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An attribute control chart for the inverse Weibull distribution under truncated life tests

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ABSTRACT

Statistical Process Control (SPC) is applied to monitor production processes in order to discover any problems or issues that may arise during the production process and to help in finding solutions for these issues. In this paper, we consider a situation in which the product's quality, as measured by its lifetime, is monitored. Since the monitoring requires life tests to be performed and this may take relatively long time, the test time is truncated at some pre-specified time t_0, chosen to be related to the product's target mean life. This results in a truncated life test. The number of failures during the life test is used as an indicator of the quality of the product. We consider the situation in which the lifetimes follow the Inverse Weibull distribution. A control chart is proposed for this specific situation, thus extending the applicability of control charts methodology to situations involving truncated life tests. Simulation techniques has been employed to obtain the quantities needed for constructing the control chart with the aim that the average run length (ARL) is close to its target value. The control chart is evaluated by obtaining the ARL values when the process is out-of-control for various values of the shift coefficient. We obtained the coefficients of the control limit and the truncation coefficient for different sample sizes and average run length target values. An example on the application of the proposed control chart is provided.

1. Introduction

Control charts are wildly used in monitoring the production process as they are useful for detecting unusual sources of variability during the production process where an investigation and a corrective action is taken to remove avoidable sources of variation and reduce the variability which leads to improving the quality of the product based on the given quality specifications. There are two sources of variation in a production process: natural or unavoidable variability and an avoidable variability like problems in machines or raw materials. The process is deemed out-of-control if the points fall outside the control limits that are calculated based on a mathematical formula or if a nonrandom pattern is detected. The majority of the current control charts assume normality, however some quality characteristics may not be normally distributed. There are several research on control charts where the process does not satisfy the normality assumption including Chang and Bai (2001), Al-Oraini and Rahim (2002), Chan and Cui (2003), Schoonhoven and Does (2009), Riaz et al. (2015) and Hwang (2021) among many others.

2. Significance of the study

In this research, we develop a control chart for monitoring the number of observed defects in a product whose lifetime follows the Inverse Weibull (IW) distribution under truncated life testing. An advantage of attribute control charts is that the expensive and time-consuming measurements that are taken for variable control charts are avoided. The np control chart is an attribute control chart in which the number of defective or nonconforming units is observed for each sample. Several studies have been done on the np control chart. Rodrigues et al. (2011), found the optimization design for the control charts by making use of double sampling. Aslam and Jun (2015) proposed np control chart for truncated life tests when the units have a Weibull distribution. Other studies have considered attribute control charts for various distributions under life test truncation including Aslam et al. (2016), Aslam et al. (2017), Shruthi and Deepa (2018), Ambreen et al. (2018), Rao (2018), Gadde et al. (2019), Gadde and Paul (2020), Adeoti and Ogundipe (2020). The work on this paper extends the previous work to the IW model.

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The IW distribution, sometimes referred to as the Frechet distribution, is a flexible distribution that can be used to describe lifetime data. It provides an alternative to the Weibull distribution that is useful when the hazard function of the lifetime random variable is nonmonotone, a situation which can't be suitably modeled by the Weibull distribution. Keller and Kamath (1982) proposed it as a useful model for explaining the deterioration of components in diesel engines. It is useful for modeling in a variety of situations, including infant mortality, and product useful life (Alkarni et al., 2020).

The reliability and quality control literature on the IW distribution contains some contributions to acceptance sampling and analysis of censored data, see, for example, Calabria and Pulcini (1994), Kundu and Howlader (2010), Singh and Tripathi (2016, 2017). However, the developments in control charts are devoted mostly to the Weibull distribution. This paper is concerned with developing attribute control charts for the IW distribution.

3. Design of the control chart

Let the lifetime T of the units follow the IW distribution with probability density function (pdf) given by

$$f(t)=\frac{\alpha}{\theta^{\alpha}}t^{-(\alpha+1)}e^{-(\theta t^{-\alpha})}, t>0, \alpha>0, \theta>0,$$

where α and θ are the shape and the scale parameters. The shape parameter α is assumed known. This assumption may follow from engineering experience we may use estimators from previous studies.

