipulatives in Solving and Posing

 Mathematical
 Problem.
 Creative
 Education,
 06(16):1718-1725.

 DOI:10.4236/ce.2015.616173
 06(16):1718-1725.
 06(16):1718-1725.

- Saadah, L. Z. K., Hobri, H., & Irvan, M. (2019). The application of problem based learning (PBL) based on lesson study for learning community (LSLC) to improve students' creative thinking skill. *Earth and Environmental Science*, 243(1):012141. DOI: 10.1088/1755-1315/243/1/012141
- Sajadi, M., Amiripour, P., & Rostamy-Malkhalifeh, M. (2013). The Examining Mathematical Word Problems Solving Ability under Efficient Representation Aspect. *Mathematics Education Trends and Research*, 1-11.
- Salim, M. & Samaan, E. (2011). The effectiveness of a training program for mathematics teachers based on a lesson study approach on the achievement and development of mathematical thinking of the preparatory stage students. (Unpublished doctoral thesis).
 Amman Arab University, Amman. http://0-search.mandumah.com.mylibrary.qu.edu.qa/Record/635713
- Salinitri, F. D., Wilhelm, S. M., & Crabtree, B. L. (2015). Facilitating Facilitators: Enhancing PBL through a Structured Facilitator Development Program. *Interdisciplinary Journal of Problem-Based Learning*, 9(1). Available at: http://dx.doi.org/10.7771/1541-5015.1509
- Saragih, S., & Habeahan, W. L. (2014). The Improving of Mathematical Problem Solving Ability and Students' Creativity by Using Problem Based Learning in SMP Negeri 2 Siantar. *Journal of Education and Practice*, 5(35), 123-132.
- Savery, J. (2006). Overview of problem-based learning: Definitions and distinctions. *Interdisciplinary Journal of Problem-Based Learning*, 1(1), 9-20. Retrieved from http://docs.lib.purdue.edu/ijpbl/vol1/iss1/
- Savery, J. R. (2015). Overview of problem-based learning: Definitions and distinctions. In
 A. Walker, H. Leary, C. E. Hmelo-Silver, & P. A. Ertmer (Eds.), *Essential Readings in Problem-Based Learning: Exploring and Extending the Legacy of Howard S. Barrows* (pp. 5-16). Retrieved from http://www.jstor.org/stable/j.ctt6wq6fh.6

Savery, J. R., & Duffy, T. M. (2001). Problem Based Learning: An Instructional Model

and Its Constructivist Framework. Bloomington: The Center for Research on Learning and Technology.

- Schoenfeld, A. H. (1989).Teaching mathematical thinking and problem solving. In L. B. Resnick and L. Klopfer (Eds.), *Toward the thinking curriculum: Current cognitive research (1989 Yearbook of the Association for Supervision and Curriculum Development)*, pp. 83-103. Washington, DC: Association for Supervision and Curriculum Development.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: MacMillan.
- Schoenfeld, A. H. (2011). Noticing matters. A lot. Now what? In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes*. New York: Routledge (in press).
- Schoenfeld, A. H. (2013). Reflections on Problem Solving Theory and Practice. The Mathematics Enthusiast, 10(2), 9-34. Retrieved from https://scholarworks.umt.edu/cgi/viewcontent.cgi?article=1258&context=tme
- Senk, L. K., & Thompson, D. R. (Eds.). (2003). Standards-based school mathematics curricula: What are they? What do students learn? Mahwah, NJ: Lawrence Erlbaum Associates.
- Shimizu, Y. (1999). Aspects of Mathematics Teacher Education in Japan: Focusing on Teachers' Roles. *Journal of Mathematics Teacher Education* 2, 107–116. https://doi.org/10.1023/A:1009960710624
- Sigurdson, S. E., Olson, A. T., & Mason, R. (1994). Problem solving and mathematics learning. *The Journal of Mathematical Behavior*, 13(4), 361-388. https://doi.org/10.1016/0732-3123(94)90001-9
- Silverman, F. L., Winograd, K., & Strohauer, D. (1992). Student-generated story problems. *Arithmetic Teacher*, *39*(8), 6-12.

