APPROXIMATE SOLUTION FOR A PROBLEM OF PROPAGATION OF ELASTIC-PLASTIC WAVE IN THE HALF-SPACE

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ABSTRACT

In this paper, we consider a half-space occupying an elastic-plastic medium. Treatments were carried out under the assumption of perpendicular load, on boundary which propagates with a constant speed D.

The assumption further involved that elastic displacement of the half-spac are in the direction of the load.

The least square method is used throughout the solution of wave equation.

INTRODUCTION

Many authors, such as, Rakhmatuline (1961), Sokoloveskii (1948), Shapiro (1946), Clifton (1963) and Berthoff (1966) have studied the problem of elastic-plastic wave propagation in the half-space.

A three-dimensional problem of elastic wave propagation has been solved by El-Dewik (1975a). The problem has been studied when an instantaneous constant load acts on the boundary of elastic half-space. The load is taken to act perpendicularly to the boundary of elastic half-space and the lateral displacements were neglected. A similar study of this problem was carried out in the case where the load is time-dependent (El-Dewik 1975b).

The two-dimensional problem was treated under the assumption that the propagation load acts perpendicularly to boundary (El-Dewik 1977), and the solution was obtained taking into account the vertical displacements, whereas the lateral displacements were neglected. A similar problem was solved (El-Dewik 1981) taking into consideration the lateral displacements as well as the vertical one.

Nevertheless, the problem of propagation of elastic-plastic wave in the half-space was recently solved (El-Dewik 1982) under the assumption that the propagation load acts perpendicularly to the boundary.

In the present we study work the three-dimensional problem of elastic-plastic wave propagation in the case when the boundary variable load moves with constant velocity.

1. BASIC EQUATION AND ITS SOLUTION

Consider a half-space occupied by an elastic-plastic medium. We assume that a load exists perpendicularly on the boundary of the half-space and that it propagates with a constant speed D.

Following the assumption made in (6), the normal component of displacement W satisfied the following wave equation

$$\frac{\partial^2 w}{\partial t^2} = a^2(e) \nabla^2 w \tag{1.1}$$

Where a is the velocity of the longitudinal wave which is a function of intensity deformation.

The initial condition is

$$w(x, y, z, o) = \frac{\partial w(x, y, z, o)}{\partial t} = 0 \tag{1.2}$$

And the boundary conditions are

$$\left(\frac{\partial w}{\partial z} \right)_{z=0} = p(x, y) \qquad 0 \leqslant r \leqslant Dt \tag{1.3}$$

$$\left(\frac{\partial w}{\partial z} \right)_{z=0} = 0 \qquad r > Dt \tag{1.4}$$

Where

$$r = \left(X^2 + y^2 \right)^{\frac{1}{2}}$$

2. THE PROPAGATION OF ELASTIC WAVE IN THE HALF-SPACE

The solution of the problem may be written in the form

$$w = \iiint_w \frac{C(\xi, \zeta, \tau) \, d\tau d\zeta d\tau}{R_0} \tag{2.1}$$

Where w is the region bounded by the surfaces

$$\mathcal{E} = t - \frac{R_O}{a} \text{ and } \mathcal{V} = o \qquad (2.2)$$

$$R_O = [(\mathcal{E} - x)^2 + (\mathcal{V} - y)^2 + z^2]^{\frac{1}{2}}$$

It is easy to prove that the expression (2.1) satisfies the wave equation (1.1) and condition

$$\left(\frac{\partial w}{\partial z}\right)_{z=o} = -2\pi \int^t C(x, y, \tau) \, dz \qquad (2.3)$$

If $\left(\dfrac{\partial w}{\partial z}\right)_{z=o} = f(x, y)$ and $C(x, y, t)$ is a continuous function then

differentiating (2.3) with respect to t we obtain $C(x, y, t) = o$

This shows that the solution (1.1) will be trivial.

To overcome this difficulty, we drop the assumption of the continuity of $C(x, y, t)$ and assume that

$$C(x, y, t) = A \delta(t) + B(x, y, t) \qquad (2.4)$$

Where $\delta(t)$ is Dirac's function and $B(x, y, t)$ is a continuous function.

