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PARABOLOID SHELL AS FOOTING

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ABSTRACT

A simplified method for the design of paraboloid shell footing base on the displacement of the shell's crown where the column axial load is transferred to the footing has been developed. A case study was presented to demonstrate the use of the proposed method and to illustrate its capabilities. The results of the proposed method confirm the ability of the shell model in determining accurate and practical results for the design of paraboloid shell footing. Base on the analytical results of this paper the paraboloid shell is more economical than the conventional flat footing on poor subsoil, and very competitive with other shell footings.

INTRODUCTION

A shell is a thin – walled curved element in which the thickness is rather small compared to the lateral dimensions and radii of curvature. Fig.1 shows a shell as a solid continuum lying between two closely spaced curved surfaces. Thickness is the distance between the two boundary surfaces. If the thickness is large, the shell will be termed as thick, otherwise, it is thin, Bairagi,N.K. (1990)

Apart from the purpose of roofing, shells have been used in a wide range of structures such as liquid containers, bunkers, silos, marine structures, etc. Therefore, there is no reason why shells should not be used in footings. Spread footings for columns, transmitting heavy loads to weak soils, tend to be massive. If mat foundations are provided they need to be excessively thick to be rigid enough to control the settlements within limits. A thin shell foundation, can provide the same rigidity as a much thicker mat foundation; for more details see Kurien (1977) .

Experience have shown that, shell foundation are economical over the conventional ones on poor subsoil because of low bearing capacities or any other reason. Constructing shell footing directly over soil bed, eliminate the chances of local buckling in thin structures. This paper presents analysis and design of a paraboloid shell as footing. The column is transferring load to the footing at the crown of that footing; this load is modeled as a concentrated load at that point. Simplified finite element analysis is used to determine deflection,forces, and moments; seeConnor(1967)and Bhattacharya (1978).

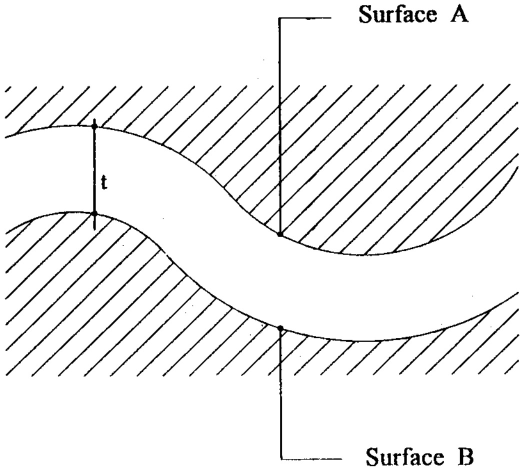

Fig. 1. Shell form

SIMPLIFIED PARABOLOID SHELL ANALYSIS

Fig. 2 shows a paraboloid shell clamped with edge beam and in plane dimension a x b. Because of symmetry the displacements u , v and rotations are zero at the center. Due to symmetry we can consider one quadrant of the shell for analysis presented by single element, that is element number 1 as shown in Fig.3. The shell being clamped, displacements and rotations at nodes 1, 2, and 3 are zero. Then we are left with one unknown that is the displacement *w* at the center. The paraboloid equation is:

$$z = \frac{2f}{ab} \cdot \left(x^2 + y^2\right) \qquad (1)$$

Where f is the total shell rise at the crown. Employing finite element shape functions we have the following equations and matrices (Ramaswamy, 1986):

