QATAR UNIVERSITY

COLLEGE OF EDUCATION

# THE IMPACT OF USING FRAYER'S MODEL IN ACQUISITION OF MATHEMATICAL CONCEPTS FOR PRIMARY STUDENTS IN QATAR

BY

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# ABSTRACT

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Title: THE IMPACT OF USING FRAYER'S MODEL IN ACQUISITION OF MATHEMATICAL CONCEPTS FOR PRIMARY STUDENTS IN QATAR

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This study aimed to investigate the impact of using Frayer's teaching Model on 3rd-grade students on the acquisition of mathematical concepts. The researcher was interested in testing the differences between students acquisitions of mathematics concepts were dependent on the teaching method.

To achieve the objectives of the study, a quasi-experimental approach was used. The researcher prepared the study tools, which were as follows: Testing the mathematical concepts in the multiplication unit during the first semester (2018-2019). The selected primary schools were purposively the study sample consisted of 100 students were chosen in the simple random who were divided into two groups one (n=50) studied mathematics using Frayer's Model, the other (n=50) used traditional teaching method. At the end of the experiment, the concepts acquisition test was administered to both groups.

The result revealed that there were statistically significant differences ($p<0.05$) between the average score of the experimental group students and the average score of students in the control group in the acquisition of mathematics concepts in favor of the experimental group.

Considering the outcome research has been to draw some conclusion from the

frayer model to the way that graphical organizers based on Frayer's Model can improve the students' ability to acquire the mathematical concepts.

The researcher recommended the need to inform mathematics teachers to modern strategies in teaching mathematics, especially such a way studied Frayer model, as an extension of this research suggested that the researcher conducting studies like this study in other levels and use other variables such as "spatial abilities" and investigate the effect of using the Frayer's Model on the spatial abilities of students.

# DEDICATION

*This Thesis is dedicated to my mother, Badriya and my sisters, Ghosoun and Roudha who have always been a constant source of support and encouragement during the challenges of my master's degree years. Also, they taught me to work hard for the things that I aspire to achieve. This work is also dedicated to my best friend, Ferdaouss Rahmouni for her inspiring ideas and continues effort during my thesis.*

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Finally, I would like to thank my family for the love, support and giving me the strength to reach the stars and chase my dreams all my life.

This thesis is only the beginning of my journey.

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# CHAPTER 1: INTRODUCTION

## 1.1 Background

Mathematics, as a subject, is a crucial part of the curriculum in almost every country across the world. Mathematics is critical for the general success of most people as it relates to numerous other subjects encountered throughout their lives. At a young age, students who typically perform poorly in Mathematics struggle in their other studies as well. This subject is so essential to future success that it can affect career advancements, production of informed or misinformed citizens, and even a person's sense of personal fulfillment. Mathematical knowledge has become even more vital in today's society as our dependence on technology increases with every passing year. This has placed a greater demand for individuals to use and interpret mathematics to make sense of complex information and situations. Because of this, it is a vital tool in various fields including social sciences, medicine, engineering, and natural science, as well as being used continuously in day-to-day activities at home, in marketplaces and offices (Neyland, 1994).

In 2002, the state of Qatar implemented systematic changes to its education system (Brewer et al., 2006). It was finally realized that education impacts the knowledge of its citizens; who are a nation's greatest natural resource. To improve the quality of education, many new changes were to be applied. In 2007, Qatar participated in the Trends in International Mathematics and Science Study (TIMSS) for the first time. Despite some improvements taking place over the previous five years, The results of Qatar remain less than the organization rate compared to other countries, with the greatest disparity in Mathematics scores. In the 2015 TIMSS results, Qatar ranked 28 out of 39 participating countries for fourth-grade math assessment (Hejaze, 2018).

The Ministry of Education and higher education has therefore sought to restructure the curricula of Mathematics across the country. This is designed to make the Mathematics curriculum in Qatar compatible with other modern practices, including changing the appearance of the subject which should allow students to develop a better understanding. This will also allow for the development of skills and habits appropriate for individuals of Qatari society, which will increase the possible success and increase their capabilities based on national values and trends. The specialists and developers who create the new mathematics curriculum are interested in mathematical structure, whereas teachers who must implement the curriculum are interested in the mathematical knowledge and how easily they can convey that knowledge to students. (Keitel,1989), Both groups must communicate to determine the best approaches for teaching concepts, generalizations, skills, and problem-solving techniques to the general population. The process of teaching the new curricula must be geared toward the student as a pivotal part of the educational process.

Overarching mathematical concepts are considered the main cornerstone for any mathematical system. These concepts include number theory, geometry, algebra, and data handling. Knowledge in mathematics means that students understand the nature of the subject as well as why and how an answer was derived. This is all without memorizing answers or formulas. A student who has truly learned a mathematical concept knows why it works, how it works and can work out the formulas and answers by him/herself. By understanding the concept itself, it is easier for the student to figure out when something has gone wrong. Acquisition of this type of knowledge, enables the student to more easily understand future concepts in a mathematical system, by allowing him/her think about and process the concept abstractly.

The teacher plays a vital role in a student's development and acquisition of

mathematical concepts. The teacher must use many different modern strategies in teaching to help their students to develop proper conceptual structure. A proper conceptual structure is characterized by understanding. They make sure that each new concept is in the appropriate position relative to the previous conceptual structure and is related to it through the processes of representation and harmonization (Piaget, 1976). Development of mathematical concepts takes place in stages beginning when a child starts school in kindergarten and progresses through all increasing levels of education. At an early stage, the student is introduced to the easiest elements creating a mathematical concept. For example, while starting to learn geometry, the student is first introduced to simple shapes such as circles, rectangles, and triangles. As the student develops, he/she is introduced to more complex shapes such as ellipses and polygons. By teaching a student first the differences between a circle and an oval, they can later more easily understand what an ellipse is.

Wilkins (2002) states that the teacher's mathematical knowledge plays a crucial role in his/her assessment of their students. Teachers who have broad and advanced knowledge of mathematical concepts, provide appropriate problems and help students develop higher-order thinking skills. Contrarily, teachers who only have specific and narrow knowledge can only produce students who think in a narrow circle and depend upon the memorization of procedures provided by the teacher. It is easier for a teacher who has acquired a greater mathematical understanding to explain the concept to his/her students. Thus, the teacher can explain the concept that will translate into the student acquiring the same mathematical knowledge. Teachers who struggle to explain a particular mathematical concept due to lack of knowledge of the concept, end up memorizing the concept and defining it to students without adequate illustrations, explanations, or examples. Students taught by such teachers are less likely to retain or

even acquire the concept being taught. Therefore, Researchers have also stressed the necessity of teachers acquiring mathematical concepts for themselves so that students can acquire the same concepts in the best form possible (Usiskin, 2001; and Toh, 2007).