From (2.4) and (2.3) we get

$$\left(\frac{\partial w}{\partial z}\right)_{z=0} = -2\pi A - 2\pi \int_o^t B(x, y, t) \, t \qquad (2.5)$$

Differentiating (2.5) with respect to t we get

$$\qquad (2.6)$$

$$B(x, y, t) = \frac{-1}{2\pi} \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial z}\right)_{z=0}$$

Substituting from 2(.5) and (2.4) in (2.1) we get

$$W = \iiint_w \frac{A \delta(t) \, d\mathcal{E} d\mathcal{V} d\mathcal{E}}{R_o} - \frac{1}{2\pi} \iiint \frac{w_z d\mathcal{E} dz}{R_o} \qquad (2.7)$$

Using the polar coordinates

$$\mathcal{C} = r \cos \theta, \quad \mathcal{E} = r \sin \theta$$

then from (2.7) and the boundary conditions we get

$$W = \frac{1}{2\pi} \int_0^{2\pi} \int_0^v \int_0^{t-R/a} \frac{[\ P_1(r,t) + \frac{\partial}{\partial t} P_2(r,t)]}{R_o} \ dr \ d\theta d\mathcal{C} \qquad (2.8)$$

Where $R_1 = (r^2 + r_o^2 + z^2 - 2r \ r_o \cos \theta)^{\frac{1}{2}}$

and $r_o{}^*$ is the root of the equation

$$r_o{}^* = R [\ t - r^* + \overset{2}{z^2} + r^2 - 2r^* \ r_o \text{Cos}]^{\frac{1}{2}} \qquad (2.9)$$

In case the load propagates with a constant velocity D the root of equation (2.9) will take the following form

$$r^* = \frac{D}{a-D} [\ at - \frac{D}{a} r_o \text{Cos}\theta \pm \{\ (at - r_o \text{Cos} \ \theta)^2 - \frac{D}{a}(at^2 - r_o^2 - z^2)\}^{\frac{1}{2}}] \qquad (2.10)$$

Now we discuss the following cases.

At D<a the disturbed region is bounded by semi-sphere and the part of the surfaces A CoC'A'. At the parts A C and A'C', the stresses will vanish, whereas at the part C0C', the stresses equal the value of the load on the boundary. Inside semi-sphere, the solution of the problem will take the form (2.8). (figure 1).

At D=a the solution of the problem has the same form in 2(.8).

At D>a, it is clear that the region ACE and BDK excluding the boundary of this region, the stresses are constant and equal to the value of the load at the boundary. At the park EK, the stresses will vanish. Inside semisphere, the solution will take the following form

$$W = \frac{1}{2\pi} \int_0^{2\pi} \int_{r_X^*}^{r2} \int_0^{r-R/a} \frac{[\ P_1(r\mathcal{E}) - \frac{\partial}{\partial t}(P_1(r, \mathcal{E}) \ rd \ rd\partial \ d\mathcal{E}}{R^1}$$

