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Ferromagnetic effects for nanofluid venture through composite permeable stenosed arteries with different nanosize particles

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In the present article ferromagnetic field effects for copper nanoparticles for blood flow through composite permeable stenosed arteries is discussed. The copper nanoparticles for the blood flow with water as base fluid with different nanosize particles is not explored upto yet. The equations for the Cu-water nanofluid are developed first time in literature and simplified using long wavelength and low Reynolds number assumptions. Exact solutions have been evaluated for velocity, pressure gradient, the solid volume fraction of the nanoparticles and temperature profile. Effect of various flow parameters on the flow and heat transfer characteristics are utilized. [http://dx.doi.org/10.1063/1.4926342]

INTRODUCTION

The study of blood flow of non-Newtonian fluids in a stenosed artery is very useful topic because of the fact that number of cadiovascular diseases in the blood vessel such as hearts attacks and strokes are the leading cause of deaths. In cardiac related problems, the effected arteries get harden as a result of accumulation of fatty substances inside the lumen. These accumulations of substances in arteries known as stenosis. For last couple of decades, researcher have become interested to model the blood flow in stenosed arteries experimentally and theoretically. A number of theoretical studies related to the blood flow through stenosis arteries have been carried out recently in which most of the studies focused in presence of mild or single stenosis as discussed by Chakravarty and Mandal.[1,2] A mathematical analysis for the flow in arteries in the presence of stenosis is studied by Mishra and Chakravarty.[3] Herschel Bulkely fluid in stenosed arteries has been examined by Sankar and Lee.[4] The mathematical modelling done by them was for pulsatile flow. They used regular perturbation technique and found analytical solutions. They have also made the comparison of their results with the Newtonian behavior of blood flow in their article. Oscillatory type blood flow through stenosed arteries with three layered is studied by Tripathi.[5] Mekheimer and Kot[6] in this regards discussed the mathematical modelling of a non-Newtonian fluid through an anisotropically tapered elastic artery with time variant overlapping stenosis. Non-Newtonian behaviour of blood is also discussed by, Mishra et al.[7] They have presented the blood flow through a composite stenosis in an artery with permeable walls. Very recently Akbar and Butt[8] presented magnetic field effects for copper suspended nanofluid venture through a composite stenosed arteries with permeable wall. Further recent literature related to the topic could be seen through. Refs. 9–12.

A Nanofluid is a fluid containing nanometer-sized particles, called nanoparticles. These fluids are engineered colloidal suspensions of nanoparticles in a base fluid. The nanoparticles used in nanofluids are typically made of metals, oxides, carbides, or carbon nanotubes. Nanofluids have their huge applications in heat transfer, like microelectronics, fuel cells, pharmaceutical processes,

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and hybrid-powered engines, domestic refrigerator, chiller, nuclear reactor coolant, grinding and space technology etc extensive literature is available which deals with the study of nanofluid and its applications.[13–21] Maxwell[22] and further developed Hamilton and Crosser model[23] to take into account irregular particle geometries by introducing a shape factor. According to this model, when the thermal conductivity of the nanoparticles is 100 times larger than that of the base fluid. They gives that tha particle can be of shape like bricks, cylinder and platelets. Keep in mind the Maxwell[22] and Hamilton and Crosser model[23] Ellahi et al.[24] discussed the shape effects of nanosize particles in Cu-H_{2o} nanofluid on entropy generation. Further related literature can be viewed through Refs. 25–27.

According to the authors knowledge the shape effects of nanosize particles for the blood flow with water as base fluid is not explored so far. To fill this gap i here discussed shape effects of nanosize particles for the blood flow through composite stenosis in arteries with permeable wall. In the next section we present formulation of the problem. Section three gives solutions of the problem. In section four we discussed physical significance of the problem through graphs and discussion. Last section contains the summary of the present work.

