Engineering Journal of the University of Qatar, Vol. 16, 2003, pp.61-72

PERFORMANCE ANALYSIS OF CODED SFH-SSMA COMMUNICATION SYSTEMS OVER MUI CHANNEL WITH NO SIDE INFORMATION

Yousef G. El Jaafreh* and Khalid A. Qaraqe**
*** Qatar University, Qatar ** Texas A&M University, USA**

ABSTRACT

This paper presents the performance analysis of a Slow Frequency Hopped Spread Spectrum Multiple Access (SFH-SSMA) communication system under conditions of Multiple Users Interference (MUI) over a communication channel with no side information. Reed Solomon (RS) code with error models characterized by probability of error for decoding are employed along with an orthogonal M-ary frequency shift keying (MFSK) modulation scheme, and a non-coherent demodulation. The tradeoff between selected communication system parameters; RS code rate (r), number of frequency slots (q) available in the frequency hopping pattern, and modulation alphabet size (M) are considered and analyzed. Incomplete Beta function approximation and Gaussian random variable approximation methods are used in the analytical model. The simulation and analytical results show that a significant improvement of the channel normalized throughput can be achieved on the expense of minimal RF bandwidth expansion factor for a given active users (k).

I. INTRODUCTION

One of many reasons for developing a cellular mobile telephone system and deploying it in major cities is the operational limitations of conventional land mobile telephone systems; namely : limited service capability and inefficient frequency spectrum utilization. There are three multiple access schemes: Frequency Division Multiple Access (FDMA) serves the calls with different frequency channels, Time Division Multiple Access (TDMA) serves the calls with different time slots, and Code Division Multiple Access (CDMA) serves the calls with different code sequences. CDMA scheme was developed mainly to increase communications systems capacity, and is uniquely designed to work in cellular systems. The primary purpose of using CDMA for higher capacity cellular communications is due to its inherited characteristics where either the same radio channel can be reused in all neighboring cells, or different code sequence uses the same radio channel to carry different traffic channels. Advantages of CDMA implemented in cellular communications can be found in reference [1]. Two papers [2,3] analyzed CDMA in depth, while other interesting literature on CDMA can be found in references [4,5]. Frequency hopping multiple access, using code division multiplexing as the main multiple access mechanism has been the subject of considerable research effort in recent years [6,7,8], leading to a viable system design, the SFH_900. [9], which was a strong contender for the GSM Pan-European digital mobile radio network. Not only does frequency hopping multiple access provide inherent frequency diversity, but also, it has the property of randomizing cochannel interference [10].

In this paper we consider SFH-SSMA communication system utilizing RS codes. When two or more transmissions simultaneously occur in the same frequency slot, an event referred to as a hit may occur. Further, the demodulator and the decoder can not obtain information on whether or not symbols in the packet have been hit. Clearly, the collision probability increases as the number of transmitters increase and the available number of frequency slots decrease. Consequently, as more users are added, the communication channel error probability will also increase, resulting in degrading channel throughput (transmitted information). Since our focus of interest will be on Multiple Users Interference (MUI), the channel may be assumed noiseless, and hence, we can attribute all bit errors to interference. Similar works on throughput, bandwidth expansion and channel capacity for FH-SSMA systems are presented in [11,12]. However, in this paper additional coded system parameters will be investigated to demonstrate the necessary tradeoffs among the following parameters; coding rate (r), number of active users (k), number of frequency slots available (q), and modulation alphabet size (M) in order to maximize system throughput and minimize bandwidth expansion (B_{exp}) for a given channel bandwidth.

II. SYSTEM AND CHANNEL MODEL

The communication system model block diagram used is shown in Fig. 1 [11,12]. In a FH-SSMA communication system, the available channel bandwidth is subdivided into a number of smaller sub-channels known as frequency slots (q) and each user pair has a frequency hopping pattern that randomly hops among all (q) frequency slots with probability 1/q independent of previous hop frequencies (i.e., identical independent distribution) [13]. A Pseudo Noise (PN) sequence generator is used as a spreading code to control the sequence of the carrier frequencies by selecting a frequency that is produced in the frequency synthesizer. Let T_s be the duration of a MFSK modulation symbol and T_h is the hop duration interval. In slow FH, which is of primary interest here, the hop rate is much less than the information symbol rate, and thus many symbols are sent on the same carrier frequency during each hop, maintaining narrow band transmission conditions within each hop, provided, that the symbol modulation band width does not exceed the coherence band width. The ratio $N_s = T_h / T_s$ is the number of data symbols per hop where $N_s \geq 1$. In this paper we consider SFH system, by assuming one symbol is transmitted per hop, with the result that each packet consist of one symbol, $N_s = 1$, and $T_s = T_h = T$. The information is transmitted by sending M tones separated by small radio frequency channel increment (Δf) Hz. We further assume that the slotted packet radio network consists of N independent operating users in which K users (transmitter-receiver pairs) wish to communicate over the communication channel at any given time as illustrated in [12]. Decoding is done independent of each of the K receivers, and thus there is no cooperation between users on either the transmitting or the receiving side. It is also assumed that any hit (full or partial), causes complete error of the symbol if all the interfering symbols occupy different tones.