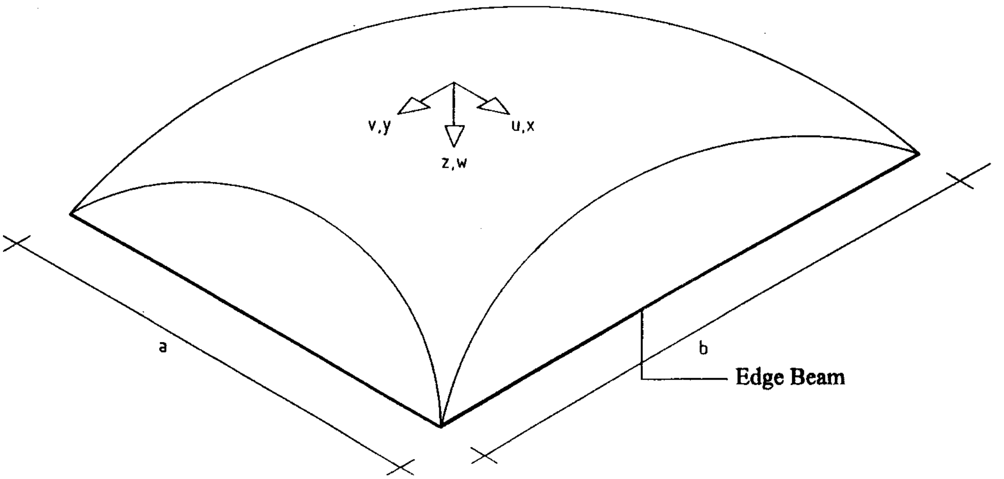

Fig. 2. Paraboloid shell

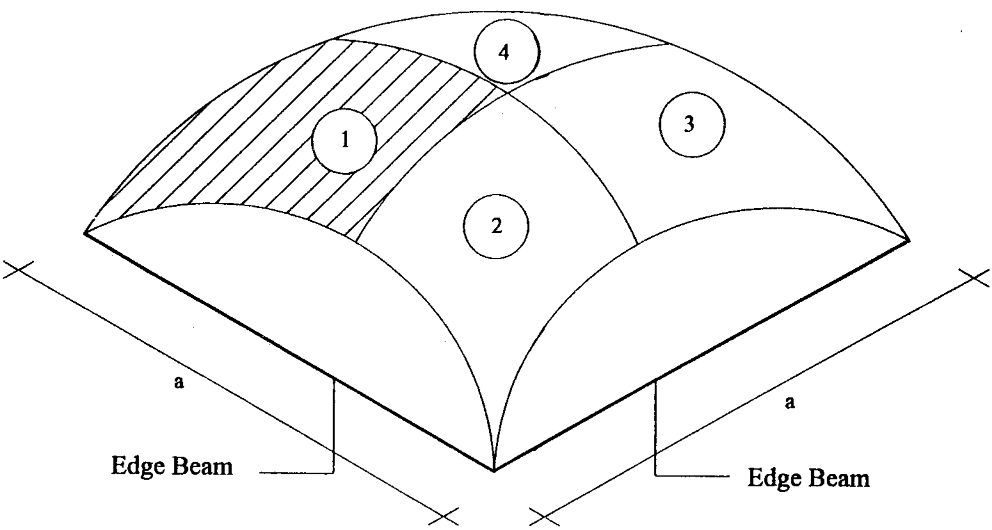

Fig. 3. Shell model

$$\chi_x = -\frac{\partial^2 w}{\partial x^2} \tag{10}$$

$$\chi_x = -6x\frac{\left(y^3 - 3yb^2 + 2b^3\right)}{a^3b^3} \tag{11}$$

$$\chi_y = -\frac{\partial^2 w}{\partial y^2} \tag{12}$$

$$\chi_y = -6y\frac{\left(x^3 - 3xa^2 + 2a^3\right)}{a^3b^3} \tag{13}$$

$$2\tau = -2\frac{\partial^2 w}{\partial x \partial y} \tag{14}$$

$$2\tau = -18\left(-x^2 + a^2\right)\frac{\left(-y^2 + b^2\right)}{a^3b^3} \tag{15}$$

$\varepsilon = B(w)$, where B is the assembly matrix

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \chi_x \\ \chi_y \\ 2\tau \end{Bmatrix} = \begin{bmatrix} -4f\left(y^3 - 3yb^2 + 2b^3\right)\cdot\frac{\left(-5x^3 - 3xy^2 + 2a^3\right)}{a^4b^4} \\ -4f\left(x^3 - 3xa^2 + 2a^3\right)\cdot\frac{\left(-5y^3 - 3yx^2 + 2b^3\right)}{a^4b^4} \\ 0 \\ -6x\frac{\left(y^3 - 3yb^2 + 2b^3\right)}{a^3b^3} \\ -6y\frac{\left(y^3 - 3xa^2 + 2a^3\right)}{a^3b^3} \\ 18\left(-x^2 + a^2\right)\frac{\left(-y^2 + b^2\right)}{a^3b^3} \end{bmatrix} (w) \tag{16}$$

$$N_y = \frac{Et}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \tag{21}$$

$$N_{xy} = 0 \tag{22}$$

$$M_x = D'\left(\chi_x + \nu\chi_y\right) \tag{23}$$

$$M_y = D'\left(\chi_y + \nu\chi_x\right) \tag{24}$$

$$M_{xy} = D'\frac{(1-\nu)}{2}2\tau \tag{25}$$

Where υ is Poisson's ratio, E is modulus of elasticity , M is moment, N is force, ε is strain, τ is stress, χ is moment curvature, t is thickness, and

$$D' = \frac{Et^3}{(1-\nu^2)12}$$

CASE STUDY

In order to demonstrate the use of the simplified method and to verify the accuracy of its results, a case study was conducted. Six paraboloid shell footings were selected with different flatness ratio (r = f / a), six flat square footing, one hypar shell footing, and one domical shell footing. Hypar and domical shell footings are shown in Figs. 4 and 5 respectively. For more details on hypar and domical shells see Ramswamy (1986) and Bairagi (1990).

