HIGH FREQUENCY CHARACTERISTICS OF FERRITE MATERIALS AND APPLICATIONS TO MICROSTRIP CIRCUITS

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ABSTRACT

Dispersion behavior of the parameters of microstrip lines printed on ferrite substrate is presented. The characteristic impedance for lines on magnetized ferrite substrates are obtained for partially magnetized substrates in the direction of wave propagation and vertically magnetized substrates with variable DC magnetic field. The obtained results are applied to analyze variable phase shifters, controlled resonators, and impedance matching.

Keywords: Microstrip line, Magnetized Ferrite, Phase shifter, Resonator, Matching.

INTRODUCTION

Ferrite devices are of key importance for mm-wave, radar applications and microwave integrated circuit MICS. This work represents a detailed analysis of the dispersion characteristics of microstrip lines on magnetic materials with external magnetization. The effective permittivity, \( \varepsilon_{\text{eff}} (f) \), and effective permeability, \( \mu_{\text{eff}} (f) \), are evaluated using accurate dispersion formula [1]. The purpose of this work is to: 1) extend the work of Kobayashi [1] to include magnetic materials, 2) apply the results to practical microwave components. Even though the analysis is based on well known formulas the results are new and useful for applications in microwave and millimeter ranges of frequency.

Ferrite materials exhibit magnetic anisotropy with the application of external DC magnetic fields. The permeability of the material has tensor form, whose elements depend on the DC magnetic fields, the saturation magnetization, and the operation frequency. Planner transmission lines containing ferrite have received considerable attention in microwave applications and millimeter-wave non-reciprocal devices, such
as circulators, isolators, phase shifters, and resonators. In this work the dispersion characteristics for ferrite parameters are obtained and the results are applied to establish variable phase shifters, broadband quarter wave, and circular resonators. Also, the results are applied to taper lines with variable terminal impedance.

ANALYSIS

Effective Relative Permittivity

The new developed dispersion formula for the effective relative permittivity, $\varepsilon_{\text{eff}}(f)$, by M.Kobayashi [1] is accurate with error less than 1%. The frequency dependent, $\varepsilon_{\text{eff}}(f)$, is given by:

$$\varepsilon_{\text{eff}}(f) = \varepsilon - \frac{\varepsilon_{\text{eff}}(0)}{1 + (f / f_{50})^m}$$

(1)

Where $\varepsilon$ is the relative permittivity of the ferrite substrate. $f$ is the operating frequency $f_{50}$ and $m$ is given in the work of Kobayashi [1]. $\varepsilon_{\text{eff}}(0)$ is the effective relative permittivity at $f = 0$ and it is given by:

$$\varepsilon_{\text{eff}}(0) = 1 + q (\varepsilon - 1)$$

(2)

Where $q$ is the effective filling fraction, which is given in the work of Kobayashi [1-3]. Figs.1a & 1b show the variation of $\varepsilon_{\text{eff}}(f)$ with frequency for different materials and different w/h.

Effective Relative Permeability

This work discusses three biasing modes of operation, which are of practical importance:
Fig. 1a Dispersion of $\varepsilon_{\text{eff}}(f)$ for different W/H for ferrite substrate material with $\varepsilon_r = 16$

Fig. 1b Dispersion of effective permittivity $\varepsilon_{\text{eff}}(f)$ for different substrate material W/H = 2
A. Substrate demagnetized

It was shown in the work of Pucel and Masse [4] that the permeability tensor element, \( \mu_d \), can be approximated by:

\[
\mu_d = \frac{1}{3} \left[ 1 + 2 \left( 1 - \left( \frac{\omega_m}{\omega} \right)^2 \right)^{1/2} \right]
\]  

(3)

Where \( \omega_m = \gamma (4\pi M_s) \) and \( \omega \) is the operating frequency, \( \gamma \) is the geomagnetic ratio [(2.8) MHz/oc] and \( (4\pi M_s) \) is the saturation magnetization. Fig. 2 illustrates the frequency dependence of \( \mu_{\text{eff}} \), which is a function of \( \omega_m \) as well.