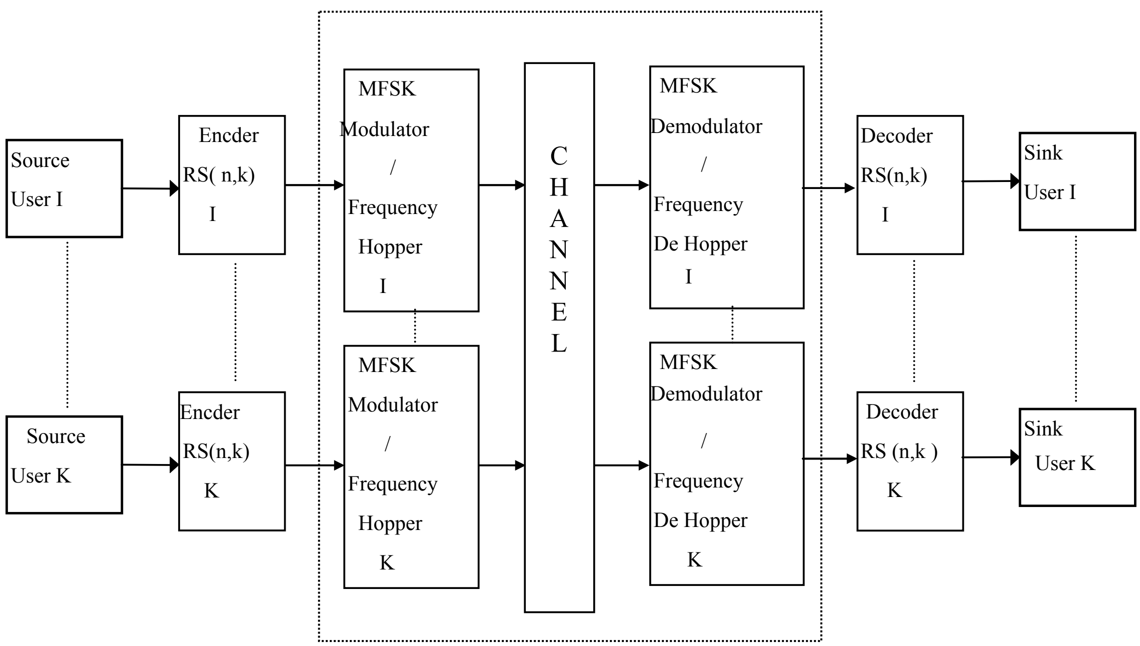


Fig.1 Communication System Model

For K active users the probability (P_r (i hit)) that i other transmitters share the same frequency slot with the transmitter under consideration is given by [13];

$$P_r(\text{i hit}) = \binom{K-1}{i} \cdot (P_h)^i \cdot (1 - P_h)^{K-1-i} \tag{1}$$

where P_h is the probability of a hit, and $\binom{K-1}{i}$ is the binomial coefficient.

For any RS symbol and M-ary FSK- FH system this has been shown to be [4];

$$P_h = \begin{cases} [1+\frac{m}{N_S}\cdot(1-\frac{1}{q})]\cdot\frac{1}{q} & \text{asynchronous FH} \\ \\ [1+\frac{m}{N_S}\cdot(1-\frac{1}{q})]\cdot\frac{1}{2q} & \text{synchronous FH} \end{cases} \tag{2.a}$$

N_s is the number of code symbols per hop ($N_s \geq 1$) and (m) accounts for the fact that each RS symbol contains, m M-ary FSK symbols. The upper bound is very close to being exact whenever $q \geq 16$ and $K \geq 2$. By using our previous assumption for $N_s = 1$, and $m = 1$, (2.a) can then be upper bounded as;

$$P_h = \begin{cases} \frac{2}{q} & \text{asynchronous FH} \\ \\ \frac{1}{q} & \text{synchronous FH} \end{cases} \tag{2.b}$$

III. PERFORMANCE ANALYSIS

When no side information is available, i.e. (the demodulator and the decoder can not obtain information on whether or not symbols in the packet have been hit), the system does not produce any erasure symbols, but instead the receiver makes hard decisions based on the non-coherent demodulator outputs. Hence, whenever a hop is hit, the modulator output is equally likely to be any one of the M-ary possible symbols, and if there is no hit, the symbols are correctly received. The modeling of the interference becomes much more difficult, and represents the worst case in the sense of pessimistic results for the achievable regions and throughputs (lower bound of throughput). The resulting discrete channel is modeled as an M-ary symmetric channel. It is assumed that in the vicinity of particular receivers there are K transmitted signals, all of which share the same channel, and whenever two or more users occupy the same frequency slot at the same time, the probability of error is M-1/M, and zero if there is no hit. For K simultaneous transmissions, the probability of the symbol error P_e (uncoded system) can be upper bounded as;

$$P_e = \frac{M-1}{M}\cdot[1-(1-P_h)^{K-1}] \tag{3}$$

When a bounded distance decoding for the RS codes with hard decision is employed; the decoder can correct up to (t) errors out of the (n) symbols. The symbol error probability for the coded system is given in [14] as;

$$P_{e,s} = \sum_{j=t+1}^{n}\binom{n}{j}\cdot(P_e)^j\cdot(1-P_e)^{n-j} \tag{4}$$

where ($P_{e,s}$) is the symbol error probability for the bonded distance decoder, given k simultaneous transmission, and assuming a memory less M-ary erasure channel.

and $t = \left\lfloor \frac{n-k}{2} \right\rfloor$. Equation (4) is valid when all RS symbols are subjected to independent errors, and it serves as an upper bound for the M-ary symbol error probability. However, it has a computational problem due to the round off error when the summation term is greater than about 12.

