Timetabling hub-and-spoke parcel distribution inter-facility network

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Abstract: This paper introduces an important problem encountered when designing hub-and-spoke parcel distribution network. This problem, which comes after generating the network design and before implementing it, is concerned with parcel timetabling. A non-linear model, composed of the shipping-time objective function and the time-precedence constraints, is developed in order to optimise the daily movements of parcels from their origins to their destinations. The resolving of the problem highly depends on the ability of the analyst to develop intelligent ways to limit the values of decision variables. The model applied to a real-world case is entered in a spreadsheet where the objective function is evaluated for the possible decision variable values; the timetable generated for a newly designed hub-and-spoke network decreased shipping time by around 30% as compared to the timetable of the existing design.

Keywords: distribution; network design; timetabling; scheduling; transport; parcel; hub; inter-facility; operations.

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1 Introduction

Parcel distribution companies have one principal activity in common, namely collecting parcels from origin customers and delivering them to destination customers. Since the quantity shipped from one origin customer to one destination customer is usually less than vehicle load, parcels have to be consolidated in certain distribution facilities (Iyer and Ratliff, 1990; Klose and Drexl, 2005; ReVelle et al., 2008; Lin and Chen, 2008). Parcel distribution involves two major problems:

1 network design problem

2 timetabling problem.

This paper is concerned with the second problem whose major input is the solution generated by the first one.

Hub-and-spoke paradigm, which is the most common consolidation system in parcel distribution, was pioneered in 1955 by Delta Air Lines at its hub in Atlanta, Georgia (Delta, 2008), in an effort to compete with Eastern Air Lines. Although the problem was addressed very early by Goldman (1969), the research on hub location actually began with the original work of O'Kelly (1987) and evolved with such contributions as those of Campbell (1994), Aykin (1995), Ernst and Krishnamoorthy (1998), Kara and Tansel (2001). More recent works include Marín (2005), Yaman and Carello (2005), Campbell et al. (2007), Cánovas et al. (2007), Wagner (2007), Alumur and Kara (2008), and Costa et al. (2008).

Parcel distribution is one of the most important areas of application of hub location problem. Related works include Chan and Ponder (1979), Hall (1989), Iyer and Ratliff (1990), Kuby and Gray (1993), Min (1996), Bruns et al. (2000), Lin (2001), Gunnarsson et al. (2006), Jeong et al. (2006), Cunha and Silva (2007), Yaman et al. (2007), and Lin and Chen (2008). However, all these works deal with a single level of facilities whereas parcel distribution processes involve two levels, namely the stations and the hubs (see next section). Few papers have started dealing with locating more than one level of facilities (Wasner and Zäpfel, 2004; Thomadsen and Larsen, 2007; Yaman, 2009; Ben-Ayed, 2010, 2012b).

All the literature listed above is related to parcel distribution network design problem (PDNDP), which is concerned with the location and allocation of the facilities in order to minimise shipping cost. Although the PDNDP is a necessary phase that precedes the parcel distribution timetabling problem (PDTP), the two problems are quite different. PDTP, which is the object of this paper, is not concerned with cost minimisation; it rather abides by the resources allocated by the PDNDP to minimise the time needed by the shipments in their journey from their origins to their destinations. The decision variables of the PDTP are the daily occurrence times of the shipment movements between the facilities; the location and types of these facilities, as well as the numbers and capacities of vehicles connecting them, are provided by the PDNDP.

Unlike PDNDP, which has been extensively addressed in the literature, PDTP is a new problem hardly known despite its theoretical and practical importance. Certainly, companies like UPS, FedEx and DHL are continuously working in their research centres on solving this problem. Without an efficient and effective timetabling, it would be impossible for UPS for example to deliver more than 15 million packages a day to 6.1 million customers in more than 200 countries and territories around the world (UPS, 2010). However, there is no evidence that such large companies are developing operational research models in generating their timetables.

The PDTP can be considered as service network design problem (SNDP), which is defined by Crainic and Kim (2007) as 'the movements through space and time of the vehicles and convoys'. SNDP involves four common sub-problems (Crainic and Rousseau, 1986; Crainic and Laporte, 1997; Grünert and Sebastian, 2000), namely:

1. selection of routes
2. traffic distribution
3. establishment of terminal policies
4. empty balancing.

PDTP can be considered as a fifth sub-problem seeing that it does not seem to fit any of the four sub-problems above. More details about SNDP can be found in Crainic (2000), Crainic and Kim (2007), and Wieberneit (2008).

Until 2011 there was no single paper in the literature that explicitly addresses the PDTP. However, there are some works that may look similar to our problem while most of them are actually unrelated. One example is the vehicle and crew scheduling problem (Freling et al., 2000, 2001; Hollis et al., 2006; Kéri and Haase, 2007; Mesquita and Paias, 2008); the timetable in this problem is an input rather than output. Another example is parcel hub scheduling problem (McWilliams et al., 2005, 2008; McWilliams, 2009); this problem, which is concerned with intra-rather than inter-facility movements, is specifically concerned with the assignment of trailers to dock doors within the hub.

The model on the latest arrival hub location problem (LAHLP), addressed by Kara and Tansel (2001), Yaman et al. (2007) and Tansel and Kara (2007), has one similarity with PDTP: it explicitly includes the decisions concerning the arrival and departure times at each node. However, LAHLP is more concerned with the design of the network than with its scheduling. The absence of clock times in the model imposes some limitations to its application in parcel scheduling, especially when shipping times exceed 24 hours:

1. it cannot take into account the daily occurrence of the events
2. it cannot include time windows
3. it cannot give different priorities to the different pairs of customers; it can only treat all pairs equally.

This paper, which aims at providing an optimisation approach to the PDTP, is organised in five sections. The next one defines the problem. Section 3 presents and discusses the construction of the model. The fourth section illustrates the application of the model to a real-world case. The paper is concluded in the last section with some recommendations for future extensions.

2 Problem definition

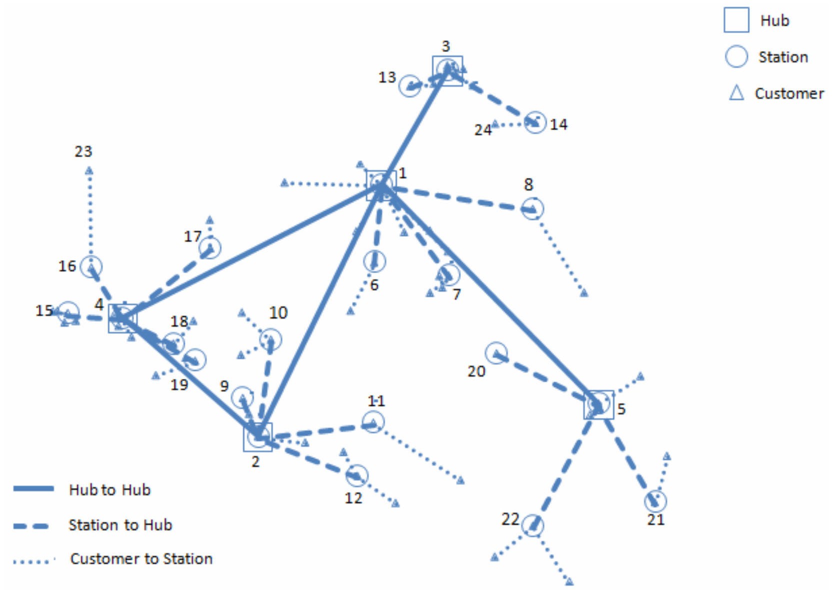
The definition of the PDTP includes the description of the process as well as the explanation of time input data and related assumptions. This section relies on both Table 1 and Figure 1. The table defines the notation of the problem, while the figure shows a hub-and-spoke network that is used to explain and illustrate the concepts. The figure shows the network design of the real-world application that will be discussed in Section 4.

Parcels collected from origin customers are typically moved by parcel distribution companies through two types of nodes, namely stations and hubs, before they are delivered to destination customers. A station, also called terminal, depot or satellite, is a facility usually located in a large city and servicing that city in addition to neighbour towns and villages. As the load between two stations is usually not large enough to justify the use of a dedicated vehicle, neighbour stations are usually assigned to a higher-level facility called hub.

Outbound shipments, collected by couriers from origin customers, are consolidated in origin stations before they are sent to origin hubs, where they are sorted and forwarded to destination hubs. The route from an origin hub to a destination hub is assumed to be a two-link path including a transit hub; when the origin and destination hubs are directly connected, the transit hub is the same as the origin hub. Shipments received by destination hubs, called inbound shipments, are sorted according to their destination stations. At each destination station, the shipments destined to a group of neighbour destination customers are loaded on a van in order to be delivered to their recipients.

