

ORIGINAL RESEARCH

Robust stabilization of LTI negative imaginary systems using the nearest negative imaginary controller

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Abstract

This paper considers the problem of robust stabilization of linear time-invariant systems with respect to unmodelled dynamics and structure uncertainties. To that end, a methodology to find the nearest negative imaginary system for a given non-negative imaginary system is presented first. Then, this result is employed to construct a near optimal linear quadratic Gaussian controller achieving desired performance measures. The problem is formulated using port-Hamiltonian method and the required conditions are defined in terms of linear matrix inequalities. The technique is presented using the fast gradient method to solve the problem systematically. The designed controller satisfies a negative imaginary property and guarantees a robust feedback loop. The effectiveness of the approach is demonstrated by a simulation on a numerical example.

1 | INTRODUCTION

Robustness is a crucial aspect of feedback control systems to cope with model uncertainties such as disturbances, unmodelled dynamics and plant parameter variations. These issues can severely affect the performance and stability of the closed-loop system. For instance, highly resonant modes in flexible systems such as aerospace systems [1–3], robot manipulators [4], atomic force microscopes [5–8]) can affect robustness and stability characteristics [9–11]. Moreover, relatively small stimuli or perturbations in the environment, such as changing temperature can result in significant impact on the system's phase and gain at a given frequency, which can lead to instability or poor performance of the closed-loop system. These dynamical systems are infinite-dimensional systems [9, 12, 13], which are hard to model and control. Instead, finite-dimensional models are used as approximation models to be used in controller synthesis [9, 13].

Negative imaginary (NI) systems theory provides powerful tools to establish robustness for dynamical systems with flex-

ible structures [11, 14–26]. The NI property can be defined, in the case of the single-input single-output (SISO), based on the properties of the imaginary part of the complex function of the frequency response. For instance, this NI property is known to be satisfied for flexible structure dynamics when force actuators are collocated with position or acceleration sensors. A fundamental result of NI systems theory is concerned with feedback-interconnected NI systems. That is, the feedback interconnection between an NI system and a strictly NI (SNI) system is stable under a given DC gain condition. Hence, if the system under control satisfies the NI property, one can design a strictly NI controller so that robust stability is guaranteed for free. This has attracted several attempts to synthesise NI controllers that preserve certain performance measures [11, 27, 28]. However, to our knowledge, there are no available results on the design of output optimal controllers for NI systems in the literature, which is the objective of this work.

We aim to design robust and optimal output feedback controllers against unmodelled dynamics and parameter uncertainties for a class of multi-input multi-output (MIMO), linear

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time-invariant (LTI) systems that satisfy the NI property. The proposed methodology consists of two steps: synthesizing an optimal linear quadratic Gaussian (LQG) controller for the given NI model and then using a developed algorithm to find the nearest NI model for the synthesized LQG controller.

The idea of finding the nearest NI model was inspired by a similar technique of finding the nearest positive real system (passive system) presented in [29–34]. In [31], some assumptions are imposed on the dimensions of the control input and the output measurement to prevent singularities. Also, [31] restricts the perturbation to the output matrix only. The methods in [32] and [33] allow perturbations on both the matrices of the control input and the output measurement, while in [35], perturbations are allowed for all system matrices. Similar results for the NI system were developed in [36], where an algorithm was developed for enforcing negative imaginary property in case of any violation during system identifications. This assumes that the underlying dynamics ought to belong to the negative imaginary system class. The method is based on the spectral properties of Hamiltonian matrices. This paper uses the result developed in [29] to develop similar results for finding the nearest negative imaginary system. One of the main advantages of this method over the other perturbation methods is that no assumptions are imposed on the given system. Also, in the positive real case, it allows for perturbations of all system matrices.

The contribution of this paper is twofold:

- (1) We propose a methodology for finding the nearest NI system for a non-NI system based on the Port–Hamiltonian formulation.
- (2) The nearest NI result is employed to find a near optimal LQG for a given negative imaginary plant.

Sufficient conditions are provided in terms of the feasibility of an LMI condition to ensure the closed-loop stability. Moreover, we present an algorithm based on fast gradient method to solve the problem systematically. The effectiveness of the approach is demonstrated by a simulation on a numerical example.

The paper is organized as follows: Section 2 presents the preliminaries used throughout the paper. In Section 3, we discuss the problem formulation of finding the nearest negative imaginary system. We present an algorithmic solution for the nearest negative imaginary system problem in Section 4.1. Finally, we discuss the optimal control design problem in Section 4.2.