Two useful approximate methods have been used in our analytical analysis;
First: the Incomplete Beta function approximation method [15,16] yielding;

$$P_{e,s} = \frac{B_{P_e}(t+1, n-t+1)}{B(t+1, n-t+1)} \tag{5}$$

where $B_{P_e}(t+1, n-t+1) = \int_0^{P_e} y^{\frac{n-k}{2}} \cdot (1-y)^{\frac{n+k}{2}+1} dy$. and

$$B(t+1, n-t+1) = \int_0^1 y^{\frac{n-k}{2}} \cdot (1-y)^{\frac{n+k}{2}+1} dy$$

Asymptotically as, n, k$\rightarrow \infty$ while r = k/n is held constant, the symbol error probability becomes

$$\underset{n.k \to \infty}{P_{e,s}} = \begin{cases} 0 & r < r_c \\ 0.5 & r = r_c \\ 1 & r > r_c \end{cases} \tag{6}$$

where the critical coding rate (r_c) at large enough values of n is $r_c = 1 - 2 \cdot P_e$
The second approximation method is obtained by using Gaussain random variable approximation [11,17,18], (with same mean and variance), yielding;

$$P_{e,s} = 1 - \Phi\left(\frac{t - n \cdot P_e}{\sqrt{n \cdot P_e \cdot (1 - P_e)}}\right) \tag{7}$$

where $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-y^2}{2}} dy$. Asymptotically as, n, k$\rightarrow \infty$ while r = k/n is held constant, the symbol error probability becomes

$$\underset{n.k \to \infty}{P_{e,s}} = \begin{cases} 0 & r < r_c \\ 0.5 & r = r_c \\ 1 & r > r_c \end{cases} \tag{8}$$

1. Normalized Throughput

To maximize the channel throughput, the optimum system parameters (K_{opt}, r_{opt}, and q_{opt}) must be obtained. The normalized throughput is given as [12];

$$W = \frac{K}{q} \cdot r \cdot P_{c,s} \tag{9}$$

where $P_{c,s} = 1 - P_{e,s} = \sum_{j=0}^{t} \binom{n}{j} \cdot (P_e)^j (1 - P_e)^{n-j}$ is the probability of a correct codeword. Asymptotically as n, k $\rightarrow \infty$, while r = k/n is held constant, $P_{c,s} \rightarrow 1$, then (9) can be written as a function of (r,q) or (K,q) to be:

$$\lim_{n,k\to\infty} W = r_c \cdot \frac{K}{q} \tag{10}$$

Optimizing W over (r), the maximum normalized throughput obtained is $W_{opt} = 0.0725$ at $r_{opt} = 0.4597$ for large enough values of q , [$1.96k \le q_{opt} \le 2k$ where $25 \le k \le 10^{13}$]. Given $r_{opt} = 0.4597$, the optimal pairs of (K,q) can be calculated, for example (16,100), (36,225), and (50,320). In general we can obtain the approximation function of q_{opt} (K) as

$$q_{opt}(K) \approx 6.32 \cdot K \tag{11}$$

For a finite length of RS code, the approximation method of the probability of a correct codeword that will be used is [12];

$$P_{c,s} \approx \Phi\left(\frac{t - n \cdot P_e}{\sqrt{n \cdot P_e \cdot (1 - P_e)}}\right) \tag{12}$$

2. Bandwidth Expansion

When the side information is not available, the total bandwidth expansion due to M-ary orthogonal modulation with non coherent detection is $\frac{M}{\log_2(M)}$ [19], and the total bandwidth expansion due to modulation and frequency hopping is given by

$$B_e = \frac{M}{\log_2(M)} \cdot \frac{1}{r} \cdot q \tag{13}$$

The minimum bandwidth expansion ($B_{e,opt}$) can be obtained by the optimum selection of the system parameters (r_{opt}, q_{opt}, M_{opt}), then $B_{e,opt}$ can be written as [19],

$$\begin{aligned} B_{e,opt} &= \min\left(\frac{M}{\log_2(M)} \cdot \frac{1}{r} \cdot q\right) \\ &= \frac{M_{opt}}{\log_2(M_{opt})} \cdot \frac{1}{r_{opt}} \cdot q_{opt} \end{aligned} \tag{14}$$

The B_e factor increases linearly as the modulation alphabet size (M) increases; (when we assume RS over GF (M^m), for m = 1, and M = n). Alternatively, whenever we can split n into two integers, m and M_{opt} ,for RS over GF (M^m), where $m = \frac{\log_2 n}{\log_2 M}$ we achieve an optimal M as a function of the RS code block length. Considering the first case that when M increases linearly with n, $B_{e,opt}$ can be obtained by calculating r_{opt}, and q_{opt}. The performance measure in this case is the achievable region of the pair (q,r) that satisfies a given constraint such as coded error probability is equal or less than code word error probability approximation obtained by using the Incomplete Beta function $P_{e,s} \le \tilde{P}_{e,s}$ at K active users.