Now we will introduce a reparameterization that will simplify the derivations later. Let $\theta=\left(\frac{1}{\Lambda}\right)^{\frac{1}{\alpha}}$. Solving for Λ, we get $\Lambda=\frac{1}{\theta^{\alpha}}$. The pdf and the cumulative distribution function (CDF) expressed in terms of Λ are given by

$$f(t)=\alpha\Lambda t^{-(\alpha+1)}e^{-(\Lambda t^{-\alpha})}, t>0, \alpha>0, \Lambda>0, \quad (1)$$

$$F(t)=e^{-(\Lambda t^{-\alpha})}, t>0, \alpha>0, \Lambda>0. \quad (2)$$

The IW distribution approaches the inverse exponential distribution when $\alpha=1$ and it approaches the Rayleigh distribution when $\alpha=2$. The hazard function is constant when $\alpha=1$ and increasing when $\alpha>1$ where the distribution can be used to model wear out failures. However it is continuously decreasing When $\alpha<1$, and the distribution can be used to model the early failures. The IW distribution can also be used studies concerning reliability maintenance (Khan et al., 2008).

The IW distribution can be transformed to the Weibull distribution by letting $x=\frac{1}{t}$ in Eqs. (1) and (2). It can be noticed that the pdf of the Weibull model is either monotonically decreasing $\alpha\leq 1$ or unimodal when $\alpha>1$, whereas the pdf of the IW distribution is unimodal $\alpha>1$ (Murthy et al., 2003).

3.1. The mean life

The lifetimes of the units follow the IW distribution with a mean life defined as

$$\mu=\Lambda^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \quad (3)$$

Where Γ is the gamma function defined as $\Gamma(\delta)=\int_0^{\infty}x^{\delta-1}e^{-x}dx, \delta>0$.

The mean in Eq. (3) in is undefined when $\alpha=1$ therefore, the shape parameter value must be greater than 1 for the mean life to be defined.

The scale parameter Λ is unknown, however it can be expressed in terms of μ as shown below

$$\Lambda=\left(\frac{\mu}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha} \quad (4)$$

3.2. Truncation time

The truncation time t_0 arise from time saving considerations in life testing. Time-censoring (type I) scheme is usually adopted. The censoring "truncation" time is denoted by t_0. In this process, the times of the experiment are fixed in advance and the item is considered "defective" if it failed before t_0. The number of defectives is plotted on the control chart to monitor the process, see AL-Marshadi et al. (2021)." In the simulation study, for each generated subgroup, we count the number of failures whose lifetimes are $\leq t_0$, where the truncation time is determined by

$$t_0=a\mu_0 \quad (5)$$

where t_0 is the truncated time, a is the truncation coefficient and μ_0 is the desired mean life of the product.

The probability of failure before t_0 is given by

$$p=e^{-(\Lambda t_0^{-\alpha})} \quad (6)$$

Substituting the value of Λ and t_0 with the formulas given in Eq. (4) and Eq. (5), we get

$$p=e^{-\left(\left(\frac{\mu}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha}(a\mu_0)^{-\alpha}\right)} \quad (7)$$

Using Eq. (7), the probability of failure of a product when the process is in control is

$$p_0=e^{-\left(\left(\frac{\mu_0}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha}(a\mu_0)^{-\alpha}\right)} \quad (8)$$

When the mean shifts to μ_1, the probability of failure of a product becomes

$$p_1=e^{-\left(\left(\frac{\mu_1}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha}(a\mu_0)^{-\alpha}\right)} \quad (9)$$

Let $\mu_1=f\mu_0$, where f is the shift coefficient, then

$$p_1=e^{-\left(\left(\frac{f\mu_0}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha}(a\mu_0)^{-\alpha}\right)} \quad (10)$$

We are interested in observing the number of failures D in each sample. If the units have failure times that are less than the truncation time t_0, then the units are considered nonconforming or defective. The number of failures in a sample of units have a binomial distribution with parameters n and p where p is given in Eq. (6). The binomial probability mass function is given by

$$p(D=x)=\binom{n}{x}p^x(1-p)^{n-x}x=0,1,2,3,.. \quad (11)$$

The lower and upper control limits are given by

$$LCL=\max\left(0, np_0-k\sqrt{np_0(1-p_0)}\right) \quad (12)$$

$$UCL = np_0 + k\sqrt{np_0(1-p_0)} \tag{13}$$

where k is the coefficient of the control limits and p_0 is the probability of "nonconforming" when the process is in control.