2 | PRELIMINARIES

The notation is standard throughout. The sets of all real and complex numbers are denoted by \mathbb{R} and \mathbb{C} , respectively. We denote the minimum and maximum eigenvalues of the real matrix A as $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$, respectively. The transpose of the matrix A is denoted by A^T while A^* refers to the complex conjugate transpose of a complex matrix A . We write

$(x, y) \in \mathbb{R}^{n_x+n_y}$ to represent the vector $[x^T, y^T]^T$ for $x \in \mathbb{R}^{n_x}$ and $y \in \mathbb{R}^{n_y}$. We denote the real part of a complex variable s as $\text{Re}[s]$. The Frobenius Norm of matrix A is written as $\|A\|_F^2$.

For the sake of convenience, we present the definitions and fundamental results of NI systems and we refer the reader to [16] for more details. Consider the following LTI system

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t),\end{aligned}\tag{1}$$

where $x \in \mathbb{R}^n$ is the plant state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the measured output and A, B, C, D are constant matrices with appropriate dimensions. The transfer function matrix $G(s) = C(sI - A)^{-1}B + D$ of system (1) is said to be strictly proper if $G(\infty) = D = 0$. The notation (A, B, C, D) will be used to denote the state space realization (1).

The following definitions establish the required conditions for the NI and SNI properties of the LTI system (1).

Definition 1 ([16]). A square transfer function matrix $G(s)$ is NI if the following conditions are satisfied:

- 1) $G(s)$ has no pole in $\text{Re}[s] > 0$.
- 2) For all $\omega > 0$ such that $s = j\omega$ is not a pole of $G(s)$,

$$j(G(j\omega) - G(j\omega)^*) \geq 0.\tag{2}$$

- 3) If $s = j\omega_0$ with $\omega_0 > 0$ is a pole of $G(s)$, then it is a simple pole and the residue matrix $K = \lim_{s \rightarrow j\omega_0} (s - j\omega_0)jG(s)$ is Hermitian and positive semidefinite.
- 4) If $s = 0$ is a pole of $G(s)$, then $\lim_{s \rightarrow 0} s^k G(s) = 0$ for all $k \geq 3$ and $\lim_{s \rightarrow 0} s^2 G(s)$ is Hermitian and positive semidefinite.

Definition 2 ([37]). A square transfer function matrix $G(s)$ is SNI if the following conditions are satisfied:

- 1) $G(s)$ has no pole in $\text{Re}[s] \geq 0$.
- 2) For all $\omega > 0$, $j(G(j\omega) - G(j\omega)^*) > 0$.

The following lemma provides a state-space characterization of NI systems in terms of linear matrix inequalities (LMIs) [37].

Lemma 1. *Let (A, B, C, D) be a minimal realization of the transfer function matrix $G(s)$ for the system in (1). Then, $G(s)$ is NI if and only if $D = D^T$ and there exists a matrix $P = P^T \geq 0$ such that the following LMI is satisfied:*

$$\begin{bmatrix} PA + A^T P & PB - A^T C^T \\ B^T P - CA & -(CB + B^T C^T) \end{bmatrix} \leq 0.\tag{3}$$

Furthermore, if $G(s)$ is SNI, then $\det(A) \neq 0$ and there exists a matrix $P > 0$ such that (3) holds.

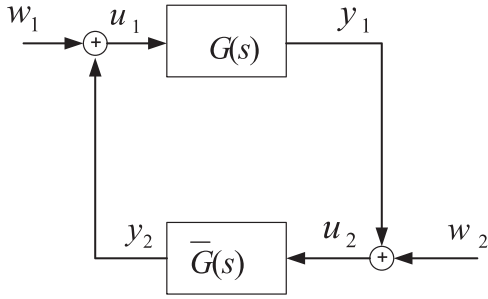


FIGURE 1 A negative-imaginary feedback control system.

One of the important results in the NI system's theory is the robustness property that emerges in the case of positive feedback interconnection between an NI and an SNI system, as shown in Figure 1. The following theorem from [11, 14] states this results.

Theorem 1. Consider an NI transfer function matrix $G(s)$ with no poles at the origin and an SNI transfer function matrix $\bar{G}(s)$, and suppose that $G(\infty)\bar{G}(\infty) = 0$ and $\bar{G}(\infty) \geq 0$. Then, the positive-feedback interconnection (see Figure 1) of $G(s)$ and $\bar{G}(s)$ is internally stable if and only if $\lambda_{\max}(G(0)\bar{G}(0)) < 1$.