FORMULATION OF THE PROBLEM

Consider an axisymmetric flow of blood through a composite stenosis in a circular tuble of finite length L , with permiable wall as shown in Fig. 1. The geometry of arterial wall with composite stenosis is described by[8]

$$\frac{R(z)}{R_0} = \begin{cases} 1 - \dfrac{2\delta}{R_0 L_0}(z-d); & d < z \le d + \dfrac{L_0}{2}, \\ 1 - \dfrac{\delta}{2R_0}\left(1 + \cos\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right); & d + \dfrac{L_0}{2} < z \le d + L_0, \\ 1; & \text{otherwise}. \end{cases} \tag{1}$$

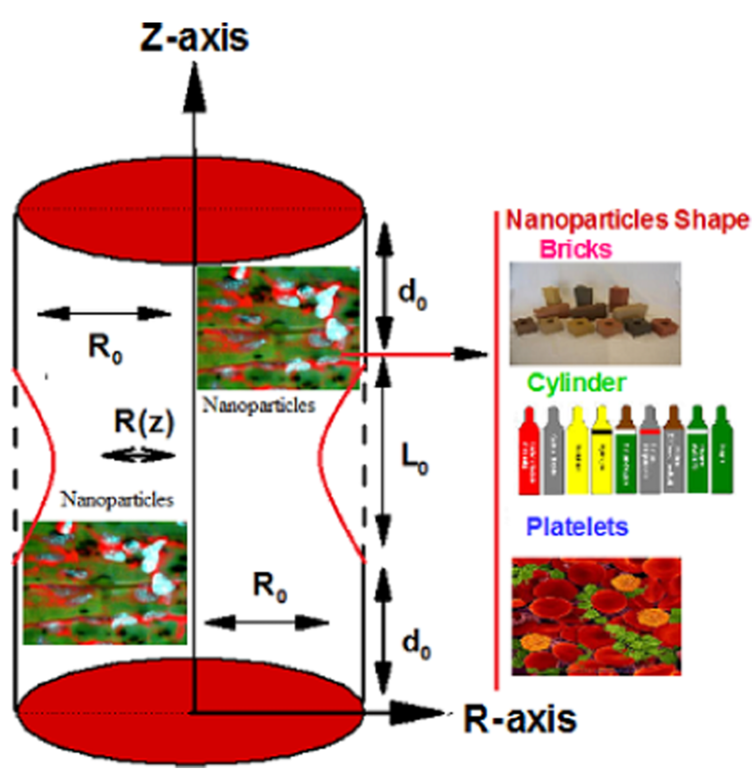


FIG. 1. Geometry of the problem.

where R(z) is the radius of the artery in the obstructed region while R_o is the radius of normal artery. L_0 d, δ are the length, location and height of the stenosis respectively.

The governing equations for an incompressible nanofluid can be written as:

$$\frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} = 0, \tag{2}$$

$$\rho_{nf}\left(v\frac{\partial v}{\partial r} + u\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu_{nf}\frac{\partial}{\partial r}\left(2\frac{\partial v}{\partial r}\right) + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right), \tag{3}$$

$$\begin{aligned}\rho_{nf}\left(v\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z}\right) =& -\frac{\partial p}{\partial z} + \mu_{nf}\frac{\partial}{\partial z}\left(2\frac{\partial u}{\partial z}\right) + \frac{\mu_{nf}}{r}\frac{\partial}{\partial r}\left[r\left(\frac{\partial v}{\partial z} + \frac{\partial u}{\partial r}\right)\right]\\ &+\rho_{nf}g\alpha\left(\overline{T} - T_0\right) - \sigma B_0^2 u,\end{aligned} \tag{4}$$

$$\left(\rho_{cp}\right)_{nf}\left(v\frac{\partial T}{\partial r} + u\frac{\partial T}{\partial z}\right) = k_{nf}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] + Q_0. \tag{5}$$

with the conditions:

$$\frac{\partial u}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0 \ \text{ at } \ r = 0, \tag{5a}$$

$$u = u_B, \quad T = T_0, \ \text{at } r = \frac{R(z)}{R_0}. \tag{5b}$$

where r and z are the coordinates. z is taken as the center line of the tube and r transverse to it, u_B slip velocity, u and v are the velocity components in the r and z directions respectively, T is the local temperature of the fluid. Further, ρ_{nf} is the effective density, μ_{nf} is the effective dynamic viscosity, $(\rho c_p)_{nf}$ is the heat capacitance, α_{nf} is the effective thermal diffusivity, and k_{nf} is the effective thermal conductivity of the nanofluid, which are defined as (see Ref. 21).