Figure 1 shows a hub-and-spoke parcel distribution network composed of 66 nodes. Nodes labelled from 1 to 5 are simultaneously customers, stations and hubs; those labelled from 6 to 22 are customers and stations; and those labelled 23 and 24 are just customers. The remaining 42 nodes are similar to nodes 23 and 24, but they are not labelled to avoid encumbering the figure. The sets of customers, stations and hubs, as obtained from the figure, can be defined, respectively, as: $\mathcal{I} = \{1, 2,\ldots,66\}$, $\mathcal{J} = \{1, 2,\ldots,22\}$ and $\mathcal{K} = \{1, 2,\ldots,5\}$. The PDTP is concerned with the daily times of the movements of the parcels between the different nodes of the network. It is important to notice that the customers in the model are whole cities and not individual customers.

Figure 1 COMP's hub-and-spoke design (see online version for colours)



The most important input data of PDTP is the network design, which can be described by the assignment sets $\mathcal{D}^{\text{CS}}$, $\mathcal{D}^{\text{SH}}$ and $\mathcal{D}^{\text{CSH}}$, and the hub-to-hub routes set $\mathcal{D}^{\text{HHH}}$ (as defined in Table 1); to illustrate, referring to Figure 1, (3, 3), (24, 14) $\in \mathcal{D}^{\text{CS}}$; (3, 3), (14, 3) $\in \mathcal{D}^{\text{SH}}$; (3, 3, 3), (24, 14, 3) $\in \mathcal{D}^{\text{CSH}}$; and (3, 1, 4), (3, 3, 1) $\in \mathcal{D}^{\text{HHH}}$. To each origin customer i_1 corresponds one and only one triplet $(i_1, j_1, k_1) \in \mathcal{D}^{\text{CSH}}$; to each destination customer i_2 corresponds one and only one triplet $(i_2, j_2, k_2) \in \mathcal{D}^{\text{CSH}}$; and to the origin-destination pair of hubs (k_1, k_2) corresponds one and only one triplet $(k_1, k_0, k_2) \in \mathcal{D}^{\text{HHH}}$. The three triplets form the unique route, denoted by the seven-tuple $(i_1, j_1, k_1, k_0, k_2, j_2, i_2)$, from the origin customer i_1 to the destination customer i_2, with j_1 being the origin station, k_1 the origin hub, k_0 the transit hub, k_2 the destination hub, and j_2 the destination station. For example, the route from customer 24 to customer 23, in Figure 1, is (24, 14, 3, 1, 4, 16, 23), and that from customer 3 to customer 3 is (3, 3, 3, 3, 3, 3, 3).

Table 1 Input data and decision variables

Network design	$\mathcal{I}$	Set of nodes, which is the set of customers and also the set of cities.
	$\mathcal{J}$	Set of stations; $\mathcal{J} \subseteq \mathcal{I}$.
	$\mathcal{K}$	Set of hubs; $\mathcal{K} \subseteq \mathcal{I}$.
	$\mathcal{D}^{\text{CS}}$	Set of couples (i, j) with customer i being assigned to station j.
	$\mathcal{D}^{\text{SH}}$	Set of couples (j, k) with j being a station assigned to hub k.
	$\mathcal{D}^{\text{CSH}}$	Set of triplets (i, j, k) with $(i, j) \in \mathcal{D}^{\text{CS}}$ and $(j, k) \in \mathcal{D}^{\text{SH}}$.
	$\mathcal{D}^{\text{HHH}}$	Set of triplets (k_1, k_0, k_2) with 1 $k_1 = k_0 = k_2$ when the origin and destination hubs k_1 and k_2 are the same 2 $k_1 = k_0 \neq k_2$ when k_1 and k_2 are directly connected 3 $k_1 \neq k_0 \neq k_2$ otherwise.
Time data	$\lambda_{i_1 i_2}$	The economic importance to the company of the origin-destination pair (i_1, i_2)
	$\rho_{i_1}^{\text{OC}}, \rho_{j_1}^{\text{OS}}, \rho_{k_1}^{\text{OH}}, \rho_{k_0}^{\text{TH}}, \rho_{k_2}^{\text{DH}}, \rho_{j_2}^{\text{DS}}$, and $\rho_{i_2}^{\text{DC}}$	Processing times at origin customer (OC) i_1, origin station (OS) j_1, origin hub (OH) k_1, transit hub (TH) k_0, destination hub (DH) k_2, destination station (DS) j_2, and destination customer (DC) i_2, respectively.
	$\tau_{ii'}$	Travel time between nodes i and i' on the hub-and-spoke parcel distribution network; a node can be a customer, a station or a hub.
	$[\alpha_i, \beta_i]$	Time window of customer i.
Decision variables	$x_{i_1 j_1}^{\text{CS}}$	Time at which courier ends pickup at customer i_1 and leaves to station j_1.
	$x_{j_1 k_1}^{\text{SH}}$	Time at which shipments leave station j_1 to hub k_1.
	$x_{kk'}^{\text{HH}}$	Time at which shipments leave hub k to hub k'; such a movement can be from an origin hub to a transit hub and/or from a transit to a destination hub.
	$x_{k_2 j_2}^{\text{HS}}$	Time at which shipments leave hub k_2 to station j_2.
	$x_{j_2 i_2}^{\text{SC}}$	Time at which courier leaves station j_2 to customer i_2.
	$u_{k_1}^{\text{OH}}$	Time at which sorting is ended at origin hub k_1.
	$u_{k_2}^{\text{DH}}$	Time at which sorting is started at destination hub k_2.

All time input data are denoted by Greek symbols to distinguish them from decision variables. There are three types of time input data: processing times, travel times and time window (see Table 1). The parcels incur a processing time at each of the seven nodes of the route (i_1, j_1, k_1, k_0, k_2, j_2, i_2); these processing times are denoted by $\rho_{i_1}^{\mathrm{OC}}$, $\rho_{j_1}^{\mathrm{OS}}$, $\rho_{k_1}^{\mathrm{OH}}$, $\rho_{k_0}^{\mathrm{TH}}$, $\rho_{k_2}^{\mathrm{DH}}$, $\rho_{j_2}^{\mathrm{DS}}$ and $\rho_{i_2}^{\mathrm{DC}}$, respectively.

In addition, between every two nodes i_1 and i_2 corresponds a travel time $\tau_{i_1 i_2}$ calculated with respect to the parcel distribution network. The travel time between the nodes 24 and 8 in Figure 1, for example, is that of the path 24-14-3-1-8, which is a long way although the two nodes are very close to each other and there may be physically a road connecting them. The travel time is the same as the one provided by a road map only when the two nodes are directly connected in the parcel distribution network; this is the case for (24, 14), (14, 3), (3, 1) and (1, 8).

Finally, there is an interval of time at which customer i can be serviced; this interval, referred to as time window, is denoted by $[\alpha_i, \beta_i]$, with α_i and β_i being two clock times. In this paper, the time displayed by a clock is converted to a number; for example midnight is the number zero, 23:00 = 11:00 pm is the number 23, and 5:30 = 5:30 am is the number 5.5. Clock times, including α_i and β_i, are thus assumed to belong to the interval [0, 24).

Four assumptions are imposed on time input data:

1 The pickup and delivery at each city i take place on the same calendar day; i.e., they start after midnight of the current day and end before midnight of the following day. In other words, the time window for each customer i is a single interval $[\alpha_i, \beta_i]$. This is verified when the pickups and delivery take place from 7 to 20, with a time window of [7, 20]; but not from 10 to 2 where the time window would be the union of the two intervals [10, 24) and [0, 2].

2 There are enough couriers to complete all pickups and deliveries of each day. In other words: $\rho_i^{\mathrm{OC}} + \rho_i^{\mathrm{DC}} < \beta_i - \alpha_i$. This assumption prevents the accumulation of uncollected and/or undelivered parcels.

3 A customer being a whole city, intra-city travel time is part of the pickup/delivery activity, and therefore the travel time between a customer and its station is also part of the pickup/delivery time whenever the customer and the station both correspond to the same city.

4 One arrival to and one departure from every node in the route (i_1, j_1, k_1, k_0, k_2, j_2, i_2) are taking place every day. Therefore, the time that a parcel spends in any of the nodes never exceeds 24 hours.

3 Model construction

In this section, we define the decision variables, the objective function and the constraints of the PDTP. We also provide the entire optimisation formulation and use an illustrative example to show the capabilities of the model.

3.1 Time decision variables

The PDTP consists of determining the daily clock times at which the parcel arrives to and departs from each of the nodes of the route (i_1, j_1, k_1, k_0, k_2, j_2, i_2). The movements of a parcel from its origin customer to its destination customer can be tracked by two types of events: departure and arrival. Let us consider three successive events:

1 departure from a previous node

2 arrival to a current node

3 departure from the current node.

The time of the first event determines that of the second one; e.g., when the parcel leaves i_1 at $x_{i_1 j_1}^{\mathrm{CS}}$ it will arrive to j_1 exactly after $\tau_{i_1 j_1}$ hours. However, the time of the second event cannot determine that of the third event because of the eventual waiting time in the node; e.g., the parcel that arrives to station j_1 is ready to leave after $\rho_{j_1}^{\mathrm{OS}}$ hours, but it does not necessarily leave as soon as it is ready to leave (it is common to have shipments wait at stations and hubs for other shipments).