Theorem 1 characterizes the conditions for the stability of the feedback interconnection of two NI and SNI systems through phase stabilization. In the case of phase stabilization, it is allowed to have arbitrarily large gains; however, the phase must be such that the Nyquist critical point is not encircled by the Nyquist plot. In other words, in the case of NI interconnected systems, the NI system has a phase lag in $[-\pi, 0]$ where the SNI system has a phase lag in $(-\pi, 0)$. Thus, the two systems in cascade have a phase lag in the interval $(-2\pi, 0)$. That is, the Nyquist plot excludes the positive-real axis.

To establish the results in this paper, we use the following lemma to formulate the NI system in terms of the Port–Hamiltonian formulation (see [38, 39]).

Lemma 2. The system given in (1) has a negative imaginary transfer function if and only if it can be written as

$$\begin{aligned} \dot{x}(t) &= (J - R)(Qx(t) - C^T u(t)), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \quad (4)$$

for some matrices Q, J, R , where,

$$Q = Q^T > 0, \quad J = -J^T, \quad R = R^T \geq 0. \quad (5)$$

3 | PROBLEM FORMULATION

The problem of robust stabilization of linear time-invariant systems with respect to unmodelled dynamics and structure uncertainties can be formulated as follows:

Given a strictly negative imaginary plant with the state-space model given in (1). Suppose that our objective is to design an

output feedback controller that satisfies the negative imaginary property, where the following quadratic cost function;

$$J_{\text{cost}} = \mathbb{E} \left[\mathbf{x}_N^T F \mathbf{x}_N + \sum_{i=0}^{N-1} (\mathbf{x}_i^T Q_i \mathbf{x}_i + \mathbf{u}_i^T R_i \mathbf{u}_i) \right], \quad (6)$$

for, $F \geq 0, Q_i \geq 0, R_i > 0$,

is minimized. In other words, the objective is to design an LQG controller that satisfies the negative imaginary property at the same time.

However, the regular LQG algorithm does not guarantee the negative imaginary property to be satisfied, and therefore, the above problem can be reformulated as follows:

Given a strictly negative imaginary plant $G(s)$ and a synthesized LQG controller $K(s)$ that satisfy the quadratic cost function given in (6); find the nearest negative imaginary transfer function $\tilde{K}(s)$ to the designed LQG controller that satisfies the negative imaginary property. This can be formally stated as follows:

Problem 1. Given an LTI MIMO strictly negative imaginary plant and a designed regular LQG controller with the following state-

space representation $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$, which satisfy the quadratic cost function (6); find the nearest (the closest) transfer function with the state-space representation $\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}$, that satisfy the NI property such that,

$$\inf_{(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})} \mathcal{F}(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}),$$

where

$$\begin{aligned} \mathcal{F}(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}) &= \|A - \tilde{A}\|_F^2 + \|B - \tilde{B}\|_F^2 \\ &+ \|C - \tilde{C}\|_F^2 + \|D - \tilde{D}\|_F^2. \end{aligned} \quad (7)$$

The above formulation has reduced the main problem to the problem of finding the nearest negative imaginary system for a given system, which in our case is a designed LGQ controller.

4 | MAIN RESULTS

As indicated in the problem formulation section, the objective of obtaining an LQG output feedback controller that satisfies the negative imaginary property is reduced to finding the nearest negative imaginary system for a given system, which in our case is a designed LQG controller. This section presents a systematic methodology for finding the nearest negative imaginary system for a given generic system. The problem of finding the nearest negative imaginary system is similar to the problem of finding the nearest positive real system (passive system) presented in [29], where the port-Hamiltonian formulation is used.

First, we introduce the following definition, which is based on Lemma 2 and compares the system described in (4) with the LTI system given in (1).

Definition 3. A system (A, B, C, D) is said to admit a port-Hamiltonian form if there exists a system as defined in (4) such that

$$A = (J - R)Q, \text{ and } B = -(J - R)C^T$$

Based on the above definition, the problem given in (1) can be reduced to the following problem:

Problem 2. Suppose an LTI system with the following state space representation $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$, find the nearest (the closest) system $\begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{C} & \tilde{D} \end{bmatrix}$, such that,

$$\inf_{(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})} \mathcal{F}(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}),$$

where,

$$\mathcal{F}(\tilde{A}, \tilde{B}) = \|A - (J - R)Q\|_F^2 + \|B - (R - J)C^T\|_F^2, \quad (8)$$

where, $\tilde{A} = (J - R)Q$, $\tilde{B} = -(J - R)C^T$.

Next, we present an algorithm for finding the nearest negative imaginary system, which will be used later to design an LQG controller that satisfies the negative imaginary property.

4.1 | Algorithm for finding the nearest negative imaginary system problem

This section proposes an algorithm to solve the problem discussed in the above section.

