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# A Fuzzy Goal Programming Model for Venture Capital Investment Decision Making

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**Abstract**—The Venture Capital decision making process involves several conflicting and imprecise criteria. The decision to invest is a difficult one with serious adverse selection risk and surrounded with uncertainty. The aim of this paper is to propose a cardinality constrained Fuzzy Goal Programming (FGP) model to deal with such a complex scenario. A FGP model does not require any assumptions on the probability distribution, better fitting with the characteristics of the Venture Capital market. The developed model is illustrated through a numerical example which uses data taken from an Italian venture capital fund.

## 1. INTRODUCTION

In some technological clusters (such as Route 128 and Silicon Valley in the USA), the Venture Capital (VC) has proven to be an essential resource for economic growth. Financing a start-up is an important driver for innovation process and it plays a relevant role in the interrelations among universities, government and industry (Colapinto, 2007, 2011a, 2011b; Colapinto and Porlezza, 2012). The VC market plays a significant role in providing capital to a wide variety of companies; however the VC market is immature and lacks transparency, thus the investment is illiquid and its success relies on the quality and the skills of the company's management team. In this context the Financial Decision Maker (FDM) has to deal with a risky scenario; thus fuzzy set theory provides a useful tool to capture the high level of uncertainty associated with any decision.

The aim of this paper is to develop a Fuzzy Goal Programming (FGP) model to support the venture capital decision making process. More precisely, we propose a cardinality constrained FGP model with integer variables that extends the works done by Aouni and La Torre (2010) and Aouni et al. (2013). The paper is organized as follows. In section 2, we will be discussing the main criteria to be considered by a Venture Capitalist (VCs) in the decision making process. Section 3 presents the main features of Fuzzy Goal Programming. In section 4

we illustrate the details of the proposed model while in section 5 we perform a numerical simulation with data coming from an Italian venture capital fund. Section 6 contains some concluding remarks.

## 2. INVESTMENT DECISION MAKING: THE PERSPECTIVES OF VENTURE CAPITALISTS

The core business of a venture capital company is to provide equity capital (usually minority interests) to unlisted companies at various stages in their development to sustain their growth, with the objective of maximizing its capital gains. The Venture Capitalists are also involved in the management of the ventures they fund (often as members of the board of directors) and provide expertise or access to other capabilities (hands-on approach) that bolstering the competitive advantage and increasing the value of start-ups that they back (Piol, 2004).

Venture capital firms raise money to invest in a portfolio of activities, made up of promising innovations and high-growth ventures projects. An investment decision is regarded as a strategic one considering its duration, its amount and its irreversible character (Zopounidis, 1999). As money and time for the support to the entrepreneurs are scarce resources, selection is a very crucial task for all VCs. The costs of due diligence and the fact that the process is time-consuming make VCs reject an

investment under a certain threshold and prefer to focus on a limited number of ventures. The VC firms generally place upper and lower limits on the size of their investments, which are closely related to the overall size of the managed fund. The market for VC investments is imperfect, because not all investors have the same information at the same point in time (information asymmetry): the target companies have little pressure to divulge information and no financial analysts are monitoring them. The diversified portfolio is made up of the best companies that arise out of matrix analysis, target selection and due diligence activities. By holding a diversified portfolio of investments, however, VCs are able to manage their risk positions. Indeed through the diversification of the financial portfolio, the overall risk of a diversified VC portfolio will not be as high as the average of its individual investments (Manigart et al., 1994). In other words, the motivations for constructing a VC portfolio are quite similar to that of constructing a financial portfolio (Markowitz, 1952).

Venture capitalists have to cope with some specific business risk factors, such as technological or market development or the development stage of the company. Moreover, the target companies often have no history (or limited track record). The decision making process is also affected by other factors, such as the individual characteristics of the managers (Laughun et al., 1980), organizational culture (Morgan, 1986), national culture (Hofstede, 1984) and institutional environment (Tyejee and Vickery, 1988).

Several studies of VC investments have been conducted previously where different tools have been applied such as: (a) descriptive methods, (b) linear statistical techniques and (c) multicriteria decision aid techniques. The descriptive studies proposed by Wells (1974), Poindexter (1976) and Tyejee and Bruno (1984) attempted to ascertain the relative importance of various criteria. Among them, Hisrich and Jankowicz (1990) identified three basic constructs, namely: (a) concept, (b) management and, (c) returns. For a VCs it is crucial the venture is a high-growth one with potential for earnings growth. The concept must rely on a feasible business idea (new product, service, or retail concept) which is able to offer a substantial competitive advantage or is in a relatively non-competitive industry. The criterion of the management team is quite predominant.