Slavin, R. E., Lake, C., & Groff, C. (2009). Effective programs in middle and high school

mathematics: A best-evidence synthesis. *Review of Educational Research*, 79(2), 839–911. https://doi.org/10.3102/0034654308330968

- Soleh, R., Susyla. D, & Syofiana. M. (2019). Developing students critical thinking ability through lesson study. *Journal of Physics Conference Series*,1320:012005. IOP Publishing. doi:10.1088/1742-6596/1320/1/01200, 1-8.
- Stacy, S. T., Cartwright, M., Arwood, Z., Canfield, J. P., & Kloos, H. (2017). Addressing the math-practice gap in elementary school: Are tablets a feasible tool for informal math practice? *Frontiers in Psychology*, 8, Article 179. https://doi.org/10.3389/fpsyg.2017.00179
- Stake, R. E. (1995). The Art of Case Study Research, Thousand Oaks, CA: Sage.
- Stanic, G., & Kilpatrick, J. (1988). Historical perspectives on problem solving in the mathematics curriculum. In R. Charles & E. Silver (Eds.), *The teaching and assessing of mathematical problem solving* (pp. 1–22). Reston, VA: National Council of Teachers of Mathematics.
- Stein, M., Remillard, J., & Smith M. (2007). How curriculum influences student learning. In F. K. Lester Jr. (Ed.), Second handbook of research on mathematics teaching and learning (pp. 319-369). Gweenwich, CT: Information Age.
- Stewart, V. (2012). A world class education: Learning from international models of excellence and innovation. Alexandria, VA: ASCD
- Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.
- Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership*, 61(5), 12-17.
- Stonewater, J. K. (2005). Inquiry teaching and learning: the best math class study. *School Science and Mathematics*, *105* (1), 36-48.
- Sumarmo, U. (2004). Independent learning: What, why and how develop among students. Bandung: Indonesia University of Education Press.

- Szetela, W., & Nicol, C. (1992). Evaluating Problem Solving in Mathematics. *Educational Leadership*, 5, 42–45.
- Takahashi, A., & McDougal, T. (2016), "Collaborative lesson research: maximizing the impact of lesson study", ZDM Mathematics Education, Vol. 48, pp. (513-526).
- Tan O.S. (2004), Enhancing thinking through problem-based learning approaches: International perspectives. Singapore, Thompson Learning, pp. 133–143
- Tee, M. Y., & Lee, S. S. (2013). Advancing understanding using Nonaka's model of knowledge creation and problem-based learning. *International Journal of Computer-Supported Collaborative Learning*, 8(3), 313-331. http://dx.doi.org/10.1007/s11412-013-9175-2
- TIMMS (2011). 2011 Trends in International Mathematics and Science Study. Retrieved from http://nces.ed.gov/timss/results11_math11.asp
- TIMSS (1999). TIMSS 1999 international mathematics report: Findings from IEA's repeat of the Third International Mathematics and Science Study at the eighth grade. International Study Center, Boston College. http://timssandpirls.bc.edu/timss1999i/pdf/T99i_Math_All.pdf
- Tracey, D., & Morrow, L. (2012). Lenses on reading: An introduction to theories and models (2nd ed.). New York, London: Guilford Press.
- Trilling, B., & Fadel, C. (2009). 21st century skills: Learning for life in our times. Jossey-Bass/Wiley.
- Tsao, Y. L. (2012). Number sense of pre-service teachers. *Research in Higher Education Journal*, *16*, 1-12.
- Tsui, A. B. M. & Law, D. Y. K. (2007). Learning as boundary-crossing in school– university partnerships. *Teaching and Teacher Education*, 23, 1289–1301.
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2015). *Elementary and middle school mathematics: Teaching developmentally* (9th ed.). Boston, MA: Pearson.

Verschaffel, L., De Corte, E., Lasure, S., Van Vaerenbergh, G., Bogaerts, H., & Ratinckx,

E. (1999). Learning to solve mathematical application problems: A design experiment with fifth graders. *Mathematical Thinking and Learning*, 1, 195-229.

- Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, Netherlands: Swets & Zeitlinger.
- Vikriyah, N. (2015). Peningkatan Kemampuan Pemecahan Masalah Matematika MelaluiModel Problem Based Learning (PBL) Pada Pokok Bahasan Trigonometri. *Skripsi*.(Online) tersedia
- Westwood, P. (2011). *Commonsense methods for children with special educational needs* (6th ed.). New York: Routledge.
- Wheatley, G. (1984). The importance of beliefs and expectations in the problem solving performance of sixth grade pupils. In J. Moser (Ed.), *Proceedings of the 6th annual meeting of PME-NA* (pp. 141–146). Madison, WI: University of Wisconsin.
- Wilder, S. (2015). Impact of Problem-Based Learning on Academic Achievement in High School: A Systematic Review. *Educational Review*, 67(4), 411-435. https://doi.org/10.1080/00131911.2014.974511
- Wilson, J. W., Fernandez, M. L. & Hadaway N. (1993). *Mathematical proble solving*. New York: MacMillan. http://jwilson.coe.uga.edu/emt725/PSsyn/PSsyn.html
- Wolcott, H. F. (1988). Ethnographic research in education. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (pp. 187-249). Washington, DC: American Educational Research Association.
- Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in Grades 4 through 8: A practice guide* (NCEE 2012-4055). Retrieved from Institute of EducationScienceswebsite:

http://ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/

Wright, T. D., Jr. (2009). Investigating teachers' perspectives on the impact of the lesson study process on their mathematical content knowledge, pedagogical knowledge, and the potential for student achievement (Order No. 3361223). Available from ProQuest Dissertations & Theses Global. (304917314). Retrieved from http://0-search.proquest.com.mylibrary.qu.edu.qa/docview/304917314?accountid=13370

- Yeo, J., & Tan, S. C. (2014). Redesigning problem-based learning in the knowledge creation paradigm for school science learning. *Instructional Science*, 42(5), 747-775. doi: 10.1007/s11251-014-9317-6.
- Yerizon, Putra, A. A., & Subhan, M. (2018) Mathematics Learning Instructional Development based on Discovery Learning for Students with Intrapersonal and Interpersonal Intelligence (Preliminary Research Stage). *International Electronic Journal of Mathematics Education*, 13(3) pp 97-101
- Zambo, R. (1996). Beliefs and practices in mathematical problem solving instruction: K8. *Research in Middle Level Education Quarterly*, 19(4), 65–82.
- Zenvenberg, R., Dole, S., & Wright, R. (2004). *Teaching mathematics in primary schools*. New South Wales, Australia: Allen & Unwin.

APPENDIX

Appendix A: Mathematical Problem Solving Test

اختبار حل المشكلات

عزيزي الطالب/ة

يهدف هذا الاختبار إلى قياس قدرتك على حل المشكلات الرياضية، وهو لغرض البحث العلمي فقط. ويتكون هذا الاختبار من (20) سؤالا، وزمن الإجابة هو (45) دقيقة.

تعليمات الاختبار:

- بالنسبة للأسئلة التي لها أربعة بدائل: يجب أن تقرأ كل سؤال بعناية قبل اختيار الإجابة الصحيحة ثم شرح طريقة الحل.
- أما فيما يخص الأسئلة التي ليس لها بدائل: يجب أن تقرأ السؤال بعناية لإيجاد الحل وكذلك شرح طريقة الحل، كما يمكنك استخدام أكثر من طريقة للحل.
 - لا تترك أي سؤال دون الإجابة عليه.

بيانات الطالب/ة

	الصف:	الاسم:
ذكر	الجنس:	المدرسة:
أنثى		
	التاريخ:	اليوم :

Q1

لدى سالي 12 خيطا مستقيما، و40 خرزة دائرية، و48 خرزة مسطحة. تحتاج سالي إلى1 خيط، و10 خرزات دائرية، و8 خرزات مسطحة لصنع 1 سوار. إذا كانت جميع أساور سالي متشابهة، فكم عدد الأساور التي يمكن لسالي أن تصنعها؟

> 40 (a 12 (b 5 (c 4 (d

وضح إجابتك.

	Q2
وجد ناتج الضرب.	
23 × 19 =	
رضح إجابتك.	
	1







ثمن زجاجة عصير التفاح QR 1.87

ثمن زجاجة عصير البرتقال QR 3.29

إذا كان لدى عبد الرحمن QR 4

كم المبلغ الإضافي الذي يحتاجه عبد الرحمن لشراء زجاجتي العصير؟

QR 1.06 (a QR 1.16 (b QR 5.06 (c QR 5.16 (d

وضح إجابتك

Q6

Q5

6 + □ = 8 + 6 ما العدد المناسب الذي يجب وضعه في المربع كي تصبح هذه العبارة العددية صحيحة؟ 17 (a 11 (b 7 (c 5 (d وضح إجابتك.



