

SHEAR BEHAVIOR OF REINFORCED HIGH-STRENGTH CONCRETE BEAMS WITHOUT SHEAR REINFORCEMENT

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ABSTRACT

Eighteen rectangular singly reinforced high-strength concrete beams without web reinforcement were tested in combined shear and flexure. The main variables were the longitudinal steel reinforcement ratio and the shear-span to effective depth ratio. The uniaxial compressive strength of concrete was about 93 MPa (13,500 psi). The experimental shear capacities were compared with the shear capacities predicted by different empirical equations presented in literatures. Two empirical equations have been proposed to better predict the shear capacity of reinforced high-strength concrete beams without stirrups.

NOMENCLATURE

- a = shear span, distance from support to the first concentrated load, mm.
- d = effective depth of beam, mm.
- d_a = maximum size of aggregate, mm.
- f_c' = compressive strength of concrete, MPa.
- f_{cu} = characteristic cube strength of concrete, MPa.
- f_y = yield strength of flexural reinforcement, MPa.
- L_n = clear span, measured face-to-face of supports, mm.
- M_u = bending moment at section considered, kNm.
- v_{cr} = diagonal cracking strength of concrete, MPa.
- v_{exp} = experimental ultimate shear strength, MPa.
- v_{pred} = theoretically predicted shear strength, MPa.
- v_u = ultimate shear strength, MPa.
- V_u = shear force at section considered, kN.
- ρ = Longitudinal reinforcement ratio, %.
- ρ_{max} = maximum steel reinforcement ratio according to ACI Code, %.

- ρ_{\min} = minimum steel reinforcement ratio according to ACI Code, %.
 τ_m = partial safety factor for shear strength.

INTRODUCTION

The use of high-strength concrete with compressive strength exceeding 41 MPa (6,000 psi) has increased steadily in recent years. Its use has preceded full information on its engineering properties. Previous studies [1-3] have revealed both similarities and dissimilarities between the properties of normal strength and high-strength concrete.

High-strength concrete loaded to failure in uniaxial compression cracks suddenly and usually will form a smooth failure surface when used with normal types of aggregate. It is observed [2,3] that the surface of a diagonal tension fracture of shear-critical beam is relatively smooth, and this might lead to a deficiency in aggregate interlock which constitutes an important component of the contribution of concrete in resisting shear.

The provisions on shear capacity in the present ACI code 318M-89 [4] are based mainly on the experimental investigations carried out on normal-strength concrete. However, article 11.1.2 of the ACI Code briefly deals with the shear capacity of high-strength concrete beams. Further research is needed to examine the suitability of the various provisions of the ACI Code for determining the shear capacity of structural members using high-strength concrete [4].

PREDICTION OF SHEAR STRENGTH

Several equations are available in the literature [5-12] for the prediction of shear capacity of both normal and high-strength concrete beams without web reinforcement.

BS 8110 Equations

For a beam without web reinforcement, the design procedure of BS 8110 [5] is based on safe lower bound nominal shear stress at collapse.

Case I : $a/d \geq 2$

$$v_c = \left[\left(\frac{f_{cu}}{25} \right)^{.33} 0.79 \left(\frac{100A_s}{bd} \right)^{.33} \left(\frac{400}{d} \right)^{1/4} \right] / \tau_m \leq 0.8\sqrt{f_{cu}} \leq 5 \text{ MPa} \quad (1a)$$

where,

f_{cu} = characteristic cube strength of concrete ≤ 40 MPa

$$\frac{100A_s}{bd} \leq 3$$

$$\frac{400}{d} \geq 1$$

τ_m = partial safety factor for shear strength = 1.25.

Case II: $a/d < 2$

For sections near a support ($a/d < 2$), section 4.4.5.8 of the BS 8110 increases the design shear stress of concrete as follows:

$$v_c = (2d/a) [Eq. 1a] \leq 0.8\sqrt{f_{cu}} \leq 5 \text{ MPa} \quad (1b)$$

ACI Equations

The ACI Code equations [4] assumes that the useful shear strength of normal-strength concrete is exhausted when inclined cracking first develops in the member. Article 11.1.2 of the ACI 318M-89 stated that the values of $\sqrt{f'_c}$ used shall not exceed 8.31 MPa. The predicted diagonal cracking stresses are as given below:

(a) Simplified Method [ACI Eq. 11.3]

where,

$$1 \leq \left(3.5 - 2.5 \frac{M_u}{v_u d} \right) < 2.5$$

Zsutty's Equations

Zsutty's equations [6] for normal-strength reinforced concrete beams are :