There are broad generic criteria and the specifics of each criterion may vary from a VCs to another. For instance, specialist VCs have specific criteria on investment size, industries in which they will be investing, geographic location of the investment, and stage of financing and development. The specialization exploits the ability of investors to influence nature and performances of the venture. However, we can identify a set of common and most used investment decision criteria (objectives). Each VCs evaluates the country in which the company is set up, the industry in which it operates and the availability and the reliability of data on which the choice will be made. The problem of selecting a country for business venturing should be

formulated by considering four perspectives as follows: (a) economic (access to financial capital, growth of real gross domestic product), (b) legal (business law, labor regulations, risks for intellectual property), (c) political (bureaucracy, lack of corruption) and (d) cultural.

In general the VCs operate in sectors where the competitive strength of a company relies on intangible assets. Hence, the cash flows method and income based methods are to be preferred rather than the traditional balance sheet method. The value of most of venture-backed companies lays in the present value of opportunities the firm will be able to reap in the future (Gatti 2004). Among the financial criteria to be considered for the VC decision making process, we can consider the following ones: (a) needed time to attend the break-even, (b) the expected rate of return and (c) the needed time to payback. According to Schaffer (1989) and Gilbert et al. (2006), the profitability seems to be a reasonable objective for the VCs. However, Storey (2000) highlighted that entrepreneurs involved in start-ups worry about the likelihood of survival of their new ventures. Consequently, VCs simultaneously consider both survival and profit maximization. From the above discussion, we will be considering the following conflicting criteria for the VCs decision-making process: (a) maximizing the portfolio return, (b) maximizing the survival rate of the entire portfolio, (c) maximizing the intellectual capital and, (d) minimizing the portfolio risk. The general risk can be explained in terms of: product risk (because the products concerned may have little or no track record in the markets as they are largely untested and usually have high obsolescence rates), technology risk (hard to assess new technology on small set of products), country impact (as highlighted previously, a few of the issues includes taxes, regulatory costs, property rights) and so on. The intellectual capital refers to knowledge which must be an asset able to be used to create wealth: intellectual property protection for newly developed products or processes offers significant benefits (e.g. in terms of commercialization activities and thus of future potential) and sets the firm in a favourable position to obtain complementary assets, skills and financing (especially from VCs). Particularly, we mean by intellectual property the patents, copyrights, methods, procedures and archives. The four mentioned objectives will be incorporated in a fuzzy GP model in the next section.

### 3. FUZZY GOAL PROGRAMMING

As mentioned at the end of the previous section, a venture capital decision making context leads to a multi-criteria decision aid problem that should be analyzed through advanced optimization techniques allowing to determine the Pareto efficient solutions. A practical method to overcome this difficulty and simplify the model is to proceed by the so-called Goal Programming (GP) approach, which is a particular Distance-Based Method. A goal refers to a criterion and a numerical level, known as a target level, which the decision maker desires to achieve on that criterion. The

GP model is a well-known aggregating methodology and its solution represents the best compromise that can be made. In this formulation the negative and positive deviations  $\delta_l^-, \delta_l^+$  between the achievement and aspiration levels or goals  $g_l$  are to be minimized.

The first formulation of a GP model was presented by Charnes et al. (1955) and in more than fifty years it has been widely applied in several fields such as accounting and financial aspect of stock management, marketing, quality control, human resources, production and operations management (Aouni et al., 2014; Charnes and Cooper, 1952, 1959; Lee, 1973; Lee and Nicely, 1974; Romero, 1991). The GP model is easy to be implemented and it can be solved through some powerful mathematical programming software such as Lindo, Lingo and CPLEX. In mathematical terms the standard GP model can be formulated as follows:

$$\text{Min } Z = \sum_{l=1}^L (\delta_l^+ + \delta_l^-)$$

Subject to

$$F_l(x) + \delta_l^- - \delta_l^+ = g_l \quad (\forall \quad l \in L);$$

$$x \in D; \tag{1}$$

$$\delta_l^-, \delta_l^+ \geq 0 \quad (\forall \quad l \in L).$$

where  $\delta_l^+$  and  $\delta_l^-$  are, respectively, the positive and the negative deviations with respect to the goals  $g_l, l=1, \dots, L$ . and  $D$  is the feasible set for the input variables  $x$  (usually supposed to be compact).