Q8					
		Figure 1	Figure 2	Figure 3	Figure 4
				ىكال.	تظهر في الأعلى سلسلة لأربعة أش
	م الأشكال).	10؟ (لا ترس	وائر في الشكل رقم	،، كم سيكون عدد الدو	في حال تم إكمال سلسلة الأشكال
					الإجابة:
					وضح إجابتك.

Q9
شير مقياس الرسم الموجود على الخريطة إلى أن كل 1سنتيمتر على الخريطة يمثل 4 كيلومترات على
رَّرض. إذا كانت المسافة بين مدينتين على الخريطة 8 سنتيمترات. كم تبلغ المسافة الفاصلة بين
مدينتين بالكيلومترات؟
2 (a
8 (b
16 (c
32 (d
ضح إجابتك.

Q10	
لاء في عبوات سعة كل منها 5 لترا، يحتاج محمد ل 37 لترا من الطلاء، كم عدد العلب التي عليه	يباع الطا شراؤها؟
5 6 7 8	(a (b (c (d
وضح إجابتك.	











Q15

إذا كان ملعب المدرسة على شكل مربع، طول ضلعه 100 meters. تمشي مريم حول حافة الملعب

كلها. ما المسافة التي تمشيها مريم؟

- 100 meters (a
- 200 meters (b
- 400 meters (c
- 10,000 meters (d

وضح إجابتك.









في السباق الثلاثي، يبدأ المتسابقون بالسباحة ثم ينتقلون إلى سباق الدراجات وأخيراً سباق الجري. يشير الجدول أدناه إلى نتائج المنافسة لكل من سارة وإيمان وليلى. لقد تم حساب مجموع الدقائق لإحدى المجموعات الثلاثة لمساعدتك.

Q19

	ڻي بالدقائق	ج السباق الثلا	نتك
ليلى	إيمان	سارة	
50	25	35	لسباحة
85	90	80	سباق الدر اجات
120	130	135	سباق الجري
		250	لمجموع:

A. إذا كانت الفائزة هي التي حققت أقل عدد من الدقائق.	
فمن الفائزة بالسباق الثلاثي؟ وما هو الوقت الذي حققته الفائزة؟	

```
الإجابة: .....
```

B. تريد ليلى إنهاء السباق الثلاثي بشكل أسرع العام المقبل. في أي رياضة بالتحديد عليها التحسن للفوز على سارة وإيمان؟

.....دقيقة

(ظلل مربعاً واحداً) [] السباحة

🗌 سباق الدراجات

🗆 سباق الجري

اذكر السبب معتمداً على المعلومات الواردة في الجدول.



Appendix B: rubric

Mathematics Problem Solving Scoring Rubric

	The student:
	• shows complete understanding of the problem by using multiple representations that is unusual
	in its mathematical precision
5	• identifies, relates, and uses all important components of the problem
	Performs computations completely and correctly
	• uses more than one appropriate strategy that should lead to a correct
	solution
	• Gives a complete and precise explanation of their solution strategy using
	words pictures and/or symbols
-	
	The student:
	 shows complete understanding of the problem by using a representation that clearly depicts the problem identifies, relates, and uses all important components of the problem
4	performs computations completely and correctly but may make one minor error
	Ises an appropriate strategy that should lead to a correct solution
	Gives an explanation of their solution strategy using words, pictures, and/or symbols
-	The students
	The student:
	 shows understanding of the problem by using a representation that clearly depicts the problem identifies relates and uses most of the important components of the problem
3	netforms computations that are generally correct but may make some minor errors
	 uses an appropriate strategy that should lead to a reasonable solution
	• gives a partial explanation of their solution strategy using words, pictures, and/or symbols
	The student:
	 shows some understanding of the problem by using a representation that gives some important
2	information about the problem
	 identifies, relates, and uses some important components of the problem
	 performs computations that contain major computational errors
	 shows some evidence of a strategy to solve the problem
	Gives a minimal explanation of their solution strategy
	The student:
	 shows limited to no understanding of the problem by using a representation that gives little or no
	significant information about the problem
	• fails to identify, relate, or use important components of the problem
	 uses an inappropriate strategy for solving the problem
	· places too much emphasis on unrelated components of the problem
	attempts an answer
	• no explanation
B	Blank (no attempt or completely erased answer)
Α	Correct answer only (no work shown)
Ι	Incorrect answer only (no work shown)