(a) Diagonal cracking strength

$$v_{cr} = 2.1 \left(f'_c \rho \frac{d}{a} \right)^{0.333} \quad (4a)$$

(b) Ultimate shear strength

Case I: $a/d \geq 2.5$

$$v_u = 2.3 \left(f'_c \rho \frac{d}{a} \right)^{0.333} \quad (4b)$$

Case II: $a/d < 2.5$

$$v_u = \left(\frac{2.5d}{a} \right) [Eq. 4b] \quad (4c)$$

Bazant's Equation

Bazant [7] presented an empirical equation to predict the shear capacity of normal-strength concrete beams as:

$$v_u = \left[10 \frac{(\rho)^{0.333}}{1 + \frac{d}{25d_a}} \right] \left(0.083\sqrt{f'_c} + 20.69 \left(\frac{\rho}{a^5} \right)^{0.5} \right) \quad (5)$$

where,

$$a = \frac{M_u}{v_u d}$$

Mphonde and Frantz Equation

Mphonde and Frantz [8] presented the following empirical equation to predict the ultimate shear capacity of normal and high-strength concrete beams with a/d of 3.6 and of 3.36%:

$$v_u = 0.336 (f'_c)^{0.333} + 0.49 \quad (6)$$

Ahmad et al. Equation

Ahmad et al. [9] proposed the following empirical equation for predicting the ultimate shear capacity of high-strength concrete beams:

$$v_u = 1.8 n \left[f'_c \rho \frac{d}{a} \right]^{0.333} \quad (7)$$

where, for $3 \leq a/d \leq 6$

$$n = 1 - 0.00265 \left[\frac{(d-135.9)^{0.85}}{(a/d)^{0.63}} \right]$$

and for $a/d \leq 3$

$$n = 1 - 0.03985 \frac{(d-135.9)^{0.8}}{(a/d)^{2.84}}$$

EXPERIMENTAL PROGRAM

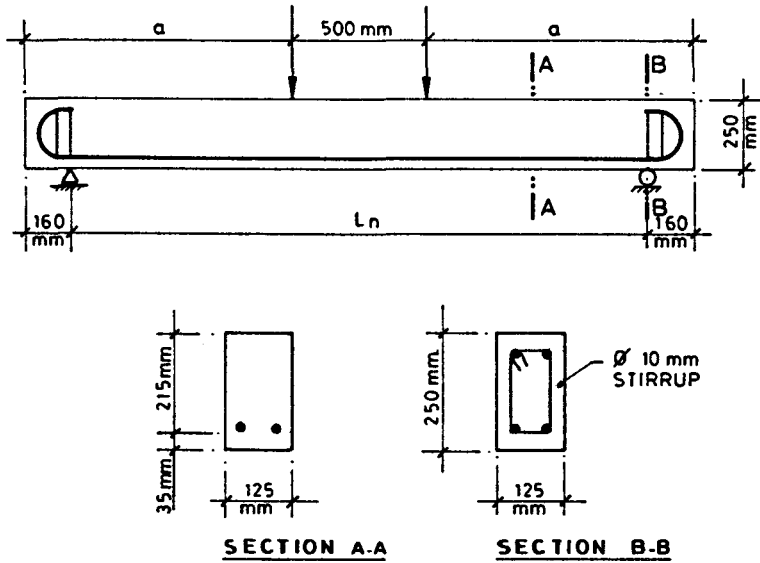
Specimen Details

Three series (18 specimens in all) of reinforced high-strength concrete beams were tested. All beams were singly reinforced and without shear (web) reinforcement. The compressive strength of concrete was about 93 MPa (13,500 psi). The variables were the shear-span/depth (a/d) ratio and the longitudinal steel ratio. The a/d ratios used were 1.0, 2.0, 2.5, 3.0, 4.0 and 6.0. Three longitudinal steel ratios of 0.37%, 2.84% and 4.58% were used for the three series of beams named L, A and M, respectively. The reinforcement ratio of beams of series L and M are close to the ACI [4] recommended minimum value ($\rho_{\min} = 1.4/F_y = 0.28\%$) and the recommended maximum value ($\rho_{\max} = 4.66\%$) respectively. The reinforcement ratio used for the beams of series A was considered to represent an average value.

The beams had a cross section of 125x250 mm (5x10 in.) and were of varied spans (Fig.1a). The beams were designated by three characters consisting of two letters and a number. The first letter "B" denoted "beam", the second number represented the a/d ratio and the third letter (L, A or M) represented the longitudinal reinforcement ratio (low, average or maximum). Thus the beam marked B-2.0-A represented a beam having an a/d ratio of 2.0 and an average longitudinal steel ratio of 2.84%.