From reviewing contemporary literature, the inadequacy of the current methods of teaching Mathematics is apparent. The effectiveness of using Frayer's Model in acquiring mathematical concepts has been well researched. It is already being applied by teachers today in all levels of schooling, as a method to "test the level of concept mastery" (Frayer, Fredrick, and Klausmeier, 1969).

Frayer's Model is based on Bruner's research in classified thinking and concept learning. This model is distinguished by using a technique that analyzes concepts by breaking them into their base components: one to teach the concept and one to measure the acquisition of the concept. Thus, it is considered an inclusive model for learning and acquisition of concepts (Al-Jazzar, 2002). Many studies have stressed the importance of using graphic organizers in teaching practices and Frayer's Model is no different.

### 1.2 Problem formulation

The Ministry of Education and Higher Education seeks to do national exams annually for both the 3$^{rd}$ and 6$^{th}$ grades in the primary stage to measure all concepts and skills learned and retained by the students. According to the 2017-2018 national results of students in Mathematics for 3$^{rd}$ grade, a significant percentage of the students did not perform well. In general, 11.3% of students failed in Mathematics with most of them being of Qatari nationality (Students in Qatar are of a different nationality). It was found that 14.1% of students failed in Algebra, 12.4% failed in data handling, and 7.6% failed the Geometry Mathematics.

According to the 2015 Mathematics TIMSS results, Qatar 4$^{th}$ grade students

have placed 439 average rates of 500. These results still reveal the poor mathematical skills of this country's 4th and 8th-grade students. Thus, the percentage of students with adequate skills and concepts in Mathematics is deficient when measured against the standard levels of other TIMSS participants. The TIMSS results also infer that the level of efficiency of the current Mathematics curricula is low, which has led to a high percentage of students not acquiring the necessary skills that would enable them to perform better in higher education.

The lack of acquisition of mathematical skills and concepts among Qatari primary school students can be attributed to several factors, including the use of teacher-centered approaches in teaching Mathematics, negative attitude towards Mathematics from both students and teachers, lack of spatial skills, and lack of practical modeling activities. There exists a problem, but the primary cause is not yet clear. The Qatar government has put in the effort to raise education standards in the state; however, poor performance in Mathematics persists despite these efforts. This implies that the efforts put in place have not addressed the underlying problem.

The researcher worked as a training specialist in Mathematics in education, she realized that most students struggle to acquire mathematical concepts. Moreover, the strategies which are used in teaching mathematics are provided ineffectively. These strategies are inefficient in helping students acquire mathematical skills such as brainstorming. The researcher believes that teaching in the primary stages should depend on a procedure based on concepts, not memorization. The current mathematical curricula focus on the skills and procedures more than concepts. Thus, if the students to learn some mathematical concepts, it is accomplished sloppily and inefficiently. Therefore, the researcher believes that preparing students for exams using better teaching methods is crucial. This study is applied to third-grade students to assess and

reinforce the mathematical concepts at this specific grade level.

### 1.3 Research Objectives

This study aims to shed light on the effectiveness of using Frayer's Model, a teaching technique based on constructivism theory, to determine its effectiveness in the acquisition of mathematical concepts for 3rd-grade students'. Thus, this study was designed to implement a new and presumably better method of teaching mathematics and assess its' results.

This study aimed to achieve the following main objective:

i. Examine the impact of the Frayer's Model on the acquisition of mathematical concepts among third graders

### 1.4 Research Questions & Hypothesis

The main objective of this research was to answer the following question:

What is the impact of using Frayer's Model in the acquisition of mathematical concepts among third-grade students in Qatar?

To find this answer, this researcher has developed the two following questions and their subsequent hypotheses:

1- Is there a statistically significant difference in the mean score between the experimental and control groups in the Multiplication Concept Test?

   Hypotheses:

$$H_0: \bar{x}_e = \bar{x}_c$$

$$H_1: \bar{x}_e \neq \bar{x}_c$$

   Where $\bar{x}_e$ is the mean score of the experimental group, and $\bar{x}_c$ is the mean score of the control group?

2- How might the use of Frayer's Model affect the student's ways to solve problems related to multiplication concept?

## 1.5 Significance of the study

The significance of the study can be viewed from two perspectives; theoretical and practical perspectives.

*Theoretical perspectives:*

i. This study may be used as a learning model based on educational ideas, which are based on the constructivism theory, and through it, the student is key in the educational process. The findings may, therefore, be used to reform the development of policy by Qatar's Ministry of Education.

ii. This study may be used to enrich the educational literature by providing academic plans and how Frayer's Model is used in teaching to acquire Mathematical concepts.

iii. The study findings may be used to sensitize primary school students in Qatar on issues that influence the acquisition of mathematical concepts by students.

*Practical part:*

i. Work done in this study on developing the academic plans by using educational Frayer's Model may be used to help teachers in the lesson planning process.

ii. The study included an exam of mathematical concepts, and the teachers may make use it during exam preparation.

iii. The findings of this research may be used to sensitize Mathematics teachers and trainers in Qatar on the need to equip Mathematics teachers with appropriate skills in acquiring mathematical concepts and mathematical problem-solving skills for use in the classroom.

### 1.6 Limitation of the study

This study was limited by:

i. *Participants*: This study was limited to a sample of 3rd-grade students at Primary government School for boys and girls.

ii. *Time*: this study was limited to the first term in the academic year 2018 – 2019.

iii. *Objectives*: This study was limited to acquire concepts of multiplication unit for the 3rd grade

iv. *Academic*: this study was limited to one dependent variable (mathematical concepts) without any other variables.

### 1.7 Definitions of terms:

i. *Concept*: A concept is an abstract idea describing some relationship within a group of facts and may be designated by some sign or symbol. (Bruner, 1956: p. 244)

ii. *Frayer's Model*: A graphical organizer used for concept analysis. This strategy emphasizes understanding concepts within the larger context of learning by requiring students, first, to analyze the items (definition and characteristics) and second, to synthesize/apply this information by thinking of examples and non-examples.

iii. *Graphic Organizer*: A Graphic Organizer (GO) is a graphical or spatial representation of Mathematical concepts.

iv. *Acquisition of Mathematical concepts*: Knowing the workings behind the answer to a mathematical problem; not memorizing formulas or answers to work out the answer.

v. *TIMSS (Trends in Mathematics and Science Study)*: A large-scale assessment

of the effectiveness of practices involving teaching and learning in Mathematics and Science. It offers an international view to help policymakers on education matters see where their schools fall relative to other schools and countries. Extensive data is collected for students in Grades 4 and 8 to access a country, school, and classroom practices for learning Mathematics and Science.