Appendix C: Interview Protocol

Interview Protocol

Interview Protocol Guide for Teachers' Interv	view:
Interviewer's Name:	Position:
Interview Date:	Interview Time:
Lesson study experience:	Highest Degree:

First, thank you for participating in this research. The aim of this interview is to collect data in order to complete my master study, entitled "The impact of using lesson study approach on students' mathematical problem-solving ability". This study seeks to examine the impact of Lesson Study as problem based learning approach used as a teaching tool to enhance students' mathematical problem solving ability. All information you share including your identity will remain confidential and will not be used in data or final study report. The interview will take approximately 40 minutes. Your participation in this study is voluntary and you may withdraw at any point. If you agree, a taped recorder will be used to ensure that data is collected accurately, besides writing notes.

Do you have any questions or comments before we start?

Interview questions

Teaching and learning strategies followed by the teacher

What are the teaching strategies you use in teaching mathematics?

How do you feel towards this teaching method of instruction in mathematics?

How did your students respond to the teaching method?

Have you noticed that the teaching method affects the students' performance? If yes, how? If no, why?

Does the teaching method offer students who have a disability or who are low achieving a chance to help them to succeed?

Does the method affect differently according to their levels and ability?

What are the advantages and disadvantages of the teaching method you apply? Can you describe the atmosphere you feel in your class during teaching and learning?

Describe the differences between your teaching methods in your math class this year and before three year.

Teaching mathematical problem solving

Do you think that the method of teaching enhances problem- solving skills in mathematics for the students? If yes, how? If no, why?

What have been difficulties for you as the math teacher when applying mathematical problem solving?

Did you observe any changes in the students' activities, relations, personalities and attitudes towards mathematics? If yes, is this new behaviour? Can you explain the reasons? If no, can you explain the reasons? What do you think helps your students learn problem solving the best? Why?

What are things that you think could be done to help students learn problem solving better? Why?

Assessment of student's problem-solving ability

What kind of evidence do you see that students are referring to the problem solving ability?

How do you see your students are performing in problem-solving ability this year versus previous years? Why?

How do you think your students persevere through math problems? Why? How does this compare to previous years?

Describe how your students solve a challenging math problem.

How do you assess your students' ability in solving mathematical problems?

How do students demonstrate their problem solving ability?

What changes have you noticed in your students' problem-solving ability?

Thank you for taking time for this interview. I will transcribe the information and email it to you. Kindly inform me if there is anything missing or needs clarifying. I greatly appreciate your help.
Appendix D: MoEHE Approval

وزارة التعليم والتعليم العالي الصادر جهات خارجيه 16031830298780 180/2020 20/10/2020 تصريح الموافقة لدخول المدارس السادة مدراء المدارس الحكومية المحترمين،،، السلام عليكم ورحمة الله وبركاته،،، نود إحاطتكم علماً بأن الباحث / الباحثون المرفق لكم بياناتهم، بصدد إجراء دراسة ميدانية في مدرستكم وعليه يرجى التكرم بتسهيل مهمة الباحث ، علماً بأن البيانات ستكون سرية ولأغراض البحث العلمي. مع الشكر لحسن تعاونكم ،،، مح/ نوف عبدالله مبارك الكعبي مدير إدارة السياسات والأبحاث التربوية 3



