

## CALCULATION OF MASS YIELD DISTRIBUTIONS IN FISSION INDUCED BY HIGH ENERGY PROTONS

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### ABSTRACT

An analytical expression is obtained for the mass-yield distribution in fission induced by high-energy protons, using a two-step model. In the first step, the incident proton penetrates the target nucleus, knocking out some of its constituting nucleons and leaving a residual nucleus in an excited state. In the second step, the residual nucleus fissions. The mass-yield distribution in the fission of this residual nucleus is described within the hypothesis of two independent fission modes, with parameters extracted from the analysis of low-energy fission. Comparison with experimental data of the mass yield in fission of  $^{238}\text{U}$  induced by 190 MeV protons shows that the agreement with experiment can be achieved only if one assumes that a considerable amount of excitation energy is removed from the residual nucleus by nucleon evaporation prior to fission.

### INTRODUCTION

High-energy proton induced reactions are often described within the framework of a two-step model (see, e.g. Hufner 1985). In the first step, the incident proton penetrates the target nucleus, collides with a number of its nucleons knocking some of them out and leaves a residual nucleus in an excited state, whose excitation energy depends on the number of proton-nucleon collisions as well as the behaviour of the struck nucleons after collision. In the second stage, the residual nucleus decays mainly by successive particle and cluster evaporation (spallation), fission or multifragmentation. Fortunately enough, the products of these decay modes are reasonably well separated in mass. Spallation produces one heavy fragment with mass number  $A_F > 2/3 A_T$ , where  $A_T$  is the target mass number, together with a number of nucleons and light nuclei (mainly alpha particles).

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Fission yields two fragments with mass numbers  $A_F \approx 1/2 A_T$ . Multi-fragmentation products have mass numbers  $A_F > 1/3 A_T$ . Therefore, the separate treatment of each of these three decay modes is not unjustified.

The first step of the proton-nucleus interaction is usually described within the intra-nuclear cascade model (e.g. Bertini *et al.*, 1976), which involves a considerable amount of numerical calculation. Abul-Magd *et al.* (1986) developed an approximate analytical expression for the excitation energy distribution of the residual nuclei after the first step of the reaction which is based on a simplified version of Glauber's multiple-scattering theory (Glauber and Matthiae, 1971). This expression was successfully applied to the study of spallation reactions by Simbel and Abul-Magd (1988) and Simbel (1990). Recently, Simbel (1989) used this expression to calculate the excitation function of fission induced by protons with incident energies in the range between 0.1 and 30 GeV. She was able to reproduce the correct behaviour of the experimental data by assuming that the fission of uranium and thorium is due to a mechanism similar to low-energy fission, while the "pseudo-fission" of lighter nuclei is due to another more violent mechanism.

In this paper, we use the two-step model to calculate the mass-yield distribution for fission induced by high-energy protons. We describe the first step of the reaction using the approximate formalism developed by Abul-Magd *et al.* (1986). The fission of the residual nucleus is described using the hypothesis of two independent modes which is widely applied for systematic analysis of mass yield curves in low-energy fission of actinides (e.g. by Ohtsuki *et al.* 1989). We compare our expression with the experimental data of fission of  $^{238}\text{U}$  induced by 190 MeV protons, reported by Becchetti *et al.* (1983).

## CALCULATION OF FISSION MASS YIELD

In this section, we derive a phenomenological description for the mass-yield distribution of the products of fission induced by high-energy protons. We assume that proton-induced fission is well described by the two-step model in which the target nucleus penetrated by the fast proton is excited in the first step and fissions in the second. The mass distribution of the reaction products is represented by

$$Y(E_i, A_F) = \int_0^{E_i} Y(E, A_F) P(E_i, E^*) \exp(-E^*/E_c) dE^* \quad (1)$$

where  $P(E_i, E^*)$  is the probability that the residual nucleus is left with excitation energy  $E^*$  after being penetrated by a proton with incident energy  $E_i$  and  $Y(E^*, A_F)$  the mass distribution of the products of fission of that nucleus. The factor  $\exp(-E^*/E_c)$  takes into account the predominance of multifragmentation at high

excitation energies (Zhang *et al.* 1987). It was introduced by Simbel (1989) in the analysis of fission excitation functions where the slope parameter  $E_c$  was fixed to the value

$$E_c = 1.8 A_0 \text{ MeV.}$$

where  $A_0$  is the mass number of the fissioning nucleus.