$$\begin{aligned}\rho_{nf} &= (1-\phi)\ \rho_f + \phi\rho_f,\ \ \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},\\ \left(\rho c_p\right)_{nf} &= (1-\phi)\left(\rho c_p\right)_f + \phi\left(\rho c_p\right)_s,\ \ \alpha_{nf} = \frac{k_{nf}}{\left(\rho c_p\right)_{nf}},\end{aligned} \tag{6}$$

Here ϕ is the solid nanoparticle volume fraction. Maxwell[22] and further developed Hamilton and Crosser model[23] to take into account irregular particle geometries by introducing a shape factor. According to this model, when the thermal conductivity of the nanoparticles is 100 times larger than that of the base fluid, the thermal conductivity can be expressed as in Refs. 24.

$$k_{nf} = k_f\left(\frac{k_s + (m+1)\ k_f - (m+1)\left(k_f - k_s\right)\phi}{k_s + (m+1)\ k_f + \phi\left(k_f - k_s\right)}\right), \tag{7}$$

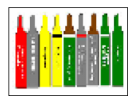
According to them, the values of shape factor are given as in Table I.

We introduce the following non-dimensional variables:

$$\begin{aligned}&\overline{r} = \frac{r}{R_0},\ \overline{z} = \frac{z}{L_0},\ \overline{v} = \frac{L_0}{\delta U}v,\ \overline{u} = \frac{u}{U},\ \overline{d} = \frac{d}{L_0},\ \overline{R} = \frac{R}{R_0},\\ &M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_f},\ G_r = \frac{g\alpha R_0^2 T_0 \rho_{nf}}{U\mu_f},\ \overline{\delta} = \frac{\delta}{R_0},\ \theta = \frac{T - T_0}{T_0},\\ &\overline{p} = \frac{UL_0\mu}{R_0^2}p,\ \beta = \frac{Q_0 R_0^2}{k_f T_0},\ \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}.\end{aligned} \tag{8}$$

where U is the velocity averaged over the section of the tube with radius R_o . Making use of these variables in eqs. (2)-(5) and applying the additional condition $\varepsilon = \frac{R_0}{L_0} = o(1)$ for the case of mild stenosis $(\frac{\delta}{R_0} << 1)$, the non-dimensional governing equations after dropping the dashes can be

TABLE I. Nanoparticle shape with their shape factor.

Nanoparticles Type	Shape	Shape Factor
Bricks		3.7
Platelets		5.7
Cylinders		4.9

written as:

$$\frac{\partial p}{\partial r} = 0, \tag{9}$$

$$\frac{dp}{dz} = \frac{1}{(1-\varphi)^{2.5}} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) - M^2 u + G_r \theta, \tag{10}$$

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta}{\partial r} \right) + \beta \left(\frac{k_s + (m+1)\ k_f + \phi \left(k_f - k_s \right)}{k_s + (m+1)\ k_f - (m+1)\ \left(k_f - k_s \right)\ \phi} \right) = 0, \tag{11}$$

where M β and G_r are the Hartmann number, heat absorption parameter and Grashof number respectively. The non-dimensional boundary conditions on velocity and temperature for permeable wall are:

$$\frac{\partial u}{\partial r} = 0, \ \frac{\partial \theta}{\partial r} = 0 \qquad \text{at} \quad r = 0, \tag{11a}$$

$$u = u_B, \ \frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{D_a}} \left(u_B - u_{porous} \right), \ \theta = 0 \ \ \text{at} \ \ r = R(z). \tag{11b}$$

Where u_B is the slip velocity to be determined and u $_{porous}$ is given as $u_{porous} = -D_a \frac{dp}{dz}$ and

$$R(z) = \begin{cases} 1 - 2\delta(z-d); & d < z \le d + \frac{1}{2}, \\ 1 - \frac{\delta}{2}\left(1 + \cos 2\pi \left(z - d - \frac{1}{2}\right)\right); & d + \frac{1}{2} < z \le d + 1, \\ 1; & \text{otherwise}. \end{cases} \tag{12}$$