### 1.8 Organization of the study

This thesis has been divided into five chapters. Chapter one is the introduction, which outlines the context of the study including the background, problem formulation, research questions, objectives, the significance of the study, and the definition of terms as previously observed.

Chapter two consists of the literature review related to the topic. This is reviewed under two subsections: Mathematical concepts and the Frayer's Model.

Chapter three lays out the study's design and the methodology used in carrying out the study. In this section, a full description of the research setting, and the participants are given. Included also, is a discussion on the data generation methods, the rationale for choosing these methods, and their execution procedures, data analysis techniques, ethical considerations, and the limitations of the study.

Chapter four presents the results and findings of the study. The research question and sub-questions are revisited, and the results and findings are presented according to the research questions.

Chapter five discusses the findings, gives conclusions of the study, lists recommendations, and offers suggestions for further research. A list of references and appendices are presented after chapter 5.

# CHAPTER 2: LITERATURE REVIEW

## 2.1 First Part (Mathematical Concepts)

The creation and development of mathematical concepts for students are the main goals when teaching mathematics at all educational stages. Acquisition of mathematical concepts is crucial for the students since, as mentioned earlier; it determines the ease of success of an individual later in life. The teacher should, therefore, equip the students with thinking skills to enable them to find alternative solutions to life's problems (Fishman, Marx, Best and Tal, 2003).

The development of mathematical concepts and skills broaden the usefulness of subjects like Algebra outside of the classroom. This means that the when students have developed the basic understandings of how problems are solved like how to manipulate an equation to solve for different variables or how basic multiplication works, the more easily they can apply those concepts to future endeavors. It is imperative that teachers develop and practice teaching strategies that enable their students to acquire and retain the use of mathematical concepts easily. Currently, methods employed by teachers today are wholly inadequate. Today's strategies like a lecture and definition-based teaching are often unable to equip the student with the ability to think critically neither nor acquire mathematical concepts. These methods would equate to closing your eyes and trying to draw what someone looks like simply based on touch. Some people may have the ability to draw this way but not many. The education of concepts is among the most difficult educational stages, so the most effective strategies and methods should be employed to teach the students with the least amount of confusion. When finished with a concept, the students should have the capability to apply whatever they have learned, creating relationships between old concepts, and the ability to more easily understand new concepts as they are introduced to them (Abu El-Ela, 2013).

### 2.1.1 Importance of Mathematical Concepts

The development of mathematical concepts and skills broaden the mathematical content of Algebra. There is a need for teachers to develop and train on strategies of teaching mathematical concepts. This need is caused by the inadequacy of students to acquire mathematical concepts. Some of the strategies of teaching mathematical concepts do not equip the student with the ability to think critically and acquire the mathematical concepts. Another reason for teachers to develop and train on strategies of teaching mathematical concepts is based on the curriculum or the educational policy (Abu El-Ela, 2013).

The education of concepts is one of the most difficult educational stages, so the strategies and proper methods of education should be used to teach the students (Mulligan & Mitchelmore, 2009, Smith, 2006). The students should be able to apply whatever they learn and through creating relations, be able to understand other concepts they are introduced to the students (Abu El-Ela, 2013).

### 2.1.2 The concept of multiplication

Smith (2006) highlighted that most of the students belonging to the elementary school grades try to memorize multiplication facts rather than developing concepts that underpin multiplication problem. The author further contended that if the concepts those underpin problem-solving for multiplication are not developed in the respective students, it directly impacts their performance in the tests that measure mathematical aptitude.

Smith (2006) addressed that the traditional educational curriculum for third-grade students emphasizes on the memorization of multiplication facts and functions rather than an understanding of the mathematical concepts that provide the basis for gaining knowledge on such facts and functions. The author voiced for the introduction

of standards-based curriculum and modality of instruction that would emphasize the development of number sense and meaning of multiplication operations that would be compatible with the general intelligence of the third-grade students. Smith (2006) also described the roadmap through which students could develop competence in such concepts. Understanding the concept of multiplication functions require the development of a language that prompts the thinking and description of the multiplicative situations and their relevant context in terms of quality and quantity. In this regard, the author emphasized on the role of visual images in developing the concept of grouping across the target population. Likewise, students should understand the units that are relevant to multiplication.

Teaching any form of concept to help a child to acquire useful information about the surrounding environment with which they could connect and reciprocate for enjoyment, pleasure, sustenance, and problem-solving. Mathematical concepts provide joy and pleasure to children irrespective of their mental ability, knowledge proficiency, or degree of maturity. Mathematical concepts help them to frame and raise questions not only about the target activities that would enable them to learn but also about those questions that kindle their logical thinking skills (Salah, 2009).

Therefore, the researcher believes that teachers must employ teaching methods to promote mathematical concepts in the teaching process, and there are common mistakes in learning mathematical concepts.

Also, the approaches to learning mathematical concepts are in line with the theory of Piaget-Brunner-Jagne.

Increasing interest in learning mathematical concepts as the basic building block in the educational ladder and the unity of building the subject matter. Some studies have indicated the importance of learning mathematical concepts (Andaerson & freebody,

1981; Nagy, 1988), (Fisher & Frey) highlighted the importance of vocabulary for mathematical achievement. while

(Fitzgreald & Graves, 2005) the study referred to the relationship between students' knowledge of vocabulary and their association with understanding

### 2.1.3 Developing Mathematical Concepts

The mathematical concept is defined as the general idea that underpins any equation, problem, or formula in the field of mathematics. The mathematical concept differs from the concept of mathematics fact which is defined as the syntax of the mathematical concept that is memorized to aid decision-making in solving the required equation, formula, or problem. It is contended that a student who acquires mathematical concepts proceed to an advanced level of learning through abstract thinking. Understanding mathematical concepts logically and appropriately minimize the need for memorizing mathematics facts. The concepts of mathematics should be first understood in terms of quality (such as shape, size, and other measurements) before moving onto quantitative attributes (Mulligan & Mitchelmore, 2009). However, to develop an innate number of senses and application of the same in mathematical concepts the students should learn to interact with their environment by exploring, manipulation, comparison, rearranging, and arranging sets of objects (Smith, 2006). Classification is a mathematical concept that involves discrimination, matching, and categorization according to attributes or attribute values. Moreover, it could be either qualitative or quantitative. The qualitative attributes in mathematics include shape and size while general number concepts represent the quantitative attributes.

The development of concepts of classification is based on discrimination or matching. Serial processing or ordering are mathematical concepts that demand logical reasoning. However, such reasoning could only develop if the concept of classification

is engraved in the respective individuals. On the other hand, conservation is a mathematical concept where the learner should recognize that how a given amount could remain same with different types of permutation and combination. Finally, a child should also learn to understand the spatial and positional concepts that are required to execute different mathematical operations (Mulligan & Mitchelmore, 2009; Smith, 2006).