Equation (1) assumes the formation of a residual nucleus with a fixed mass number. In reality, one expects a mass distribution of residual nuclei after the first step of the reaction. Fortunately, only a small number of nucleons are knocked out in the first stage of the interaction of fast protons with nuclei. Intra-nuclear cascade calculations by Cugnon (1987) have clearly demonstrated this fact. We shall thus assume that the residual nucleus has a sharp mass distribution peaked at

$$A_0 = A_T - \langle \nu \rangle$$

where  $\langle \nu \rangle$  is the mean number of proton-nucleon collisions given in Glauber's theory by

$$\langle \nu \rangle = A_T \sigma_t^{\text{PN}} / \sigma_R \quad (2)$$

where  $\sigma_t^{\text{PN}}$  is the effective proton-nucleon total cross section and  $\sigma_R$  is the total reaction cross section of protons with the target nucleus.

For the probability of formation of a residual nucleus with excitation energy  $E^*$ , we use the expression

$$P(E_i, E^*) = N [ \exp(-E^* / \langle E^* \rangle) - \exp(-E_i / \langle E^* \rangle) ] \quad (3)$$

where  $\langle E^* \rangle$  is the mean excitation energy of the residual nucleus, expressed in terms of the mean number of collisions by

$$\langle E^* \rangle = (\langle \nu \rangle - 1) E_0 \quad (4)$$

and  $N$  is a normalization factor given by

$$N^{-1} = \langle E^* \rangle - (E_i + \langle E^* \rangle) \exp(-E_i / \langle E^* \rangle). \quad (5)$$

Equation (3) is a slight modification that takes into account the fact that the excitation energy does not exceed the incident energy of the expression obtained on the basis of a simplified version of Glauber's theory, in which the probability of  $\nu$  collision is approximated by an exponential function and the angular distribution of nucleon-nucleon scattering is described by a Gaussian function of the momentum transfer (Abul-Magd *et al.* 1986). It has been successfully applied by Simbel and Abul-Magd (1988) to study the excitation functions of spallation reactions and by Simbel (1989) in the analysis of the excitation functions of proton-induced fission. The quantity  $E_0$  in Eq. (4) is the average excitation energy deposited in the target

nucleus in a single proton-nucleon collision. Abul-Magd *et al.* (1986) expressed this quantity as a sum of two terms:

$$E_o^{el} = E_o^{el} + 1/2 \frac{\sigma_{in}^{PN}}{\sigma_t^{PN}} (m_{\Delta} - m). \quad (6)$$

The first term in Eq. (6) is

$$E_o^{el} = \langle q^2 \rangle / 2 m$$

where  $\langle q^2 \rangle$  is the mean square momentum transfer per collision. It depends on the energy of the incident proton which changes during the transversal of the nucleus. At incident energies below 0.5 GeV where the proton-nucleon elastic scattering is almost isotropic,  $E_o^{el}$  is approximately given by

$$E_o^{el} \cong \frac{E_i}{\langle v \rangle} [1 - \exp(-\langle v \rangle / 4)]; \quad E_i < 0.5 \text{ GeV} \quad (7)$$

On the other hand, when the incident energy is in the multi-GeV region, the energy loss per collision (100 MeV) is small enough to neglect the variation of the proton energy in the calculation of  $\langle q^2 \rangle$ . In that region, when the elastic nucleon-nucleon differential cross section is taken proportional to  $\exp(-bq^2)$ , one obtains

$$E_o^{el} \cong 1/(2 \text{ mb}); \quad E_i > 2 \text{ GeV}$$

The second term in Eq. (6) in which  $\sigma_{in}^{PN}$  is the inelastic proton-nucleon cross section takes into account the inelastic interaction of the projectile with the target nucleon by assuming that this interaction proceeds through the formation of a delta resonance of mass  $m_{\Delta}$ , while  $m$  is the nucleon mass. This assumption may be valid at incident energies below 10 GeV otherwise the formation of more massive resonances must be taken into account. The factor 1/2 accounts for the fact that if the delta isobar is the faster product, it decays outside the nucleus and does not contribute to its excitation energy.