The minimum bandwidth expansion occurs when r converges to r_{opt}, and obtaining a minimum q as a function of the number of active users. The coding rate as a function of K, and q can be obtained by applying DeMoive laplace limit theorem as

$$r(K,q) = 1 - [2 \cdot P_e + 2 \cdot \beta \cdot \sqrt{\frac{P_e \cdot (1 - P_e)}{n}}] \tag{15}$$

where $\beta = \Phi^{-1}\left(1 - \hat{P}_{e,s}\right)$

and $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \cdot e^{-t^2/2}\, dt$

and asymptotically, the optimum number of frequency slots is

$$\begin{aligned} r_{opt}(K,q) &= 2 \cdot P_e - 1 \\ &= 2 \cdot (1 - \frac{2}{q})^{K-1} - 1 \end{aligned} \tag{16.a}$$

or $$K_{opt}(r,q) = 1 + \frac{\ln((1+r)/2)}{\ln(1 - 2/q)} \tag{16.b}$$

by optimizing (16.a) over q, the optimal q (K) can be found as

$$q_{opt}(K) = (1 - 2^{-\frac{1}{2(K-1)}})^{-1} \tag{17}$$

substituting (17) in to (16.a) results in an $r_{opt}(K,q_{opt})$, and by implementing a simple computer algorithm to find the minimum q, that minimizes B_e, at each given n, as it converges to the desired r_{opt}, it was found that ;

$$q_{min} = q_{opt} + A \cdot (K - 1) \tag{18}$$

Thus the minimum q occurs at $q_{opt} \geq q_{min}$ for $0 < r(K, q) \leq 1$, and A is a positive real number given by table I.
Table I.Values of A that minimize the number of frequency slots for a given optimal code rate

Table I

n	r_{opt}	A
32	0.393	8.52
128	0.406	5.35
256	0.432	4.95
$n \to \infty$	0.460	3.47

The second approach for minimizing B_e when introducing the alphabet size, M, is analyzed here only with respect to the possibility of reducing the codeword error probability. When $P_{e,s}$ is reduced, a minimum B_e can be obtained by selection of the optimal M. We can split n into two integers m and M_{opt} according to $m = \frac{\log_2(n)}{\log_2(M)}$ and recalling from (2.a) that the probability of hit is defined as

$$P_h \le (1 + \frac{m}{N_s}) \cdot \frac{1}{q} \text{ Asynchronous FH} \tag{19}$$

Since it was assumed before that N_s =1, and by substituting (14) in (19), it was found that P_h to be:

$$\begin{aligned} P_h &= \frac{(1+m)\cdot M}{B_e \cdot r \cdot \log_2 M} \\ &= \frac{(\log_2 n + \log_2 M)\cdot M}{B_e \cdot r \cdot (\log_2 M)^2} \end{aligned} \tag{20}$$

Since the errors occur due to MUI, the system can be optimized by minimizing P_h with respect to M, resulting in M_{opt} as a function of n of the RS code, with optimal value at $2 \le M_{opt}(n) \le 8$. For example for n = 16, m = 2, $M_{opt} = 4$ and for n = 256, m = 4, $M_{opt} = 4$ and so on. By using this optimal alphabet size value in P_h, we minimize $P_{e,s}$ and the maximum number of active users occurs when $M=M_{opt}$.

IV. RESULTS AND DISCUSSION

Results presented in this section are for the cases considered and analyzed in previous sections. The probability of a codeword error $P_{e,s}$ versus the code rate r when the simultaneous active users equal 16 (K=16) is illustrated in Fig. 2 for values of different codes lengths (n = 32, 64, 128, 256 and for $n \to \infty$). From this figure, it is clear that for the instant when $r \le r_c$, $P_{e,s}$ is close to zero, and when $r > r_c$, $P_{e,s}$ is close to one. The asymptotic case for $P_{e,s}$ approaches a unit step function.

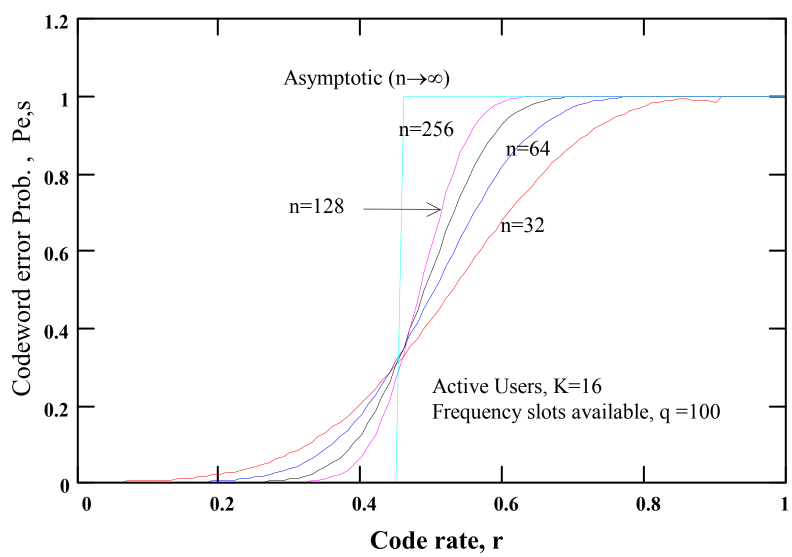


Fig.2. Symbol error probability ($P_{e,s}$) versus coding rate (r)