عينة البحث	معلمات الرياضيات وطلبة المرحلة الابتدائية
ملخص للبحث	تشير الدراسات إلى أهمية تنمية مهارات حل المشكلات الرياضية باعتبارها أحد مهارات القرن الواحد
	والعشرين كما تشير الدراسات والمعايير العالمية للرياضيات إلى أن أفضل طريقة لتنمية القدرة على حل
	المشكلات هي اتباع نهج التعلم القائم على حل المشكلات في تعليم وتعلم الرياضيات. وتهدف هذه الدراسة
	إلى الكشف عن أثر نهج تحليل الدرس القائم على حل المشكلات في تنمية مهارات حل المشكلات الرياضية
	لدى طلبة المرحلة الابتدائية في دولة قطر. وسيتم اتباع المنهج الوصفي التحليلي، كما سيتم استخدام
	المقابلة شبه المقننة مع المعلمات بالإضافة إلى اختبار حل المشكلات الرياضية واستبيان الطلبة لجمع
	البيانات.
أهداف البحث	تهدف هذه الدراسة إلى استقصاء أثر نهج (تحليل الدرس) المستند إلى التعلم القائم على حل المشكلات في
	تعزيز قدرة طلبة المرحلة الابتدائية على حل المشكلات الرباضية في دولة قطر.
أسئلة البحث	"ما أثر استخدم نهج تحليل الدرس في القدرة على حل المشكلات الرياضية لدى طلبة المرحلة الإبتدائية في دولة قطر؟"
منهج البحث وأدواته	تعتمد هذه الدراسة على المنهج الوصفي التحليلي والذي يعتمد على دراسة الظاهرة كما هي في الواقع
	ويهتم بوصفها وصفأ دقيقا ويعبر عنها كميأ ونوعياً. وللإجابة على سؤال الدراسة سيتم استخدام اختبار
فقات المطلوبة:	حل المشكلات واستبيان الطلبة وكذلك المقابلة شبه المقننة مع المعلمات.
خطاب من الجهة المشرفة على الباحد	.د
أداة البحث المراد تطبيقها ، بحيث تكو	ن مكتملة و محكمة.



تسهيل مهمة الباحث في المدارس

					1 and		بيانات الباحث
			ن	عبد القوي الأشوإ	عبد المجيد	هناء	اسم الباحث
				التربية	لة قطر/ كلية	جامع	الجهة المشرفة (جامعة / كلية)
			كلية التربية	لتطوير التربوي /	نز الوطني لا	المرك	جهة العمل
	77555457	الهاتف النقال	1		55929	151	هاتف العمل أو المنزل
			ha18053	10@student	.qu.edu	i.qa	البريد الالكتروني (الوظيفي أو الجامعي)
				htmh-04@h	otmail.o	com	البريد الالكتروني (الشخصي)
2 7	8 8	8	6 0	0 2	8	3	الرقم الشخصي
		l	والتقييم	م وطرق التدريس	ا ستير المناهع	ماجس	التخصص الجامعي
				and the second		and a	بيانات الأعضاء المشاركين بالبحث
	33162988			عصام برهم	د. اريج ه	.1	
ا لإدخال نص.	انقر أو اضغط هن	115:11 . 2:1 . 11				.2	المشاركين
ا لإدخال نص.	انقر أو اضغط هن	الهالف (للغان	ص.	مغط هنا لإدخال نم	انقر أو اض	.3	المعاع المساردين
ا لإدخال نص.	انقر أو اضغط هذ		س.	مغط هنا لإدخال نم	انقر أو اض	.4	
				ا لإدخال نص.	ر اضغط هذ	انقر أو	أسماء أخرى
T. S. Santa							بيانات البحث
	باضية	ي حل المشكلات الر	ى قدرة الطلبة في	ا تحليل الدرس علم	ستخدام نهج	أثر ا	عنوان البحث (بالعربي)
The Impact Problem-Sol	of Using Les ving Ability	son Study Ap	proach on S	tudents' Math	nematica	ıl	عنوان البحث (بالإنجليزية)
	(یذکر) : نا لإدخال نص	أخرض آخر انقر أو اضغط ه	کر) :	لبات علمية (يذ	إنهاء منط ة ماجستير	🛛 اِ	الغرض من إجراء البحث
			لة الابتدائية	يات وطلبة المرحا	بات الرباض	معلم	اسم الفنة المستهدفة
	250 تقريباً	العدد		a	لة الابتدائي	المرحا	المرحلة الدراسية للفنة المستهدفة
			بامعة قطر	لتعليم العالي - ج	ة التعليم وا	وزارة	الجهة المستفيدة من نتانج البحث
2021-07- 01	الزمن المتوقع لاكتمال البحث	2021- 05-01	تاريخ نهاية تطبيق أدوات البحث	2020-10- 25	غ بدء ن أدوات ن	تاريخ تطبيق البحث	الفترة الزمنية التي ينفذ فيها البحث
				1			