An alternative formulation of the GP model is the so called Fuzzy Goal Programming model. The concept of fuzzy set was introduced by Zadeh (1965) and, since this fundamental paper, many researchers have used this approach to determine optimal solutions in multicriteria decision making contexts. Just to mention a few of them, Bellman and Zadeh (1970) presented some applications of fuzzy sets to different decision-making contexts whilst Zimmerman (1976, 1978) proposed a Fuzzy Linear Programming model with both single and multiple objectives. Narasimhan (1980) proposed a FGP technique for modelling the fuzziness related to the aspiration levels, and Yang et al. (1991) formulated a FGP with a nonlinear membership function.

According to Zadeh (1965), the notion of fuzzy set is an extension of the classical definition of a set. In classical set theory, each element of a universe  $X$  either belongs to a set  $A$  or not, whereas in fuzzy set theory an element belongs to a set  $A$  with a certain degree of membership. A fuzzy subset  $A$  of  $X$  is defined through a membership function  $\mu_x(A)$  which expresses the degree of membership of  $x$  to  $A$ . A fuzzy set  $A$  in  $X$  is thus uniquely characterized by its membership function  $\mu_x(A)$ , which associates with each point in  $X$  a nonnegative finite real number which usually belongs to the interval  $[0, 1]$ . Thus, the nearer the

value of  $\mu_x(A)$  to 1, the higher the degree of ‘belongingness’ of  $x$  to  $A$ .

Given a set of  $L$  objective functions  $F_l$ , a general FGP with integer variables can be formulated as follows: Find  $x$  which satisfies

$$F_l(x) \cong \tilde{F}_l, \quad l = 1 \dots L$$

$$C_j(x) \leq a_j, \quad j = 1 \dots J$$

$$h_k(x) = b_k, \quad k = 1 \dots K \tag{2}$$

$$x_i \geq 0 \text{ and integer, } i = 1 \dots N$$

where:

- $F_l(x)$  is the  $l^{th}$  objective function,
- $C_j(x)$  is the  $j^{th}$  inequality constraint,
- $h_k(x)$  the  $k^{th}$  equality constraint,
- $\tilde{F}_l$  is the  $l^{th}$  fuzzy goal.

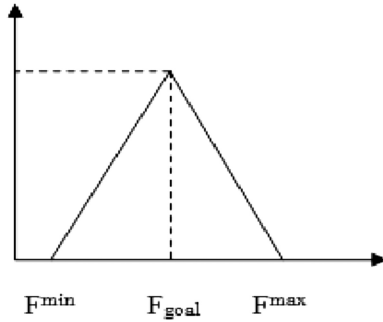
In the above formulation, the symbol ‘ $\cong$ ’ indicates the fuzziness of the objective. It describes in mathematical terms the notion of approximation and the DM will accept values slightly greater than (or less than)  $\tilde{F}_l$  up to a fixed tolerance  $\Delta_l$ . The  $j^{th}$  system constraint  $C_j(x) \leq a_j$  and the  $k^{th}$  system constraint  $h_k(x) = b_k$ , describe the feasible set. As proposed by Yang et al. (1991), in the following we consider a triangular membership function  $\mu_{[F_l(x)]}$  for the  $l^{th}$  fuzzy goal  $\tilde{F}_l$ . This choice better fits a VC investment because, as said before, the return on investment is only one of the relevant criteria; the constrains of a VC deal make the survival and the value creation as much relevant as the return.

This type of membership function is usually chosen because of its ease in defining the maximum and minimum limit of tolerance of each fuzzy goal with respect to its central value. The triangular membership function  $\mu_{[F_l(x)]}$  is shown in Fig 1 and is defined as follows:

$$\mu(F_l(x)) = \begin{cases} \frac{F_l(x) - F_l^{MIN}}{F_l^{GOAL} - F_l^{MIN}} & \text{if } F_l^{MIN} \leq F_l(x) \leq F_l^{GOAL} \\ \frac{F_l^{MAX} - F_l(x)}{F_l^{MAX} - F_l^{GOAL}} & \text{if } F_l^{GOAL} \leq F_l(x) \leq F_l^{MAX} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where:

- $F_l^{MIN}$  is a minimum limit of tolerance for  $\tilde{F}_l$ ,
- $F_l^{MAX}$  is a maximum limit of tolerance for  $\tilde{F}_l$ .
- $F_l^{GOAL}$  is the average between  $F_l^{MIN}$  and  $F_l^{MAX}$



**Figure 1:** The triangular membership function

In the standard GP formulation the aspiration levels (goals) are supposed to be precise and deterministic. However, in practice, there are many financial decision-making situations in which the DM is not able to determine precisely the value of each goal. The use of fuzzy goals seems to be more realistic and related to the uncertainty of the objectives involved in the financial decision-making contexts. The original version of the Fuzzy Goal Programming (FGP) model was developed in the 1980s to deal with such situations. In FGP the approach is different from the classical GP model as it is considered that the goals are represented by fuzzy sets whose membership functions provide the satisfaction degree regarding the achievement of the targets.

The FGP model with integer variables we consider in the following paragraph can be formulated as follows (see also [Yang et al. 1991](#)):

$$\begin{aligned}
 & \text{Max } \lambda \\
 \text{Subject to:} & \\
 & \lambda \leq \frac{F_l(x) - F_l^{MIN}}{F_l^{GOAL} - F_l^{MIN}} \quad l = 1, \dots, L \\
 & \lambda \leq \frac{F_l^{MAX} - F_l(x)}{F_l^{MAX} - F_l^{GOAL}} \\
 & C_j(x) \leq a_j, \quad j = 1, \dots, J \\
 & h_k(x) = b_k, \quad k = 1, \dots, K \\
 & x_i \geq 0 \quad \text{and} \quad \text{integer}, \quad i = 1, 2, \dots, N
 \end{aligned}
 \tag{4}$$

#### 4. THE MODEL

In the VC investment decision-making process the DM does not have sufficient information related to the different criteria: this uncertainty and lack of information can be efficiently

described using fuzzy sets and the FGP model. In our model the following criteria will be considered:

- a)  $F_1$ : provides the return of the investment,
- b)  $F_2$ : assigns the survival rate of the investment,
- c)  $F_3$ : gives the intellectual capital rate,
- d)  $F_4$ : the investment risk.

We propose the following FGP with integer variables:

$$\text{Maximize } \lambda$$

Subject to:

$$\begin{aligned}
 \lambda & \leq \mu_{[F_1(x)]} \\
 \lambda & \leq \mu_{[F_2(x)]} \\
 \lambda & \leq \mu_{[F_3(x)]} \\
 \lambda & \leq \mu_{[F_4(x)]} \\
 Ax & \leq G \\
 \text{supp}(x) & \leq M \\
 x_i & \in \{0, 1\}.
 \end{aligned}
 \tag{5}$$

With respect to the general FGP model presented in Eq. (4), the following specific aspects of the model have been modified:

- the system of inequalities  $Ax \leq G$  describes all possible financial constraints related to the specific decision-making context, including a budget constraint,
- the inequality  $\text{supp}(x) \leq M$  limits the number of investments that can be activated in the financial portfolio,
- the integer variable  $x_i$  can take only two possible values (either 0 or 1), and it shows the value 1 if the  $i^{\text{th}}$  asset is included in the financial portfolio and the value of 0 if it is not selected.

The presence of the function  $\text{supp}(x)$  makes this model belonging to the class of nonlinear integer optimization programs. This family of programs has been extensively analyzed in literature from both computational and complexity perspectives and it has been shown to belong to the class of NP-hard problems. For instance, in [Bienstock \(1996\)](#) a computational study of a family of mixed-integer quadratic programming problems is presented; in [Chang et al. \(2000a, 2000b\)](#) the authors analyze some heuristics for cardinality constrained portfolio optimization; in [Maringer and Kellerer \(2003\)](#), [Li et al. \(2006\)](#), [Shaw et al. \(2008\)](#), [Soleimani et al. \(2009\)](#), [Anagnostopoulos and Mamanis \(2011\)](#), the authors study a mean-variance cardinality constrained portfolio optimization problem; in [Fieldsend et al. \(2004\)](#), [Bertsimas and Shioda \(2009\)](#), some algorithms for cardinality-constrained quadratic optimization are illustrated and discussed; in [La Torre \(2003\)](#) the author proposed a smooth approximation of the function  $\text{supp}(x)$ .