Materials and Casting

Grade 60 deformed steel bars were used as the flexural reinforcement. The yield strengths of the 8 mm (#3), 22 mm (#7) and 28 mm (#9) bars used in this study were determined to be 500 MPa (72,500 psi), 460 MPa (66,700 psi) and 470 MPa (68,200 psi), respectively. Ordinary Portland cement type I, a natural desert sand having a fineness modulus of 3.1, crushed Basalt of maximum size 10 mm (3/8 in.) light grey densified microsilica (20% by weight of cement) were used. A superplasticizer was used and the mixing time was increased to facilitate uniform mixing of the ingredients.



a/d	1	2	2.5	3	4	6
BEAM SPAN L_n (mm)	930	1360	1575	1790	2220	3080
L_n/d	4.3	6.33	7.33	8.33	10.33	14.33

(a) DETAILS OF TEST BEAMS

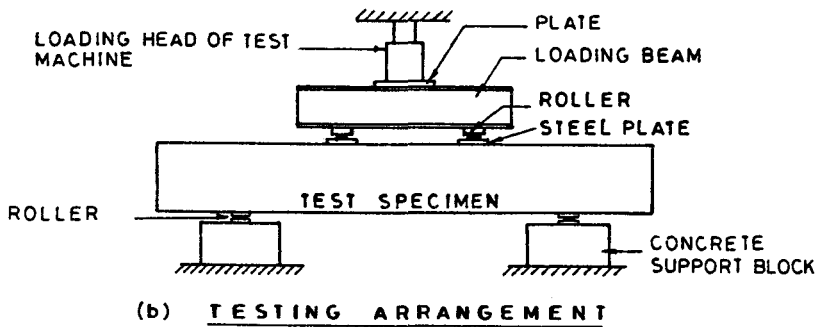


Fig. 1: Details of test beams and testing arrangement

Six 150x300 mm (6x12 in.) cylinders and three 150x150x530 mm (6x6x21 in.) beams were cast as control specimens. The uniaxial compressive strength of concrete as well as the splitting tensile strength were based on the average of test results of three control cylinders each, while the modulus of rupture was determined from the average of test results of three control beams. All test beams and control specimens were cast and cured under similar conditions of temperature and humidity.

Test Procedure

The tested beams were simply supported and were subjected to two point loads as shown in Fig. 1b. The distance between the two point loads was kept constant at 500 mm (19.69 in.) and the loads were applied by means of a 400 kN (90 kips) hydraulic testing machine. Special bearing assemblies were designed to facilitate the application of loads to the test specimens. Two internal strain gauges were glued to the longitudinal steel bars to measure the tensile strain at mid-span. The load was applied in fifteen to twenty five increments until failure of the specimen. After each increment of load, strain gauge readings, mid-span deflection and development and propagation of cracks on the surface of the beam were recorded.

EXPERIMENTAL RESULTS

Eighteen high-strength reinforced concrete beams were tested. The material properties of the tested beams are presented in Table 1 while Table 2 shows the experimental shear strengths v_{exp} and the predicted shear strengths v_{pred} obtained by applying the different empirical equations.

General Behavior

Shear in reinforced concrete beams without stirrups subjected to flexure and shear is resisted mostly by concrete up to the cracking load [13,14]. Beyond this stage, shear is resisted by aggregate interlock, dowel action of the main longitudinal reinforcing bars and the uncracked part of concrete cross section. The resistance due to aggregate interlock depends primarily on the surface roughness at the crack and on the crack width. Examination of the failure surfaces of the tested beams in this investigation revealed that the cracked surfaces were distinctly smooth indicating a possibility of a decrease in the shear resistance contributed by the aggregate interlock.

Table 1: Material Properties of Beams

Beam (1)	a/d (2)	Longitudinal Reinforcement		f'_c (MPa) (5)	f_r (MPa) (6)	f'_{sp} (MPa) (7)
		% (3)	(As) (4)			
B-1-L	1.0	0.37	2 ϕ 8	91.0	9.08	4.9
B-2-L	2.0	0.37	2 ϕ 8	90.1	9.11	5.10
B-2.5-L	2.5	0.37	2 ϕ 8	92.0	9.03	5.16
B-3-L	3.0	0.37	2 ϕ 8	90.4	9.05	5.10
B-4-L	4.0	0.37	2 ϕ 8	91.7	9.10	5.25
B-6-L	6.0	0.37	2 ϕ 8	89.5	8.90	5.05
B-1-A	1.0	2.84	2 ϕ 22	94.8	9.87	6.73
B-2-A	2.0	2.84	2 ϕ 22	94.9	9.83	6.70
B-2.5-A	2.5	2.84	2 ϕ 22	93.7	10.10	4.84
B-3-A	3.0	2.84	2 ϕ 22	91.5	9.80	5.10
B-4-A	4.0	2.84	2 ϕ 22	92.3	10.33	6.07
B-6-A	6.0	2.84	2 ϕ 22	92.0	9.99	6.30
B-1-M	1.0	4.58	2 ϕ 28	93.0	9.50	4.98
B-2-M	2.0	4.58	2 ϕ 28	90.4	8.20	5.16
B-2.5-M	2.5	4.58	2 ϕ 28	91.2	9.32	5.12
B-3-M	3.0	4.58	2 ϕ 28	90.1	9.41	5.10
B-4-M	4.0	4.58	2 ϕ 28	90.5	8.31	5.24
B-6-M	6.0	4.58	2 ϕ 28	90.3	8.11	5.12