Table 1 shows the design parameters and results for the study case . Table 2 shows a comparison among the displacements ($w1$) computed using the proposed method for the shell model with displacement ($w2$) computed using computer program STAAD – PRO for the paraboloid shell shown in Fig. 6. The results of the analysis that are presented in Table 1 are plotted in Fig. 7.

Table 1. Design parameters and results

Footing No.	Footing Type	Footing Size m X m	$r = \frac{f}{a}$	Thickness mm	Concrete Volume m^3	Axial Load kN	Soil Presure kN / m^2
1	Square	4.5 X 4.5	0	310	6.3	500	50
2	Paraboloid	"	1 / 7	250	4.29	"	"
2	"	"	1 / 7	300	5.14	"	"
2	"	"	1 / 6	250	4.39	"	"
2	"	"	1 / 6	300	5.27	"	"
2	"	"	1 / 5	250	4.56	"	"
2	"	"	1 / 5	300	5.47	"	"
2	"	"	**1 / 4**	**250**	**4.85**	"	"
2	"	"	1 / 4	300	5.82	"	"
3	Square	5 X 5	0	320	8	700	50
4	Paraboloid	"	1 / 7	250	5.29	"	"
4	"	"	1 / 7	300	6.35	"	"
4	"	"	1 / 6	250	5.42	"	"
4	"	"	1 / 6	300	6.5	"	"
4	"	"	1 / 5	250	5.63	"	"
4	"	"	1 / 5	300	6.75	"	"
4	"	"	1 / 4	250	5.98	"	"
4	"	"	1 / 4	300	7.18	"	"

Table 1. (Continued) Design parameters and results

Footing No.	Footing Type	Footing Size m X m	$r = \frac{f}{a}$	Thickness mm	Concrete Volume m^3	Axial Load kN	Soil Presure kN / m^2
5	Square	7 X 7	0	400	19.6	1,200	50
6	Paraboloid	"	1 / 7	250	10.37	"	"
6	"	"	1 / 7	300	12.44	"	"
6	"	"	1 / 6	250	10.62	"	"
6	"	"	1 / 6	300	12.75	"	"
6	"	"	1 / 5	250	11.03	"	"
6	"	"	1 / 5	300	13.23	"	"
6	"	"	1 / 4	250	11.73	"	"
6	"	"	1 / 4	300	14.07	"	"
7	Square	8.5 X 8.5	0	570	41.2	1,500	50
8	Paraboloid	"	1 / 7	250	15.29	"	"
8	"	"	1 / 7	300	18.35	"	"
8	"	"	1 / 6	250	15.66	"	"
8	"	"	1 / 6	300	18.79	"	"
8	"	"	1 / 5	250	16.26	"	"
8	"	"	1 / 5	300	19.51	"	"
8	"	"	1 / 4	250	17.29	"	"
8	"	"	1 / 4	300	20.75	"	"

Table 1. (Continued) Design parameters and results

Footing No.	Footing Type	Footing Size m X m	$r = \frac{f}{a}$	Thickness mm	Concrete Volume m^3	Axial Load kN	Soil Presure kN / m^2
9	Square	4 X 4	0	400	6.4	800	75
10	Hypar	"	1 / 4	200	3.2	"	"
11	Paraboloid	"	1 / 4	200	3.06	"	"
12	Square	5.5 X 5.5	0	500	15.13	1,000	50
13	Domical	"	1 / 4	150	4.5	"	"
14	Paraboloid	"	1 / 4	150	4.34	"	"

Table 2. Displacement comparison

Footing No.	$r = \frac{f}{a}$	$w1$ mm	$w2$ mm	Thickness mm	Concrete Strength N / mm^2
2	1 / 4	1	1	250	20
2	1 / 4	0.98	0.75	300	"
2	1 / 4	0.8	0.8	250	30
2	1 / 4	0.98	0.75	300	"
2	1 / 5	1.53	1.3	250	20
2	1 / 5	1.5	1	300	"
2	1 / 5	1.3	1.04	250	30
2	1 / 5	1.2	0.8	300	"
2	1 / 6	2.1	1.5	250	20
2	1 / 6	2	1.1	300	"
2	1 / 6	1.8	1.3	250	30
2	1 / 6	1	1.7	300	"
2	1 / 7	2.9	2	250	20
2	1 / 7	2.8	1.3	300	"
2	1 / 7	2.4	1.5	250	30
2	1 / 7	2.31	1.1	300	"
4	1 / 4	1.4	1.5	250	20
4	1 / 4	1.3	1.1	300	"
4	1 / 4	1.1	1.2	250	30
4	1 / 4	1	0.9	300	"
4	1 / 5	2.1	1.9	250	20
4	1 / 5	2	1.4	300	"
4	1 / 5	1.74	1.6	250	30
4	1 / 5	1.7	1.1	300	"
4	1 / 6	3.1	2.3	250	20
4	1 / 6	3	1.7	300	"
4	1 / 6	2.5	1.9	250	30
4	1 / 6	2.4	1.4	300	"