### 2.1.4 Acquisition of Mathematical Concepts

Mulligan & Mitchelmore (2009) highlighted the importance of pattern and structure in the early development of mathematical concepts in school- going children. current education research has emphasized on the structured development of mathematical thinking in young students. It is speculative that early skills in algebra, multiplicative-reasoning, and spatial knowledge structure help to develop mathematical competence. Warren (2005) stated that virtually all concepts in mathematics are dependent on pattern and structure. The author further stated that the power of mathematics lies in relations and transformations that give rise to patterns and generalizations and abstracting the patterns based on structural knowledge which is the goal of mathematical learning (Warren, 2005, p.305). Mulligan & Mitchelmore (2009) reported that students belonging to second to the fifth grade who were considered "low achievers" exhibited poorly organized pictorial and iconic representations that lacked structure compared to their "high achievers" counterparts who implemented abstract notations with clear and well- developed structures from the beginning. The authors further stated that individuals who exhibit strong imagery and knowledge develop deep and conceptual understanding of the mathematical concepts.

For example, students exhibiting low numerical achievement focused on descriptive and idiosyncratic images because they concentrated on the non-

mathematical concepts and surface features of the visual cues that represented during teaching. On the contrary, visualization skills that are based on the identification of patterns and structures are significantly correlated with mathematical achievement.

The repeating patterns are considered important because they often recur in measurements (that involve the alteration of similar special units) and multiplication tasks (that involve the iteration of similar numerical units). In each pattern, there is a characteristic regular fashion. The way of organization of mathematical patterns is referred to as its structure. However, mathematical structures are often expressed in the form of generalization about the numerical, spatial, or logical relation that is always true for a certain domain. The awareness of grid patterns related to mathematical structures facilitates the learning of various mathematical concepts. For example, understanding of grid patterns helps to develop competence in division and multiplication functions. It is further contended that learning mathematical concepts through three dimensions ensure an improved sensorimotor experience that could be either used to enhance future skills in mathematics or for immediately relating them to a set of rules that are functionally interlinked with each other.

Various studies have either implicitly or explicitly explored the role of patterns and structures in developing mathematical knowledge in young children. The number of concepts and processes that are mostly studied include counting, subtilizing, numerations, and partitioning. Hunting (2003) conducted a study on partitioning that showed students could change their focus from counting unique and independent items to structural grouping-based counting that was fundamental for the development of number knowledge across them. Van-Nes (2008) also showed that spatial structuring exhibits a strong correlation in developing number sense across kindergarten students.

Mulligan & Vergnaud (2006) highlighted the importance of recognizing word structures and structural relationships based on equivalence, associatively, and inversion in solving functions such as addition and subtraction. On the other hand, studies conducted on multiplication and division functions have reflected that composite structure underpins the proficiency in multiplicative reasoning. Likewise, Mulligan & Mitchelmore (2009) also reported that the intuitive models that are deployed to solve common word problems did not impose too many challenges.

The selection of these schemas was deduced from the calculation procedures that the participants deployed while solving the respective problems. In another study, English (1999) explored the fundamental understanding of combinatorial problems across 10-year-old students. To recall, combinatorial problems is another multiplicative field where there is a requirement of knowledge of mathematical structures. Although most of the participants were able to solve the problems, they exhibit limitations in explaining the two-dimensional structure of the problem. Moreover, the study participants also failed to identify the cross-multiplication attributes.

On the other hand, various authors have explored the effectiveness of spatial structuring in developing mathematical concepts. Battista (1999, p. 418) defined spatial structuring as the mental operation of constructing an organization or the forms of an object or a set of objects. Such structuring determines the object's nature, shape, and composition by identifying its spatial components, relation, and the combination between these components, and establishing the interrelationship between the components of the new object." In one study, Outhred & Mitchelmore (2000) explored the development of spatial structuring through rectangular depictions and arrays across elementary school students.

The authors showed that most of the students were able to construct the row-

by-column structure of the geometrical forms while they are in Grade-4. Moreover, the respective students also exhibited proficiency in an equal-groups structure that is required for counting the rows and the layers in multiples. Studies involving early algebra reflect that young students could develop generalizations and abstract mathematical concepts on mathematical structures if they are provided appropriate opportunities (Carraher et al., 2006).

On the other hand, contemplating studies conducted by Blanton & Kaput (2005) highlighted that the concerned stakeholders could also develop competence on functional thinking if they are appropriate extended opportunities, mathematical modeling provides appropriate opportunities for students to develop their mathematical knowledge on the implementation of patterns and structure in problem-solving exercises.

Although different authors have independently reviewed the principles of pattern and mathematical structure in developing mathematical concepts, only a few studies have explored the effectiveness of integrated principles in developing mathematical concepts across students. On the contrary, studies on mathematical modeling tend to integrate various domains. The advantage of such learning frameworks involves the creation of explicit knowledge while their disadvantage involves the development of prototypes in students on the common structural understandings. Based on these speculations, Mulligan & Mitchelmore (2009) explored the feasibility of a general learning framework based on the awareness of mathematical patterns and structures (AMP) across a diverse range of concept domains that prompt early learning of mathematical functions. Different authors have highlighted the importance of structural theories in mathematical development. Piaget highlighted that different stages of cognitive development such as sensorimotor, pre-

operational, concrete, and formal operational stages play a significant role in developing mathematical concepts. The SOLO (Structure of Observed Learning) was developed based on Piaget's concept of cognitive development.

### 2.1.5 Learning Models for Mathematical Concepts

Different types of learning models are implemented for enhancing the understanding of mathematical concepts both at the beginner and advanced levels. The major learning models that are witnessed across any field of education are the Constructivism model, Connectivism or Cooperative model, Behaviorism Model, and Cognitivist model. In the "Constructivism model," the learner develops upon their personal experiences and remains active and social throughout the learning process. The constructivist model can engage learners more actively in the learning process. The constructivist model is based on the philosophy of improving teamwork, collaboration, scaffolding, peer-grading, and self-guidance in ensuring effective learning. In the "Connectivism model" or "Cooperative Model," the learner develops self-directed learning through different nodes such as content, source, individuals, and groups within their known boundaries. The Connectivism model of learning is based on the philosophy of self-directed quest, sharing of content and spontaneous learning through benchmarking. In the "Cognitivism model," the learning is primarily promoted by the short-term and long-term memory of the respective individuals.