EXACT SOLUTIONS

Solving eqs. (9)-(11) using undeterminant coefficient method (UCM) together with boundary conditions (11a), (11b), we get the exact solutions for velocity and temperature as follows:

$$\theta(r, z) = \left(\frac{R^2 - r^2}{4} \right) \left(\frac{k_s + (m+1)\ k_f + \phi \left(k_f - k_s \right)}{k_s + (m+1)\ k_f - (m+1)\ \left(k_f - k_s \right)\ \phi} \right) \beta. \tag{13}$$

$$u(r,\ z) = \frac{1}{4M^4(1-\varphi)^{\frac{5}{2}}}\left[\frac{\begin{array}{c}(-4G_r\beta\frac{k_{nf}}{k_f} + M^2(1-\varphi)^{\frac{5}{2}}\left(-4\frac{dp}{dz} + G_r\beta\frac{k_{nf}}{k_f}\left(R^2-r^2\right)\right) + \\ 2\left(-2\alpha G_r\beta\frac{k_{nf}}{k_f} + M^2(1-\varphi)\left(-2\frac{dp}{dz}\alpha + G_r\beta\frac{k_{nf}}{k_f}\sqrt{D_a}\right) + 2M^4\frac{dp}{dz}\alpha D_a(1-\varphi)\right) I_0\left(M(1-\varphi)^{\frac{5}{4}}r\right)\end{array}}{-\alpha I_0\left(M(1-\varphi)^{\frac{5}{4}}R\right) + M(1-\varphi)^{\frac{5}{4}}\sqrt{D_a}I_1\left(M(1-\varphi)^{\frac{5}{4}}R\right)}\right], \tag{14}$$

The flux Q can be calculated as:

$$Q = 2\int_0^R r u dr, \tag{15}$$

which implies

$$\frac{dp}{dz} = \frac{1}{8M^2(1-\varphi)^{\frac{5}{2}}R}\left[\begin{array}{c}\dfrac{M(1-\varphi)^{\frac{5}{4}}\alpha\left(8M^4Q(1-\varphi)^{\frac{5}{2}} + G_r\frac{k_{nf}}{k_f}\beta R^2\left(8-M^2(1-\varphi)^{\frac{5}{2}}R^2\right)\right) I_0\left(M(1-\varphi)^{\frac{5}{4}}R\right)}{\left(-M(1-\varphi)^{\frac{5}{4}}R\alpha I_0\left(M(1-\varphi)^{\frac{5}{4}}R\right) + \left(M^2R(1-\varphi)^{\frac{5}{2}}\sqrt{D_a} + 2\alpha\left(1-M^2D_a\right)\right) I_1\left(M(1-\varphi)^{\frac{5}{4}}R\right)\right)} \\ \dfrac{+\left(-16G_r\frac{k_{nf}}{k_f}\beta\alpha R + M^4(1-\varphi)^5\left(G_r\frac{k_{nf}}{k_f}R^4\beta - 8M^2Q\right)\sqrt{D_a}\right) I_1\left(M(1-\varphi)^{\frac{5}{4}}R\right)}{\left(-M(1-\varphi)^{\frac{5}{4}}R\alpha I_0\left(M(1-\varphi)^{\frac{5}{4}}R\right) + \left(M^2R(1-\varphi)^{\frac{5}{2}}\sqrt{D_a} + 2\alpha\left(1-M^2D_a\right)\right) I_1\left(M(1-\varphi)^{\frac{5}{4}}R\right)\right)}\end{array}\right]. \tag{16}$$

Impedance resistance λ is calculated as:

$$\lambda = \frac{\Delta p}{Q},$$

since the flow rate Q is constant for all sections of tube

$$\lambda = \frac{1}{Q}\int_0^L\left(-\frac{\partial p}{\partial z}\right)dz,$$

$$\lambda = \int_0^d F(z)\,dz + \int_d^{d+1/2} F(z)\,dz + \int_{d+1/2}^{d+1} F(z)\,dz + \int_{d+1}^{L} F(z)\,dz, \tag{17}$$

where

$$F(z) = -\frac{1}{Q}\frac{dp}{dz}.$$

RESULTS AND DISCUSSION

Here in this section we have presented graphical results with physical interpretation. The velocity profile u(r,z) is plotted against the radial axis r for different values of the slip parameter α , in Fig. 2 for different shape particles. It can be clearly observed that the velocity profile is symmetric for all the parameters and it decreases as r approaches to zero. Velocity increases as r moves away from zero as shown in Fig 2. When we increase slip parameter α than there will be more resistance between blood and arteries than blood flow slowly and velocity profile decreases see Fig.2. It is also noticed that velocity is high for Platelets type nanoparticles as compare to other type of nanoparticles.