Table 2. (Continued) Displacement comparison

Footing No.	$r = \frac{f}{a}$	$w1$ mm	$w2$ mm	Thickness mm	Concrete Strength N / mm²
4	1 / 7	4.1	3	250	20
4	1 / 7	4	2.1	300	"
4	1 / 7	3.4	2.5	250	30
4	1 / 7	3.3	1.6	300	"
6	1 / 4	2.41	3.3	250	20
6	1 / 4	2.4	2.4	300	"
6	1 / 4	2	2.7	250	30
6	1 / 4	1.9	1.9	300	"
6	1 / 5	3.7	4	250	20
6	1 / 5	3.6	3	300	"
6	1 / 5	3.1	3.4	250	30
6	1 / 5	3	2.5	300	"
6	1 / 6	5.33	5.1	250	20
6	1 / 6	5.3	3.7	300	"
6	1 / 6	4.4	4.1	250	30
6	1 / 6	4.3	3	300	"
6	1 / 7	7.3	6	250	20
6	1 / 7	7	4.4	300	"
6	1 / 7	5.9	4.9	300	30
6	1 / 7	5.8	3.6	"	"
8	1 / 4	3	3.5	"	20
8	1 / 4	2.4	2.8	"	30
8	1 / 5	4.5	4.4	"	20
8	1 / 5	3.7	3.6	"	30
8	1 / 6	6.5	5.3	"	20
8	1 / 6	5.3	4.3	"	30
8	1 / 7	8.8	6.3	"	20
8	1 / 7	7.2	5.2	"	30