![Fig. 2 Dispersion of permeability \( \mu_d \) when the substrates are demagnetized for different saturation magnetization](image)

B. Partially magnetized substrate

The Ferrite substrate is magnetized longitudinally along the direction of propagation. Green and Rado [4-5] experimentally give the permeability tensor elements. In the case of lossless uni-axial Ferrite:
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$$\mu = \mu_d + (1-\mu_d)(4\pi M/4\pi M_s)3/2$$  \hspace{1cm} (4)

$$k = \gamma (4\pi M)f$$  \hspace{1cm} (5)

$$\mu_z = \mu_d \left[ 1 - 4\pi M / 4\pi M_s \right]^{5/2}$$  \hspace{1cm} (6)

The Ferrite is considered to be isotropic medium with a scalar permeability $\mu_{\text{eff}}$. Derived from the tensor equation:

$$\frac{\mu}{\mu_0} = \begin{bmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & \mu_{\text{err}} \end{bmatrix}$$  \hspace{1cm} (7)

The effect of the shape ratio (w/h) is introduced as a correction factor [4] for the effective permeability using curve fitting:

$$\mu_{\text{eff}} = (\mu^2 - k^2) / \mu a$$  \hspace{1cm} (8)

Where

$$a = 1 - \frac{1}{7} \sqrt{\left(\frac{h}{w}\right)\left(\frac{k}{\mu}\right)^2 \ln \left[ 1 + \frac{\mu}{\mu^2 + k^2} \right]}$$  \hspace{1cm} (9)

The above expression for tensor $\mu$ and $\mu_{\text{eff}}$ are based on the fact that, longitudinal magnetization leads to transverse magnetic field (H). Fig.3 shows the dispersion behavior of $\mu_{\text{eff}}$ when the substrate is ferrite with $4\pi M_s = 1780\text{G}$ for different ratios of the applied magnetization. $\mu_{\text{eff}}$ turns negative when $K$ exceeds $\mu$ due to low frequency $\omega$ and at increased ratio (r). The waves can not propagate in ferrite medium with negative $\mu_{\text{eff}}$ and the electromagnetic energy will be absorbed by the ferrite and consumed in the material resonance (ferromagnetic resonance).
C. Vertically magnetized substrate

For vertically magnetized uni-axial Ferrite substrate, the effective permeability is given in equation (9), where $\mu$ and $k$ are defined by Patton [6] by:

$$\mu = 1 + \frac{\omega_m(\omega_b + \omega)}{(\omega_b + \omega)^2 - \omega^2}$$  \hspace{1cm} (10)

$$k = \frac{-\omega_m \omega}{(\omega_b + \omega)^2 - \omega^2}$$  \hspace{1cm} (11)
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\[ \omega + \omega = \gamma (H_o + H_a) \]  \hspace{1cm} (12)

\[ \omega_m = \delta (4\pi M_s) \]  \hspace{1cm} (13)

Where \( H_o \) and \( H_a \) are the internal and the anisotropy fields respectively.

Fig. 4 shows the frequency response of \( \mu_{\text{eff}} \) for vertically magnetized substrates with different DC magnetization. It must be noticed that there is a cutoff in the curves with vertical magnetization. The propagation cutoff occurs when the effective permeability \( \mu_{\text{eff}}(f) \) is below a certain threshold.

![Dispersion of permeability](image)

**Fig. 4** Dispersion of permeability \( \mu_{\text{eff}}(f) \) for vertically magnetized substrate at different magnetization ratios \( \varepsilon_r = 16, 4\pi M_s = 1780\text{G}, \ W/H = 2 \)
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permeability $\mu_{\text{eff}}$, turns negative after reaching a certain maximum positive value. This inversion or cut off happens at certain critical frequency $\omega$. For $H_0 = 0$, the critical frequency is given by

$$\omega < (\omega_a + \omega_m) \quad \text{or} \quad \omega^2 < (\omega_a \omega_m + \omega_m^2)$$

(14)

In the absence of internal field $H_0$, $\mu_{\text{eff}}$ inversion is controlled by the values of saturation magnetization $\omega_m$, applied magnetization $H_a$ and the operating frequency. For a specific ferrite sample $H_a$ and $\omega$ can be chosen for obtaining certain value of $\mu_{\text{eff}}$. For example if $4\pi M_s = 1780$ G and the anisotropy frequency ($\gamma H_0$) is twice that of $\gamma(4\pi M_s)$ i.e. $(\omega_a / \omega_m = 2)$ maximum $\mu_{\text{eff}}$ is obtained around 12 to 12.5 GHz, while same condition is obtained at 17 to 17.5 GHz if the ratio $(\omega_a / \omega_m = 3)$, as illustrated in Fig 4. It is important to keep the effective fields $(H_a + 4\pi M_s)$ small in order to avoid ferromagnetic resonance at which increased losses occur (the condition in equation (11)). The characteristic impedance $Z_0(f)$ is given by:

$$Z_0(f) = Z_{01}(f) \sqrt{\frac{\mu_{\text{eff}}(f)}{\varepsilon_{\text{eff}}(f)}}$$

(15)

Where $Z_{01}(f)$ is the characteristic impedance when the substrate is air.