Since the arrival time to any node can be obtained from the departure time of the previous node, the decision variables can be limited to leaving time variables. There are five groups of decision variables corresponding to five types of departures:

1 $x_{i_1 j_1}^{\mathrm{CS}}$, from customer to station

2 $x_{j_1 k_1}^{\mathrm{SH}}$, from station to hub

3 $x_{kk'}^{\mathrm{HH}}$, from hub to hub including $x_{k_1 k_0}^{\mathrm{HH}}$ and $x_{k_0 k_2}^{\mathrm{HH}}$

4 $x_{k_2 j_2}^{\mathrm{HS}}$, from hub to station

5 $x_{j_2 i_2}^{\mathrm{SC}}$, from station to customer

Moreover, there are two more groups of variables that are assigned to each hub *k*. The first one u_k^{OH} is the time of completing outbound sorting at hub *k*, which is also the earliest time at which shipments can depart from *k* to the other hubs; *k* in this case is an origin hub. The second group of variables u_k^{DH} is the time of starting inbound sorting at hub *k*, which is also the latest time at which shipments arrive to *k* from the other hubs; *k* in this case is a destination hub.

All decision variables are clock times converted to numbers belonging to the interval [0, 24), as explained at the end of the previous section. Moreover, all events take place on a daily basis and at the same time; for example, deciding that the parcels from station 13 to hub 3 leave at 19:00 (19 with our notation) means that such a decision applies to all working days of the week.

3.2 Shipping-time objective function

The total shipping time from an origin customer i_1 to a destination customer i_2 is the total time spent on the route (i_1, j_1, k_1, k_0, k_2, j_2, i_2). Such time involves three components, namely:

1. the travel time $\tau_{i_1 i_2}$ on the route
2. the processing time $\rho_{i_1 i_2} = \rho_{i_1}^{\mathrm{OC}} + \rho_{j_1}^{\mathrm{OS}} + \rho_{k_1}^{\mathrm{OH}} + \rho_{k_0}^{\mathrm{TH}} + \rho_{k_2}^{\mathrm{DH}} + \rho_{j_2}^{\mathrm{DS}} + \rho_{i_2}^{\mathrm{DC}}$ at the nodes of the route
3. the waiting time at the nodes.

The first two components are known values (input data); but the third one is a dependent decision variable that we denote by $z_{i_1 i_2}$. The waiting time is the time that a shipment spends at a node in excess of the processing time; it can also be defined as the time elapsing from the instant at which the parcel is ready to depart until the instant at which it departs.

The time elapsing from a clock time x_1 to another clock time x_2 is the difference $x_2 - x_1$ only when $x_2 - x_1 \geq 0$; for example, the time elapsing from 15 to 22 is 22 – 15 = 7 hours, but that from 22 to 15 is not 15 – 22. Since the events of the timetable are repeated every 24 hours, the time spent in a facility (processing plus waiting) never exceeds 24 hours; thus we can use the function mod(x, 24) to find the time spent in a facility based on the time of entering the facility and the time of leaving it. The function mod(x, 24) is defined as:

$$\mathrm{mod}(x, 24) = x - 24\left\lfloor \frac{x}{24} \right\rfloor$$

where $\left\lfloor \frac{x}{24} \right\rfloor$ is the largest integer less than or equal to $\frac{x}{24}$ (Cormen et al., 2001). The function mod($x' - x$, 24) provides the time elapsing from the clock time x until the clock time x'; for example, when the parcel enters the facility at 22 and leaves it at 15, the time it spends at this facility is provided by mod(15 – 22, 24) = –7 – 24 × (–1) = 17 hours. This is valid only when the two events take place within the same 24 hours, which is guaranteed by assumption 4 in Section 2. The same function can be applied to find the clock time x' that exceeds or precedes another clock time x by t hours; $x' = \mathrm{mod}(x + t, 24)$ with t being any real number; the clock time exceeding 22 by 3 hours is mod(22 + 3, 34) = 1, and that preceding 1 by 3 hours is mod(1 – 3, 24) = 22.

The objective of the PDTP is to minimise the averaged shipping time for all origin-destination pairs of customers. Each pair is weighted by its economic importance to the company (the revenue is a possible weight); we denote such a weight by $\lambda_{i_1 i_2}$ The objective function can therefore be formulated as: min $\sum_{i_1 \in \mathcal{I}} \sum_{i_2 \in \mathcal{I}} \lambda_{i_1 i_2} \times \left(\tau_{i_1 i_2} + \rho_{i_1 i_2} + z_{i_1 i_2}\right)$

The quantities $\tau_{i_1 i_2}$ and $\rho_{i_1 i_2}$ being constant, the objective function can be equivalently written as:

$$\min \sum_{i_1 \in \mathcal{I}} \sum_{i_2 \in \mathcal{I}} \lambda_{i_1 i_2} z_{i_1 i_2} \tag{1}$$

Without loss of generality, we can assume that the waiting time is at the facilities only; the arrival to the origin station and the departure from the destination station can be chosen in such a way that there is no waiting at the customer. For each origin-destination pair (i_1, i_2), the variable $z_{i_1i_2}$ is therefore the sum of five waiting times, namely:

1 at the origin station $(z_{j_1}^{\text{OS}})$

2 at the origin hub $(z_{k_1}^{\text{OH}})$

3 at the transit hub $(z_{k_0}^{\text{TH}})$

4 at the destination hub $(z_{k_2}^{\text{DH}})$

5 at the destination station $(z_{j_2}^{\text{DS}})$.

The objective function (1) can therefore be written as:

$$\min \sum_{(i_1,j_1,k_1)\in\mathcal{D}^{CSH}} \sum_{(i_2,j_2,k_2)\in\mathcal{D}^{CSH}} \sum_{(k_1,k_0,k_2)\in\mathcal{D}^{HHH}} \lambda_{i_1i_2} \times \left(z_{j_1}^{OS} + z_{k_1}^{OH} + z_{k_0}^{TH} + z_{k_2}^{DH} + z_{j_2}^{DS} \right) \quad (2)$$

Each of the five waiting times is the time elapsing from the instant at which the parcel is ready to leave the facility until the instant at which it leaves it. To illustrate, let us focus on $z_{j_1}^{\text{OS}}$. The arrival time to origin station j_1 is $\text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1}, 24)$ and the processing time at that station is $\rho_{j_1}^{\text{OS}}$. The time at which the shipment is ready to leave the station is therefore $\text{mod}(\text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1}, 24) + \rho_{j_1}^{\text{OS}}, 24) = \text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24);$ this equality is possible because $\text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24)$ and $\rho_{j_1}^{\text{OS}}$ have the same sign (both positive). The actual departure time being $x_{j_1k_1}^{\text{SH}}$, the waiting time $z_{j_1}^{\text{OS}}$ at the station j_1 is the time elapsing from $\text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24)$ to $x_{j_1k_1}^{\text{SH}}$, i.e.:

$$z_{j_1}^{\text{OS}} = \text{mod}\left(x_{j_1k_1}^{\text{SH}} - \text{mod}\left(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24 \right), 24 \right) \quad (3)$$

It is worth mentioning that because the quantities $-\text{mod}(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24)$ and $x_{j_1k_1}^{\text{SH}}$ have different signs:

$$\text{mod}\left(x_{j_1k_1}^{SH} - \text{mod}\left(x_{i_1j_1}^{\text{CS}} + \tau_{i_1j_1} + \rho_{j_1}^{\text{OS}}, 24 \right), 24 \right) \neq \text{mod}\left(x_{j_1k_1}^{\text{SH}} - x_{i_1j_1}^{\text{CS}} - \tau_{i_1j_1} - \rho_{j_1}^{\text{OS}}, 24 \right)$$

The same principle can be applied to the other four types of nodes:

$$z_{k_1}^{\text{OH}} = \text{mod}\left(x_{k_1k_0}^{\text{HH}} - \text{mod}\left(x_{j_1k_1}^{\text{SH}} + \tau_{j_1k_1} + \rho_{k_1}^{\text{OH}}, 24 \right), 24 \right) \quad (4)$$

$$z_{k_0}^{\text{TH}} = \text{mod}\left(x_{k_0k_2}^{\text{HH}} - \text{mod}\left(x_{k_1k_0}^{\text{HH}} + \tau_{k_1k_0} + \rho_{k_0}^{\text{TH}}, 24 \right), 24 \right) \quad (5)$$

$$z_{k_2}^{\text{DH}} = \text{mod}\left(x_{k_2j_2}^{\text{HS}} - \text{mod}\left(x_{k_0k_2}^{\text{HH}} + \tau_{k_0k_2} + \rho_{k_2}^{\text{DH}}, 24 \right), 24 \right) \quad (6)$$

$$z_{j_2}^{\text{DS}} = \text{mod}\left(x_{j_2i_2}^{\text{SC}} - \text{mod}\left(x_{k_2j_2}^{\text{HS}} + \tau_{k_2j_2} + \rho_{j_2}^{\text{DS}}, 24 \right), 24 \right) \quad (7)$$