The problem (2) can be written as follows:

$$\inf_{J, R, Q} \|A - (J - R)Q\|_F^2 + \|B - (R - J)C^T\|_F^2,$$

$$\text{such that } J^T = -J, Q = Q^T > 0, \text{ and } R^T = R \geq 0. \quad (9)$$

The projected gradient method (FGM) presented in [40] and [29] is used to solve the problem in (9).

As indicated in [29], the projected gradient method is much faster and hence better to use compared to the standard projected gradient method [41].

The steps can be summarized as follows:

- Compute the gradient as follows:

$$\|A - (J - R)Q\|_F^2 \Rightarrow \nabla_{J_1} = -2Q\|A - (J - R)Q\|,$$

ALGORITHM 1 Fast gradient method (FGM) with restart [29]

Input: The function under the minimization $f(x)$, the set \mathcal{X} , an initial starting point $x \in \mathcal{X}$, step length lower bound $\underline{\gamma}$, and a parameter $\alpha_1 \in (0, 1)$,

Output: A visible solution x to the given minimization problem $\operatorname{argmin}_{x \in \mathcal{X}} f(x)$.

```

1:  $y = x$ ; initial step-length  $\gamma > \underline{\gamma}$ .
2: for  $k = 1, 2, \dots$  do
3:   % Save the last iteration in memory.
4:    $\hat{x} = x$ .
5:   % Project onto  $\mathcal{X}$ 
6:    $x = \mathcal{P}_{\mathcal{X}}(y - \gamma \nabla f(y))$ .
7:   % Make sure that the function  $f$  has decreased, and if not, decrease  $\gamma$ .
8:   while  $f(x) > f(\hat{x})$  and  $\gamma \geq \underline{\gamma}$  do
9:      $\gamma = \frac{2}{3}\gamma$ .
10:     $x = \mathcal{P}_{\mathcal{X}}(y - \gamma \nabla f(y))$ .
11:   end while
12:   % Reinitialize  $\gamma$  if  $\gamma$  reaches the lower bound.
13:   if  $\gamma < \underline{\gamma}$  then
14:     Restart the fast gradient:  $y = x$ ;  $\alpha_k = \alpha_1$ .
15:     Reinitialize  $\gamma$  at the last value for which it allowed a decrease in  $f$ .
16:   else
17:      $\alpha_{k+1} = \frac{1}{2}(\sqrt{\alpha_k^4 + 4\alpha_k^2 - \alpha_k^2})$ ,  $\beta_k = \frac{\alpha_k(1-\alpha_k)}{\alpha_k^2 + \alpha_{k+1}}$ .
18:      $y = x + \beta_k(x - \hat{x})$ .
19:   end if
20:    $\gamma = 2\gamma$ .
21: end for

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$$\|A - (J - R)Q\|_F^2 \Rightarrow \nabla_{R_1} = 2Q\|A - (J - R)Q\|,$$

$$\|A - (J - R)Q\|_F^2 \Rightarrow \nabla_Q = -2(J - R)\|A - (J - R)Q\|,$$

$$\|A - (R - J)C^T\|_F^2 \Rightarrow \nabla_{J_2} = 2C^T\|A - (R - J)C^T\|,$$