The effect of the magnetization on the line impedance is illustrated in Figs (5a) and (5b). These show that using magnetic material, with applied anisotropy can control the impedance value of the same line. Application of variable line impedance for matching purposes will be discussed in section (3).
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Fig. 5a The impedance for partially magnetized substrate $W/H = 2, 4\pi M_s = 1780\text{G}, \varepsilon_r = 16$

Fig. 5b Characteristic impedance $Z$(ohm) of a line $W/H = 2$. The substrate is vertically magnetized at different magnetization ratios, $\varepsilon_r = 16, 4\pi M_s = 1780\text{G}$
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APPLICATIONS

1. Rectangular and Circular Resonators on Ferrite substrate

Rectangular quarter wavelength and circular resonators are fabricated on ferrite substrates for building antenna arrays in the lower range of UHF. Antennas are built on dielectric substrates working in the GHz range and the bandwidth is 1%. As the frequency is lower, the size of the antenna becomes larger. The resonant frequency in terms of the quarter wave resonant length \( l_r \) is given by [7].

\[
f_r = \frac{c}{4l_r \sqrt{\mu_{eff} \varepsilon_{eff}}}
\]  

(16)

Where \( c \) is the speed of light in vacuum, the bandwidth is \( \Delta f_r / f_r = 1/Q \)

Where \( Q \) = total loss factor (dielectric, magnetic, conductor, and radiation losses). The input resistance \( R_{in} \) seen by the coaxial feed through the ground at a distance \( d \) from one of the ends is given by [7].

\[
R_{in} = 4/\pi \left( \frac{f_r}{\Delta f_r} \right) Z_{o1} \sqrt{\frac{\mu_{eff}}{\varepsilon_{eff}}} \sin^2 \left[ 2\pi d \frac{C_{eff}}{\lambda_o} \right]
\]  

(17)

\( Z_{o1} \) is the characteristic impedance of the microstrip line having shape ratio \( w/h \), when \( \mu_r = \varepsilon_r = 1 \). For the circular resonator, the resonant frequency in terms of radius \( r_o \) is give by [8].

\[
f_r = \frac{C_{n} \varepsilon_{nm}}{(2\pi r_o C_{eff})}
\]  

(18)

where

\[
C_{eff} = \sqrt{\mu_{eff}(f) \varepsilon_{eff}(f)}
\]  

(19)
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nm is the mth zero of the derivative of Bessel function of order n. For a given \( r/h \) the resonator is manufactured on ferrite substrate with applied dc magnetic field. The resonant frequency can be changed according to the type of magnetization and its density. And the same resonator can be used as an antenna element with variable resonant frequency, from Fig.3 the resonant frequency can be changed up to 100 times above its original value when the substrate is magnetized in the direction of propagation. Meanwhile, the resonant frequency can be reduced to any ratio (100 times for example) if the substrate is vertically magnetized as illustrated from fig.4. The input resistance and the bandwidth will be altered by \( \mu_{\text{eff}}(f) \) variation according to equations (16) and (17).

2. Variable Phase Shifter and Slow wave operation:

A 50 Ohm line is built on a Ferrite substrate with \( \varepsilon_r = 16 \), \( W/h = 0.2 \), the phase shift constant \( \beta(f) \) is approximated by

\[
\frac{\beta(f)}{\beta_0(f)} = \sqrt{\mu_{\text{eff}}(f)} \varepsilon_{\text{eff}}(f) \sqrt{1 + \tan^2 \delta + 1}
\]

(20)

where

\[
\beta_0(f) = \frac{\omega \sqrt{\varepsilon_0 \mu_0}}{\sqrt{2}}
\]