When replacing $z^{\mathrm{OS}}_{j_1}, z^{\mathrm{OH}}_{k_1}, z^{\mathrm{TH}}_{k_0}, z^{\mathrm{DH}}_{k_2}$, and $z^{\mathrm{DS}}_{j_2}$ by their formulas obtained from (3), (4), (5), (6) and (7), respectively, the objective function (2) can be written as:

$$\min \sum_{\substack{(i_1, j_1, k_1)\in\mathcal{D}^{\mathrm{CSH}} \\ (i_2, j_2, k_2)\in\mathcal{D}^{\mathrm{CSH}} \\ (k_1, k_0, k_2)\in\mathcal{D}^{\mathrm{HHH}}}} \lambda_{i_1 i_2} \times \begin{pmatrix} \mathrm{mod}\left(x^{\mathrm{SH}}_{j_1 k_1} - \mathrm{mod}\left(x^{\mathrm{CS}}_{i_1 j_1} + \tau_{i_1 j_1} + \rho^{\mathrm{OS}}_{j_1}, 24\right), 24\right) \\ + \mathrm{mod}\left(x^{\mathrm{HH}}_{k_1 k_0} - \mathrm{mod}\left(x^{\mathrm{SH}}_{j_1 k_1} + \tau_{j_1 k_1} + \rho^{\mathrm{OH}}_{k_1}, 24\right), 24\right) \\ + \mathrm{mod}\left(x^{\mathrm{HH}}_{k_0 k_2} - \mathrm{mod}\left(x^{\mathrm{HH}}_{k_1 k_0} + \tau_{k_1 k_0} + \rho^{\mathrm{TH}}_{k_0}, 24\right), 24\right) \\ + \mathrm{mod}\left(x^{\mathrm{HS}}_{k_2 j_2} - \mathrm{mod}\left(x^{\mathrm{HH}}_{k_0 k_2} + \tau_{k_0 k_2} + \rho^{\mathrm{DH}}_{k_2}, 24\right), 24\right) \\ + \mathrm{mod}\left(x^{\mathrm{SC}}_{j_2 i_2} - \mathrm{mod}\left(x^{\mathrm{HS}}_{k_2 j_2} + \tau_{k_2 j_2} + \rho^{\mathrm{DS}}_{j_2}, 24\right), 24\right) \end{pmatrix} \tag{8}$$

3.3 *Time-precedence constraints*

The only functional constraints in the model are time-precedence constraints, which establish the mathematical relationships between the events defined by the decision variables. The departure from an origin station, for example, interacts with the arrival times from the origin customers as well as the departure times from the origin hub to the other hubs. However, since one station services multiple customers, the effect of a station (in terms of departure decisions) on a customer is higher than the effect of the customer on that station; similarly, the effect of a hub on a station is higher than the effect of the station on that hub. The most influential decisions are those related to the hubs. In the rest of this section we show how the hub-to-hub departure times $x^{\mathrm{HH}}_{kk'}$ determine all other decision variables, namely $x^{\mathrm{CS}}_{i_1 j_1}, x^{\mathrm{SH}}_{j_1 k_1}, x^{\mathrm{HS}}_{k_2 j_2}$, and $x^{\mathrm{C}}_{i_2}$.

Every vehicle from an origin station j_1 has to arrive to its assigned hub k_1 early enough to have its shipments processed before the departures of the earliest vehicle from k_1 to the other hubs. This is equivalent to saying that the shipments from j_1 have to arrive to k_1 and be processed at k_1 before the earliest departure $u^{\mathrm{OH}}_{k_1}$ from k_1 to the other hubs. This means that the departure $x^{\mathrm{SH}}_{j_1 k_1}$ from origin station j_1 to origin hub k_1 has to precede $u^{\mathrm{OH}}_{k_1}$ by $\rho^{\mathrm{OH}}_{k_1} + \tau_{j_1 k_1}$ hours; i.e.:

$$x^{\mathrm{SH}}_{j_1 k_1} = \mathrm{mod}\left(u^{\mathrm{OH}}_{k_1} - \rho^{\mathrm{OH}}_{k_1} - \tau_{j_1 k_1}, 24\right) \tag{9}$$

$u^{\mathrm{OH}}_{k_1}$ can be calculated as follows. Let $\mathcal{I}^*_{k_1}$ be the set of the important (large) cities i^*_1 serviced by the stations j^*_1 assigned to k_1, and $\pi^{\mathrm{OC}}_{i^*_1}$ be the earliest end-of-pickup time at city i^*_1. Let sup be the supremum (least upper bound) function. The processing, at hub k_1, of the outbound shipments of the important cities cannot be completed earlier than

$$\sup\left\{\pi^{\mathrm{OC}}_{i^*_1} + \tau_{i^*_1 j^*_1} + \rho^{\mathrm{OS}}_{j^*_1} + \tau_{j^*_1 k_1} + \rho^{\mathrm{OH}}_{k_1} : i^*_1 \in \mathcal{I}^*_{k_1}, \left(i^*_1, j^*_1, k_1\right) \in \mathcal{D}^{CSH}\right\};$$

the value of $u^{\mathrm{OH}}_{k_1}$ is just the corresponding clock time:

$$u^{\mathrm{OH}}_{k_1} = \mathrm{mod}\left(\sup\left\{\pi^{\mathrm{OC}}_{i^*_1} + \tau_{i^*_1 j^*_1} + \rho^{\mathrm{OS}}_{j^*_1} + \tau_{j^*_1 k_1} + \rho^{\mathrm{OH}}_{k_1} : i^*_1 \in \mathcal{I}^*_{k_1}, \left(i^*_1, j^*_1, k_1\right) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$$

$u_{k_1}^{\mathrm{OH}}$ is substituted in formula (9) to obtain $x_{j_1k_1}^{\mathrm{SH}}$, which is substituted in formula (10) below to find the departure time $x_{i_1j_1}^{\mathrm{CS}}$ from customer i_1 to origin station j_1. The pickup end time $x_{i_1j_1}^{\mathrm{CS}}$ is equal to $\mathrm{mod}(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1}, 24)$, i.e., precedes $x_{j_1k_1}^{\mathrm{SH}}$ by $\rho_{j_1}^{\mathrm{OS}} + \tau_{i_1j_1}$, only if $\mathrm{mod}(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1}, 24)$ and the corresponding pickup start time $\mathrm{mod}(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1} - \rho_{i_1}^{\mathrm{OC}}, 24)$ both belong to time window interval $[\alpha_{i_1}, \beta_{i_1}]$. Otherwise, $x_{i_1j_1}^{\mathrm{CS}}$ takes the value β_{i_1}, thereby having the pickup performed at the latest possible time:

$$x_{i_1j_1}^{\mathrm{CS}} = \begin{cases} \mathrm{mod}\left(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1}, 24\right) \text{ if } \begin{cases} \mathrm{mod}\left(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1}, 24\right) \in [\alpha_{i_1}, \beta_{i_1}] \\ \mathrm{mod}\left(x_{j_1k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1j_1} - \rho_{i_1}^{\mathrm{OC}}, 24\right) \in [\alpha_{i_1}, \beta_{i_1}] \end{cases} \\ \beta_{i_1} \qquad\qquad\qquad\qquad\qquad \text{otherwise} \end{cases} \tag{10}$$

Formulas (9) and (10) show how the decision variables $x_{i_1j_1}^{\mathrm{CS}}$ and $x_{j_1k_1}^{\mathrm{SH}}$ are determined by the hub-to-hub earliest departure times $u_{k_1}^{\mathrm{OH}}$. Formulas (11) and (12) below show in a similar way how the variables $x_{k_2j_2}^{\mathrm{HS}}$, $x_{j_2i_2}^{\mathrm{SC}}$ and $x_{i_2}^{\mathrm{C}}$ are determined by the hub-to-hub latest arrival times $u_{k_2}^{\mathrm{DH}}$, which are calculated as follows. Let $\mathcal{I}_{k_2}^*$ be the set of important (large) cities i_2^* serviced by the stations j_2^* of k_2, and $\pi_{i_2^*}^{\mathrm{DC}}$ be the latest start-of-delivery time allowed in city i_2^*. Let inf be the infimum (greatest lower bound) function. Processing of important cities' inbound shipments at hub k_2 must be completed no later than $\inf\{\pi_{i_2^*}^{\mathrm{DC}} - \tau_{i_2^*j_2^*} - \rho_{j_2^*}^{\mathrm{DS}} - \tau_{j_2^*k_2} : i_2^* \in \mathcal{I}_{k_2}^*, (i_2^*, j_2^*, k_2) \in \mathcal{D}^{\mathrm{CSH}}\}$; $u_{k_2}^{\mathrm{DH}}$ is the corresponding clock time:

$$u_{k_2}^{\mathrm{DH}} = \mathrm{mod}\left(\inf\left\{\pi_{i_2^*}^{\mathrm{DC}} - \tau_{i_2^*j_2^*} - \rho_{j_2^*}^{\mathrm{DS}} - \tau_{j_2^*k_2} : i_2^* \in \mathcal{I}_{k_2}^*, (i_2^*, j_2^*, k_2) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right).$$