The Cognitivism model is guided by the philosophy that visualizing tools and other aids that improve memorization skills improve learning in concerned stakeholders. In the Behaviorism Model, the learner remains primarily passive and learns through external processes such as positive reinforcement. The Behaviorism model of learning is based on the philosophy that the exhibition of certain behavior (routine drills and practice) improves learning skills (Laz & Shafei, 2014). However,

the authors reflected that the constructivist model is more pertinent for developing mathematical concepts in beginners.

Laz & Shafei (2014) highlighted that advancements in technology, communications, and information have radically revolutionized the field of teaching and learning. The authors speculated that the necessities, future challenges, and liberalization mandate greater attention to the fundamental knowledge of the theories that underpin mathematical concepts. The authors further stated that modern mathematics is based on the pillars of understanding and skill levels that translate into the competence of students. Moreover, the learning of mathematical concepts is the central line to convey information through diverse means easy to direct and understand. Hence, the study of mathematical concepts needs frequent adjustments in the theoretical frameworks as well as the strategic adjustments that improve the learning of such concepts.

The constructivist model of teaching and learning involve four phases; the phase of the call, the phase of exploration and innovation, the phase of proposal explanations and solutions, and the phase of the decision. During the first phase, the students are invited to learn in different ways. The respective individuals are asked certain questions amongst which some are thought-provoking in nature. However, the questions so asked should be framed according to their level of knowledge that was developed in the previous interaction. The second phase involves challenging the capability of the concerned stakeholders to search for the correct answers through observation, measurement, experiments, and teamwork. The third phase involves the reciprocation of findings and their interpretations. During the third phase, students are encouraged to clear their misconceptions through scientific and logical concepts. The final phase represents the decision point whereby the students learn to implement the learned

concepts through practical application. The constructivist learning model is beneficial because it helps the learner to focus on the educational processes by discovering and integrating with the teaching activities. The model also helps the learner to develop competence in debate and dialogue with their peers or mentors upon the conceptions and misconceptions. The constructivist learning approach in mathematics has received wide recognition owing to its compatibility with the understanding of the subject of mathematics. Mathematics is a unique subject where learning focuses on concepts and general rules that are linked with each other and in a tangible manner. As a result, the teaching models are predictive and follow a distinct prototype that helps to transfer the concepts of mathematics across a wide target population in a uniform manner.

### 2.1.6 Challenges of Mathematical Concepts

Mulligan & Mitchelmore (2009) highlighted the importance of mathematical representation systems in developing mathematical concepts. Thomas et al. (2002) suggested that children prefer pictorial, iconic, and symbolic representations for gaining competence in mathematical concepts. Godino & Batanero (1996) challenged the reductionist theory of conceptualizing mathematical concepts because such theory does not incorporate the facets of social and cultural aspects that confound the understanding of mathematical concepts. The researchers confirmed that the students should "understand" mathematics for developing proficiency and expertise in their mathematical skills. The authors further elaborated the need for teaching and learning mathematics through concepts that help the students to understand mathematics logically and rationally. Although the emphasis of “understanding mathematics” has been traditionally bestowed on institutional perspectives, the dominant psychological approach of understanding mathematics from the perspective of students is often neglected.

The psychological context of understanding mathematics was supported by the "cognitive revolution" theory of Vygotsky. Vygotsky emphasized that "the analytic and genetic priority of sociocultural factors when an attempt to understand individual psychological processes requires conceptualization of mathematical knowledge and their understanding." The authors mentioned the dilemma of discerning between the acts of understanding and processes while relating sound understanding of mathematical situations (as defined by the concept, theory, and problem) to the sequence of acts that are required to overcome the obstacles related to the situation.

Sieprinska (1994) stated that it is possible to identify meaningful acts for understanding complex mathematical concepts through historic-empirical approaches. On the other hand, weak and ineffective traditional teaching methods such as verbal prompting, conning, and verbal recall of counting and number skills make it even difficult for the concerned stakeholders to develop their competence on mathematical concepts.

As a result, the lack of comprehension of constructing and appraising mathematical concepts reduces the competence of the concerned stakeholders in mathematics. Although children with intellectual disability have compromised cognitive milestones compared to their healthy counterparts that make it difficult to develop mathematical concepts across them, Alnajdi (2001) stated that senses of an individual are like windows through which knowledge and information percolate within an individual which eventually helps to develop the concept.

Concept formation is thus driven by the sensorimotor experience of the learner Children with an intellectual disability find it difficult to acquire competence in mathematical concepts. The disability model could help to identify the cognitive skills that need to be sensitized or sharpened in healthy children for developing their

competence on mathematical concepts equally as in Yahia and Obeid (2005) studies Which confirmed by Khalifa (2006) endorsed the findings of bath researchers because acquisition of mathematical concepts is a mental function that needs the ability of a child to recognize the features of a concept first before developing abstract notions. In this regard, Saleh (2018) highlighted the importance of guided discovery as a method for enabling students with intellectual disability to develop a pre-academic mathematical concept in school-going children of KSA.

The major difficulty faced by the concerned stakeholders is in abstract and word problems. Moreover, these children also face difficulty in moving from one mathematical rule to another or correlate between the rules that they have learned. Therefore, the inability of children to recognize and grasp abstract concepts imposes limitations in developing mathematical concepts across the concerned stakeholders. These findings suggest that learners exhibit various challenges in acquiring mathematical concepts. The challenges are mostly intrinsic whereby the concerned individual exhibit poor semantic and judgmental skills in learning a mathematics concept. On the contrary, cognitive development and exposure to inappropriate learning methods also contribute to poor development of mathematical concepts in the concerned individuals. Hence, such learning tools and methods are desirable that would help to optimize the interface between teachers and students to improve the developmental of mathematical concepts in the respective students.

### 2.1.7 Studies related to Mathematical Concepts

Heindel (1998) contended that the Cognitivism model is more suitable for developing mathematical concepts in advanced learners. The author showed the relationship between student characteristics and mathematics test scores when the stakeholders were encouraged using spreadsheets. It was contended that spreadsheets

are valuable cognitive tools for 7th-grade math students that live in the development of mathematical concepts through active learning irrespective of the socioeconomic conditions of the students. While Mahryar (2003) study also developed innovative approaches to developing mathematical concepts on a group of high school students in Australia. The study used the experimental method using interviews, multimedia and internet in the promotion of concepts. The results of the study showed that students enjoy the participation of mathematics classes after the application of innovative methods by 73% and the high rate of academic achievement of students from the previous year.