$$\|A - (R - J)C^T\|_F^2 \Rightarrow \nabla_{R_2} = -2C^T\|A - (R - J)C^T\|,$$

or simply, for a given term in the objective function, $f(X) = \|AX - B\|_F^2$ the gradient is $\nabla_Y f(Y) = 2A^T(AY - B)$.

- Project onto the feasible set of matrices Q and R that satisfy both conditions, $Q = Q^T > 0$, and $R^T = R \geq 0$.

The FGM Algorithm 1, which is presented in [29], is used to compute the matrices Q, R .

Similar to the implementation in [29], positive weights, w_i were added to the objective function terms in order to give opportunity for a different importance of each term if needed. Therefore, the objective function can be written as follows:

$$\mathcal{F}(\tilde{A}, \tilde{B}) = w_1\|A - (J - R)Q\|_F^2 + w_2\|B - (R - J)C^T\|_F^2.$$

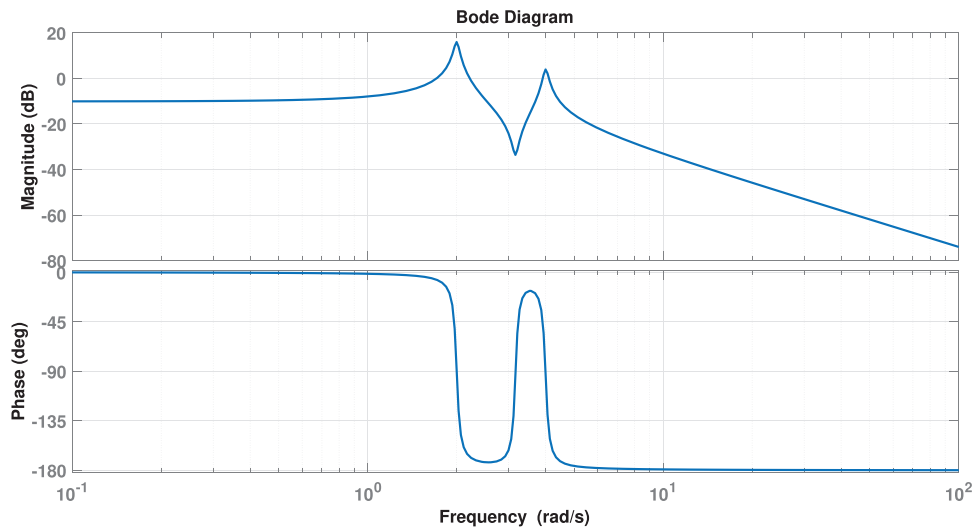


FIGURE 2 Bode plot of the negative imaginary system given in (12), where $N = 2$.

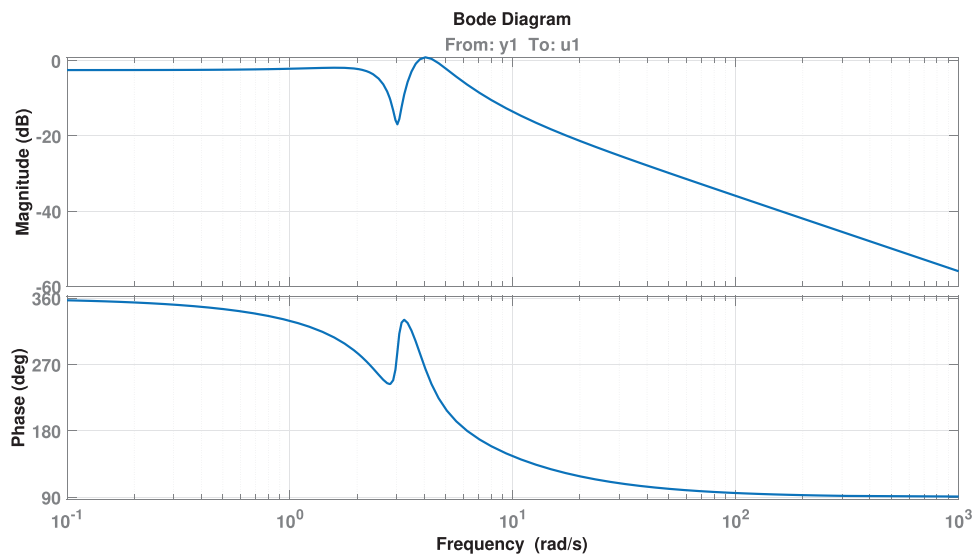


FIGURE 3 Bode plot of the designed controller given in (13).

Parameter settings in our implementation are similar to the parameter settings that were used in [29]. For instance, the step length is calculated as $\gamma = 1/L$ where $L = \|\mathcal{Q}\|_2^2$. Furthermore, in the initialization step, two different initializations were used.

- The first initialization,

$$\mathcal{Q} = I_n, \quad J = (A - A^T)/2, \quad R = \mathcal{P}_{>0}((-A - A^T)/2),$$

where the notation $\mathcal{P}_{>0}(X)$ stands for the projection of a matrix X on the cone of positive semi-definite matrices.

- The second initialization is an LMI-based: Since the given system is not an NI system, the LMI given in (3) has no solution. However, a solution P to the nearby LMIs should be a good initialization for the matrix \mathcal{Q} . We propose the

following to relax LMIs (3):

$$\min_{\delta, P} \delta^2$$

$$\text{such that } \begin{bmatrix} -PA - A^T P & -PB + A^T C^T \\ -B^T P + CA & (CB + B^T C^T) \end{bmatrix} + \delta I_{n+m} \geq 0, \tag{10}$$

4.2 | Optimal control design

In this section, the nearest NI problem, which was presented in the previous subsection, will be used in order to design a near-optimal controller for a given NI plant.

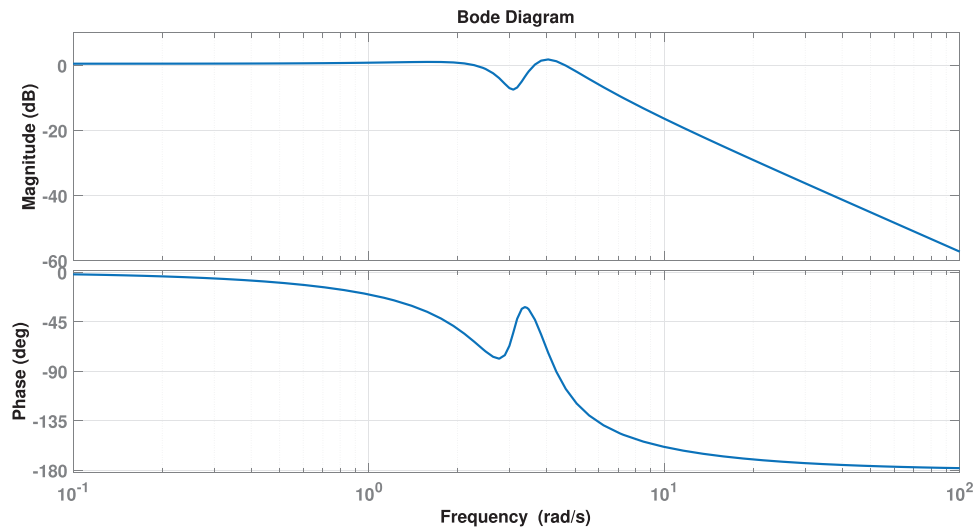


FIGURE 4 Bode plot of the designed Linear–quadratic–Gaussian controller given in (13).

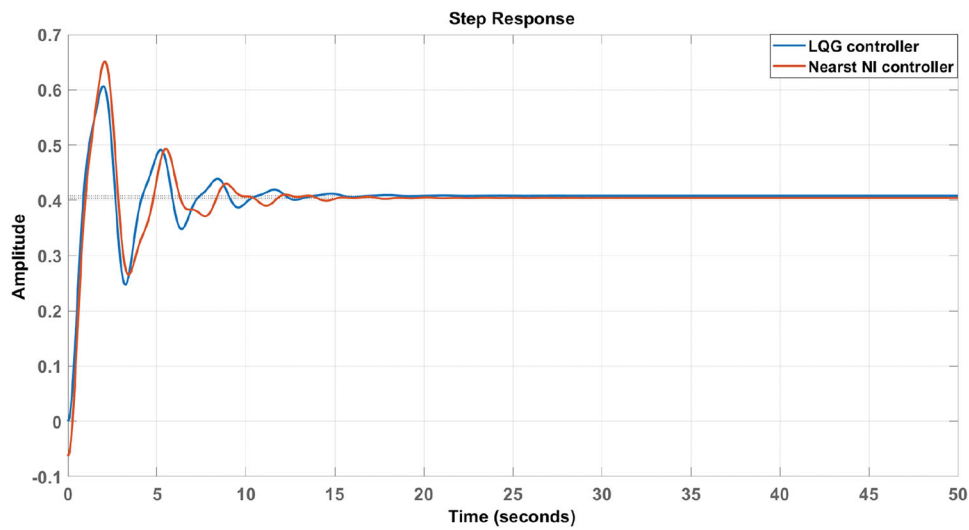


FIGURE 5 Step response of the Linear–quadratic–Gaussian controller and the Nearest negative imaginary controller for the system given in (12), where $N = 2$.