The normalized throughput (transmitted information) versus the number of simultaneous users and coding rate is illustrated in Fig. 3. It is noted that W_{opt} occurs at $r< r_{opt}$, and it has a triangular shape at $r = r_{opt}$ as $n\to \infty$. The actual value of W_{opt} is 0.0736 at q = 100, and K=16 as $n\to \infty$, but for a large enough values of q, W_{opt} is 0.0724 and the difference in W value is due to the approximation function between q and K. Also, it should be noted that Fig.3, provides the optimum code rate for a finite RS code, with n = 32, 64, 128, 256 and the maximum normalized throughput that can be achieved at each RS code block length.

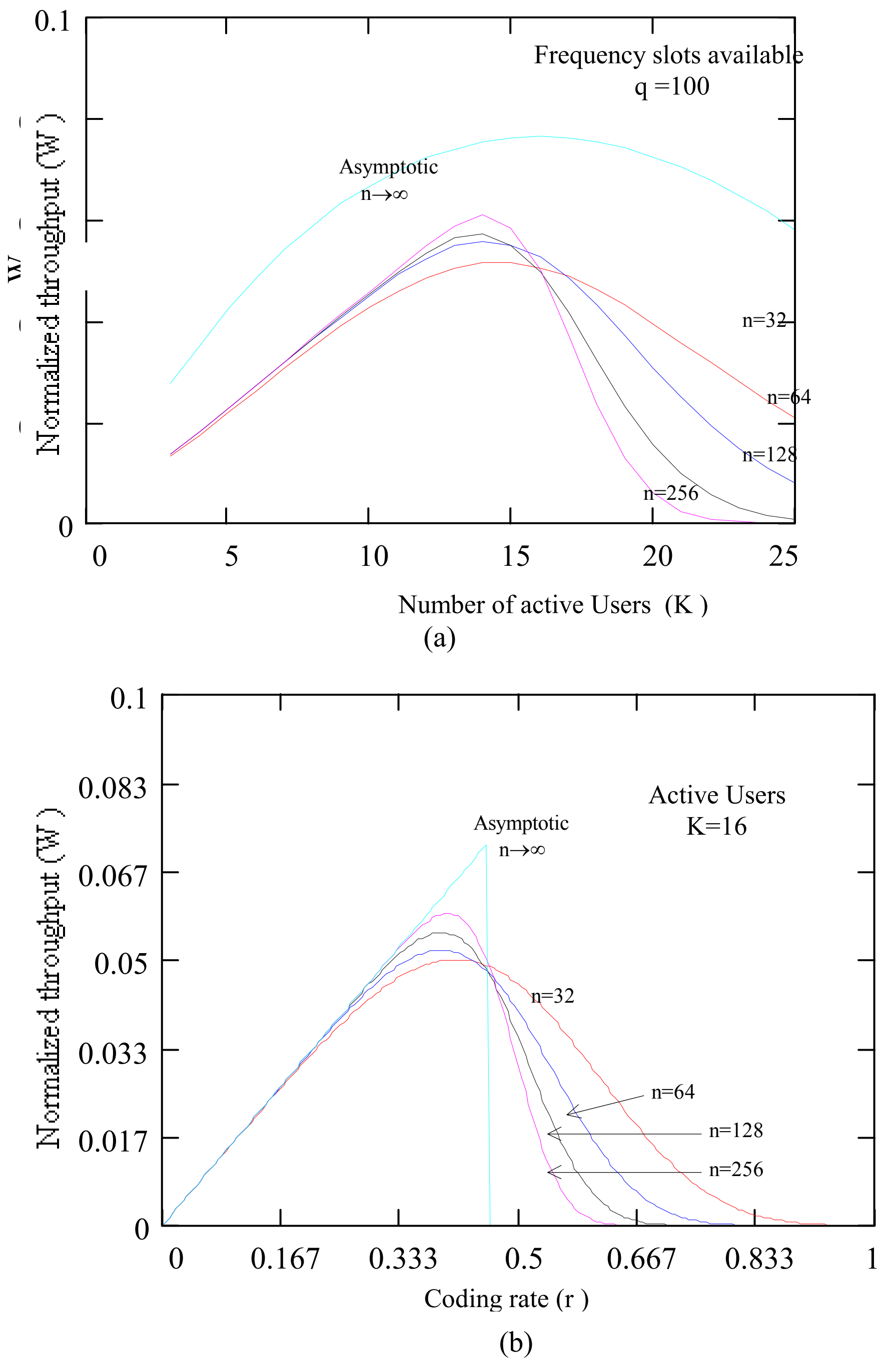
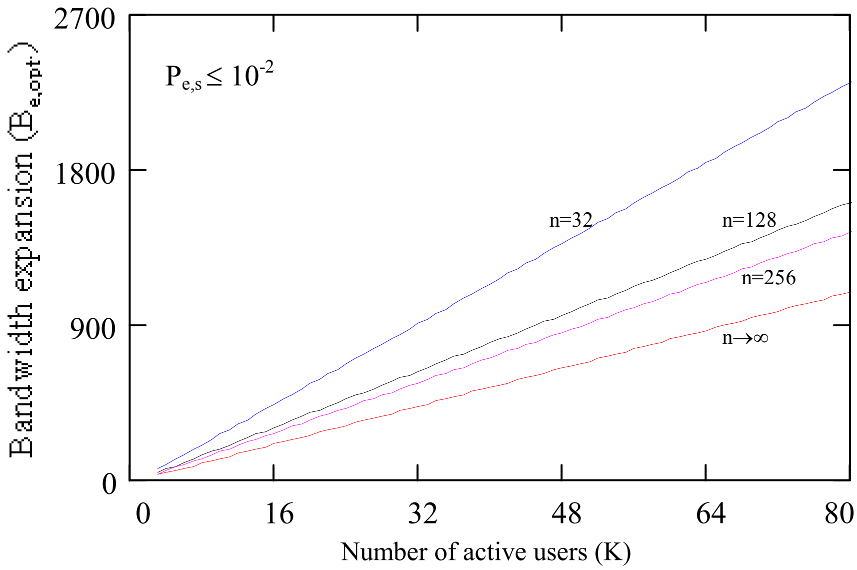