(21)

tan (\( \delta \)) stands for both the dielectric and the magnetic tangent losses. Applying DC magnetic field with different magnetization ratios, the normalized phase constant \( \beta(f)/\beta_0(f) \) will be changed as illustrated in figs. 6a and 6b. In Fig.6a at center frequency \( f_c = 15.5 \) GHz the phase shift ratio \( \beta(f)/\beta_0(f) \) changes from 3 to 10 according to the magnetization ratio. At magnetization ratio \( r \leq 1 \), the bandwidth is 7 GHz at center frequency 16 GHz. The percentage error in phase shift ratio \( \beta/\beta_0 \) is 4% within the bandwidth. When the substrate is vertically magnetized (Fig.6b) the phase shift ratio is higher than that when magnetization is in the direction of propagation. The center frequency will be different and the bandwidth is much smaller. Different center frequencies can be achieved for the same phase shift ratio if the magnetization ratio (r) is changed. Results obtained in Figs. 6a and 6b are
Fig. 6(a) Phase shifter with $W/H = 2$, $4\pi M_s = 1780$G, $\varepsilon_r = 16$ for partially magnetized substrate with different magnetization ratios

Fig. 6b Phase shifting using vertical magnetization at different magnetization ratios $\varepsilon_r = 16$, $4\pi M_s = 1780$G, $W/H = 2$
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used also for slow wave operation with different slow wave factors \( \beta (f) / \beta_0 (f) \).

3. Controlled Taper line

A taper line, which is printed on ferrite substrate with \( \varepsilon_r = 16 \) is used to match between 50 to 25 \( \Omega \) terminal impedance. If the substrate is partially magnetized then \( \mu_{\text{eff}} \) is less than 1 and the impedance of the two terminals will decrease as shown in Fig.5a. The terminal impedance can be changed according to the ratio \( (r) \) from 25 \( \Omega \) to 5 \( \Omega \). If the substrate is vertically magnetized and \( \mu_{\text{eff}} \) is greater than 1 then the terminal impedance will increase according to the ratio of magnetization as shown in Fig.5b. Fig.7 illustrates the reflection patterns when the 25 to 50 ohm taper line is partially magnetized such that \( r = 3 \) at center frequency 12 GHz, the line will turn to match between 5 to 25 \( \Omega \). Hence, terminal impedance can be changed for this line up to 80\% of its original value depending on the ratio \( (r) \). The propagating waves along the taper line suffers from continuous reflections due to variations in section impedance \( Z(x, f) \). The characteristic impedance depends not only on the position \( (x) \) but also on \( \varepsilon_{\text{eff}} (f,x) \) and \( \mu_{\text{eff}} (f,x) \), that will vary with line width variation. The phase constant \( \beta \) along the taper is a function of \( x \) and \( f \) and also \( \varepsilon_{\text{eff}} \) varies. Applying the small reflection theorem [9] the total reflection coefficient for a line length \( (L) \) is evaluated by the modified formula:

\[
\Gamma_{in}(f) = \frac{1}{2} \int_{0}^{L} \exp(-2\beta x) \frac{d}{dx} \ln[Z] \, dx
\]  \hfill (22)

The reflection patterns shown in Fig.7 are calculated for exponential taper line before and after magnetization, i.e. when the terminal impedances were 25-50 ohms and after being changed to 15 - 30 ohms by applying DC magnetization ratio of 2 with center frequency of 10 GHz.
Fig. 7 Reflection patterns for taper line matching between 25-50Ω, which is converted to match between 15 - 30Ω

CONCLUSION

Accurate dispersion analysis of Microstrip lines on magnetized ferrite substrates has useful applications for phase shifters, rectangular and circular resonators at microwave frequencies which are required for monolithic, broad band hybrid MICS, and variable matching sections which can be controlled by the level of applied DC magnetic field. In this way fine adjustment of center frequency for resonators is achieved. Manufacturing phase shifters on ferrite substrates can easily do changing the phase shift for phased array antenna. Variations of terminal impedance of a taper line makes the matching process more flexible and at the same time realizing very
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Small impedance on microstrip lines is available using magnetized ferrite substrates. Achieving such small valued impedance is a difficult task on dielectric substrates because the line width "w" must be very large compared to substrate height "h" and wide lines will excite higher order modes. These small values of impedance are required to match impedance between microwave active devices with small input impedance and circuits having high output impedance. The results are also applied to matching techniques between special values of terminal impedance's $Z_1$ and $Z_2$, which are not standard such as laser diodes and microwave active devices. Hence, the magnetized ferrite substrates are challenging materials for many applications in microwave and mm-wave range of frequency.

REFERENCES


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