The departure from destination hub k_2 to destination station j_2 takes place $\rho_{k_1}^{\mathrm{DH}} + \tau_{k_1j_2}$ hours after $u_{k_2}^{\mathrm{DH}}$; i.e.:

$$x_{k_2j_2}^{\mathrm{HS}} = \mathrm{mod}\left(u_{k_2}^{\mathrm{DH}} + \rho_{k_2}^{\mathrm{DH}} + \tau_{k_2j_2}, 24\right) \tag{11}$$

The departure from destination station j_2 to destination customer i_2 occurs $\tau_{k_2j_2} + \rho_{j_2}^{\mathrm{DS}}$ hours after $x_{k_2j_2}^{\mathrm{HS}}$, i.e., at $\mathrm{mod}(x_{k_2j_2}^{\mathrm{HS}} + \tau_{k_2j_2} + \rho_{j_2}^{\mathrm{DS}}, 24)$, only if the resulting delivery start time $\mathrm{mod}(x_{k_2j_2}^{\mathrm{HS}} + \tau_{k_2j_2} + \rho_{j_2}^{\mathrm{DS}} + \tau_{j_2i_2}, 24)$ and the corresponding delivery end time $\mathrm{mod}(x_{k_2j_2}^{\mathrm{HS}} + \tau_{k_2j_2} + \rho_{j_2}^{\mathrm{DS}} + \tau_{j_2i_2} + \rho_{i_2}^{\mathrm{DC}}, 24)$ are both in the time window internal $[\alpha_{i_2}, \beta_{i_2}]$. Otherwise, the delivery takes place at the earliest possible time (i.e., α_{i_2} of following day), in which case the departure from the station would take place at $\mathrm{mod}(\alpha_{i_2} - \tau_{i_2j_2}, 24)$:

$$x_{j_2 i_2}^{\mathrm{SC}} = \begin{cases} \mathrm{mod}\left(x_{k_2 j_2}^{\mathrm{HS}} + \tau_{j_2 k_2} + \rho_{j_2}^{\mathrm{DS}}, 24\right) & \text{if} \begin{cases} \mathrm{mod}\begin{pmatrix} x_{k_2 j_2}^{\mathrm{HS}} + \tau_{j_2 k_2} \\ +\rho_{j_2}^{\mathrm{DS}} + \tau_{i_2 j_2}, 24 \end{pmatrix} \in [\alpha_{i_2}, \beta_{i_2}] \\ \mathrm{mod}\begin{pmatrix} x_{k_2 j_2}^{\mathrm{HS}} + \tau_{j_2 k_2} + \rho_{j_2}^{\mathrm{DS}} \\ +\tau_{i_2 j_2} + \rho_{i_2}^{\mathrm{DC}}, 24 \end{pmatrix} \in [\alpha_{i_2}, \beta_{i_2}] \end{cases} \\ \mathrm{mod}\left(\alpha_{i_2} - \tau_{i_2 j_2}, 24\right) & \text{otherwise} \end{cases} \tag{12}$$

The only variables not identified by the formulas above are $x_{kk'}^{\mathrm{HH}}$; these variables are determined by u_k^{OH} and $u_{k'}^{\mathrm{OH}}$. Indeed $x_{kk'}^{\mathrm{HH}}$ must take place no earlier than u_k^{OH} and no later than $\mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)$. There are two cases: either $u_k^{\mathrm{OH}} \le \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)$ or $u_k^{\mathrm{OH}} > \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)$; the second case applies when u_k^{OH} occurs before midnight and $u_{k'}^{\mathrm{DH}}$ occurs after midnight. In the first case $x_{kk'}^{\mathrm{HH}} \in \left[u_k^{\mathrm{OH}}, \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)\right]$ and in the second case $x_{kk'}^{\mathrm{HH}} \in \left[u_k^{\mathrm{OH}}, 24\right) \cup \left[0, \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)\right]$:

$$x_{kk'}^{\mathrm{HH}} \in \begin{cases} \left[u_k^{\mathrm{OH}}, \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)\right] & \text{if } u_k^{\mathrm{OH}} \le \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right) \\ \left[u_k^{\mathrm{OH}}, 24\right) \cup \left[0, \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right)\right] & \text{if } u_k^{\mathrm{OH}} > \mathrm{mod}\left(u_{k'}^{\mathrm{DH}} - \tau_{kk'}, 24\right) \end{cases} \tag{13}$$

Finally, it is worth mentioning that the distribution company often fails to provide the set of important customers $\mathcal{I}_k^*$, in which case u_k^{OH} and u_k^{DH} have no fixed values and have to be dealt with as decision variables. The closest customer to hub *k* is the city *k* where the hub is located, and the corresponding station is also *k*; since $\tau_{kk} = 0$, the earliest possible occurrence of u_k^{OH} is $\mathrm{mod}\left(\pi_k^{\mathrm{OC}} + \rho_k^{\mathrm{OS}} + \rho_k^{\mathrm{OH}}, 24\right)$. The latest possible occurrence is when shipments are collected from all customers, routed to corresponding stations, processed at stations, routed to hub *k* and processed at *k*: $\mathrm{mod}\left(\sup\left\{\pi_i^{\mathrm{OC}} + \tau_{ij} + \rho_j^{\mathrm{OS}} + \tau_{jk} + \rho_k^{\mathrm{OH}} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$. We introduce u_k^{OHE} and u_k^{OHL}, defined below, to simplify the obtained formula:

$$\begin{aligned} u_k^{\mathrm{OHE}} &= \mathrm{mod}\left(\pi_k^{\mathrm{OC}} + \rho_k^{\mathrm{OS}} + \rho_k^{\mathrm{OH}}, 24\right) \\ u_k^{\mathrm{OHL}} &= \mathrm{mod}\left(\sup\left\{\pi_i^{\mathrm{OC}} + \tau_{ij} + \rho_j^{\mathrm{OS}} + \tau_{jk} + \rho_k^{\mathrm{OH}} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right) \\ u_k^{\mathrm{OH}} &\in \begin{cases} \left[u_k^{\mathrm{OHE}}, u_k^{\mathrm{OHL}}\right] & \text{if } u_k^{\mathrm{OHE}} \le u_k^{\mathrm{OHL}} \\ \left[u_k^{\mathrm{OHE}}, 24\right) \cup \left[0, u_k^{\mathrm{OHL}}\right] & \text{if } u_k^{\mathrm{OHE}} > u_k^{\mathrm{OHL}} \end{cases} \end{aligned} \tag{14}$$

Similarly, the earliest occurrence of u_k^{DH} is $\mathrm{mod}\left(\inf\left\{\pi_i^{\mathrm{DC}} - \tau_{ij} - \rho_j^{\mathrm{DS}} - \tau_{jk} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$ and the latest is $\mathrm{mod}\left(\pi_k^{\mathrm{DC}} - \tau_{kk} - \rho_k^{\mathrm{DS}} - \tau_{kk}, 24\right)$; we denote the first quantity as u_k^{DHE} and the second one as u_k^{DHL}:

$$u_k^{\mathrm{DHE}} = \operatorname{mod}\left(\inf\left\{\pi_i^{\mathrm{DC}} - \tau_{ij} - \rho_j^{\mathrm{DS}} - \tau_{jk} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$$
$$u_k^{\mathrm{DHL}} = \operatorname{mod}\left(\pi_k^{\mathrm{DC}} - \tau_{kk} - \rho_k^{\mathrm{DS}} - \tau_{kk}, 24\right) \tag{15}$$
$$u_k^{\mathrm{DH}} \in \begin{cases} \left[u_k^{\mathrm{DHE}}, u_k^{\mathrm{DHL}}\right] & \text{if } u_k^{\mathrm{DHE}} \le u_k^{\mathrm{DHL}} \\ \left[u_k^{\mathrm{DHE}}, 24\right) \cup \left[0, u_k^{\mathrm{DHL}}\right] & \text{if } u_k^{\mathrm{DHE}} > u_k^{\mathrm{DHL}} \end{cases}$$