Appendix E: QU-IRB Approval



Appendix F: Students' Assent form

الطلبة	موافقة	نموذج
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أنا هناء الأشول طالبة دراسات عليا من جامعة قطر. أقوم بدراسة لمعرفة (أثر نهج تحليل الدرس على قدرة طلبة المرحلة الابتدائية في حل المشكلات الرياضية). أطلب منك المشاركة في الدراسة البحثية.

في هذه الدراسة ستقوم بإجراء اختبار خلال الحصة الدراسية (لا يتعدى زمن الاختبار 45 دقيقة) وأثناء الدوام الرسمي لقياس قدرتك على حل المشكلات الرياضية. سنحافظ على خصوصية جميع إجاباتك، ولن نعرضها على (معلمك أو والديك أو ولي أمرك). لن يرى إجابتك سوى الأشخاص الذين يعملون على الدراسة من جامعة قطر.

لن تحدث لك أي مشاكل نتيجة المشاركة في هذه الدراسة، ولكن قد تحتاج للتفكير في أسئلة الاختبار. وقد تنزعج أيضًا إذا رأى الآخرون إجاباتك، لذلك سنحاول منع أي شخص من رؤية ما تكتبه.

يمكنك أن تشعر بالرضا حيال مساعدتنا على تحسين تعلم الطلبة الآخرين الذين قد يواجهون مشاكل في تعلم الرياضيات. **عليك معرفة أنه:**

- سواء قررت المشاركة أو لا فقرارك لن يؤثر على علامتك المدرسية أو على علاقتك بمعلمك.
- ليس عليك المشاركة في هذه الدراسة إذا كنت لا ترغب في ذلك. لن تتورط في أي مشكلة مع (جامعة قطر أو معلمك أو المدرسة) إذا قلت لا.
 - يمكنك التوقف عن المشاركة في الدراسة في أي وقت. (إذا كان هناك سؤال لا تريد الإجابة عليه، فاتركه فارغًا).
 - وقّع على هذا النموذج فقط إذا:
 - فهمت ما ستفعله لهذه الدراسة.
 - تمت الإجابة على جميع أسئلتك.
 - تحدثت إلى والديك / ولي الأمر حول هذا المشاركة في الدراسة.
 - الموافقة على المشاركة في هذا البحث.

إذا كان لديك أي تساؤل أو رغبت بالحصول على مزيد من التفاصيل فيمكنك الاتصال على الهاتف الخاص بالباحثة: 77555457، أو على البريد الإلكتروني: <u>HA1805310@qu.edu.qa</u> ولك جزيل الشكر...

	التاريخ:	نوقيع الخاص بك :	الإسم: ال
	التاريخ:	توقيع الباحثة:	الباحثة المشاركة هناء الاشول
nstitutional Review Board (IRB) APPROVED			