## 5. NUMERICAL EXAMPLE

In order to illustrate the proposed model, we will consider some data from an anonymous Italian venture capital fund, fictionally named Venture Capital Partners (VCP). The activity sector of this company is related to information technology, communication and media. The fund manages a Euro 500 million fund. The size of investment is usually between 1 million and 7 million of euros (Table 1), and typically it holds minority shares between 15 % and 35 % in any one company.

Europe has been chosen as reference market because of the potential growth in many areas and the geographical proximity. The VCs deal quite frequently, with the complex problem of capital budgeting, in the case of a high technology company that lacks a sufficient number of comparables/peers, thus the degree of uncertainty is high. This is the case of VCP.

Table 2 shows the VCP portfolio: we use fictional names for all venture-backed companies and we provide a brief description of their business activity.

Table 3 reports the investment criteria: in the matrix for each company we indicate the relative investment return rate, the survival rate, the intellectual capital index and the investment risk

rate. The investment return rate expresses the profitability and the capability to create value by the company. The survival rate represents the possibility to remain in operation after one year. The intellectual capital index expresses the company's capability to create value, and here we focus on patents, copyrights, methods, procedures and archives. The highest the investment risk is, the most risky the company is; this rate is affected by the kind of product, industry, country and so on.

The goal levels  $g_i$  for each criterion are as follows:  $g_1 = 2.82$ ,  $g_2 = 5.63$ ,  $g_3 = 1.8$ , and  $g_4 = 0.5$ ; and the tolerances are  $\Delta_1 = 1$ ,  $\Delta_2 = 1.5$ ,  $\Delta_3 = 0.8$ , and  $\Delta_4 = 0.07$ . We suppose that the number of investments  $M$  is equal to 7, and the available budget is equal to 10 millions of euros. Let us define  $x_i$  as follows:

$$x_i = \begin{cases} 1 & \text{if VCs invest in company } i \\ 0 & \text{otherwise.} \end{cases}$$

In this example, triangular membership functions as given in Eq. (3) are used.

The triangular membership functions of the four fuzzy goals (namely the return of the investment, the survival rate, the

TABLE 1.  
Portfolio data

Company	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Investment (mln)	6	3	1.05	1.78	3.18	0.51	5.24	6.64	2	0.79	3	5.17	4.71	3.36	2.56

TABLE 2.  
Venture Backed companies

Company	Focus
1 Space Newco S.A.	Reseller of hosting space and domain registration.
2 Mortgage Newco S.A.	Offer both mortgage quotes and links to developers of buy-to-let property investment.
3 Info NewCo Ltd.	On line financial information.
4 Adv Newco S.r.l.	Internet advertising
5 Mmania Newco Ltd.	M-Commerce and E-commerce for the UK mobile market.
6 E-Finance Newco S.p.A.	Web design services and Internet financial information.
7 Together Newco	On line group buying in Europe.
8 Mphone Newco S.p.A.	Distributors of mobile phone in Germany.
9 Invest Newco SA	On line trading service.
10 Mobile Newco Inc.	New technology into web-enabled or SMS-enabled mobile phones.
11 IVP Newco S.r.l.	Internet Video Producer: from creativity, to shooting, editing and streaming.
12 Security Newco SpA	Provider of Security Solution applicable for e-commerce transactions.
13 Egrocery Newco	On line grocery.
14 Ecom Newco Inc.	Leading ecommerce outsourcing platform.
15 eHotel Newco	Internet specialized tour packages.

TABLE 3.  
Investment criteria

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Investment return rate	.15	.33	.2	.1	.18	.2	.3	.15	.15	.09	.15	.18	.2	.15	.23
Survival rate (1Year)*	.84	.95	.93	.94	.93	.94	.95	.9	.94	.93	.94	.94	.94	.93	.9
Intellectual capital	.1	.1	.1	0	.2	.2	.1	.1	.2	.5	.4	.5	.2	.2	.4
Investment risk rate	.07	.05	.03	.07	.03	.07	.05	.04	.07	.03	.07	.07	.07	.03	.04

intellectual capital rate and the investment risk) are constructed as given in (6) - (9), the triangular membership functions for the four objectives are shown in Fig 2.