3.4 PDTP model

The entire formulations (8) to (15) of the PDTP is as follows:

$$\min \sum_{\substack{(i_1, j_1, k_1) \in \mathcal{D}^{\mathrm{CSH}} \\ (i_2, j_2, k_2) \in \mathcal{D}^{\mathrm{CSH}} \\ (k_1, k_0, k_2) \in \mathcal{D}^{\mathrm{HHH}}}} \lambda_{i_1 i_2} \times \begin{pmatrix} \operatorname{mod}\left(x_{j_1 k_1}^{\mathrm{SH}} - \operatorname{mod}\left(x_{i_1 j_1}^{\mathrm{CS}} + \tau_{i_1 j_1} + \rho_{j_1}^{\mathrm{OS}}, 24\right), 24\right) \\ + \operatorname{mod}\left(x_{k_1 k_0}^{\mathrm{HH}} - \operatorname{mod}\left(x_{j_1 k_1}^{\mathrm{SH}} + \tau_{j_1 k_1} + \rho_{k_1}^{\mathrm{OH}}, 24\right), 24\right) \\ + \operatorname{mod}\left(x_{k_0 k_2}^{\mathrm{HH}} - \operatorname{mod}\left(x_{k_1 k_0}^{\mathrm{HH}} + \tau_{k_1 k_0} + \rho_{k_0}^{\mathrm{TH}}, 24\right), 24\right) \\ + \operatorname{mod}\left(x_{k_2 j_2}^{\mathrm{HS}} - \operatorname{mod}\left(x_{k_0 k_2}^{\mathrm{HH}} + \tau_{k_0 k_2} + \rho_{k_2}^{\mathrm{DH}}, 24\right), 24\right) \\ + \operatorname{mod}\left(x_{j_2 i_2}^{\mathrm{SC}} - \operatorname{mod}\left(x_{k_2 j_2}^{\mathrm{HS}} + \tau_{k_2 j_2} + \rho_{j_2}^{\mathrm{DS}}, 24\right), 24\right) \end{pmatrix}$$

For $(i_1, j_1, k_1), (i_2, j_2, k_2) \in \mathcal{D}^{\mathrm{CSH}}$ and $(k_1, k_0, k_2) \in \mathcal{D}^{\mathrm{HHH}}$:

$$x_{j_1 k_1}^{\mathrm{SH}} = \operatorname{mod}\left(u_{k_1}^{\mathrm{OH}} - \rho_{k_1}^{\mathrm{OH}} - \tau_{j_1 k_1}, 24\right)$$

$$x_{i_1 j_1}^{\mathrm{CS}} = \begin{cases} \operatorname{mod}\left(x_{j_1 k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1 j_1}, 24\right) & \text{if } \begin{cases} \operatorname{mod}\left(x_{j_1 k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1 j_1}, 24\right) \in \left[\alpha_{i_1}, \beta_{i_1}\right] \\ \operatorname{mod}\left(x_{j_1 k_1}^{\mathrm{SH}} - \rho_{j_1}^{\mathrm{OS}} - \tau_{i_1 j_1} - \rho_{i_1}^{\mathrm{OC}}, 24\right) \in \left[\alpha_{i_1}, \beta_{i_1}\right] \end{cases} \\ \beta_{i_1} & \text{otherwise} \end{cases}$$

$$x_{k_2 j_2}^{\mathrm{HS}} = \operatorname{mod}\left(u_{k_2}^{\mathrm{DH}} + \rho_{k_2}^{\mathrm{DH}} + \tau_{k_2 j_2}, 24\right)$$

$$x_{j_2 i_2}^{\mathrm{SC}} = \begin{cases} \operatorname{mod}\left(x_{k_2 j_2}^{\mathrm{HS}} + \tau_{k_2 j_2} + \rho_{j_2}^{\mathrm{DS}}, 24\right) & \text{if } \begin{cases} \operatorname{mod}\begin{pmatrix} x_{k_2 j_2}^{\mathrm{HS}} + \tau_{k_2 j_2} \\ + \rho_{j_2}^{\mathrm{DS}} + \tau_{j_2 i_2}, 24 \end{pmatrix} \in \left[\alpha_{i_2}, \beta_{i_2}\right] \\ \operatorname{mod}\begin{pmatrix} x_{k_2 j_2}^{\mathrm{HS}} + \tau_{k_2 j_2} + \rho_{j_2}^{\mathrm{DS}} \\ + \tau_{j_2 i_2} + \rho_{i_2}^{\mathrm{DC}}, 24 \end{pmatrix} \in \left[\alpha_{i_2}, \beta_{i_2}\right] \end{cases} \\ \operatorname{mod}\left(\alpha_{i_2} - \tau_{j_2 i_2}, 24\right) & \text{otherwise} \end{cases}$$

$$x_{k_1k_0}^{\mathrm{HH}} \in \begin{cases} \left[u_{k_1}^{\mathrm{ED}}, \mathrm{mod}\left(v_{k_0}^{\mathrm{LA}} - \tau_{k_1k_0}, 24\right)\right] & \text{if } u_{k_1}^{\mathrm{ED}} \leq \mathrm{mod}\left(v_{k_0}^{\mathrm{LA}} - \tau_{k_1k_0}, 24\right) \\ \left[u_{k_1}^{\mathrm{ED}}, 24\right) \cup \left[0, \mathrm{mod}\left(v_{k_0}^{\mathrm{LA}} - \tau_{k_1k_0}, 24\right)\right] & \text{if } u_{k_1}^{\mathrm{ED}} > \mathrm{mod}\left(v_{k_0}^{\mathrm{LA}} - \tau_{k_1k_0}, 24\right) \end{cases}$$

$$x_{k_0k_2}^{\mathrm{HH}} \in \begin{cases} \left[u_{k_0}^{\mathrm{ED}}, \mathrm{mod}\left(v_{k_2}^{\mathrm{LA}} - \tau_{k_0k_2}, 24\right)\right] & \text{if } u_{k_0}^{\mathrm{ED}} \leq \mathrm{mod}\left(v_{k_2}^{\mathrm{LA}} - \tau_{k_0k_2}, 24\right) \\ \left[u_{k_0}^{\mathrm{ED}}, 24\right) \cup \left[0, \mathrm{mod}\left(v_{k_2}^{\mathrm{LA}} - \tau_{k_0k_2}, 24\right)\right] & \text{if } u_{k_0}^{\mathrm{ED}} > \mathrm{mod}\left(v_{k_2}^{\mathrm{LA}} - \tau_{k_0k_2}, 24\right) \end{cases}$$

$$u_k^{\mathrm{OHE}} = \mathrm{mod}\left(\pi_k^{\mathrm{OC}} + \rho_k^{\mathrm{OS}} + \rho_k^{\mathrm{OH}}, 24\right)$$

$$u_k^{\mathrm{OHL}} = \mathrm{mod}\left(\sup\left\{\pi_i^{\mathrm{OC}} + \tau_{ij} + \rho_j^{\mathrm{OS}} + \tau_{jk} + \rho_k^{\mathrm{OH}} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$$

$$u_k^{\mathrm{OH}} \in \begin{cases} \left[u_k^{\mathrm{OHE}}, u_k^{\mathrm{OHL}}\right] & \text{if } u_k^{\mathrm{OHE}} \leq u_k^{\mathrm{OHL}} \\ \left[u_k^{\mathrm{OHE}}, 24\right) \cup \left[0, u_k^{\mathrm{OHL}}\right] & \text{if } u_k^{\mathrm{OHE}} > u_k^{\mathrm{OHL}} \end{cases}$$

$$u_k^{\mathrm{DHE}} = \mathrm{mod}\left(\inf\left\{\pi_i^{\mathrm{DC}} - \tau_{ij} - \rho_j^{\mathrm{DS}} - \tau_{jk} : (i,j,k) \in \mathcal{D}^{\mathrm{CSH}}\right\}, 24\right)$$

$$u_k^{\mathrm{DHL}} = \mathrm{mod}\left(\pi_k^{\mathrm{DC}} - \tau_{kk} - \rho_k^{\mathrm{DS}} - \tau_{kk}, 24\right)$$

$$u_k^{\mathrm{DH}} \in \begin{cases} \left[u_k^{\mathrm{DHE}}, u_k^{\mathrm{DHL}}\right] & \text{if } u_k^{\mathrm{DHE}} \leq u_k^{\mathrm{DHL}} \\ \left[u_k^{\mathrm{DHE}}, 24\right) \cup \left[0, u_k^{\mathrm{DHL}}\right] & \text{if } u_k^{\mathrm{DHE}} > u_k^{\mathrm{DHL}} \end{cases}$$

Although all the constraints in the formulation above are equalities, most of them are non-linear, which makes it impossible to obtain an equivalent unconstrained optimisation problem. To illustrate the role played by these constraints, let us assume that our network consists of only the route (23, 16, 4, 1, 3, 14, 24) in Figure 1, with:

1 $[\alpha_{23}, \beta_{23}] = [\alpha_{24}, \beta_{24}] = [8, 18]$

2 $\tau_{23,1} + \rho_{16}^{\mathrm{OS}} + \rho_4^{\mathrm{OH}} + \rho_1^{\mathrm{TH}} = 13$

3 $\tau_{1,24} + \rho_3^{\mathrm{DH}} + \rho_{14}^{\mathrm{DS}} = 4$

4 $\rho_{23}^{\mathrm{OC}} = \rho_{24}^{\mathrm{DC}} = 3.$

This means that if there is no waiting, then the time elapsing from leaving customer 23 until leaving hub 1 is 13 hours, and the time elapsing from leaving hub 1 until leaving customer 24 is 7 hours (4 + 3).