The effects of varying parameter β and the stenosis height δ for the temperature profile θ are shown in Figs. 3(a), 3(b). It can be seen that temperature is maximum at r =0 Fig. 3(a) shows that the rise heat absorption parameter β definitely increases temperature rapidly and when we rise stenosis height, then the temperature declines. It is also noticed that the change in temperature for Platelets type nanoparticles is more rapid as compare to the other type of nanoparticles.

The wall shear stress τ has been plotted against the axial distance z and the stenosis height δ . It has been observed Fig. 4(a) – 4(c) that the wall shear stress τ increases with the stenosis height δ from z = 0 to its peak value at z = 0.5 and then decreases back to its minimum value at z = 1. Also the wall shear stress increases for different values of the slip parameter α and the Darcy number D_α with rapid change for bricks type nanoparticles as compared to other type of nanoparticles.

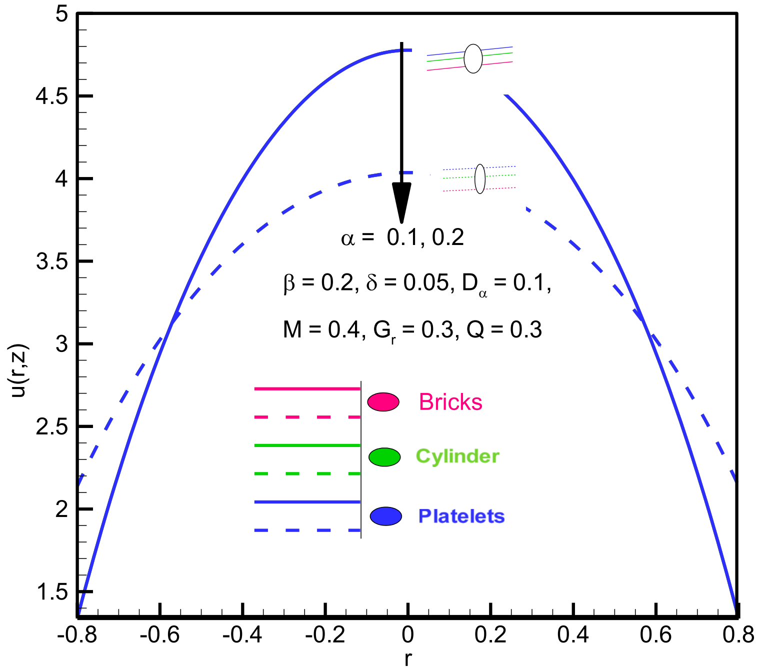


FIG. 2. Velocity profile against the radial axis for different values of α.

The impedance λ is plotted against the the slip parameter α and the Darcy number D_α for different values of the stenosis height δ in Figs. 5(a) and 5(b) It can be observed graphically that the impedance resistance is inversely proportional to both the Darcy number and the slip parameter for different values of stenosis height. Impedance resistance decreases with the increase in stenosis height. It is also noticed that the change in impedance resistance for platelets type nanoparticles is more rapid as compare to the other type of nanoparticles.

Streamlines have been plotted in Figs. 6(a)-6(c). It is observed that the number of trapped bolus increases for bricks shape nanoparticles as compared to the other shape of nanoparticles. But the size of bolus increases for the case of platelets shape of nanoparticles as compared to the bricks and cylinder shape particles.

