Applied Mathematical Sciences, Vol. 8, 2014, no. 103, 5105 - 5114
HIKARI Ltd, www.m-hikari.com
http://dx.doi.org/10.12988/ams.2014.46428

Fuzzy Linear Programming for Supply Chain Management in Steel Industry

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Abstract

Linear programming is one of the frequently applied tools in supply chain management. However, managers and decision makers may lack information about exact values of most of the parameters used in the optimization models. Fortunately, fuzzy linear programming comes up with a powerful tool to deal with this kind of incomplete data. In this paper, the flexible approach of fuzzy linear programming is proposed and used to solve supply chain management of steel manufacturing company. This approach reformulated some constraints from conventional linear programming to fuzzy linear programming and provides alternative solutions to decision makers. The results obtained indicate that the fuzzy linear programming gives more flexibility to the decision maker to achieve some aspiration level in order to choose what he considers as the best optimal solution.

Subject Classification: 65K05, 90Cxx

Keywords: Fuzzy Linear Programming; Supply Chain Management

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1 Introduction

Many problems arose concerning the supply chain management with imprecise data. Usually supply chain management problems are formulated as conventional linear programming problems with the objective of optimizing supply, process and/or demand. However, most of the parameters used in the real world are not precise. Even if the results obtained using linear programming is accepted, neither the expected revenues nor the constraints can be characterized by uncertainty. The source of uncertainty are classified into three parameters; demand, process/manufacturing and supply. The most important aspects in dealing with imprecise information is the use of membership functions, the value of the membership function represents the degree of satisfaction (preference) on the constraints. The feasible alternative with the maximum degree of satisfaction is chosen as the optimal solution of the production scheduling problem.

In many cases the available materials to be supplied may not be known precisely or a change may occur in some range because he can get additional materials from suppliers, or the available time may also change. IN these cases Fuzzy Linear Programming (FLP) is the best approach to use instead of the conventional LP. In this paper, we will consider the flexible approach to formulate the FLP. In this approach, the decision variables are crisp numbers while the right hand side coefficient are characterized by uncertainties. The proposed FLP will be implemented on a real supply chain management problem in steel industry.

The theory of fuzzy mathematical programming was first proposed by Tanaka et al. [6] based on the fuzzy decision to address the uncertainty of the parameters in the problems with fuzzy objective functions and constraints. The formulation of FLP was first introduced by Zimmermann [8]. He built a crisp model of the problem and obtained its results by an existing algorithm. He then used the results and fuzzified the problem by evaluating admissible deviations for the goal and constraints of the subjective constants, then he represented the maximization of the minimization of the deviations on the constraints by defining an equivalent crisp problem using auxiliary variable. He used Bellman and Zadeh's [7] interpretation that a fuzzy decision is a union of goals and constraints.

2 Supply Chain of Steel Industry

Over the last two decades supply chain management (SCM) has attracted ever increasing attention as a consequence of the pressure on organizations in competitive market place to create and deliver value to customers. According to Min and Zhou [4], a supply chain is an integrated system which synchronizes a series of inter-related business processes. Petrovic et al. [5] was the earlier contributors to describe the fuzzy modeling and simulation of a supply chain in an uncertain environment. Carlsson and fuller [2] proposed a fuzzy logic approach to reduce bullwhip effect.

The motivation of this research is to provide an alternative way for optimizing supply chain management in an uncertain environment, which at the end helps the decision makers to take their decision and implement the results obtained. The main aims of this research is to implement FLP on solving supply chain in steel industry, more precisely we are interested in developing a fuzzy linear programming formulation to describe the supply chain, from suppliers to consumers and implement it for solving a supply chain management problems. We then incorporate existing data in the proposed model in order to enhance its validity and ensure practical and optimal solutions.

The overall components of the supply chain problem can be summarized as follows Diabat et al. [3]:

- The number of suppliers that are responsible for supplying the raw materials.
- The cost of raw materials to be supplied.
- The inventory level of any item from suppliers that enters in to inventory. The lower and upper inventory level, minimum and maximum purchased unit, holding cost and duration in inventory are to be considered.
- The products sold; its minimum sold, maximum sold, lower and upper inventory level are to be considered.
- Production plant. Its receiving capacity, production capacity and time before shipping out.
- Customers, which includes distributors, products sold, products lead time/materials and selling price.

Figure 2.1 shows the main elements/stages in a supply chain network from raw materials to customers.