Modes of Failure

Shear failures in beams are characterized by the occurrence of inclined cracks [13]. In some cases inclined cracking is immediately followed by member-failure and in other cases the inclined cracks stabilize and substantially greater shear force can be applied before the members fail. In the present investigation,

Table 2: Experimental and Predicted Shear Strengths of Reinforced High-Strength Concrete Beams without Stirrups

Beam	Experi- mental Shear Stress at Failure	Predicted Shear Strength (MPa)								Experimental Mode of Failure
		BS-8110	ACI Eq. 11.3	ACI Eqs. 11.6 & 11.30	Zsutty	Bazant	Ahmed	Proposed Equations		
	(MPa)	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5	Eq. 7	Eq. 9	Eq. 10	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
B-1-L	2.130	-	-	-	-	-	-	-	-	Flexure
B-2-L	1.139	-	-	-	-	-	-	-	-	Flexure
B-2.5-L	0.845	-	-	-	-	-	-	-	-	Flexure
B-3-L	0.735	-	-	-	-	-	-	-	-	Flexure
B-4-L	0.563	-	-	-	-	-	-	-	-	Flexure
B-6-L	0.342	-	-	-	-	-	-	-	-	Flexure
B-1-A	7.098	2.444	1.385	4.152	7.990	9.596	-	7.303	7.332	Arch-Rib
B-2-A	3.348	1.222	1.385	1.672	3.174	3.185	1.622	3.653	2.913	Arch-Rib
B-2.5-A	2.065	1.222	1.385	1.381	2.347	2.585	1.658	2.905	2.153	Shear Comp.
B-3-A	2.047	1.222	1.385	1.348	2.191	2.275	1.622	2.395	2.011	Shear Comp.
B-4-A	1.897	1.222	1.385	1.308	1.996	2.026	1.492	1.803	1.833	Diag. Tens.
B-6-A	1.494	1.222	1.385	1.267	1.743	1.868	1.316	1.200	1.599	Diag. Shear
B-1-M	10.270	2.486	1.385	4.151	9.290	13.658	-	7.534	8.531	Arch-Rib
B-2-M	4.650	1.243	1.385	1.972	3.656	4.110	1.869	3.719	3.356	Arch-Rib
B-2.5-M	2.548	1.243	1.385	1.501	2.724	3.246	1.924	2.987	2.500	Shear Comp.
B-3-M	2.130	1.243	1.385	1.448	2.557	2.805	1.889	2.476	2.343	Shear Tens
B-4-M	2.048	1.243	1.385	1.383	2.323	2.429	1.735	1.861	2.132	Diag. Tens
B-6-M	1.909	1.243	1.385	1.317	2.023	2.196	1.239	1.239	1.861	Diag. Shear

Shear Behavior of Reinforced High-Strength Concrete Beams

the amount of flexural tensile steel did have a quantitative rather than a qualitative effect and influenced the magnitude of load at which a particular mode of failure occurred instead of the mode of failure itself. The observed modes of failure (Table 2 and Fig. 2) are discussed below:

Mode I - Flexural Failure:

Very shallow beams ($a/d > 6$) usually fail in the flexural mode of failure. All beams of series L having a low value of the longitudinal steel ratio ($\rho=0.37\%$) failed in flexure, irrespective of their a/d ratio. The first crack formed due to flexural tension at the section of maximum moment. The beams failed suddenly with little warning due to the failure of the longitudinal reinforcements (Fig.2a). This confirmed the findings of Alvaro [15] that by providing a very low percentage of longitudinal reinforcement ratio it is possible to avoid the shear failure for beams even with small a/d ratios and not shear reinforcement.

Mode II - Flexure-Shear and Diagonal Tension Failure:

In a moderately slender beam ($4 \leq a/d \leq 6$), flexure-shear-crack occurs when an inclined crack in the web of a beam is developed as an extension of a previously formed flexural crack; one of the flexural cracks may continue to propagate through the beam with further load until at some stage it becomes unstable and extends through the beam causing what is called a "diagonal tension failure". The flexure-shear-crack causing failure was observed at the tip of a flexural crack (initiating crack) near the support and propagated simultaneously towards the load-point and towards the support along the tensile reinforcements causing failure of the beam (Fig. 2b).

Mode III - Shear-Tension and Shear-Compression Failures:

Beams having a/d ratios of 2.5 and 3.0 exhibited this mode of failure. In the region of combined moment and shear of some of the beams, secondary cracks propagated backward along the longitudinal tensile reinforcement from the inclined crack (due to dowel action) causing a loss of bond. As the main reinforcement began to slip, the wedge action of the bar deformations contributed to splitting of the concrete and a further propagation of the crack, resulting in anchorage failure of the reinforcement called "shear-tension failure". Beams B-3-A and B-3-M were observed to have shear-tension failures (Fig. 2c). In beams B-2.5-A and B-2.5-M the concrete above the inclined crack failed by crushing causing "shear-compression failure".