The copper nanoparticles for the blood flow with water as base fluid with different nanosize particles is discussed. Key points of the performed analysis are as follows:

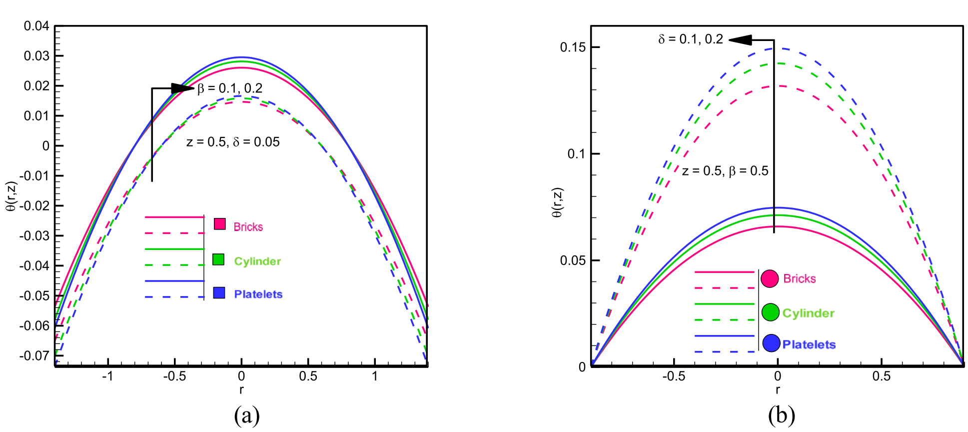


FIG. 3. Temperature profile for different values of (a) β and (b) δ.

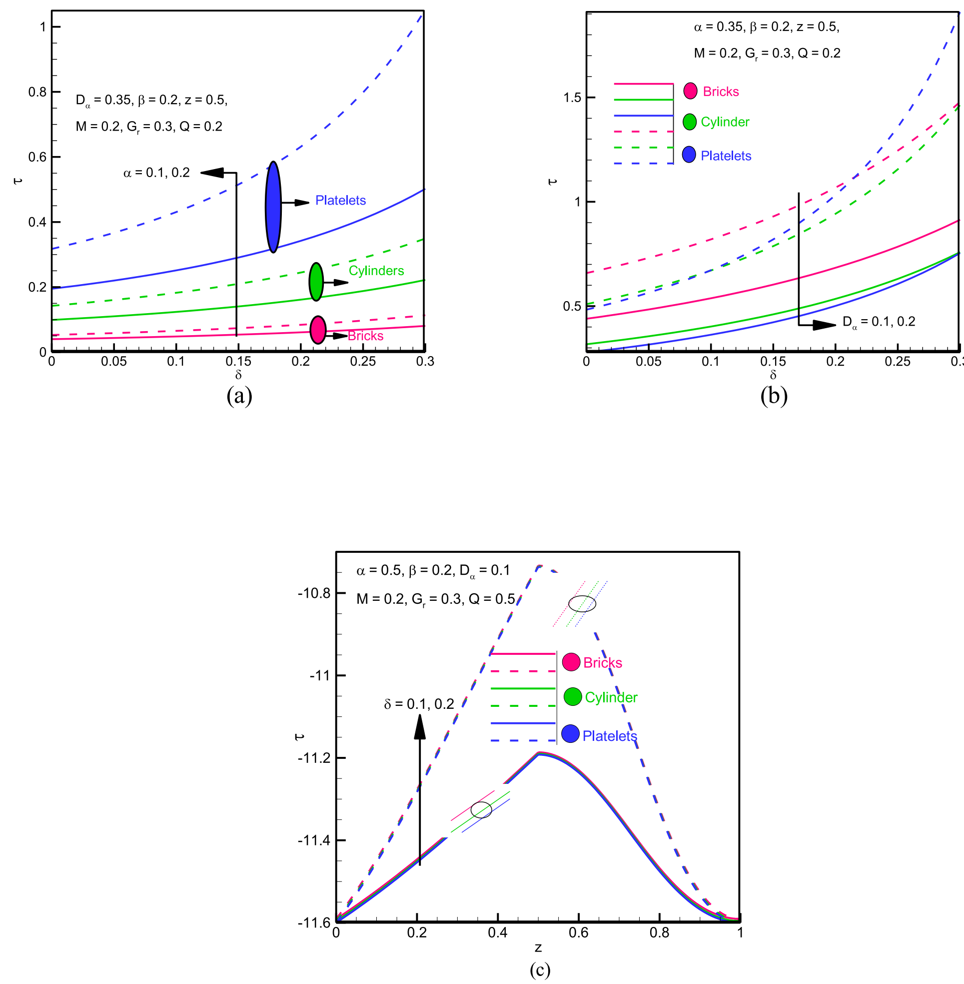


FIG. 4. Wall shear stress for different values of (a) α. (b) D_α and (c) δ.