Smith (2006) conducted a prospective study and included participants from two different elementary schools in the United States. The sample was intentionally biased by the authors to select the students with high proficiency. These individuals were made to learn mathematical concepts through standard-based teaching curriculum that emphasized on building mathematical concepts based on the multiplication properties those underpin multiplication fact and functions. The author implemented four different questions to the study participants that initiated with the conventional number sentences with which fourth-grade. The results indicated that the traditional group participants (fourth-grade students) provided 100% accurate answers when the multiplication function required small numbers, while they performed poorly in terms of reaction time for response for multiplication involving larger quantities.

(Mosley & Perry, 2009) Help to develop mathematical concepts for children before entering school and their ages (0-5) years. The study included 64 teachers. The study used the experimental approach to study the students' ability to learn mathematical concepts through playing and employing interviews with the presentation of sections of the study video in two categories of males and females. The study stressed

the existence of neglect in learning mathematical concepts in the early stages.

Mulligan & Mitchelmore (2009) explored the feasibility of SOLO in developing mathematical concepts on multiplication and division functions. It is reflected that AMP might act through two modalities; cognitive modality and meta-cognitive modality. On the other hand, spatial structuring is considered the key to such mathematical operations. They explored whether AMP could be widely applied across different student scenarios and mathematical operations. The study showed that PS competence reflects that the mathematical representations lacked evidence of numerical and spatial structures. During this phase, the numerical and spatial structures were appropriately and legibly represented by the study participants.

Laz & Shafei (2014) explored the role of constructivist learning models in teaching and concurrent development of mathematical concepts among students. The authors randomly allocated the participants into two experimental groups. One experimental group (n=44). The study showed that the mean marks in the statistical concepts test were significantly higher in the group that learned such concepts through the constructivist model compared to their counterparts who learned the same concepts through the traditional learning models (15 versus 26.5, $p<0.05$).

Garima & Narang (2016) elucidated the importance of computer-assisted instructions as a teaching tool in improving mathematical concepts at the secondary school level. Most often, CAI is used in combination with other teaching methods for enhancing the performance of students that require mathematical concepts. The authors acknowledged innovative teaching methods in the field of mathematical concepts. The result shows that the experimental group (n=50) get more score compared to their control counterparts (n=50) who are instructed through traditional teaching modalities (36.05 versus 33.6, $p<0.05$). The study was conducted across students of class-9

students studying in various secondary schools of a Tehsil (locality) in India.

Shaltout & Fatani (2017) explored the effectiveness two infographic types of teaching (interactive and static) in developing mathematical concepts among female second-grade students belonging to the schools of the Kingdom of Saudi Arabia (KSA). The authors undertook a quasi-experimental approach for evaluating the research questions that were undertaken in their study. The study uses animated infographics that were designed based on the technology help easy and practical accessible curriculum teaching (THEPACT) protocol. An ANOVA test was undertaken to report the findings of the study. Turkey's HSD outputs reflected that the experimental group that received teaching through the interactive and static infographic modes outperformed their comparators who received learning through the traditional mode in the mathematical achievement test. Therefore, the authors highlighted the necessity of teaching mathematical concepts through such teaching methods that improve the engagement and rationalize the thinking skills of the second-grade students.

Ibrahim (2017) explored the effectiveness of cooperative learning in developing mathematical concepts students presenting with mild intellectual disability (SPMID). The participants (n=8) consisted of mild intellectual disability and were from KSA. The participants were randomly assigned to two study groups. The students belonging to one group studied mathematical concepts through cooperative learning while their counterparts in the control group learned through conventional teaching. The photo- mathematical concept test (PMCT) was implemented to assess their mathematical concepts before and after implementation of the learning intervention. Ibrahim (2017) showed that participants belong to the cooperative learning group significantly outperformed their counterparts who received teaching on mathematical concepts through the traditional methods ($p<0.05$).

The guided discovery method is a teaching modality that depends on the activity of the learner based on the directions of a teacher to reach targeted educational goals. The method is based on a problem-solving approach and is highly effective in disseminating and percolating complex teaching and learning skills. In their study, Saleh (2018) explored the effectiveness of guided discovery method in developing pre-academic mathematical concepts in school going children (n=20) presenting with intellectual disability. The participants were randomly allocated into two groups. One group was destined to receive guided discovery sessions while the other group was not exposed to such sessions. During the guided discovery sessions, the teacher extended immediate reinforcements and continuous encouragement to the participants. The experimental group significantly outperformed the control group in the pre-academic mathematical ability test ($p<0.01$). However, the concepts that were presented through these sessions started from easy ones to the hard ones. The definition of "easy" referred to concepts that were immediately the existing knowledge of the participants before the study was initiated or with which one enters the academic curriculum.

### 2.1.8 Analysis of previous studies

After examining many of the previous studies, we find that the similarity between the models and the methods used to enhance the mathematical concepts of the students, where the structural model - Cognitive discovery - Cognitivist Model - use computers in education - two types of planning programs - Solo program. Also, studies use a variety of evaluation tools, including tests - personal interviews – questionnaires. The researcher benefited from previous studies through Firstly focus on the study of the concepts of multiplication among students. Secondly the application of the test tool. What has been added to the current study?

1. Using the Frayer model to impact the suitable terms of mathematical concepts.

2 - Application of the model in the early stage’s students of the third level

**2.2 Second Part (Frayer’s Model)**

Many different educational models have appeared in the classroom; some of these models include the Model of Education Course, Hilda Taba’s Model, Gagne's Model, Model of Structural Learning, Frayer’s Model, the Conceptual Change of Posner, Bybee’s Model and his colleagues and Bruner’s Model. The previous models proved their effectiveness in teaching the concepts to students. However, educationalists recommended the necessity of making the teachers use additional models and methods that provide the teacher with the mathematical knowledge and the need to reinforce the modern teaching methods.

Given what was discussed about the importance of mathematical concepts, different strategies, and the methods employed in teaching mathematics based on what is required from teaching, the requirements of the curricula, and challenges and difficulties faced when implementing the different strategies; educators have their work cut out for them.

Using diagrams, as in Frayer’s Model, can provide a beneficial tool to direct students through visualization of the relationships between the concepts and examples. Also, the graphical organizer provides each student with a written summary of what was learned. The application of Frayer’s Model was developed from research conducted by Frayer, Frederick, and Klausmeier at the University of Wisconsin in 1969. Frayer developed a model defined as Frayer’s Model that aims to more easily and effectively teach students new concepts. The design of this model was intended to test the level of concept mastery (Brooke, 2017). Appendix A shows a schema developed that can be used to teach unfamiliar concepts which entailed groups of information about the concept.