$$\mu_{[F_1(x)]} = \begin{cases} \left[ \frac{F_1(x) - 1.82}{1} \right], & \text{if } 1.82 \leq F_1(x) \leq 2.82 \\ \left[ \frac{3.82 - F_1(x)}{1} \right], & \text{if } 2.82 \leq F_1(x) \leq 3.82 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

$$\mu_{[F_3(x)]} = \begin{cases} \left[ \frac{F_3(x) - 1}{0.8} \right], & \text{if } 1 \leq F_3(x) \leq 1.8 \\ \left[ \frac{2.6 - F_3(x)}{0.8} \right], & \text{if } 1.8 \leq F_3(x) \leq 2.6 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

$$\mu_{[F_2(x)]} = \begin{cases} \left[ \frac{F_2(x) - 4.13}{1.5} \right], & \text{if } 4.13 \leq F_2(x) \leq 5.63 \\ \left[ \frac{7.13 - F_2(x)}{1.5} \right], & \text{if } 5.63 \leq F_2(x) \leq 7.13 \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

$$\mu_{[F_4(x)]} = \begin{cases} \left[ \frac{F_4(x) - 0.43}{0.07} \right], & \text{if } 0.43 \leq F_4(x) \leq 0.5 \\ \left[ \frac{0.57 - F_4(x)}{0.07} \right], & \text{if } 0.5 \leq F_4(x) \leq 0.57 \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

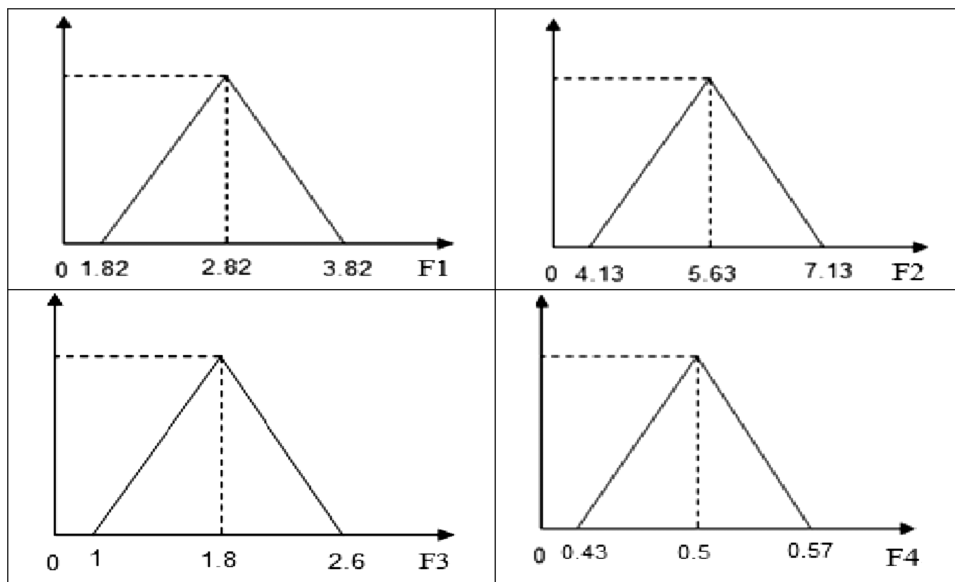


Figure 2: Membership function of different objectives

The model that we will be considering involves the following criteria: the investment return, the survival index, the intellectual capital, and the investment risk.

The model can be formulated as follows:

Maximize  $\lambda$

Subject to:

$$\lambda \leq -(0.9x_1 + 0.99x_2 + 0.21x_3 + 0.178x_4 + 0.5724x_5 + 0.102x_6 + 1.572x_7 +$$