Since there is only one route, the parcels have no reason to wait at the hubs; they can be processed as soon as they arrive, and leave as soon as they are processed. The only eventual waiting in this case is at the stations, where the parcels may be held to have them picked-up and/or delivered during time window. The total waiting time, as visualised in Figure 2(a), is the amount by which $x_{1,3}^{\mathrm{HH}}$ exceeds $\mathrm{mod}(x_{23,16}^{\mathrm{CS}} + 13, 24)$ plus the amount by which x_{24}^{DC} exceeds $\mathrm{mod}(x_{1,3}^{\mathrm{HH}} + 7, 24)$:

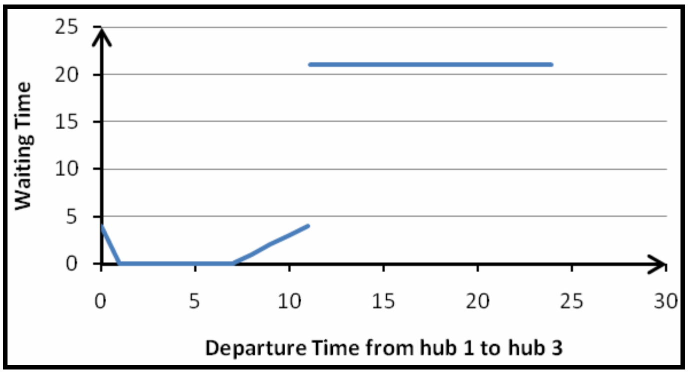
$$\begin{aligned} z_{23,24} &= z_{16}^{\mathrm{OS}} + z_{14}^{\mathrm{DS}} \\ &= \mathrm{mod}\left(x_{1,3}^{\mathrm{HH}} - \mathrm{mod}\left(x_{23,16}^{\mathrm{CS}} + 13, 24\right), 24\right) + \mathrm{mod}\left(x_{24}^{\mathrm{DC}} - \mathrm{mod}\left(x_{1,3}^{\mathrm{HH}} + 7, 24\right), 24\right) \end{aligned}$$

The quantity $x^{\text{CS}}_{23,16}$ is equal to $\text{mod}(x^{\text{HH}}_{1,3} - 13, 24)$ if both $\text{mod}(x^{\text{HH}}_{1,3} - 13, 24)$ and $\text{mod}(x^{\text{HH}}_{1,3} - 16, 24)$ belong to [8, 18]; otherwise it takes the value 18. Similarly, x^{DC}_{24} equals $\text{mod}(x^{\text{HH}}_{1,3} + 7, 24)$ if both $\text{mod}(x^{\text{HH}}_{1,3} + 7, 24)$ and $\text{mod}(x^{\text{HH}}_{1,3} + 4, 24)$ belong to [8, 18]; otherwise it equals 8 + 3 = 11.

Thus, $x^{\text{CS}}_{23,16}$ and x^{DC}_{24} are determined by $x^{\text{HH}}_{1,3}$, which means that the waiting time $z_{23,24} = z^{\text{OS}}_{16} + z^{\text{DS}}_{14}$ can be expressed as a function of the only variable $x^{\text{HH}}_{1,3}$. This function, depicted in Figure 2(b), shows that for $x^{\text{HH}}_{1,3} \in [1, 7]$, the value of the variable $z_{23,24}$ is the lowest (zero), while it can as high as 21 hours when $11 < x^{\text{HH}}_{1,3} < 24$ The solution is not trivial and becomes much more complex when more nodes and routes are added.

Figure 2 Illustrative example, (a) time-precedence relationships (b) effect of hub-to-hub departure time on waiting time (see online version for colours)

(a)



(b)

The example illustrates some of the complexities involved in the model; in particular, the ‘mod’ function and the ‘if’ condition make the model hard to solve by commercial optimisation software. However, such formulas are easy to deal with by any computer programming language. Moreover, the only decision variables are $x^{\text{HH}}_{kk'}$; all other variables are dependent variables determined by $x^{\text{HH}}_{kk'}$. In practice, the number of $x^{\text{HH}}_{kk'}$ variables is not huge, and usually the number of values that can be taken by each variable $x^{\text{HH}}_{kk'}$ is limited.

This suggests that explicit enumerations can be an effective solution method to this problem, although it is definitely not the ideal one. The idea is to have a computer program that tries all possible values of $x^{\text{HH}}_{kk'}$ to obtain the values of the other variables and then the value of the objective function. The optimal values of $x^{\text{HH}}_{kk'}$ are the ones that give the lowest total waiting time. As will be illustrated in the following application, the major challenge facing the analyst, when applying this method, is in finding intelligent ways to reduce the number of possible values of $x_{kk'}^{\mathrm{HH}}$.

4 Application to a real-world case

The PDTP introduced in this paper is applied to a real world case study related to a parcel distribution company whose real identity will be concealed in order to preserve its private information, as desired by its managerial body. Hence, in the remainder of this section, the company shall be referred to as 'COMP'. This company, with 1,000 employees, offers a large variety of parcel and cargo services for both domestic and international parcels. It is basically a ground carrier with more than 98% of its parcel weight being routed by ground. The inter-facility process of COMP had initially been devised for a small network. The rapid growth of the company since its establishment, 15 years ago, has been creating greater complexities to its operations, and has been making it increasingly difficult for management to satisfy franchisor's service standards. In striving to cope with this situation, the Chairman of the company has called for redesigning the network, thereby laying the ground for expansion while enhancing the safety of the parcels and minimising the shipping cost and time. A three-phase project has been conducted to respond to these needs (Ben-Ayed, 2011, 2012a); the first phase is the redesign of the network, the second phase is its timetabling, and the third phase is its implementation.

The network design generated by the first phase is shown in Figure 1; it involves 66 customers, 22 stations and five hubs. This section is concerned with timetabling this design. Around two thirds of the customers are too small to be serviced by dedicated couriers; these customers are divided into small groups with each group forming a tour serviced by a single inter-city courier (Goncalves et al., 2009). Let us denote by $i_1, i_2, \ldots, i_m$ the cities of a tour i. The tour i is treated as a single customer whose pickup time (ρ_i^{OC}) and delivery time (ρ_i^{DC}) are both equal to the duration of the tour; assuming that the tour starts and ends at station j:

$$\begin{aligned}\rho_i^{\mathrm{OC}} &= \rho_i^{\mathrm{DC}} \\ &= \tau_{ji_1} + \rho_{i_1}^{\mathrm{OC}} + \rho_{i_1}^{\mathrm{DC}} + \tau_{i_1 i_2} + \rho_{i_2}^{\mathrm{OC}} + \rho_{i_2}^{\mathrm{DC}} + \tau_{i_2 i_3} + \ldots + \tau_{i_{m-1} i_m} + \rho_{i_m}^{\mathrm{OC}} + \rho_{i_m}^{\mathrm{DC}} + \tau_{i_m j}\end{aligned}$$

Since the pickup and the delivery are performed simultaneously in the tour i, we add for that tour the requirement that pickup and delivery be ended simultaneously, i.e., $x_{ij}^{\mathrm{CS}} = \mathrm{mod}\left(x_{j_2 i_2}^{\mathrm{SC}} + \tau_{j_2 i_2} + \rho_{i_2}^{\mathrm{DC}}, 24\right)$.

The relationships (8) to (15) established for the different types of nodes and different types of movements have been entered in a spreadsheet with the specific input data of COMP. The number of hub-to-hub direct connections in COMP's network being 17 (see Figure 1), if we assume half-an-hour increment, i.e., $x_{kk'}^{\mathrm{HH}} \in \{0, 0.5, 1, \ldots, 23, 23.5\}$, the number of times the formulas (8) to (15) have to be evaluated is $48^{17} \cong 3.8 \times 10^{28}$, which means that computation time may be thousands of years. It was therefore mandatory to limit the values of $x_{kk'}^{\mathrm{HH}}$. Discussions with sales and marketing personnel have revealed that the schedule should give top priority to the three largest metropolitan cities. These cities together are generating greater revenue than all the rest of the cities combined.