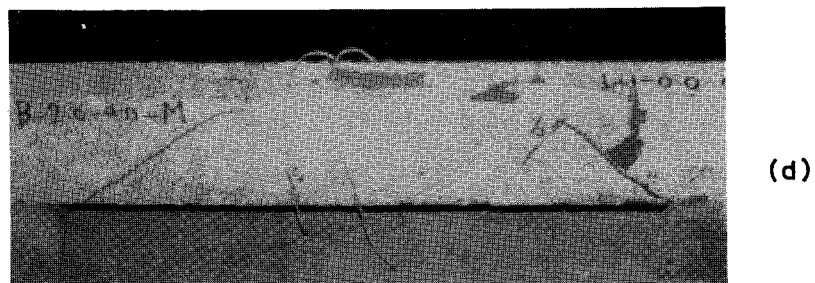
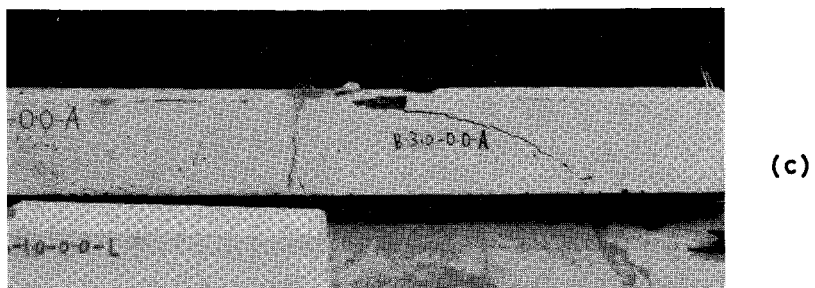
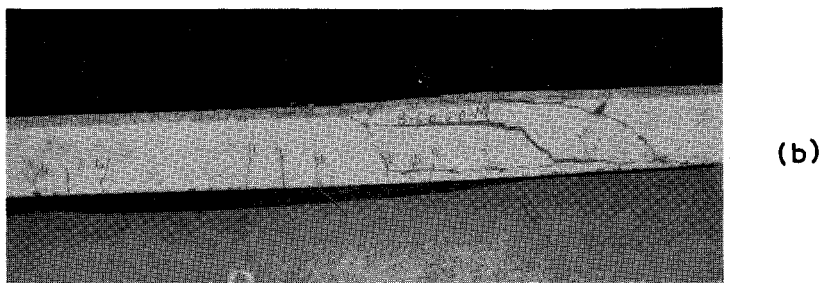


Fig. 2: Typical crack patterns for beams

Mode IV - Arch-Rib Failure:

For a relatively deeper beam, significant principal compression and tension exist respectively along and across the line joining the load and the reaction, and inclined cracks occur immediately in-side the reaction plates. Thereafter, the beam without web reinforcement transforms into a tied-arch which may fail in a number of ways [13]. Beams of series A and M having a/d ratios of 2 and 1 exhibited failures of mode IV and the failures resulted from the crushing of the arch-rib (Fig. 2d).

Shear Strength

Table 2 and Fig.3 show that the experimental shear strength decreased with an increase of a/d ratio and the a/d ratio of 2.5 was found to represent a transition zone (Fig.3). For a given applied load on a beam, a larger a/d ratio would cause a larger bending moment in the shear-span resulting in greater flexural stresses and

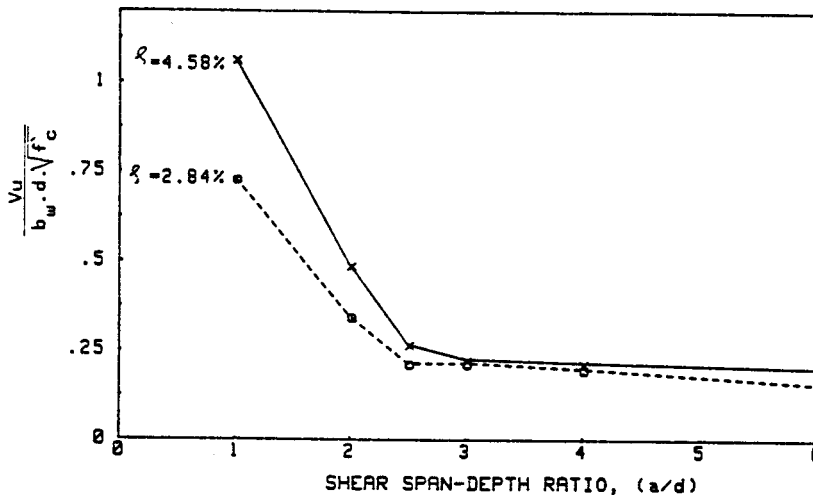


Fig. 3: Effect of longitudinal steel ratio and shear-depth (a/d) ratio on the experimental shear strength

increased depth of penetration of the flexural cracks and increase the probability that a flexural crack would turn into an inclined one. However, the influence of increased a/d ratio on the shear strength was observed to be dependent also on the longitudinal reinforcement ratio. An increase of a/d ratio from 4.0 to 6.0 resulted in shear strength reductions of 6.8% and 21.3% for beams of series M ($\rho = 4.58\%$)

and A ($\rho=2.84\%$), respectively. This significant reduction in strength is attributed to the yielding of the longitudinal steel close to failure which permitted the crack to increase in both length and width and hence adversely affected the dowel action.