We shall assume that the fission of the residual nucleus is reasonably well described within the hypothesis of two decay mode. According to this hypothesis, the fission mass-yield curve is expressed as a linear combination of a symmetric and asymmetric component (Ohtsuki *et al.* 1989):

$$Y(E^*, A_F) = f_s(E^*) y_s(A_F) + f_a(E^*) y_a(A_F) \quad (8)$$

where  $f(E^*)$  is the probability of either symmetric or asymmetric fission at excitation energy  $E^*$  and  $y(A_F)$  is the fractional yield of the fragment mass  $A_F$  in each fission mode. The subscript "a" denotes asymmetric fission and "s" denotes symmetric. Following Ohtsuki *et al.* (1989), we assume that the function  $y(A_F)$  does not depend on  $E^*$  and  $f(E^*)$  does not depend on  $A_F$ , although earlier

systematics of the mass yield curves, e.g. by Mariama and Ohnishi (1974), indicate a slight dependence of each function on the other variable. The two decay mode hypothesis has also been used in the analysis of kinetic energy distribution of fission fragments, e.g. by Itkis *et al.* (1985). These analyses have clearly demonstrated that symmetric fission is the only fission mode at high excitation energies in all nuclei independent of their nucleon composition and that asymmetric fission is important only at low excitation energies. We shall thus arbitrarily assume that the probability of asymmetric fission is reasonably approximated by a Fermi step function of excitation energy hoping that the detailed excitation-energy dependence of  $f_a$  does not change many of our conclusions. We therefore write

$$f_a(E^*) = [1 + \exp(\frac{E^* - E_t}{\Delta})]^{-1} \quad (9)$$

$$f_s(E^*) = 1 - f_a(E^*) \quad (10)$$

where  $E_t$  is a certain critical excitation energy characterising the transition from the asymmetric to the symmetric fission mode, and  $\Delta$  is the excitation-energy range within which this transition takes place.

Substituting Eqs. (3) and (8-10) into Eq. (1), we obtain

$$Y(E_i, A_F) = F_s(E_i) y_s(A_F) + F_a(E_i) y_a(A_F) \quad (11)$$

where

$$F_{a(s)}(E_i) = N \int_0^{E_i} f_{a(s)}(E^*) \left( e^{-E^*/\langle E^* \rangle} - e^{-E_t/\langle E^* \rangle} \right) e^{-E^*/E_c} dE^* \quad (12)$$

The integrals in Eq. (12) are familiar in statistical mechanics and can be evaluated in a closed form when  $E_i \gg E_t \gg \Delta$  (Kittel 1958):

$$\int_0^{E_i} \frac{e^{-E^*/E}}{1 + e^{\frac{E^* - E_t}{\Delta}}} dE^* \cong E \left[ 1 - \frac{\pi \Delta}{E} \operatorname{cosec} \left( \frac{\pi \Delta}{E} \right) e^{-E_t/E} \right] \quad (13)$$