Appendix G: Parents' Consent Form

إخطار موافقة عنوان الدراسة أثر استخدام نهج تحليل الدرس على قدرة طلبة المرحلة الابتدائية في حل المشكلات الرياضية عزيزي ولي الأمر السلام عليكم ورحمة الله وبركاته، نبذة عن الدراسة تشير الدراسات إلى أهمية تنمية مهارات حل المشكلات الرياضية باعتبارها أحد مهارات القرن الواحد والعشرين كما تشير الدراسات والمعايير العالمية للرياضيات إلى أن أفضل طريقة لتنمية القدرة على حل المشكلات هي اتباع نهج التعلم القائم على حل المشكلات في تعليم وتعلم الرياضيات. وتهدف هذه الدراسة إلى الكشف عن أثر نهج تحليل الدرس القائم على حل المشكلات في تنمية مهارات حل المشكلات الرياضية لدى طلبة المرحلة الابتدائية في دولة قطر. <mark>وتتكون عينة الدراسة من 30 معلم ومعلمة من</mark> معلمي ومعلمات الرياضيات للصفين الرابع والخامس بدولة قطر وطلابهم في المدراس الابتدائية للعام الدراسي 2021- 2020 الذين تتراوح أعمارهم بين 8- 12 سنة، وستكون المشاركة في هذه الدراسة تطوّعية. ويستبعد من هذه الدراسة أي معلم أو معلمة لا يعمل في المدارس الابتدائية الحكومية القطرية، وكذلك يستبعد من الدراسة الطلبة الذين تقل أعمارهم عن 8 سنوات، وأيضاً الطلبة الذين تزيد أعمارهم عن 12 سنة. الاعتماد: اعتُمدت هذه الدراسة من قبل مكتب المراجعة المؤسسية في جامعة قطر، ورقم الاعتماد ابنك / ابنتك مدعوين للمشاركة في دراسة بحثية تهدف هذه الدراسة إلى استقصاء أثر نهج (تحليل الدرس) المستند إلى التعلم القائم على حل المشكلات في تعزيز قدرة طلبة المرحلة الابتدائية على حل المشكلات الرياضية في دولة قطر. إن مشاركة ابنك / ابنتك في هذه الدراسة هي مشاركة تطوعية كلياً، وله حرية الرفض أو التوقف في أي وقت، ولن يتأثر بأي حال إذا ما قرر التوقف. إن عدم الرغبة في المشاركة في الدراسة و / أو الانسحاب من الدراسة لن يؤثر بأي شكل من الأشكال على علاقة الطالب بالمعلم كما لن يؤثر على تقييم درجات الطالب في المادة الدراسية. وبالمثل، فإن المشاركة في الدراسة لن تؤثر بأي حال من الأحوال على العلاقة بين الطالب والمعلم كما لن تؤثر على تقييم درجات الطالب في المادة الدراسية.

ستتم الدراسة من خلال إجراء استبيان (لا يتعدى زمن الاستبيان 15 دقيقة) أثناء الدوام الرسمي لقياس قدرة ابنك/ ابنتك في حل المشكلات الرياضية.

إن جميع البيانات سوف تعامل بسرية تامة وسوف تستخدم لأغراض البحث العلمي فقط وكذلك فإن بيانات الدراسة سيتم تخزينها لمدة 5 سنوات بعد الانتهاء من الدراسة قبل إتلافها.

إذا رغبت بالحصول على ملخص لنتائج الدراسة بعد إتمامها فيمكنك الاتصال على الهاتف النقال الخاص بالباحثة:

77555457، أو على البريد الإلكتروني: HA1805310@qu.edu.qa

أو التواصل مع المشرف الأكاديمي على الدراسة، الدكتورة أريج برهم على هاتف المكتب44035172 أو البريد الالكتروني:

areejbarham@qu.edu.qa

ولك جزيل الشكر... هناء الاشول، الباحثة المشاركة

	التاريخ: التاريخ:	بيان الموافقة يرجى تحديد اختيارك أدناه: - لقد قرأت المعلومات المذكورة أعلاه - أنت توافق طوعاً على مشاركة ابنك / ابنتك في هذه الدراسة. - أن عمر ابنك / ابنتك أقل من 18 سنة - أن عمر ابنك / ابنتك أقل من 18 سنة توقيع ولي الأمر:
Lind Grander Institutional Review Board (IRB) APPROVED		

Appendix H: Teacher's Interview Consent Form



	بيان الموافقة
ن هذه الدراسة وطبيعتها وأنا أشارك فيها طواعية. أفهم أنه	أوافق على المشاركة في الدراسة البحثية. أنا أفهم الغرض مز
	يمكنني الانسحاب من الدراسة في أي وقت.
	ں د لا
	أوافق على تسجيل صوت المقابلة.
	o نعم
	٥ لا
التاريخ	توقيع المشارك:
التاريخ	توقيع الباحثة المشاركة:
	لأي معلومات إضافية أرجو التواصل على:
- المشرف على الرسالة: أ.د. أريج عصام برهم	 الاسم: هناء عبد المجيد الأشول
- البريد الإلكتروني: areejbarham@qu.edu.qa	- البريد الإلكتروني: ha1805310@qu.edu.qa
رقم المكتب: 44035172	 رقم الهاتف النقال: 66002269
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Institutional Review Board (IRB)	
APPROVED	