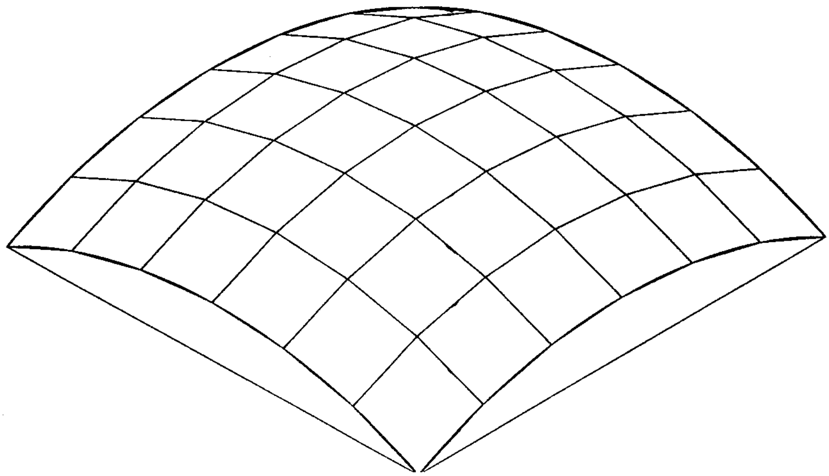

Fig. 6. Staad-pro paraboloid shell

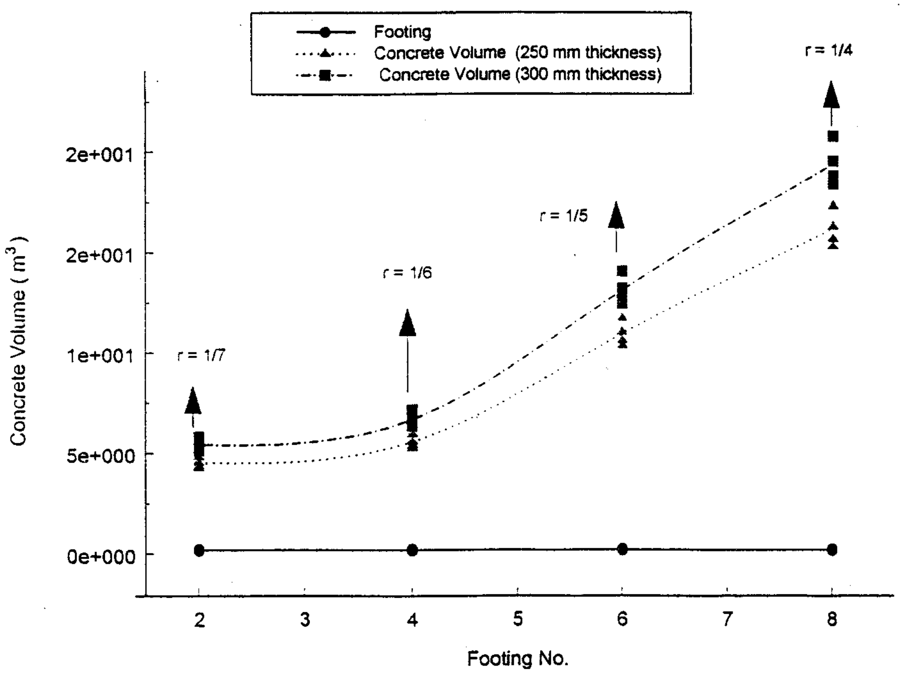

Fig. 7. Flatness ratio & concrete volume

The displacements – flatness ratio curves for different footing thickness, flatness ratio, and concrete strength are shown in Figs. 8 through 11. In table 2 the results of the proposed method is larger than the results of the STAA – PRO, which indicates the proposed method is slightly conservative, because the STAAD – PRO provides a more accurate results by using too many elements.

In selecting one element for shell footing analysis, one has to work out the trade off between accuracy and the complexity and cost of the design. Generally in footings the factor of safety is high, so the exact accuracy is not required. The results determined by the shell model is very practical and safe for shell footing of thickness of 250 mm and higher since this thickness is the minimum requirement by most codes of design.