$$0.996x_8 + 0.3x_9 + 0.0711x_{10} + 0.45x_{11} + 0.9306x_{12} +$$

$$0.942x_{13} + 0.504x_{14} + 0.5888x_{15}) + 3.82$$

$$\lambda \leq (0.9x_1 + 0.99x_2 + 0.21x_3 + 0.178x_4 + 0.5724x_5 +$$

$$0.102x_6 + 1.572x_7 + 0.996x_8 + 0.3x_9 + 0.0711x_{10} +$$

$$0.45x_{11} + 0.9306x_{12} + 0.942x_{13} +$$

$$0.504x_{14} + 0.5888x_{15}) - 1.82$$

$$\lambda \leq -(0.56x_1 + 0.63x_2 + 0.62x_3 + 0.627x_4 +$$

$$0.62x_5 + 0.63x_6 + 0.6x_7 +$$

$$0.63x_8 + 0.62x_9 + 0.627x_{10} + 0.627x_{11} + 0.627x_{12} +$$

$$0.627x_{13} + 0.62x_{14} + 0.6x_{15}) + 4.75$$

$$\lambda \leq (0.56x_1 + 0.63x_2 + 0.62x_3 +$$

$$0.627x_4 + 0.62x_5 + 0.63x_6 + 0.6x_7 +$$

$$0.63x_8 + 0.62x_9 + 0.627x_{10} + 0.627x_{11} + 0.627x_{12} +$$

$$0.627x_{13} + 0.62x_{14} + 0.6x_{15}) - 2.75$$

$$\lambda \leq -(0.125x_1 + 0.125x_2 + 0.125x_3$$

$$+ 0.125x_4 + 0.25x_5 + 0.25x_6 +$$

$$0.125x_7 + 0.125x_8 + 0.25x_9 + 0.625x_{10} + 0.5x_{11} + 0.625x_{12} +$$

$$0.25x_{13} + 0.25x_{14} + 0.5x_{15}) + 3.25$$

$$\lambda \leq (0.125x_1 + 0.125x_2 + 0.125x_3 +$$

$$0.125x_4 + 0.25x_5 + 0.25x_6 +$$

$$0.125x_7 + 0.125x_8 + 0.25x_9 + 0.625x_{10} +$$

$$0.5x_{11} + 0.625x_{12} + 0.25x_{13} + 0.25x_{14} + 0.5x_{15}) - 1.25$$

$$\lambda \leq -(6x_1 + 2.14x_2 + 0.45x_3 + 1.78x_4 + 1.36x_5 +$$

$$0.51x_6 + 3.74x_7 + 3.794x_8 + 2x_9 + 0.3386x_{10} +$$

$$3x_{11} + 5.17x_{12} + 4.71x_{13} + 1.44x_{14} + 1.462x_{15}) + 8.14$$

$$\lambda \leq (6x_1 + 2.14x_2 + 0.45x_3 + 1.78x_4 + 1.36x_5 +$$

$$0.51x_6 + 3.74x_7 + 3.794x_8 + 2x_9 + 0.3386x_{10} +$$

$$3x_{11} + 5.17x_{12} + 4.71x_{13} + 1.44x_{14} + 1.462x_{15}) - 6.142$$

$$6x_1 + 3x_2 + 1.05x_3 + 1.78x_4 + 3.18x_5 +$$

$$0.51x_6 + 5.24x_7 + 6.64x_8$$

$$+ 2x_9 + 0.79x_{10} + 3x_{11} + 5.17x_{12} +$$

$$4.7x_{13} + 3.36x_{14} + 2.56x_{15} \leq 10$$

$$\sum_{i=1}^{15} x_i \leq 7$$

$$x_i \in \{0, 1\} \quad i = 1, 2, \dots, 15$$

The number of investments  $M=7$ , and the budget=10 millions of euros. LINGO provides the following solution  $x_1=0$ ,  $x_2=1$ ,  $x_3=1$ ,  $x_4=0$ ,  $x_5=0$ ,  $x_6=1$ ,  $x_7=0$ ,  $x_8=0$ ,  $x_9=1$ ,  $x_{10}=1$ ,  $x_{11}=0$ ,  $x_{12}=0$ ,  $x_{13}=0$ ,  $x_{14}=0$ ,  $x_{15}=1$ . From the above result we conclude that the number of investment is equal to 6 companies (namely Mortgage Newco S.A., Info NewCo Ltd., E-Finance Newco S.p.A., Invest Newco SA, and Mobile Newco) and the budget which should be invested is equal to 9.91 millions of Euros.

## 6. CONCLUSION

The aim of this paper was to propose a FGP model with integer variables for venture capital investments. This decision making context shows a high level of fuzziness and uncertainty. Fuzzy sets seem to be the right mathematical tool to handle the complexity of such a financial decision-making situation. The proposed model aggregates simultaneously the following four conflicting and incommensurable objectives: (a) the investment return, (b) the survival rate, (c) the intellectual capital rate, and (d) the investment risk. The obtained financial portfolio is one of the best compromises. We have illustrated our model through a numerical example based on data provided by an Italian VC company investing in the field of information



technology, communication and media. The advantage of using the FGP approach with respect to the stochastic GP relies on the possibility to describe the uncertainty and volatility of financial assets without knowing and estimating the probability distribution function. In fact, it is well known that in many financial decision making contexts it is quite complex to determine the family of probability distribution and to estimate the value of the unknown parameters.

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