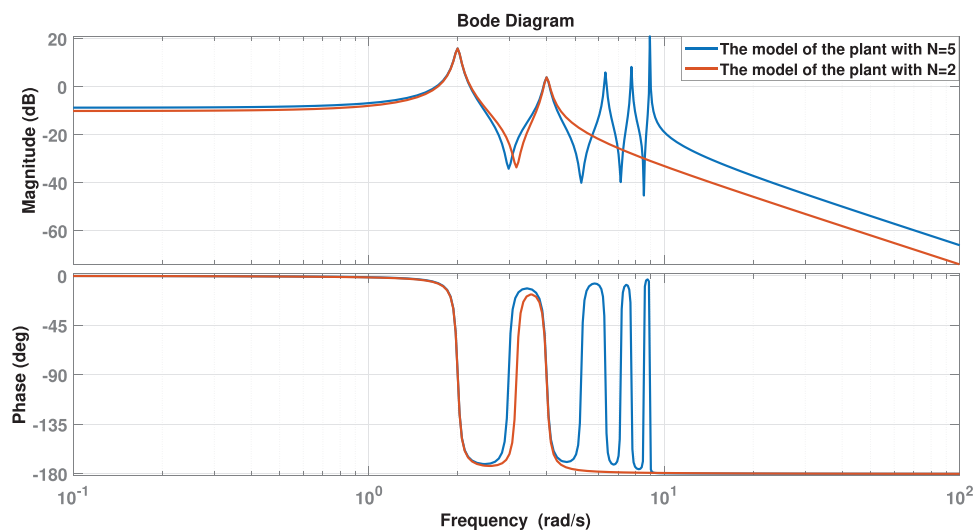


FIGURE 6 Bode plot of the plant model given in (12), with different number modes.

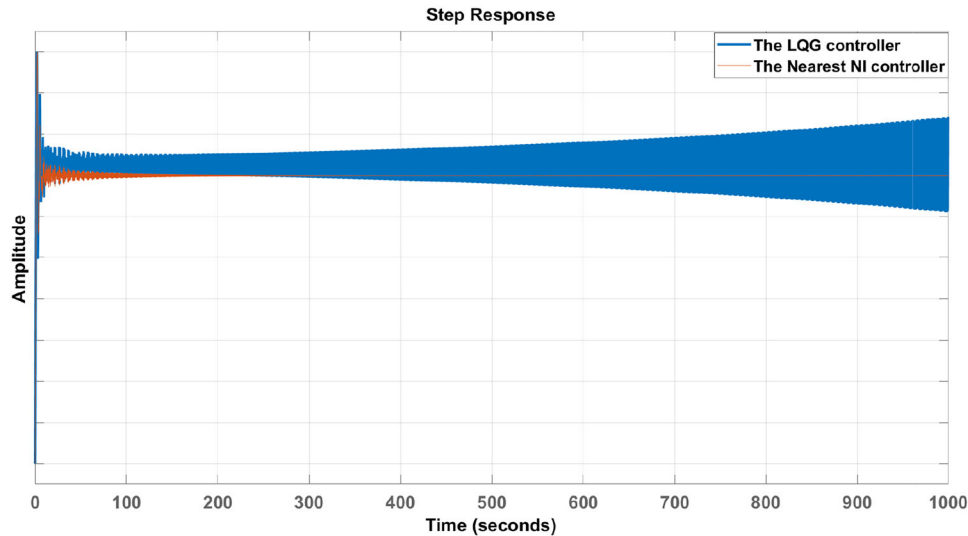


FIGURE 7 The step response of the designed Linear–quadratic–Gaussian (LQG) controller (13) and the nearest negative imaginary controller plant model given in (12), with $N = 5$.

Suppose that we want to design a controller for a given NI plant, $G(s)$, with the state space representation given in (1). Suppose also that we decided to use any standard control synthesis methodology, such as LQG or H_∞ , to design a controller that satisfied a particular performance measure. It is unlikely that the designed controller will satisfy the NI property, and therefore, a robustness property will not be guaranteed. Hence, we can use the nearest NI problem, which was presented in the previous subsection, to find the nearest NI controller to the designed one. The following steps summarize the NI-control design, assuming that an LQG is used in the design.

- Given an LTI NI plant in the form (1), with the transfer function matrix $G(s) = C(sI - A)^{-1}B + D$.
- Design an LQG controller $K(s) = C_c(sI - A_c)^{-1}B_c + D_c$, which minimizes the following cost function:

$$J(u) = \int_0^\infty \{x^T Q_c x + 2x^T N_c u + u^T R_c u\} dt. \quad (11)$$

- Use the methodology presented in this paper to find the nearest NI controller $\bar{G}(s) = \bar{C}(sI - \bar{A})^{-1}\bar{B} + \bar{D}$, to the designed LQG controller $K(s)$.

The new modified controller $\bar{G}(s)$ is a near-optimal controller that satisfied the NI property.

Remark 1. The DC gain condition $\lambda_{\max}(G(0)\bar{G}(0)) < 1$, can be included in the optimization process of finding the nearest NI controller. The DC gain of the NI controller can be calculated as follows:

$$\begin{aligned} \bar{G}(0) &= -\bar{C}(\bar{A})^{-1}\bar{B} + \bar{D}, \\ &= \bar{C}(J - R)Q^{-1}(J - R)\bar{C}^T + \bar{D}, \\ &= \bar{C}Q^{-1}\bar{C}^T + \bar{D}. \end{aligned}$$

In the iterations of finding the matrix Q , particularly in the projection iteration, the matrix Q is scaled to satisfy the DC gain condition. The scaling factor that preserves the DC gain condition is:

$$Q_{new} = \alpha Q_{old}$$

where in the single-input single-output case,