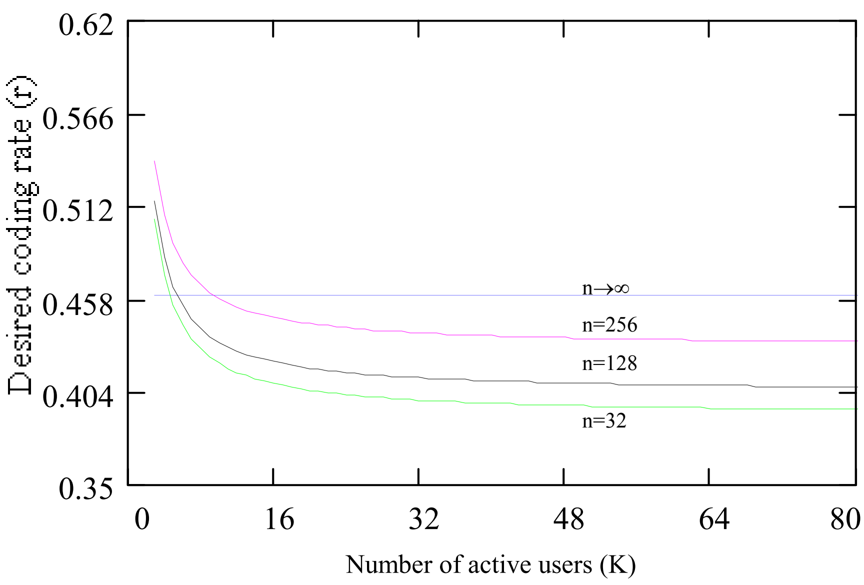
Fig. 3.(a) Normalized throughput (W) versus No. of active users (K) ,
and (b) Normalized throughput (W) versus coding rate (r).

The optimal bandwidth expansion ($B_{e,opt}$) and the optimal convergence coding rate (r_{opt}) as a function of the number of active users (K) is illustrated in Fig. 4. [$B_{e,opt}(K)$, $r_{opt}(K)$, and $q_{opt}(K)$ are obtained under the criteria of

$P_{e,s} \le \bar{P}_{e,s} = 10^{-2}$ satisfying the finite and asymptotic cases for the RS code length. For the asymptotic case and for $25 \le K \le 10^{14}$, $13.306K \le B_{e,opt} \le 13.692K$, $r_{opt} \to 0.465$, and $6.121K \le q \le 6.37K$. But when $K \to 10^{13}$, then $r \to 0.460$, and $B_{e,opt}$ approaches 13.796].



(a)



(b)

Fig. 4. (a) Optimal bandwidth expansion ($B_{e,opt}$.) versus No. of active users (K), and (b) Optimal desired coding rate (r) converges as a function to No. of active users (K).

The comparison between selecting M=2 (binary FSK), M = n (M-ary FSK) and $M=M_{opt}$ (split n as parallel FSK) is illustrated in Fig. 5. Whenever $P_{e,s}$ is minimized the normalized throughput definitely will be maximized as illustrated in Fig. 6.