where $r^*{}_1$ and $r^*{}_2$ are respectively corresponding to the negative and the positive signs in the expression (2.10). (figure 2).
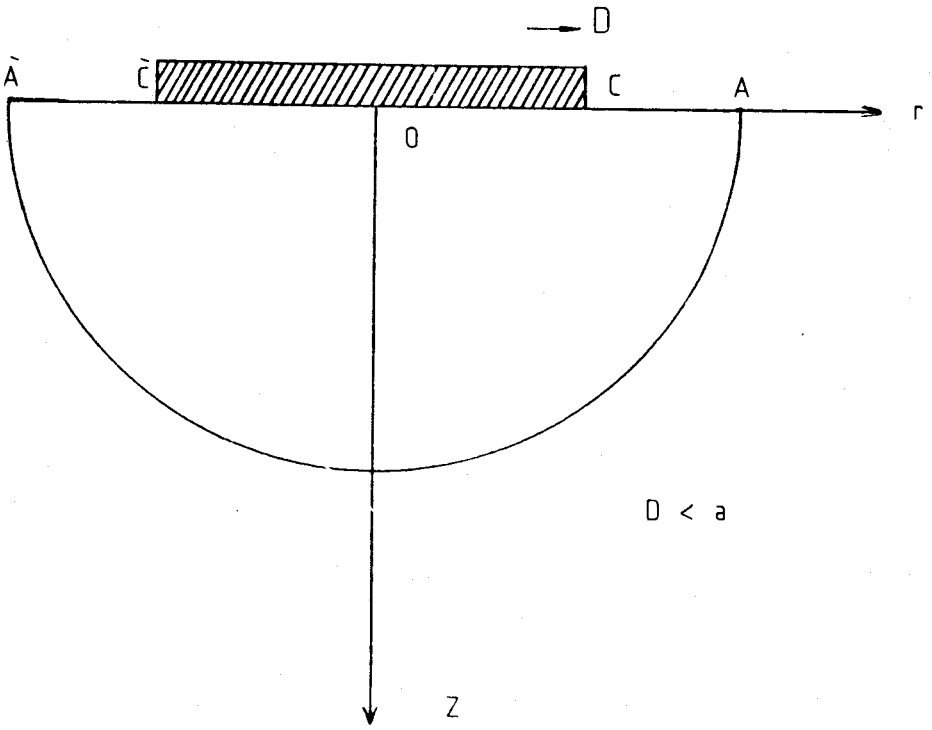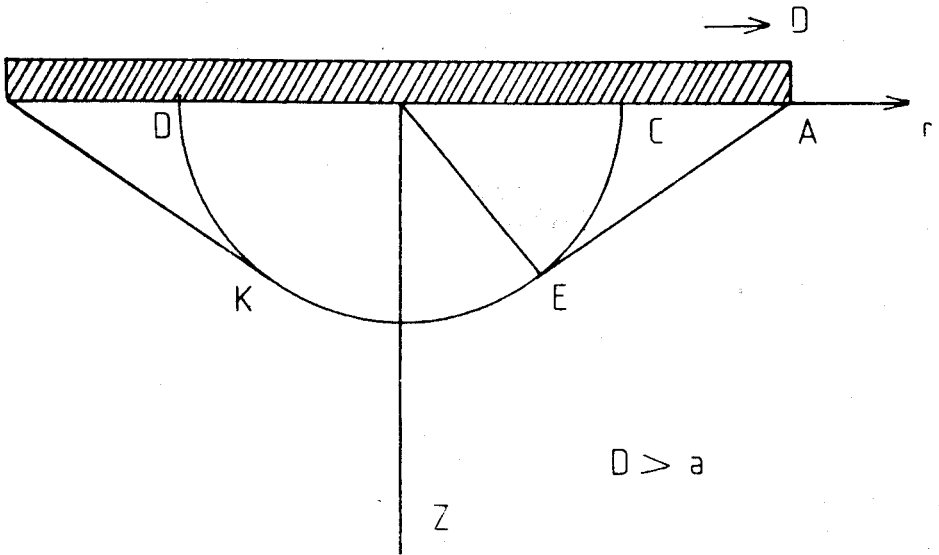
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Fig. 1

Fig. 2

3. THE PROPAGATION OF ELASTIC-PLASTIC WAVE IN THE HALF-SPACE

In this case, if we take into consideration Prantel's diagram, then elastic-plastic wave propagation may be illustrated as shown in figure (3). In the first region ABCB'A'A the solution takes the form $W = W_1 + W_2$. In the second region between the surface ABCB'A' and circumference $x^2 + y^2 = a^2_2 t^2$ exists only the elastic solution W_2.

The solution for W_1 and W_2 can be written in the following form

$$W_1 = \iiint_{w_1} \frac{C(\tau, \xi, t)\, d\tau\, d\xi\, d\gamma}{R_1} \tag{3.1}$$