Each of the three cities is simultaneously a customer, a station and a hub. The three cities are labelled as nodes 1, 2 and 3 in the figure, and the other two hubs of the design are labelled 4 and 5. The hub-to-hub departures that need to be fixed are: $x_{1,1}^{\mathrm{HH}}$, $x_{1,2}^{\mathrm{HH}}$, $x_{1,3}^{\mathrm{HH}}$, $x_{1,4}^{\mathrm{HH}}$, $x_{1,5}^{\mathrm{HH}}$, $x_{2,1}^{\mathrm{HH}}$, $x_{2,2}^{\mathrm{HH}}$, $x_{2,3}^{\mathrm{HH}}$, $x_{2,4}^{\mathrm{HH}}$, $x_{3,1}^{\mathrm{HH}}$, $x_{3,2}^{\mathrm{HH}}$, $x_{3,3}^{\mathrm{HH}}$, $x_{4,1}^{\mathrm{HH}}$, $x_{4,2}^{\mathrm{HH}}$, $x_{4,4}^{\mathrm{HH}}$, $x_{5,1}^{\mathrm{HH}}$, and $x_{5,5}^{\mathrm{HH}}$.

COMP requires that the pickup end time at the three metropolitan cities be μ, which is a value provided by the company:

$$x_{1,1}^{\mathrm{CS}} = x_{2,2}^{\mathrm{CS}} = x_{3,3}^{\mathrm{CS}} = \mu$$

Since $\tau_{kk} = 0$, u_k^{OH} will be $\rho_k^{\mathrm{OS}} + \rho_k^{\mathrm{OH}}$ hours after μ, for $k = 1, 2, 3$; i.e., $u_k^{\mathrm{OH}} = \mathrm{mod}\left(\mu + \rho_k^{\mathrm{OS}} + \rho_k^{\mathrm{OH}},\ 24\right)$. When one of these three hubs is simultaneously origin and destination, the departure time can take place as early as the end time of the processing of shipments at origin hub:

$$x_{1,1}^{\mathrm{HH}} = u_1^{\mathrm{OH}} = \mathrm{mod}\left(\mu + \rho_1^{\mathrm{OS}} + \rho_1^{\mathrm{OH}}, 24\right)$$
$$x_{2,2}^{\mathrm{HH}} = u_2^{\mathrm{OH}} = \mathrm{mod}\left(\mu + \rho_2^{\mathrm{OS}} + \rho_2^{\mathrm{OH}}, 24\right)$$
$$x_{3,3}^{\mathrm{HH}} = u_3^{\mathrm{OH}} = \mathrm{mod}\left(\mu + \rho_3^{\mathrm{OS}} + \rho_3^{\mathrm{OH}}, 24\right)$$

Besides, cities 2 and 3 being the two ends of a straight-line expressway passing by 1 (see Figure 1), the travel time between them is the longest and thus the departures between them have to be carried out at the earliest time, i.e.:

$$x_{2,3}^{\mathrm{HH}} = x_{2,2}^{\mathrm{HH}}$$
$$x_{3,2}^{\mathrm{HH}} = x_{3,3}^{\mathrm{HH}}$$

Unlike hub 1, which is the transit hub between every two hubs not directly connected, hubs 2 and 3 are transit hubs only to themselves; as a result there is no benefit in delaying the departure of their shipments to the other hubs:

$$x_{2,1}^{\mathrm{HH}} = x_{2,4}^{\mathrm{HH}} = x_{2,2}^{\mathrm{HH}}$$
$$x_{3,1}^{\mathrm{HH}} = x_{3,3}^{\mathrm{HH}}$$

However, hub 1 may have to delay a departure to one hub in order to wait for the arrival of a vehicle from another hub. The value of x_{1,k_2}^{HH}, for $k_2 \in \{2, 3, 4, 5\}$, is selected among several possibilities. In particular, the departure from 1 to 2 cannot be earlier than $x_{1,1}^{\mathrm{HH}}$, and has to be such that the resulting arrival does not exceed the arrival from 3 to 2. Same applies between 1 and 3. We take all possible values between the two limits with an increment of half an hour:

$$x_{1,2}^{\mathrm{HH}} \in \left\{x_{1,1}^{\mathrm{HH}}, \mathrm{mod}\left(x_{1,1}^{\mathrm{HH}} + 0.5, 24\right), \mathrm{mod}\left(x_{1,1}^{\mathrm{HH}} + 1, 24\right), \ldots, \mathrm{mod}\left(x_{3,2}^{\mathrm{HH}} + \tau_{3,2} - \tau_{1,2}, 24\right)\right\}$$
$$x_{1,3}^{\mathrm{HH}} \in \left\{x_{1,1}^{\mathrm{HH}}, \mathrm{mod}\left(x_{1,1}^{\mathrm{HH}} + 0.5, 24\right), \mathrm{mod}\left(x_{1,1}^{\mathrm{HH}} + 1, 24\right), \ldots, \mathrm{mod}\left(x_{2,3}^{\mathrm{HH}} + \tau_{2,3} - \tau_{1,3}, 24\right)\right\}$$

The departure from 1 to either 4 or 5 may take any of the values of $x_{1,2}^{\mathrm{HH}}$ and $x_{1,3}^{\mathrm{HH}}$:

$$x_{1,4}^{\mathrm{HH}}, x_{1,5}^{\mathrm{HH}} \in \left\{x_{1,2}^{\mathrm{HH}}, x_{1,3}^{\mathrm{HH}}\right\}$$

The departures from 4 to 1 and from 5 to 1 should be such that the resulting arrivals (to 1) occur at the same time as the arrival from 2. Such departures do not depend on the arrival from 3 (to 1) because the shipments from 3 arrive to 1 always before those from 2:

$$x_{4,1}^{\mathrm{HH}} = \mathrm{mod}\left(x_{2,1}^{\mathrm{HH}} + \tau_{2,1} - \tau_{4,1}, 24\right)$$
$$x_{5,1}^{\mathrm{HH}} = \mathrm{mod}\left(x_{2,1}^{\mathrm{HH}} + \tau_{2,1} - \tau_{5,1}, 24\right)$$

The departures from 4 and 5 to their adjacent hubs occur at the same time as those to 1:

$$x_{4,2}^{\mathrm{HH}} = x_{4,4}^{\mathrm{HH}} = x_{4,1}^{\mathrm{HH}}$$
$$x_{5,5}^{\mathrm{HH}} = x_{5,1}^{\mathrm{HH}}$$

When plugging in the processing and travel time data of COMP, we obtain six possible values for $x_{1,2}^{\mathrm{HH}}$, 14 for $x_{1,3}^{\mathrm{HH}}$, 2 for $x_{1,4}^{\mathrm{HH}}$, and 2 for $x_{1,5}^{\mathrm{HH}}$; all the other variables $x_{k_1 k_2}^{\mathrm{HH}}$ have one possible value. The total number of possible solutions is therefore 6 × 14 × 2 × 2 = 336. The objective function is evaluated for each of the 336 possible solutions and the one that gives the smallest value is selected.

When compared to the existing timetable, the proposed one has revealed an expected decrease in shipping time by 26% for express and 35% for deferred. The new timetable is developed to a new design based on hub-and-spoke network while the existing timetable was developed to an old design not based on hub-and-spoke network. However, most of the shipping time improvement is resulting from the new timetable rather than from the new design; the reason is that the timetabling problem is concerned with minimising shipping time while the design problem is concerned with minimising shipping cost.

5 Conclusions

This paper describes the problem of timetabling parcel distribution hub-and-spoke network. The problem is not one of the common sub-problems of SNDP and is also different from vehicle and crew scheduling problem, and from parcel hub scheduling problem. The scarcity, not to say inexistence, of works related to this problem, is not in line with its potential features. In addition to its theoretical importance discussed in the paper, the problem has significant practical importance. Any progress made in solving this problem is a great help not only to such large parcel-distribution companies as UPS and FedEx, but also to the numerous small companies not having the required resources to optimally solve their timetabling problem by themselves. Oxford Economic Forecasting (2005) estimates that parcel delivery industry directly supports 1.25 million jobs in the world (larger than the refining of oil) in addition to another 1.4 million jobs that it supports indirectly.

In this paper, the PDTP is introduced, formulated and applied to a real world case. The constructed model, consisting of time-precedence constraints and time-shipping objective function, is a complex non-linear programme that could be solved by reducing the number of its variables. However, despite the success achieved in formulating and solving the problem, the work presented in this paper is only a first step in a long research journey. The next step is to come up with a formulation that can be solved in a more systematic way. A significant progress has been made in this regard with the recent work of Hamzaoui and Ben-Ayed (2011), who built on this research to provide a mixed integer programming formulation for the basic case (single product, no transit hubs and no small customers). Ben-Ayed and Hamzaoui (2012) went one step further by investigating the multi-product case and the multi-objective nature of the problem.

Additionally, there are several aspects of the problem that were not investigated in this paper despite their relevance to the problem. One of these aspects is resource sharing, which imposes more restrictions on the times of the movements; in particular, pickup and delivery are usually performed by the same courier at two different times, and a trip between a hub and its station is usually carried out by the same vehicle. Another aspect is the route choice between hubs that are not directly connected; while such routes are assumed in this paper to be fixed by the design, their selection by the timetabling model can lead to better results. Finally, our assumption that all processing times are fixed is a simplification of reality; there are several unforeseeable events that may considerably influence the time spent on nodes.

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