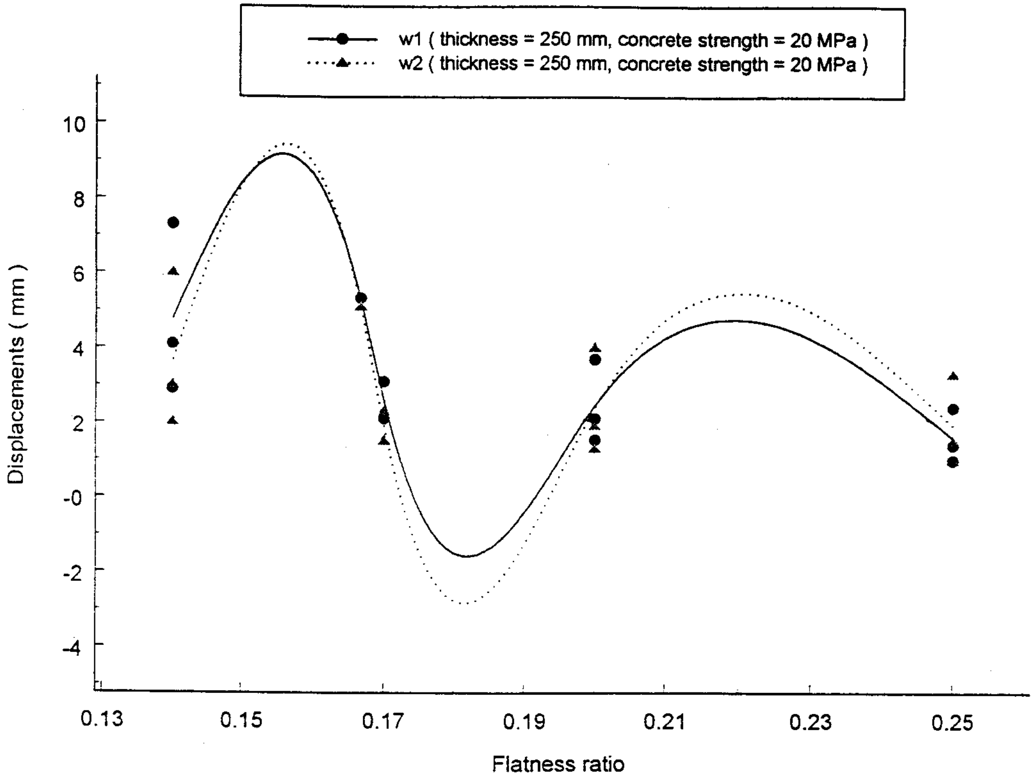

Fig. 8. Flatness ratio vs. displacements

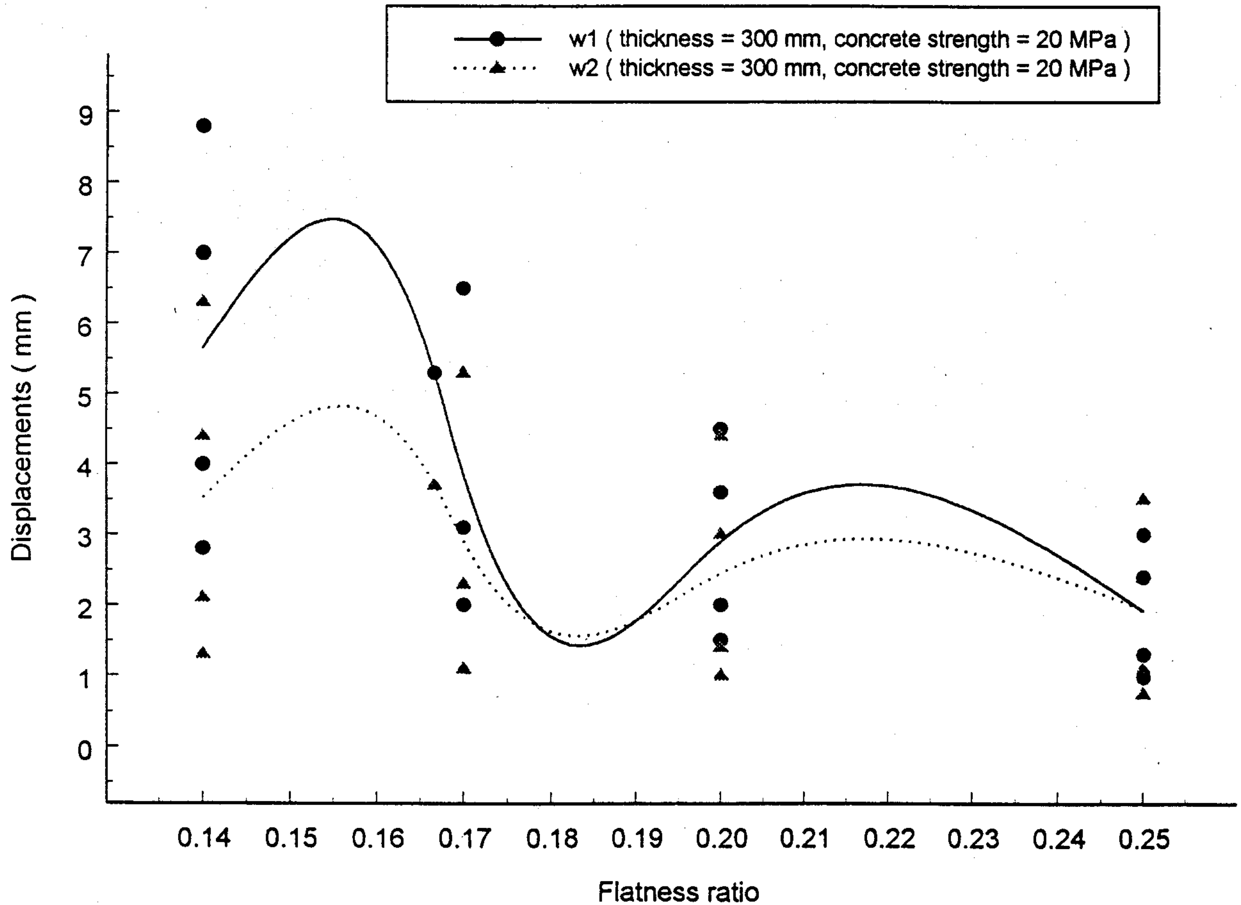

Fig. 9. Flatness ratio vs. displacements

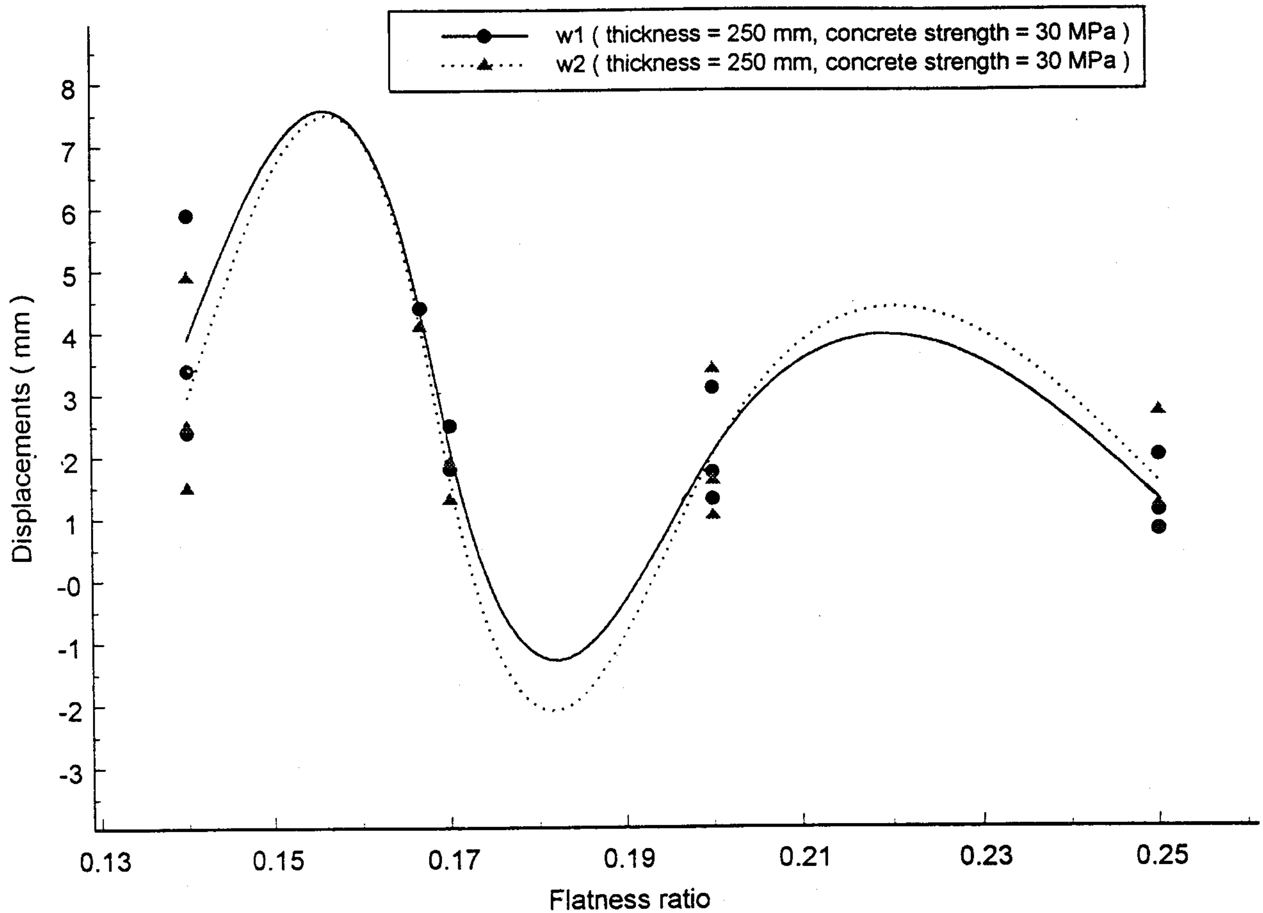

Fig. 10. Flatness ratio vs. displacements

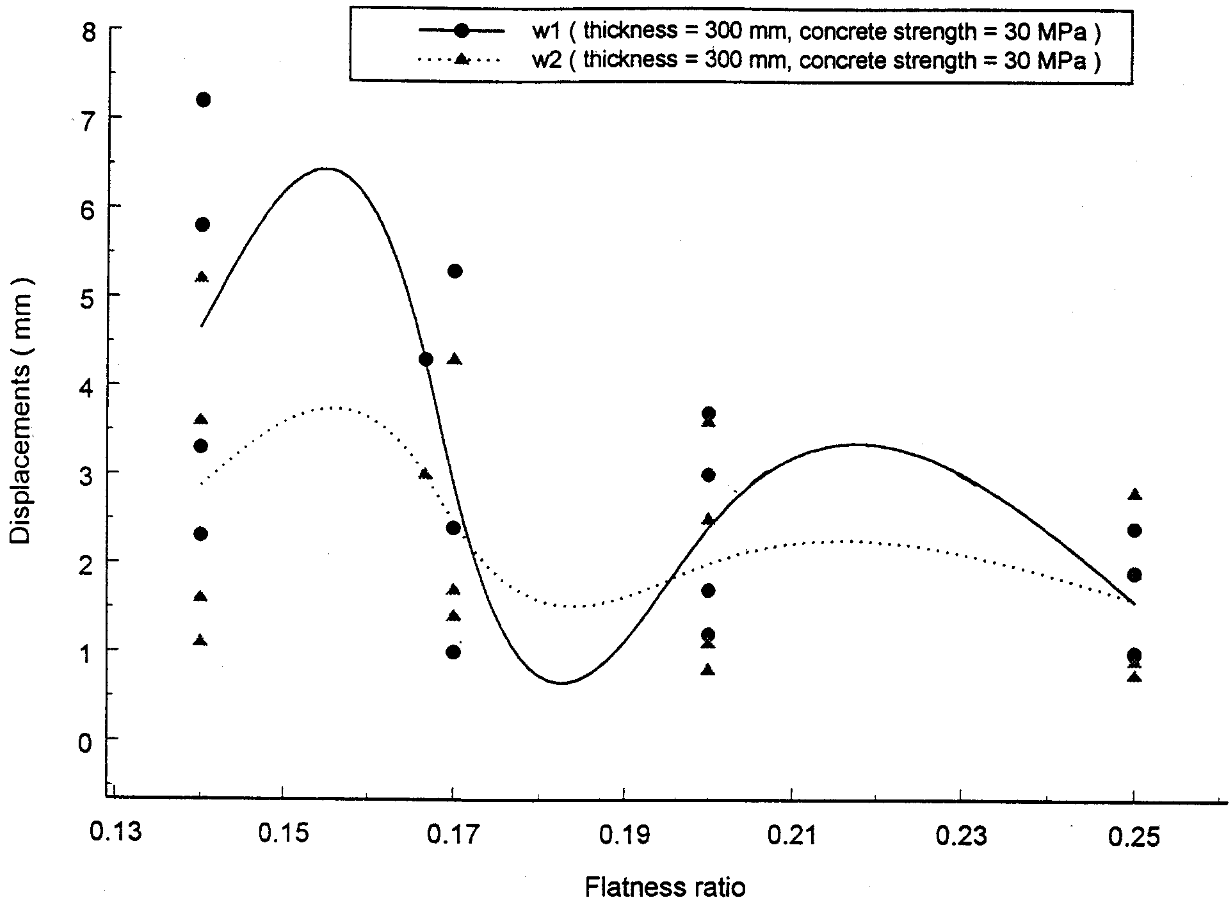

Fig. 11. Flatness ratio vs. displacements

PROGRAM DESCRIPTION

A computer program has been written using the mathematical package Mathcad to implement the computational procedure of the proposed method. Fig. 12 shows the sheet of the Mathcad program for footing number 2, flatness ratio of 0.25, concrete strength of 20 MPA, and footing thickness of 300 mm. The input data for the program consist of the dimension a, total shell rise at the crown f, Poisson's ratio υ shell footing thickness t, axial column load P, and concrete strength. The output consists of the displacement of the shell crown under the axial load w, *forces* N, and moments M.

INPUT DATA:

Shell footing dimension:	$a \equiv 180$	in	$a' = 4.5$	m