The beams with $a/d < 2.5$ exhibited greater resistance to shear than the longer beams. This may be a consequence of the tied-arch action of beams as explained earlier. Also, the longitudinal reinforcement ratio had greater influence on the shear strength of beams whose a/d ratios were smaller than 2.5. An increase of ρ from 2.84% to 4.58% raised the shear strength by 38.9% to 44.7% for beams of a/d ratios of 2 and 1, respectively.

Table 2 shows a comparison of the experimental shear capacities of the tested beams with those predicted by the different empirical equations [4-9] presented earlier. The following may be construed from the comparative results.

1. BS 8110 [5] always gives a lower bound estimate of the shear capacity of the tested beams. The high conservatism of the equation is mainly due to limiting the maximum compressive strength of concrete to 40 MPa and the effect of a/d ratio.
2. ACI Eq.11.3 [4] always gives a lower bound estimate of the shear strength of the tested beams and is highly conservative for a/d ratios less than 2.5. The equation neglects the influences of a/d and on the shear capacity of beams.
3. ACI Eq.11.6 or ACI Eq.11.30 [4] gives a conservative prediction of the shear strength of the tested beams. The test results indicate that the influence of a/d on the shear strength is not adequately accounted for in the equations.
4. Zsutty's Equations [6], in general, give good prediction of the shear strength of the tested beams.
5. Bazant's Equation [7] gives an over-estimated prediction of shear strength of beams for all values of a/d .
6. Mphonde and Frantz's Equation [8] is valid only for beams with a/d values of 3.6 and of $\rho=3.36\%$ and therefore could not be considered for this investigation.
7. Ahmed et al. Equation [9] provides good estimates of the shear strengths of the tested beams.

PROPOSED EQUATIONS

Based on the test results modifications of the ACI equation [4] and Zsutty's equations [6] are proposed for predicting the shear capacity of high-strength concrete beams.

Modified ACI Equation

The ACI equation (Eq.3) is proposed to be modified for predicting the shear capacity of high-strength concrete beams without web reinforcement by emphasizing the contribution of a/d ratio and the longitudinal steel ratio as follows:

$$v_u = 0.7\sqrt{f'_c} \frac{d}{a} + 17.2 \rho \frac{d}{a} \quad (8)$$

Modified Zsutty's Equations

The proposed modifications of Zsutty's equations (Eq.4) for predicting the shear capacity of high-strength concrete beams are as follows:

$$\text{Case I: For } \frac{a}{d} > 2.5$$

$$v_u = 2.1 \left(f'_c \rho \frac{d}{a} \right)^{0.33} \quad (9a)$$

$$\text{Case II: For } \frac{a}{d} \leq 2.5$$

$$v_u = \left[\frac{2.5}{a/d} \right] [Eq.9a] \quad (9b)$$

The proposed modified equations gave good predictions of shear capacities of the tested beams for the different flexural reinforcement ratios and for $a/d \geq 2$ and gave conservative predictions for $a/d < 2$ as shown in Table 2 and Fig.(4). The proposed equations were also verified with the results of experimental