However, Eq. (11) is not suitable for calculating the fission mass yield because  $F_a + F_s \neq 1$  and hence the mass distribution is not normalized. It is more convenient to use the following "renormalized" distribution:

$$\tilde{Y}(E_i, A_F) = \tilde{F}_s(E^*) y_s(A_F) + \tilde{F}_a(E^*) y_a(A_F) \quad (14)$$

where

$$\tilde{F}_a(E^*) = F_a / (F_s + F_a)$$

$$= \tilde{N} [ E_1 \{ 1 - \frac{\pi \Delta}{E_1} \operatorname{cosec} \left( \frac{\pi \Delta}{E_1} \right) \exp (-E_i/E_1) \} \\ - E_c \exp (-E_i/\langle E^* \rangle) \{ 1 - \frac{\pi \Delta}{E_c} \operatorname{cosec} \left( \frac{\pi \Delta}{E_c} \right) \exp (-E_i/E_c) \} ]$$

$$\tilde{F}_s(E^*) = 1 - \tilde{F}_a(E^*)$$

$$\tilde{N}^{-1} = E_1 (1 - e^{-E_i/E_1}) - E_c e^{-E_i/\langle E^* \rangle} (1 - e^{-E_i/E_c})$$

$$E_1^{-1} = \langle E^* \rangle^{-1} + E_c^{-1}$$

### COMPARISON WITH EXPERIMENT

In this section, we apply Eq. (14) to calculate the mass yield in the fission of  $^{238}\text{U}$  induced by 190 MeV protons and compare the result of calculation with the experimental data reported by Becchetti *et al.* (1983). At this energy, most of the proton-nucleon collisions are purely elastic so that we can put  $E_o = E_o^{\text{el}}$  in Eq. (4) and calculate  $E_o^{\text{el}}$  by using Eq. (7). Taking the proton-proton and proton-neutron cross sections from the compilation by Schubert (1980), we find that the isospin-averaged proton-nucleon cross section in  $^{238}\text{U}$  at this energy is 35.7 mb. However, the incident energy under consideration is not high enough to neglect the effect of Pauli principle, which forbids the momentum transfer to a bound nucleon that is less than a minimum value to bring it outside the "Fermi sea." We shall take the effect of Pauli principle by following Goldberger (1948), who shows that the effective cross section of a proton with a nucleon in a nuclear matter with Fermi energy  $E_F$  is related to the free proton-nucleon cross section  $\sigma_t$  by

$$\sigma_t^{\text{PN}} = (1 - 7 E_F / 5E_i) \sigma$$

With  $E_F = 38$  MeV we obtain  $\sigma_t^{\text{PN}} = 25.7$  mb, so that the mean number of collisions from eq. (2) becomes  $\langle \nu \rangle = 3$  when we set  $\sigma_R = \pi R_o^2$  with  $R_o = 1.3 A_T^{1/3} \times 10^{-13}$  cm. Substituting this value into Eqs. (4) and (7) we obtain  $\langle E^* \rangle = 66.8$  MeV.

In this analysis, the fragment mass yield in symmetric and asymmetric fission is expressed as follows:

$$Y_s(A_F) = (2/\sqrt{\pi \Delta_s}) \exp \{ - (A_F - A_o/2)^2 / \Delta_s^2 \} \\ Y_a(A_F) = (1/\sqrt{\pi \Delta_a}) [ \exp \{ - (A_F - A_1)^2 / \Delta_a^2 \} + \\ \exp \{ - (A_F - A_o + A_1)^2 / \Delta_a^2 \} ] \quad (15)$$

The mass yield distribution of the residual nucleus has then five parameters, namely  $E_t$ ,  $\Delta$ ,  $A_1$ ,  $\Delta_s$ ,  $\Delta_a$ . We fix these parameters by an analysis of the experimental data on the mass yield of the fission of  $^{239}\text{Pu}$  induced by neutrons in the energy range 0.17 – 7.9 MeV. Fig. 1 shows a comparison between the predictions of Eqs. (8-10) and (15) and the experimental data reported by Gindler *et al.* (1983), with the following choice of the parameters:

$$\begin{aligned}
 E_T &= 14.0 \text{ MeV} & \Delta &= 2.3 \text{ MeV} & A_1 &= 136 \\
 \Delta_s &= 20 & \Delta_a &= 8 & &
 \end{aligned}$$