2.1 Formulating the Problem as Conventional LP

The supply chain problem has been formulated as a conventional LP problem model Alawneh et. al [1]. However, the purchase of raw materials is not always specific, therefore any raw material constraints can be considered as fuzzy with uncertain at the right hand side. In this case, we can use flexible approach to reformulate the aforementioned constraints to meet the target of the decision

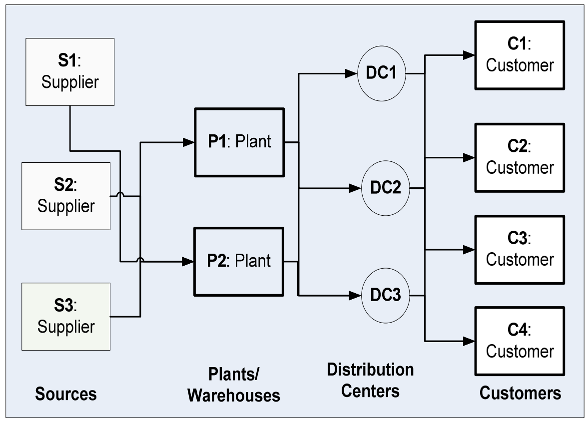

Figure 2.1: Supply chain network.

makers. We reformulate the first two constraints in to Fuzzy constraints and study the effect of the new solution to maximize the company's profit.

consider 4 types of products; namely product1, product2, product3, and product4, 6 customer sites; customer1, customer2,..., customer6 and 5 suppliers, namely supplier1, supplier2,..., supplier5. Furthermore, there are 4 processing units, namely Direct Reduction [DR], the Electric Furnace [EF], the Continues Casting [CC] and the Rolling Mill [RM]. In the model we consider the time to be in hour or days, in order to account for duration in the inventory. The flow chart below is provided to illustrate the proposed model.

The decision variables of the proposed model are expressed in tons as follows:

$ordo_i$ the order of oxide pellets from supplier i
$ords_1$ the order of scrap from supplier 1
$ords_2$ the order of scrap from supplier 2
o_i the amount of oxide pellets from supplier i that enters DR
s_1 the amount of scrap from supplier 1
s_2 the amount of scrap from supplier 2
io_i the level of oxide pellets from supplier i that enters inventory
is_1 the level of scrap from supplier 1 that enters inventory
is_2 the level of scrap from supplier 2 that enters inventory
a_1 the amount of high pure iron that exits DR and enters EF
a_2 the amount of scrap that enters EF
b the amount of molten steel that exits EF and enters CC
d the amount of product3 that exit CC and enter RM

f_j the amount of finished product that will be sent to costumer j.

Other parameters involved in the model are as follows:

dr capacity of direct reduction.
ef capacity of electric furnace.
cc capacity of continuous casting.
rm capacity of rolling mill.
p_j price per unit.
c_i unit cost
cs_1 cost of scrap from supplier 1.
cs_2 cost of scrap from supplier 2.
ic_i holding cost from supplier i.

The objective function is expressed as follows:

$$\max Z = \sum_{j=1}^{n} p_j.f_j - \sum_{i=1}^{m} c_i.ordo_i - [s_1.cs_1 + s_2.cs_2] - \sum_{i=1}^{m} ic_i.io_i - [ic_3.is_1 + ic_4.is_2] \tag{1}$$

subject to the following constraints:

$$minord \leq \sum_{t} ordo_i \leq maxord \tag{2}$$

$$minscrap \leq \sum_{t} ords_i \leq maxscrap \tag{3}$$

$$a_1 = \sum_{i=1}^{m} dr_{1i}.o_i + dr_{13}.s_1 + dr_{14}.s_2 \tag{4}$$

$$a_2 = \sum_{i=1}^{m} dr_{2i}.o_i + dr_{23}.s_1 + dr_{24}.s_2 \tag{5}$$

$$b = ef_1.a_1 + ef_2.a_2 \tag{6}$$

$$d = cc.b \tag{7}$$

$$fp = rm.d \tag{8}$$

$$f_1 + f_2 \leq fp \tag{9}$$

$$io_i = o_i - ordo_i \tag{10}$$

$$is_i = s_i - ords_i \tag{11}$$

$$\sum_{t} o_1 + o_2 \leq dr \tag{12}$$

$$\sum_{t} a_1 + a_2 \leq ef \tag{13}$$

$$\sum_t b \leq cc \tag{14}$$

$$\sum_t d \leq rm \tag{15}$$

The objective of the model aims at maximizing the profit over one year, which consist of the following terms: the income from selling the finished products to consumers, minus the cost of purchasing the raw materials, minus the cost of holding raw materials in inventory.
The first two constraints guarantee that the orders placed for raw materials from all suppliers will be within the given range of minimum and maximum quantities. From these information the first two constraints can be considered as fuzzy constraints with uncertain at the right hand side.