$$\alpha = (CQ_{old}^{-1}C^T + D)G(0) + \epsilon,$$

with a small $\epsilon > 0$.

5 | EXAMPLE

In this section, we present an example to illustrate the design approach presented in this paper.

It is well known that mechanical structures with co-located force actuators and position sensors yield negative imaginary systems [11]. Naturally, these systems are infinite-dimension systems, whereas their models are not. This makes the control design for such systems challenging particularly in the case where the synthesis methodology does not take into account the robustness issue. Therefore, our method shows a big advantage over optimal control methodologies.

To illustrate this fact, consider the following lightly damped flexible structure LTI second-order system with a co-located force actuation and position measurement with the following structure:

$$G(s) = \sum_{n=1}^N \frac{1}{s^2 + 2\zeta_n \omega_n s + \omega_n^2}, \quad (12)$$

where ω_n is the natural frequency and ζ_n the damping factor. Suppose that we want to design an LQG controller for the

system given in (12). Since this model represents an infinite-dimension system, a finite model is chosen to design the controller. We chose $N = 2$ with $\omega_1 = 2, \omega_2 = 4$ and $\zeta_1 = \zeta_2 = 0.02$ for the model parameters. This implies that the model gives the transfer function given in Figure 2.

With appropriate LQG parameters, the controller is given as follows:

$$LQG(s) = \frac{-1.593s^3 + 9.84s^2 - 12.58s + 93.76}{s^4 + 3.847s^3 + 26.66s^2 + 46.86s + 125.1}. \quad (13)$$

The bode plot of the designed LGQ controller as given in Figure 3 shows that it is not an NI controller, since the phase is not in the $(0, -\pi)$.

Now, applying our method of finding the nearest NI controller to the LQG controller given in (13), we get the following transfer function:

$$NILQG(s) = \frac{13.75s^2 + 6.77s + 132.5}{s^4 + 3.847s^3 + 26.66s^2 + 46.86s + 125.1} \quad (14)$$

The bode plot in Figure 4 of the controller given in (14) shows that it satisfies the NI property.

In this example, we apply the FGM algorithm defined in (1) with the initialization given in (10). This gives a nearby standard NI system with error

$$\begin{aligned} \|A - \hat{A}\|_F^2 + \|B - \hat{B}\|_F^2 + \|C - \hat{C}\|_F^2 \\ + \|D - \hat{D}\|_F^2 = 0.6430. \end{aligned}$$

In terms of relative error for each matrix, we have