The probability of stating that the process is in control when it is truly in control is obtained using the binomial distribution formula in Eq. (11)

$$p_{in}^0 = p(LCL \le D \le UCL|p_0) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} {p_0}^d (1-p_0)^{n-d} \tag{14}$$

The probability of stating that the process is in control when the mean life of the product has shifted to μ_1 is given by

$$p_{in}^1 = p(LCL \le D \le UCL|p_1) = \sum_{d=LCL+1}^{UCL} \binom{n}{d} {p_1}^d (1-p_1)^{n-d} \tag{15}$$

The ARL is the mean of the geometric random variable representing the number of samples needed to obtain a sample out – of – control. It follows from the values p_{in}^0 and p_{in}^1 given in Eqs. (14) and (15) that the in-control ARL is given by

$$ARL_0 = \frac{1}{1-p_{in}^0}$$

and the out-of -control ARL are given by

$$ARL_1 = \frac{1}{1-p_{in}^1}$$

Note that, from Eq. (8), the value of μ_0 does not affect the value of p_0 when the process is in-control and hence it does not affect the average run length. However, if the process is not in-control, Eqs. (9) and (10) show that p_1 and the average run length depend on μ_0 through the ratio $f = \mu_1/\mu_0$. The smaller the ratio, the larger is p_1 and, consequently, the smaller is the average run length.

4. Simulation study

A simulation study was conducted where samples of units of size n = 25 and n = 30 were generated for 40 subgroups where the units' failure times follow the IW distribution with α = 1.1, α = 2 and α = 3, unknown scale parameter and a target mean life $\mu_0 = 1000$. The values of the truncation coefficient (a) and the control chart coefficient (k) were determined based on three target values: $r_0 = 260$, $r_0 = 370$ and $r_0 = 470$ for the in-control ARL.

The ARL_1 values are obtained for different mean values with shift coefficient values f as shown in Tables 1 and 2 below.

The ARL value is obtained as the average (over all simulation replications) of the run length of samples in-control before obtaining the first out-of-control sample.

5. Analysis of the simulation results

From the two tables above, we observe that the out-of-control ARLs are small when there is a small shift in the mean and the values increase as the mean shift increases. We noted that the simulated values of the ARL are very close to the target values when the shift parameter $f = 1$, this case corresponds to the in-control situation.

It was found that the ARL values are larger when the shift parameter f has smaller values. This is because smaller values of the shift parameter mean larger difference between the process and the target mean and therefore smaller ARL is observed for detecting shift in the process mean from the target.

We noted also that larger values of α result in a smaller value of ARL_1. That is, the control chart becomes more sensitive to deviations from the target mean when the shape parameter is large.

It appears that larger values of k, result in larger values of ARL_1. This is expected because, if we consider the role of k in Eqs. (12) and (13), we see that larger of values of k give wider intervals for the in-control status and this increases the possibility that the process stays in-control, thus giving a larger average run length.

It appears that larger values of (a) result in a faster decrease ARL_1. This is because values of the truncation constant a are chosen such that ARL_0 equals a predetermined constant r_0. It is related to the truncated test time t_0 and the desired mean life of the product μ_0 by the relation $t_0 = a\mu_0$. The larger the value of a, the longer is the test time t_0 compared to μ_0 and hence a larger probability of failure during the life test which implies a shorter run, especially when the process is out of control.

6. An illustrative example

To implement an industrial application of the proposed control chart, we assume that the lifetime of the electronic product in which the producer wants to improve its quality follows the IW distribution with $\alpha = 1.1$ for example. The product's target mean life is assumed to be $\mu_0 = 1000$ h and the target in control ARL $r_0 = 260$. From Table 2, n = 30, $a = .130$, $k = 3$ and $p_0 = 0.49$ from Eq. (8). The control limits based on Eq.

Table 1. ARL_1 of the chart for IW distributed lifetimes with $\mu_0 = 1000$ h when n = 25.