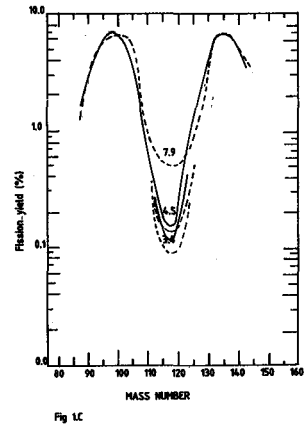
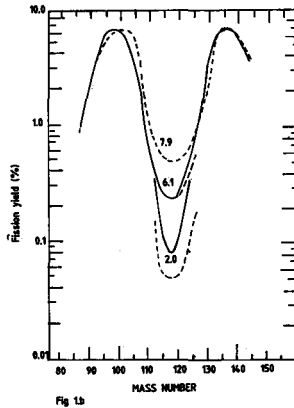
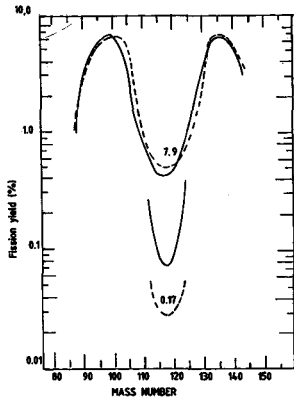


Fig. 1: Mass yield distribution in fission of  $^{238}\text{Pu}$  induced by neutrons of energies  
 (a) 7.9, 0.2 eV.  
 (b) 6.1, 2.0 eV.  
 (c) 4.5, 3.4 eV.

The solid lines are calculated using Eqs. (8-10) and (15) while the dashed lines are the experimental data reported by Gindler *et al.* (1983).

With these parameters we applied Eq. (14) to calculate the mass yield distribution in the fission of  $^{238}\text{U}$  induced by 190 MeV protons. Fig. 2 shows the result of calculation by a dashed-dotted histogram and the experimental data of Becchetti *et al.* (1983) by a solid histogram. This figure shows that, while the theoretical distribution is dominated by a symmetric component, the experimental data clearly suggests the predominance of the asymmetric mode of fission of the residual nuclei.

Since asymmetric fission is important at low excitation energies we may conclude that the experimental data suggest that the mean excitation energy of the fissioning nucleus is less than the value given by Eq. (4). This can be the case if the residual nucleus emits few nucleons before it fissions. We can estimate the mass number from the experimental data to be twice the mean mass number of the fragments.