$$\begin{aligned} \frac{\|A - \hat{A}\|_F}{\|A\|_F} = 5.5917e^{-18}\%, \quad \frac{\|B - \hat{B}\|_F}{\|B\|_F} = 0.0631\%, \\ \frac{\|C - \hat{C}\|_F}{\|C\|_F} = 0\%, \quad \frac{\|D - \hat{D}\|_F}{\|D\|_F} = 0\%. \end{aligned}$$

The step response of the closed feedback interconnection of the plant is given in (12) and both the designed LQG (13) controller and the nearest NI controller (14) are given in Figure 5. It is clear that the response is very similar.

The more interesting part of this example is when we add more non-modelled modes to the plant, that is, $N = 5$ as shown in Figure 6. This means that we include unmodelled dynamics in the plant, which was regarded as uncertainty.

As shown in Figure 7, the designed LGQ (13) will become unstable if we consider the five-mode plant. However, the nearest NI controller still stabilizes the system with acceptable performance. This is due to the negative imaginary property of both the controller and the plant.

6 | CONCLUSION AND FURTHER RESEARCH

In this paper, we have investigated the robust stabilization of negative imaginary systems by finding the near-optimal

negative imaginary controller. The approach is based on a port-Hamiltonian formulation and can be systematically applied by solving LMI conditions. The effectiveness of the technique has been verified by simulation. It has been shown in simulation that the nearest negative imaginary controller produces a very similar response to the nominal LQG controller while ensuring robustness with respect to unmodelled dynamics.

Future research directions include more control design approaches such as H_∞ control. Also, a deeper analysis of the convergence of the optimization algorithm is needed in order to make the results more attractive. Furthermore, the results in this paper can be extended to the class of positive real systems as well.

AUTHOR CONTRIBUTIONS

Mohamed Mabrok: Conceptualization, formulating the problem, developing the results and the simulation. Writing the first draft of the paper. Mahmoud Abdelrahim: Part of the simulation, writing, review and editing. All authors have read and agreed to the published version of the manuscript.

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CONFLICT OF INTEREST STATEMENT


The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

No data was used.

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