1. It can be clearly observed that the velocity profile is symmetric for all the parameters and it decreases as r approaches to zero.

2. It is also noticed that velocity is high for Platelets type nanoparticles as compare to other type of nanoparticles.

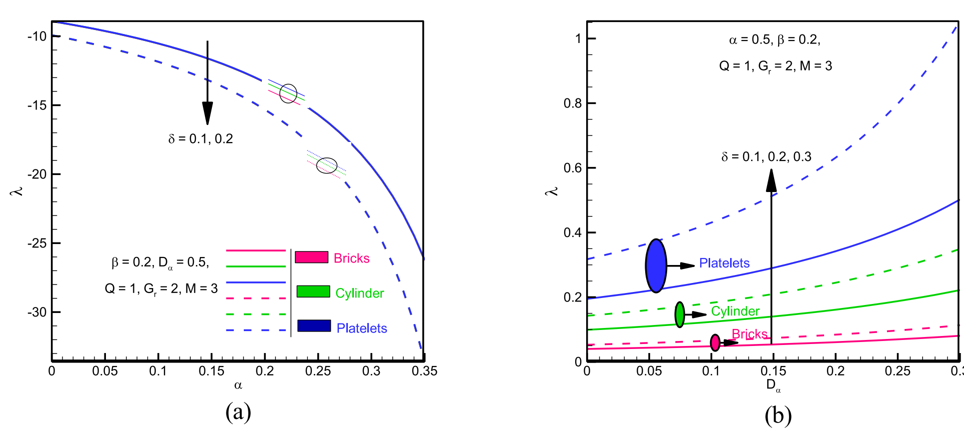


FIG. 5. Impedance resistance for different values of (a) δ with α and (b) δ with D_α.

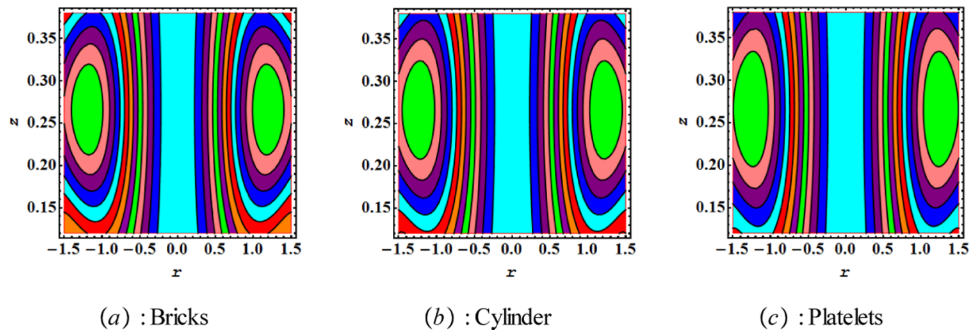


FIG. 6. Streamlines for different nanosize particles for $\alpha = 0.1$, $\beta = 0.2$, $D_\alpha = 0.3$, $Q = 0.5$, $M = 2$, $G_r = 2$.

3. It can be seen that the rise heat absorption parameter β definitely increases temperature rapidly and when we rise stenosis height, then the temperature declines.

4. It is also noticed that the change in temperature for Platelets type nanoparticles is more rapid as compare to the other type of nanoparticles.

5. The wall shear stress increases for different values of the slip parameter α and the Darcy number D_α with rapid change for bricks type nanoparticles as compared to other type of nanoparticles.

6. It can be observed graphically that the impedance resistance is inversely proportional to both the Darcy number and the slip parameter for different values of stenosis height. Impedance resistance decreases with the increase in stenosis height.

7. It is also noticed that the change in impedance resistance for platelets type nanoparticles is more rapid as compare to the other type of nanoparticles.

8. It is observed that the number of trapped bolus increases for bricks shape nanoparticles as compared to the other shape of nanoparticles. But the size of bolus increases for the case of platelets shape of nanoparticles as compared to the bricks and cylinder shape particles.

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