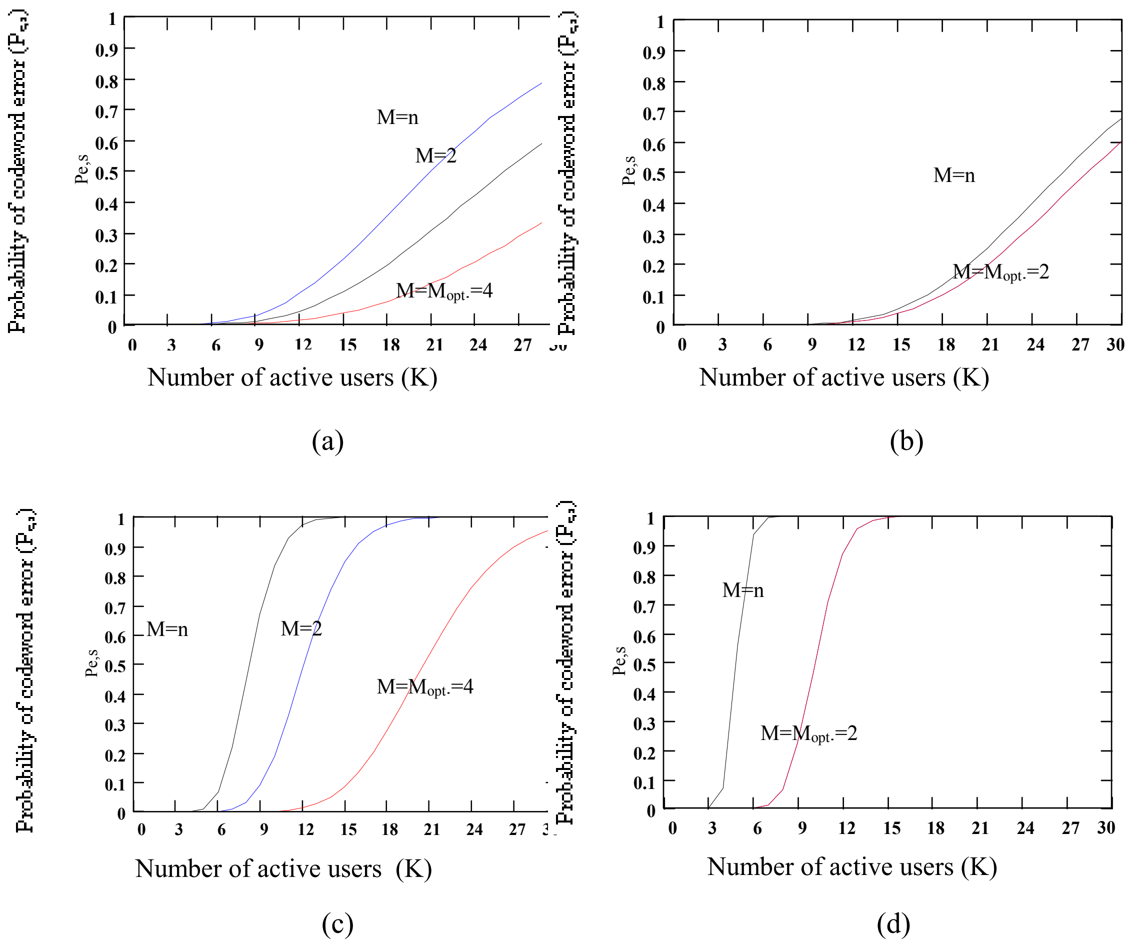


Fig. 5. Probability of codeword error ($P_{e,s}$) versus No. of active users (K) , for various RS Codes lengths, (n), at Fixed bandwidth, (B), with Frequency slots available, q=100.

(a) RS Codes Lengths, n=16,
(b) RS Codes Lengths, n=32
(c) RS Codes Lengths, n=64,
(d) RS Codes Lengths, n=128.