2.2 Reformulation of the First Two Constraint as Fuzzy

Let B_1 be the minimum purchased oxide pellets then the maximum purchased oxide pellets is $B_1 + \theta_1$, again let B_2 be the minimum purchased scrap then the maximum purchased scrap is $B_2 + \theta_2$ θ_1 and θ_2 can be obtained by subtracting minimum from the maximum. the constraints (1) and (2) can be reformulated as follows:

$$\sum_t ordo_i = \tilde{B_1}, \tilde{B_1} \in [B_1, B_1 + \theta_1] \tag{16}$$

$$\sum_t ords_i = \tilde{B_2}, \tilde{B_2} \in [B_2, B_2 + \theta_2] \tag{17}$$

θ_1 and θ_2 are the maximum purchased oxide pellets and maximum purchased scrap tolerance respectively

2.3 Estimation of Membership Function

The estimation of the membership functions for the constraints for the first constraints, linear type is assumed.

$$\mu_1(\sum_t ordo_i) = \begin{cases} 0 & \text{if } \sum_t ordo_i < B_1 \text{ or } \sum_t ordo_i > B_1 + \theta_1; \\ 1 & \text{if } B_1 < \sum_t ordo_i < B_1 + \theta_1; \\ 1 - \frac{(\sum_t ordo_i - B_1)}{\theta_1} & \text{if } B_1 \leq \sum_t ord_i \leq B_1 + \theta_1 \end{cases}$$

for the second linear type is also assumed. The membership function for the first constraint is give depicted in Figure 2.2.

$$\mu_2(\sum_t ords_i) = \begin{cases} 0 & \text{if } \sum_t ords_i < B_2, \sum_t ords_i > B_2 + \theta_2 \\ 1 & \text{if } B_2 < \sum_t ords_i < B_2 + \theta_2; \\ 1 - \frac{(\sum_t ords_i - B_2)}{\theta_2} & \text{if } B_2 \leq \sum_t ords_i \leq B_2 + \theta_2 \end{cases}$$

where $\theta_1, \theta_2 \geq 0$. Likewise the membership function for the second constraint is given by Figure 2.3.

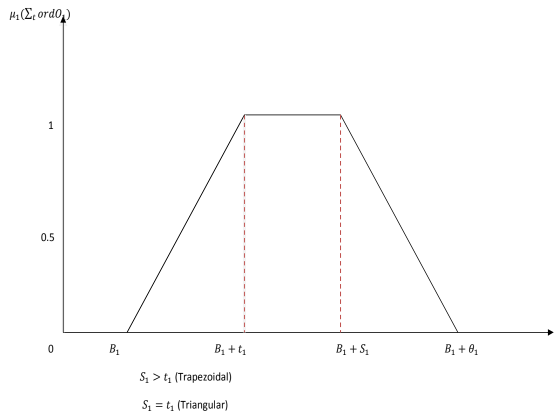

Figure 2.2: Membership function for the first constraint

2.4 Converting the Problem in to Parametric Linear Programming

the fuzzy linear programming can be converted to convectional linear programming as follows:

$$\max Z = \sum_{j=1}^{n} p_j.f_j - \sum_{i=1}^{m} c_i.ordo_i - [s_1.cs_1 + s_2.cs_2] - \sum_{i=1}^{m} ic_i.io_i - [ic_3.is_1 + ic_4.is_2] \quad (18)$$

subject to ;

$$\sum_t ordo_i = B_1 + \theta_1(1-\alpha)$$
$$\sum_t ords_i = B_2 + \theta_2(1-\alpha)$$

where $\alpha \in [0,1]$
the other crisp constraints remain the same (i.e. 3 to 14),
setting $\gamma = 1 - \alpha$, the constraints can be transformed to the following form

$$\sum_t ordo_i = B_1 + \theta_1\gamma$$
$$\sum_t ords_i = B_2 + \theta_2\gamma$$

All the decision variables satisfy the non-negativity and $\gamma \in [0,1]$ is a parameter.