### 2.2.1 Distinguishing feature of Frayer's Model

The distinguishing feature of Frayer's Model from other models is in its graphic organizer. According to Monroe and Pendergrass (1997), the graphic organizer represents the working of the brain in arranging the information. The graphic organizer allows the student to give an overview of various parts of a concept. Such an overview makes it viable for students to develop new and unfamiliar concepts as well to think critically and clarify the relationship between concepts (Teacher Resource Guide, 2006). Frayer's model plays a significant role in raising the academic attainment of the students (Nahampun and Sibarani, 2014, Trask, 2011). The model ensures the student can analyze a concept, synthesize the concept, and finally apply the information acquired. In new mathematical concepts, Frayer's Model is a vital tool in ensuring the student grasp the meaning of a new concept and understands it. It is contended that concept development is the key to the understanding of mathematical concepts (Russell, Waters & Turnet, 2013).

Frayer's Model enables students to understand similarities and relationships between concepts through visual, graphical aid (Clark, 2007). The teacher can use this model to confirm the information, which is provided for the students as mathematical concepts (Macceca, 2007). It depends on analyzing the concept for its main characteristics and providing supportive examples for the concept and examples that are not applied to the concept and to facilitate the idea of the used model as described in Figure 1 on the following page:

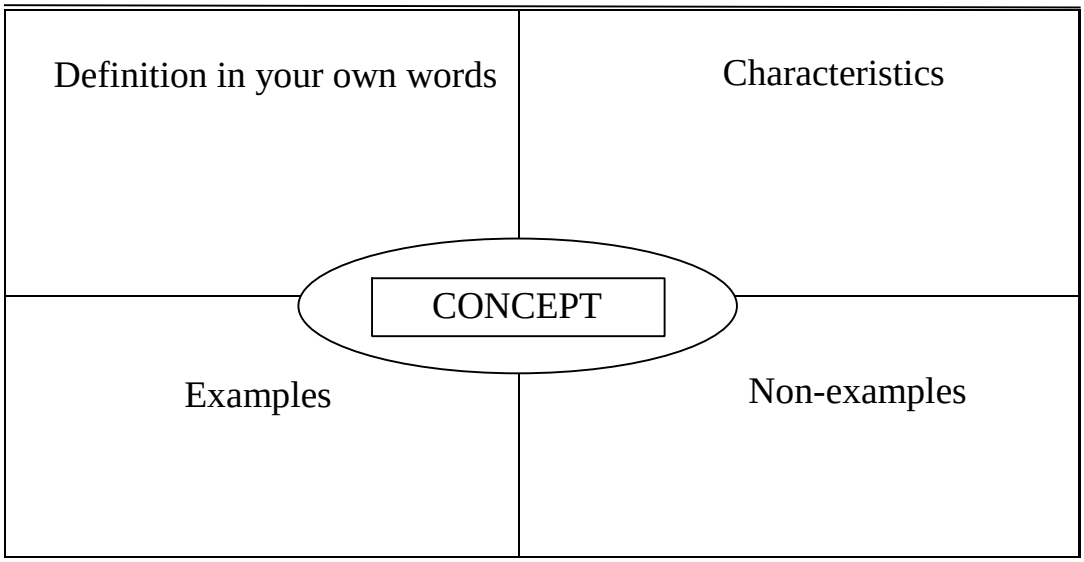



Figure 1. Frayer's Model

Some previous studies have illustrated the advantages of the Fryer model, which helps to think critically (Teacher resources guide 2006, Trask 2011). Building and understanding relationships and distinguishing between characteristics. It also helps to increase student achievement and the extent to which they acquire mathematical concepts (Nahampun & Sibarani 2014, Trask 2011). As well as increasing student motivation towards learning (Karjala, 2010). It is a successful tool for teaching confusing and abstract concepts and developing education (Ilter, 2015).

### 2.2.2 Characteristics of the Frayer's Model

The framework behind Frayer's Model is made up of a concept, its definition, characteristics of the concept, examples, and non-examples. This model enables students to have a greater understanding of a mathematical concept and the contexts in which the concept can and cannot be applied. It allows students to demonstrate their understanding and to construct meaning by providing examples and non-examples from the text or even from their own lives and experiences (Doty, Cameron & Barton, 2003). For this study, students in the experimental group will be exposed to this framework

and fill in each section according to the current lesson they are being taught, namely the basics of multiplication. This way they can easily write out their interpretation of the definition, the characteristics of multiplication as well as an example and a non-example. Thus, they will essentially be creating their study guide for learning multiplication they can refer to in the future as well.

### 2.2.3 Phases of the Frayer's Model

A graphic organizer, as used in Frayer's Model, ensures the student thinks about a concept in an organized manner. The first step is the definition, then characteristics, examples, and non-examples. According to the graphic organizer, the definition is written in the top left square, the characteristics are written in the top right square, the examples are written in the bottom left square, and the non-examples in the bottom right square. Definition of the concept should be developed by the student rather than obtained from a dictionary. Characteristics entail features or elements of the concept that are essential or that form the basis of the concept. Examples and non-examples ensure the students are thinking about the concept. Frayer's Model offers a thought process and structure that provides students with an opportunity to develop a deep understanding of concepts they are taught.

By focusing on the vocabulary that describes a complex concept that is difficult for the student to understand, they can use other concepts that they already know to better develop their understanding of both the new and old concepts at one time. The Frayer's Model ensures the student understands the concept which is essential in the learning process.

It is recommended that when introducing a concept, the teacher should ask questions that require individual thought and brainstorming such as questions like "what is a polygon?" or "what is a matrix?" (Marzano, 2013). It is of much importance

that all students take part in brainstorming and thinking up examples, which is in line with 21st-century pedagogical skills. Students are then required to provide important characteristics and examples based on the mathematical concept, and at the same time provide non-important characteristics and non-examples of the concept. Naturally, the teacher begins by modeling, that is using overhead transparency or by recording suggested concepts or example on the board (Marzano, 2013). Numerous questions are bound to emerge from the class if the teacher is encouraging.