$$W_2 = \iint_{\sigma} \frac{A(\tau, \xi)d\tau\, d\xi}{R^1} + \iiint_{W_2} \frac{B(\tau, \zeta, t)d\xi}{R^1 \mid} \tag{3.2}$$
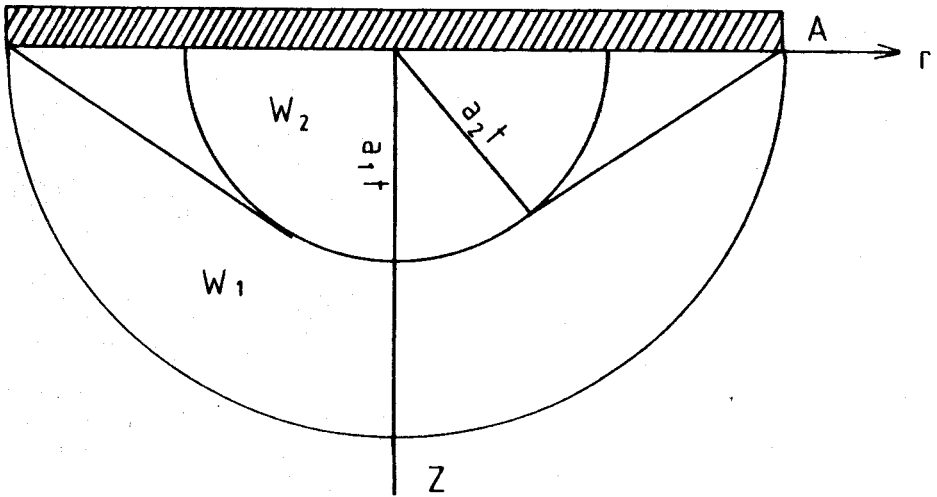

Fig 3

Where

w$_1$ is the region bounded by the surface

$$\tau = t - \frac{R}{a(e)} \text{ and } t = 0$$

and

w$_2$ is the same region of w$_1$ after replacing a(e) with a
while

σ is the common region of circles $(\xi-x)^2 + (\xi-y)^2 = a^2t^2$ and the $\xi^2+\xi^2 = R^2(t)$.

The unknown functions A, B and C can be determined from the boundary condition

$$-2\pi A(x,y) - 2\pi \int^{t} C(x,y,\tau) \, d\tau - 2\pi \int^{t} B(x,y,\tau) \, d\tau = P. \tag{3.3}$$

$$-2\pi.A(x,y) - 2\pi \int_{o}^{t} C(x,y,\tau) - 2\pi \int_{o}^{t} B(x,y,\xi) \, dz = 0 \tag{3.4}$$

On ABCBA we have

$$[\in_i] \text{ ABCBA} = \xi \tag{3.5}$$

Where $\in_s$ is the limit value deformation between the elastic and plastic region and $\in_i$ is the intensitive deformation.

In the present case, $\in_i$ is reduced to the following form

$$\in_i = (\frac{2}{3})^{\frac{1}{2}} [(\frac{\partial w}{\partial x})^2 + (\frac{\partial w}{\partial v})^2 + (\frac{\partial w}{\partial z})^2]^{\frac{1}{2}} \tag{3.6}$$

The relations (3.3), (3.4) and (3.5) are enough to determine A, B and C. Let us discuss the case when

$$A(\xi,\xi) = b_o + b_1 \, \xi$$

i.e. the case when the boundary load is symmetrical.

The unknown functions $b(\tau)$ and b_1 ($\tau 1$ are to be determined by using the least square method.

Let $\in i$ differ from $\in s$ by infinitesimal quantity

i.e. $\in_i - \in_s = e(b_o b_1)$

$e_2 = \int_o^t (\in_i - \in_s)^2 \, d t$

Using (3.6), we get

$$e^2 = \frac{2}{3} \int_o^t [(\frac{\partial w}{\partial x})^2 + (\frac{\partial w}{\partial y})^2 + (\frac{\partial w}{\partial t})^2 - \Sigma_s]^2 \, d t$$

To minimise this error, its first derivative with respect to b_o and b must vanish

Thus, we get $\dfrac{\partial e^2}{\partial b_o} = e^2 = 0$ (3,7)

Solving relation (3.7) for b_0 and b_1 we get

$b_0 = 1, 07_s$

$b_1 = 0.\dfrac{53}{(at)^2}$

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الحل التقريبي لحل مسألة انتشار الموجات المرنة اللدنة
في نصف الفراغ المرن اللدن

فوزى شعبان الدويك.

يعالج هذا البحث مشكلة انتشار الموجات المرنة ـ اللدنة في نصف الفراغ الذي يشغله وسط مرن ـ لدن وقد عولجت هـذه المسألـة تحت افتراض ان الحمـل عـلى السطح يكون انتشاره في الأتجاه العمود عليه ويسـرعة ثـابتة . كـذلك أخـذنا في الاعتبار الازاحات في اتجاه الحمل بينما اهملنا الازاحات المستعرضة ثم حصلنا على حل تحليلي للازاحات والاجهادات عند اي نقطة داخل المنطقة المضطربة .