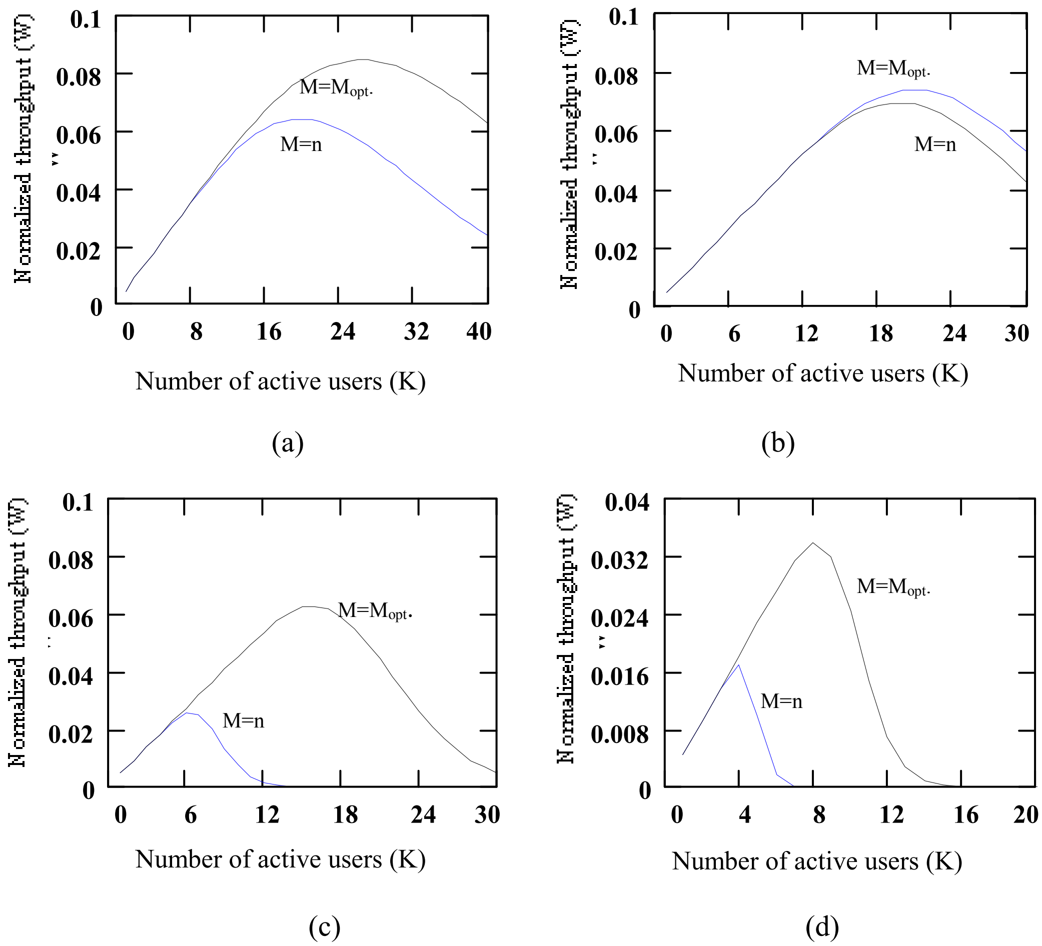


Fig. 6. Normalized throughput (W) versus No. of active users (K), for various RS Codes lengths, (n), at Fixed bandwidth, (B), with Frequency slots available, q=100.

(a) RS Codes Lengths, n=16,
(b) RS Codes Lengths, n=32
(c) RS Codes Lengths, n=64,
(d) RS Codes Lengths, n=128.

V. CONCLUSIONS

This paper has investigated optimum parameter selection of coded FH-SSMA in multi users interference with no side information available . Three parameters; code rate (r), number of frequency slots (q) available in the frequency hopping, and modulation alphabet size (M) are considered. A significant improvement of the channel normalized throughput can be achieved on the expense of minimal bandwidth expansion factor for a given K active users. Results obtained are supported by both analytical evaluation and computer simulation.

REFERENCES

1. Lee,W.C.,1993. Mobile Communications Design Fundamental . John Wiley & Sons, Inc.
2. Githousen, K., Jacobs,M., Padovani,R., Viterbi,A., and Weaver,L., 1993. One The Capacity of a Cellular CDMA System. IEEE Trans. Veh. Tech., Vol.40, pp.303-312.
3. Pickholtz,R., Milstein,B., and Schilling,D., 1991. Spread Spectrum for Mobile Communications. IEEE Trans.Veh. Tech., Vol .40, pp.313-322.
4. Huth,G., 1977. Optimization of Coded Spread Spectrum System Performance. IEEE Trans \ Commun. Com-25, pp.763-770.
5. Lee,W.C., 1991.Overview of Cellular CDMA. IEEE Trans.Veh.Tech.,Vol.40, pp.291-302.
6. Verhulst D, Mouly,M., 1985. Slow Frequency Hopping Techniques for Future digital Mobile Systems. Proc. 1st. Nordic Seminar on Digital Land Mobile Radio, Finland.
7. Verhulst,D., Mouly,M.,1984. Cellular Efficiency with Slow Frequency Hopping. IEEE Trans. Veh. Tech., Vol. 33, pp.179-190.
8. Dornstetter, J., and Verhulst,D.,1987. Cellular Efficiency with Slow Frequency Hopping. IEEE J. Selected Areas Commun, Vol.SAC-5, pp.835-848.
9. Verhulst,D.,1986. Spectrum Efficiency Analysis of the Digital System SFH-9000. Proc. 2nd. Nordic Seminar on Digital Land Mobile Radio, Finland.
10. Steel,R.,1996. Mobile Radio Communications. John Wiley& Sons Inc. Ch.7.
11. Sang W. Kim and W. Stark,W., 1989. Optimum Rate Reed-Solomon Codes for Frequency- Hopped Spread-Spectrum Multiple-Access Communication Systems. IEEE Trans.Comm., Vol. 37, No.2,pp.138-144.
12. Manjunath. V. Hegde and Wayne. E., 1990. Capacity of Frequency-Hop Spread-Spectrum Multiple-Access Communication Systems," IEEE Trans. Commun., Vol. 38, No.7,pp.1050- 1059.
13. Simon, M.K., Omura, J.K., Scholtz, R.A., and Levitt,B.K., 1985. Spread Spectrum Comms. Vol. I, II and III. Rockville MD: Computer Science Press.
14. Kim,S.W, Y.H. Lee, Y.H., and Kim, S., 1994. Bandwidth Tradeoffs Among Coding, Processing Gain and Modulation in Frequency-Hopped Multiple Access Communication, IEE Proc. Commun., vol. 141, No. 2, pp.63-69.
15. Michelson, A. M and Levesque, A.H., 1985. Error-Control Techniques for Communication. New York, John Wiley.
16. Geraniotis, E. A and M. B. Pursley,M.B., 1982. Error Probability for Slow- Frequency- Hopped Spread-Spectrum Multiple-Access Communications over Fading Channels. IEEE Trans. Commun., vol. COM-30, No. 5, pp. 996-1009.
17. Abramowitz, M. and I. A. Stegun, I.A., 1970. Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables. U.S Department of Commernce, National Bureau of Standard Applied Mathematics Series (AMS55), 9 th printing.
18. Press, W., Flannery,P., Teukolsky, S.A., and. Vetterling, W.T., 1989. Numerical Recipes. The Art of Scientific Computing." New York, Cambridge Univ. Press.
19. Papoulis, A., 1991. Probability, Random Variables and Stochastic Processes. New York , McGraw-Hill.
20. Proakis, J. G., 1989. Digital Communications. New York: McGraw-Hill.