The actual data are from the case company and they are summarized in Tables 2.1, 2.2 and 2.3.

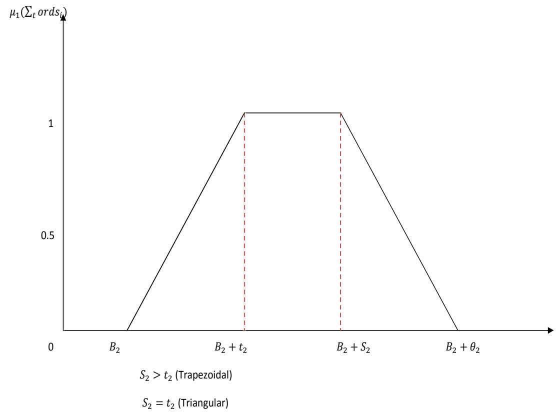

Figure 2.3: Membership function for the second constraint

Table 2.1: Products

Product Type	Min Sold (Ton)	Unit Price (USD/Ton)	Max Sold (Ton)	Low Inv. Level (Ton)	Upper Inv. Level (Ton)	Raw Material Per Unit
Product1	120,000	420	230,000	70,000	150,000	1.45
Product2	160,000	420	350,000	30,000	60,000	1.45
Product3	350,000	620	450,000	45,000	80,000	1.02
Product4	1,550,000	725	1,637,000	25,000	50,000	1.01

3 Results and Analysis

After reformulation of Steel Company from conventional LP to FLP by converting supply constraints in to fuzzy constraints, we implemented the model in the commercial software GAMS, using available data as parameters and introducing additional parameter that belong to the interval $[0, 1]$ called maxtol in the model, it is possible to conduct an analysis regarding the optimal levels of raw materials to be purchased and finished products to be supplied to customers. Assuming that the demand is fixed for each customer and based on this demand the amount of raw materials to be supplied will be decided on, taking in to consideration the processes and the capacity of the units. According to the designed model in the software, there are five suppliers considered and six customers, minimum and maximum inventory levels, as well as unit holding price are drawn from the data sheet and included in the model. The additional two constraints are included to ensure that maxtol is between $[0, 1]$.

Table 2.2: Demand table in Tons

Customer/Product	Cust. 1	Cust. 2	Cust. 3	Cust. 4	Cust. 5	Cust. 6
product1	994,342	148,389	74,881	55,491	353,573	81,250
product2	76,981	11,488	5,797	4,297	27,400	6,290
product3	102,642	15,318	7,730	5,728	36,498	8,387
product4	224,529	33,507	16,909	12,530	79,839	18,347

Table 2.3: Price table (USD/Tons)

Customer/Product	Cust. 1	Cust. 2	Cust. 3	Cust. 4	Cust. 5	Cust. 6
product1	725	533	805	805	730	690
product2	420	320	460	470	430	410
product3	420	308	450	460	420	400
product4	620	455	687	687	640	624

Table 3.1 summarizes the amount of oxide pellets to be purchased from supplier i in tons.

Table 3.1: Oxide pellets to be supplied

suppliers	amount of oxide pellets (Tons)
supplier 1	500,000
supplier 2	486,120
supplier 3	500,000
supplier 4	500,000
supplier 5	500,000

Table 3.2 summarizes the level of scrap to be purchased in tons, inventory of oxide pellets, inventory of scrap and the total time required for given demand to be met.

Finally, the objective value (profit) is 1,567,858,997.00 USD for a fixed demand.

4 Conclusion

In this paper, we have discussed formulating a supply chain management of Steel manufacturing company as a fuzzy linear programming by reformulating some of the constraints. It pays more emphasis on the supply of raw materials, it has been considered as an uncertain. This model has been tested by using data from actual

Table 3.2: Required inventory level

inventory	level
Scrap to be purchased in tons	227,350 Tons
inventory of oxide pellets	918,433 tons
Inventory of scrap	50,000 Tons
Total time required for given demand to be met.	31,670 Tons

steel Company. The flexible approach of fuzzy formulation is more effective than the conventional LP formulation model. Furthermore, the model provides an alternative to managers and decision makers. The General Algebraic Modeling System (GAMS software) has been used to solve the supply chain problem in this paper.

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Received: June 7, 2014