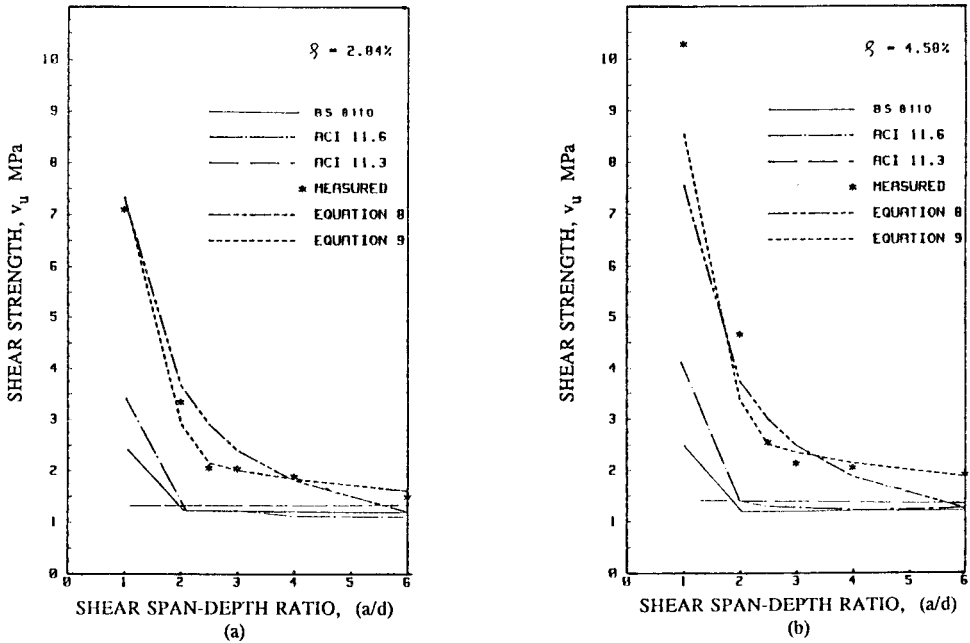


Fig. 4: Comparison of the experimental and predicted shear strength for beams with (a) $\delta=2.84\%$, and (b) $\delta=4.58\%$

investigations of other researchers [8,9,16] and were found to give good predictions of shear capacities (Figs. 5, 6 and 7). The histograms in Figs. (7a) and (7b) show the ratios of the experimental/predicted shear strengths using the proposed modified equations. Eighty one observations were considered in these histograms which include data obtained from the present investigation as well as from the previously published results [8,9,16]. It is observed that Eq. 9(a and b) gives better predictions of the shear strengths than Eq.8. The cumulative density functions for both equations are shown in Fig. (7c).

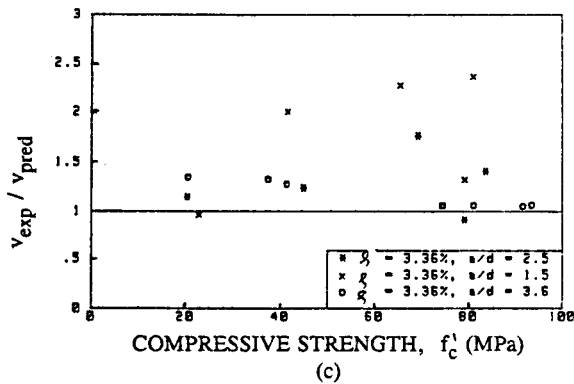
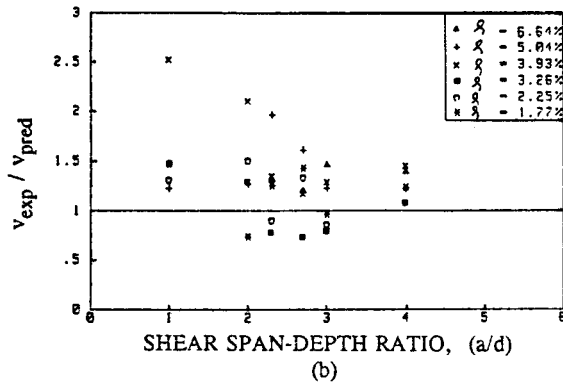
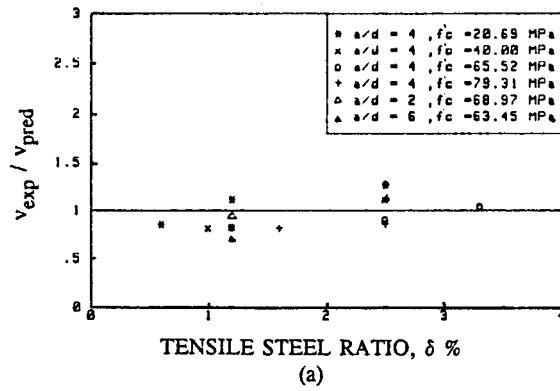


Fig. 5: Verification of the proposed equation [Eq. (8)] with works presented by (a) Mphonde et al. [8], (b) Ahmad et al. [9] and (c) Elzanaty et al. [16]