Total shell rise:	$f \equiv 45$	in	$f' = 1.125$	m
Poissons ratio:	$v \equiv 0.15$			
Shell thickness:	$t \equiv 12$	in	$t' = 0.3$	m
Concrete strength:	$fc \equiv 2901$	psi	$fc \equiv 20$	$\frac{N}{mm^2}$
Column axial load:	$P \equiv 1000$	lb	$P' = 4.448 \cdot 10^3$	N

OUTPUT :

Shell displacement:	$w = 0.039$	in	$w' = 9.801 \cdot 10^{-4}$	m
Shell forces:	$Nx = -5.87 \cdot 10^3$	$\frac{lb}{in}$	$N'x = -1.045 \cdot 10^6$	$\frac{N}{m}$
	$Ny = -5.87 \cdot 10^3$	$\frac{lb}{in}$	$N'y = -1.045 \cdot 10^6$	$\frac{N}{m}$
	$Nxy = 3.905 \cdot 10^3$	$\frac{lb}{in}$	$N'xy = 6.951 \cdot 10^5$	$\frac{N}{m}$
Shell moments:				
	$Mx = -1.174 \cdot 10^3$	$\frac{lb \cdot in}{in}$	$M'x = -5.411 \cdot 10^3$	$\frac{N \cdot m}{m}$
	$My = -1.174 \cdot 10^3$	$\frac{lb \cdot in}{in}$	$M'y = -5.41 \cdot 10^3$	$\frac{N \cdot m}{m}$
	$Mxy = -2.343 \cdot 10^3$	$\frac{lb \cdot in}{in}$	$M'xy = -1.08 \cdot 10^4$	$\frac{N \cdot m}{m}$

Fig. 12. Mathcad program sheet for paraboloid sheet footing

CONCLUSION

A simplified method for the design of paraboloid shell footing base on the displacement of the shell's crown where the column axial load is transferred to the footing has been developed. The method, which deals with the design of the paraboloid shell footing, uses the finite element method to obtain the crowns displacement of the shell; then the remaining forces and moments determined base on the displacement magnitude. The deflection of the plate under the column's load is very important in shell footing design and must be determined. The results of the proposed method confirm the ability of the shell model in determining accurate and practical results for the design of paraboloid shell footing. Base on the analytical results of this paper, a high flatness ratio will lead to a small displacement and a large volume of concrete; therefore the designer should make the right choice to control safety and cost.

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