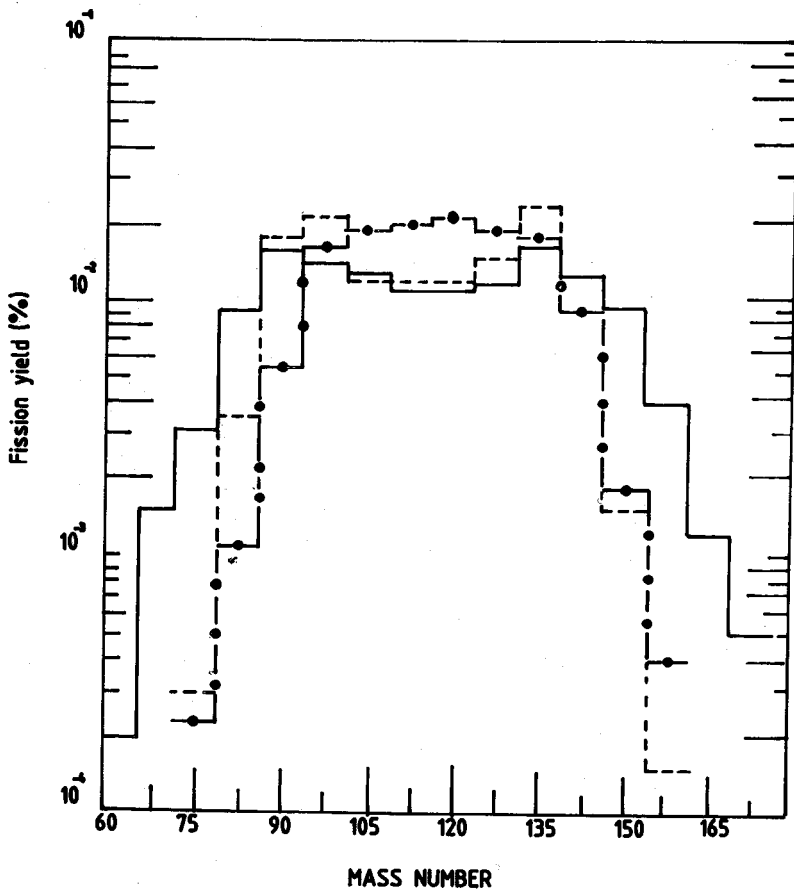


Fig. 2: Mass yield distribution in fission of  $^{238}\text{U}$  induced by protons of energy 190 MeV. The dashed-dotted line is calculated using Eq. (14) for a mean excitation energy of 66.8 MeV while the dashed line is calculated with  $\langle E^* \rangle = 16.8$  MeV. The solid line represents the experimental data of Becchetti *et al.* (1983).

We then find that  $A_0 = 230$ . On the other hand, we have estimated the mass of the residual nucleus by  $A_T - \langle \nu \rangle = 235$ . This means that, on average, the residual nucleus emits five nucleons before it fissions. If each nucleon emission removes 10 MeV of excitation energy as (heavy-ion, xn) reaction data suggest (Neubert 1973), then the mean excitation energy of the fissioning nucleus will be 16.8 instead of 66.8 MeV. The dashed histogram in Fig. 2 is calculated using Eq. (14) and taking  $\langle E^* \rangle = 16.8$  MeV. We see from the figure that this curve is in good agreement with the experimental data.



## CONCLUSION

We have developed an analytical description of the fragment mass distribution in fission induced by high-energy protons based on the two-step model. The first step in which the target nucleus is excited by the proton penetration is treated using an approximation of Glauber's multiple scattering theory proposed by Abul-Magd *et al.* (1986). The second step in which the residual nucleus fissions is described using the hypothesis of two decay modes. Comparison between the mass yield distribution obtained in the present calculation and the experimental data on fission of  $^{238}\text{U}$  induced by 190 MeV protons suggests that the fissioning nucleus has a mean excitation energy considerably less than the mean excitation energy deposited in the first step of the reaction. Agreement with experiment is achieved by assuming that before the residual nucleus fissions, it emits a number of nucleons carrying away more than 70% of its excitation energy. This conclusion may serve as an invitation for a detailed study of fission induced by high-energy protons in which the competition with neutron emission is carefully taken into account. The first step in this direction has already been taken by Il'inov *et al.* (1980).

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## حساب توزيع الكتل الناتجة من الانشطار المستحث بالبروتونات ذات الطاقة العالية

محمد راغب عيسى و عادل يحيى أبو المجد

في حساب توزيع الكتل الناتجة من الانشطار المستحث بالبروتونات ذات الطاقة العالية ، تم استخدام دالة تحليلية تعتمد على فكرة النموذج ذو المرحلتين . في المرحلة الأولى من هذا النموذج يتم اختراق البروتونات لنواه الهدف ويقوم بطرد بعض مكوناتها ثم يتركها في حالة استتاره . أما المرحلة الثانية فهي انشطار النواه المستتاره .

يوصف توزيع الكتل الناتجة من الانشطار بنموذج يعتمد على افتراض استقلالية مرحلتي الانشطار ، وباستخدام بعض الثوابت اللازمة والتي تم الحصول عليها في دراسة نتائج التجربة للانشطار عند طاقة منخفضة .

بمقارنة توزيع الكتل مع نتائج التجربة لانشطار اليورانيوم ( $^{238}\text{U}$ ) بالبروتونات عند طاقة ( 190 Gev ) تبين أنه للحصول على نتائج تتفق مع التجربة ، فلا بد من الافتراض بأن معظم طاقة الاستتاره المضافة للنواه قد تم التخلص منها عن طريق تبخر بعض مكوناتها قبل الانشطار .