n = 25									
	α = 1.1			α = 2			α = 3		
	a = .127	a = .131	a = .121	a = .45	a = .454	a = .46	a = .630	a = .637	a = .642
	k = 3	k = 3.1	k = 3	k = 3.4	k = 3.5	k = 3.5	k = 4.5	k = 4.1	k = 5
r_0	260	370	470	260	370	470	260	370	470
Shift (f)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
.1	1.01	1.01	1.00	1	1	1	1	1	1
.2	1.19	1.17	1.08	1	1	1	1	1	1
.3	1.84	1.75	1.45	1.00	1.00	1.00	1	1	1
.4	3.66	3.33	2.48	1.00	1.00	1.00	1.00	1.00	1.00
.5	8.85	7.65	5.26	1.04	1.08	1.06	1.00	1.00	1.02
.6	24.86	20.38	13.20	1.33	1.57	1.50	1.09	1.08	1.33
.7	78.30	60.80	37.93	2.83	4.19	3.72	2.06	1.89	3.98
.8	257.20	193.92	120.42	11.98	23.07	18.65	12.93	10.36	46.46
.9	510.07	479.8	365.22	101.92	251.77	186.9	388.82	261.72	2477.05
1	271.83	360.07	470.54	284.63	379.86	471.14	261.74	365.07	471.34

Table 2. ARL_1 of the chart for IW distributed lifetimes with $\mu_0 = 1000$ h when n = 30.

n = 30									
	α = 1.1			α = 2			α = 3		
	a = .130	*a* = .133	*a* = .136	*a* = .43	*a* = .456	*a* = .444	*a* = .615	*a* = .641	*a* = .63
	k = 3	*k* = 3.1	*k* = 3.1	*k* = 3	*k* = 3	*k* = 3	*k* = 3	*k* = 3	*k* = 3
r_0	260	370	470	260	370	470	260	370	470
Shift (*f*)	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
.1	1.00	1.00	1.00	1	1	1	1	1	1
.2	1.05	1.12	1.11	1	1	1	1	1	1
.3	1.33	1.64	1.59	1	1	1	1	1	1
.4	2.19	3.22	3.00	1.00	1.00	1.00	1	1	1
.5	4.60	7.95	7.13	1.00	1.00	1.01	1.00	1.00	1.00
.6	11.79	23.64	20.28	1.10	1.09	1.13	1.00	1.00	1.00
.7	35.38	81.01	66.46	1.80	1.68	1.95	1.07	1.06	1.09
.8	118.68	301.57	239.60	6.14	5.04	7.09	2.10	1.89	2.24
.9	329.12	721.79	678.45	49.00	33.51	59.82	17.84	12.53	19.69
1	253.26	365.86	463.08	263.84	372.72	467.00	258.28	369.35	467.33

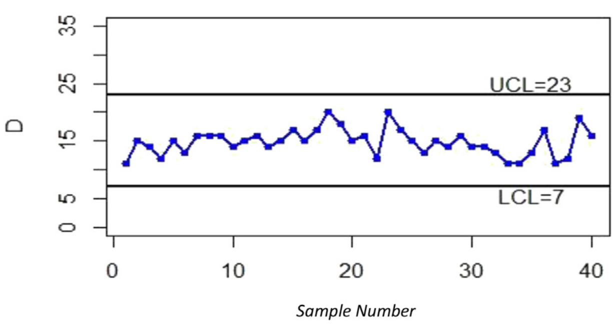

Figure 1. Proposed control chart for IW distributed lifetimes with $\alpha = 1.1$, $\mu_0 =$ 1000 hours and sample size n = 30.

(12) and Eq. (13) are given by LCL = 7 and UCL = 23. The inspection time is 130 h.

The proposed procedure is as follows

I. 40 subgroups each having a sample of size 30 are put on a life test for 130 h and the number of defective units D is plotted as shown in Figure 1 below. The unit is considered defective if it fails in less than 130 h.
II. The process is in-control if $7 \leq D \leq 23$. Otherwise, it is out-of-control.

7. Conclusions and further research

We proposed a control chart that monitor the quality of products having IW distributed lifetimes where the lifetimes are truncated to save the test monitoring time. The proposed control chart is then evaluated by the ARL values obtained through a simulation study where different values of the sample size, shape parameter and target in-control ARL have been considered. The implementation of this control chart has been explained through an illustrative example based on a typical industrial application. For further research, one can consider applying the proposed control charts on other important lifetime distributions. Another possibility is to consider accelerated testing designs and develop suitable control charts for such situations. As another area of further development, we may consider situations in which the duration of the production run is short or some sources are fixed and limited, see Tran et al. (2021). In this case a modified, short run, control chart is needed, and its performance can be assessed by using the truncated average run length.

Declarations

Author contribution statement

Ayman Baklizi, Sawsan Abu Ghannam: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

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Data availability statement

No data was used for the research described in the article.

Declaration of interest's statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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