Shear Behavior of Reinforced High-Strength Concrete Beams

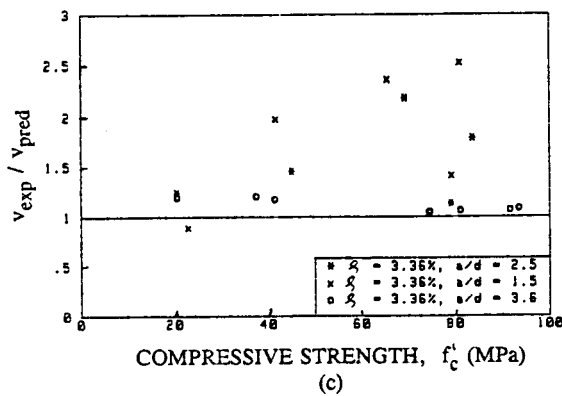
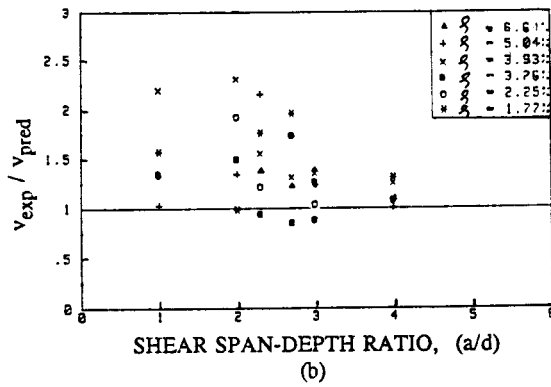
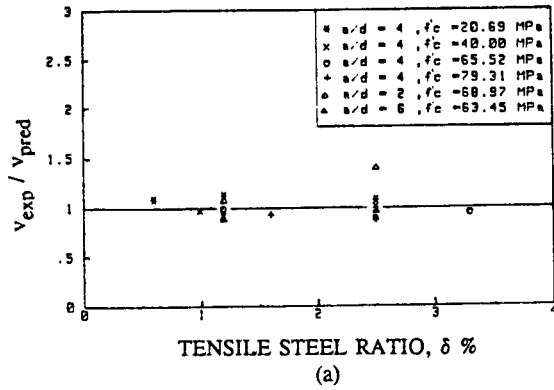


Fig. 6: Verification of the proposed equation [Eq. (9)] with works presented by (a) Mphonde et al. [8], (b) Ahmad et al. [9] and (c) Elzanaty et al. [16]

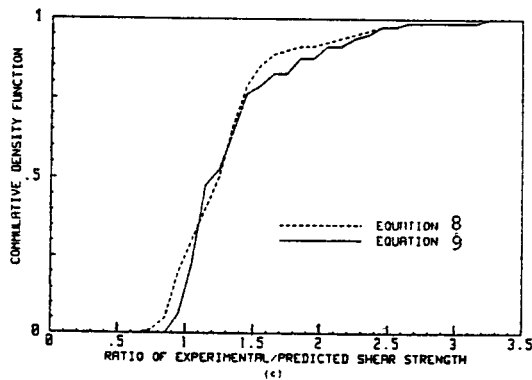
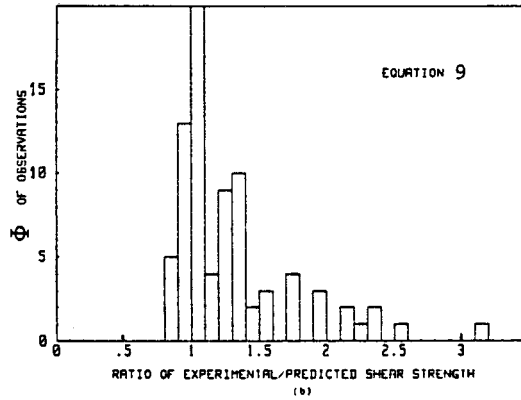
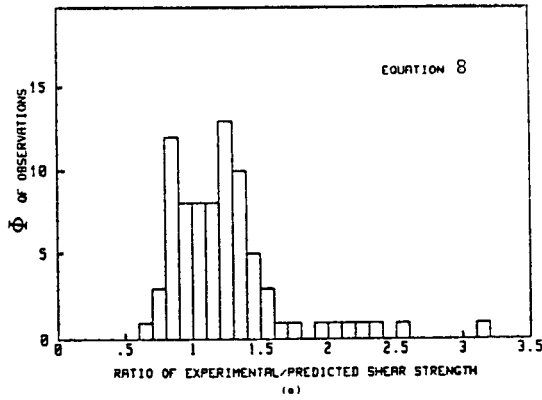


Fig. 7: Statistical analysis of the experimental to predicted shear strength by means of (a) histogram using Eq. 8, (b) histogram using Eq.9 and (c) cumulative density function using Eqs. 8 and 9

CONCLUSIONS

From the results of eighteen singly reinforced high-strength concrete beams without shear reinforcement tested under flexure and shear, the following conclusions are drawn :

1. The shear strength of the tested beams increases with a decrease in a/d ratio, and the increase becomes more significant for a/d ratios lower than 2.5.
2. Beams of low reinforcement ratios ($\rho \approx \rho_{\min}$) fail in flexure irrespective of their a/d values.
3. BS 8110 and ACI Code equations give conservative prediction of the shear strength of high-strength concrete beams and present highly conservative prediction for beams with a/d values of less than 2.5. For low reinforcement ratios ($\rho \approx \rho_{\min}$), flexural failure occurs and the predictions of shear capacity by BS 8110 and the ACI equations bear no significance.
4. Modifications in the ACI Code equation and Zsutty's equations have been proposed to predict the shear capacity of reinforced high-strength concrete beams without stirrups. Barring beams with very low reinforcement ratios, the proposed modified equations predict shear strength which agree well with not only the present test results but also with other published values.

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