Clark (2007) gives a procedure for using the Frayer's Model. According to Clark (2007), in implementing Frayer's Model in teaching, the teacher should first distribute copies of the graphic organizer. The student is then required to input the concept at the center of Frayer's Model graphic organizer. The concept may take the form of a phrase or a single word. The class is supposed then to define the concept to the teacher. Students use their textbooks and other resources to develop a definition that is clear, concise, and easy for them to understand. The teacher then helps the students to establish key features and characteristics of the concepts. Finally, as a class, the students should determine what constitutes the concept and what does not constitute the concept. The teacher should allow the class to give examples and have discussions with their classmates on the examples given. After the students are comfortable with this strategy, the teacher can then allow them to work individually, in pairs, or in groups to work on different mathematical concepts. Frayer and her colleagues originally outlined a seven-step procedure as follows (Greenwood, 2010):

i. Define the new concept, discriminating the attributes relevant to all instances of the concept
ii. Discriminate the relevant from irrelevant properties of the concept
iii. Provide an example of the concept

iv. Provide a non-example of the concept

v. Relate the concept to a subordinate concept

vi. Relate the concept to a superordinate concept

vii. Relate the concept to a coordinate term

Another procedure of the Frayer's Model can be applied by providing each student with the Frayer's Model student page. After each student has the student page, the teacher explains to the students that this method of teaching would enable them to understand the meaning of a concept. The teacher then asks the students to come up with their definition of a concept and put it down in the top left box of the Frayer's Model student page in their own words. Students are then instructed by the teacher to write down the characteristics of the concept in the top right box of the student page. Based on their experiences, the students should then work in pairs to think of examples and non-examples of the concept. The teacher asks the students to develop examples and non-examples from whatever they have learned previously and try to make a connection with the concept they are learning at the time. The students can then present their models as they explain to other groups. As various groups present their models to their classmates, the teacher is supposed to be informally assessing the students' understanding of the concept and clarify wherever is necessary. Another procedure for using the Frayer's Model can, therefore, be developed as follows (Urquhart & Frazee, 2012):

i. Assign the concept to be studied

ii. Explain all the attributes of Frayer's Model to be completed

iii. Model for students using the Frayer's Model with an easy concept that the students are familiar with

iv. Have students work in pairs and complete their model diagram using the assigned concept

v. Once the diagram is completed, have students have their work with other students

At its simplest, while teaching mathematical concepts, the teachers would point out sections of the student textbook where the concept is defined or applied. The teacher should provide illustrations using charts or objects to explain their concepts further. Direct teaching is considered the most efficient way of introducing a concept, but it is not an entirely effective method. By doing this, students do not feel the need to pay attention to what the teacher is saying. This is very common, especially if the routine of the explanations becomes too tedious and regular. To ensure the effectiveness of this teaching method, the presentation and explanation need to be varied and exciting. Heavy involvement by the students is also a requirement if this teaching method is to be effective. This way of teaching vocabulary should be interspersed with other approaches to developing mathematical concepts (Rusell, Waters & Turner, 2013). This method allows students being active and highly motivated, which is a key advantage it has over other processes. According to Roe and Smith (2012), active involvement is the best way for students to learn new concepts.

Frayer's Model ensures that students taught using this method have a better and deeper understanding of concepts they are taught and are not simply memorizing definitions of the concepts. According to Cohen and Cowen (2008), based on their results from employing the Frayer's Model, students have an increased understanding of new vocabulary and a more complex and deeper understanding of concepts. According to Greenwood (2010), following the Frayer's Model, the student follows a process of defining a word or a concept, gives characteristics of the concept, and

provides both examples and non-examples of the concept. This allows the student to develop a deeper understanding of the concept compared to if the concept was only defined to them.

### 2.2.4 Studies Related to Frayer's Model

Once a new model is created and implemented, a need arises in the educational process to understand the impact of acquiring the mathematical concepts of the students. Researchers all over the world have conducted studies by implementing the Frayer's Model at different levels of education aiming to evaluate the effectiveness of this model. Most of these studies have focused on the acquisition of mathematical concepts through vocabulary enhancement.

Monroe and Pendergrass (1997) explained that "the human brain naturally organizes information into categories determined by experience, illustrating the reason for the success of graphic organizers that demonstrate conceptual relationships" (p.4). According to Barton (1997), Brunn (2002), Gillis and MacDougall (2007) and Monroe (1997), "The Frayer model reveals relationships of similarity and difference between concepts, which has been shown to create deep connections and understandings that would be retained by students and retrieved for future learning experiences." Frayer's Model has been used in a different educational setting and is believed to be effective in the acquisition of concepts.

Monroe (1997)**,** conducted a study among fourth-grade students to evaluate the difference in impact between two models of vocabulary instruction- an integrated graphics organizer model (discussion model) and a definition-only model. The integrated graphics organizer model was a combination of Frayer's discussion model and a modified Concept of Definition graphic organizer. In the definition-only model, the students are to note, in writing, the definition of terms or concepts after an oral

review. The study involved an elementary school in a rural area. The study involved two classes of fourth graders as participants. The population of the area was primarily middle class and Caucasian. The rationale for picking the fourth graders, according to Monroe and Pendergrass, was that there is a vocabulary explosion at this level of study since students are beginning to read in the content areas of their syllabus. For the fourth grade, the teacher/researcher taught a Measurement unit consisting of ten lessons in the standard system, the metric system, area, and perimeter as the curriculum for this experiment.

The students were assessed using mathematical writing. Writing is considered a valid method of testing the understanding of concepts by students. The aim of mathematical writing was to assess the acquisition of concepts using the two models. Monroe and Pendergrass found out that the students taught using the integrated model (CD-Frayer model) showed better acquisition of mathematical concepts. A key implication of Monroe and Pendergrass's is that the use of CD-Frayer model in teaching mathematical vocabulary is effective.

(Alsamei, 2003) study the effectiveness of the use of the Frayer model in the acquisition of mathematical concepts and generalizations for students of the fourth level of primary in Yemen, where the study consisted of two groups, the experimental group of 80 students were taught using the model Frayer and in contrast the group included the fingerprint of 78 students used the usual method In the study, the results of the study proved to be superior to the experimental group that used the Fryer model during the testing of mathematical concepts.

The study was conducted by (Al-Wazzan, 2009) to investigate the effect of Frayer's model on the acquisition of mathematical concepts among primary school students. This study was applied to the fifth level students in Baghdad. The study

sample consisted of 69 students. The study sample consisted of two experimental groups, the results of the study showed that there were statistically significant differences between the average scores of the students in the two groups in the concept acquisition test for the experimental group.

Brooke (2017) conducted a study aimed at improving the self-efficacy of math learners using a direct and focused approach to vocabulary clarification. Brooke defines self-efficacy as "how students feel as math learners. It may affect their willingness to experiment with questions or attempt new scenarios, and their overall enjoyment of Mathematics courses" (Brooke, 2017, p.2). Brooke mentions that Mathematics is a language in and of itself as it contains vocabulary specific to the content and the use of unique symbols. It is therefore essential to teach mathematics through an approach that is focused on giving clear instructions to the students. This would enable vocabulary development, which in turn results in the acquisition of concepts. Brooke uses another approach that is modified—the Collaborative Four-Square Frayer Model (CFSF Model). The Collaborative Four-Square Frayer Model integrated sections of different models including the original Frayer Model, the Four-Square Strategy, and the Integrated CD-Frayer Model. This study also incorporated the use of technology in developing students' vocabulary. Google Docs© was used to display the CFSF Model. Using this technology offered different advantages. In general, it improved the effectiveness of the whole teaching model. The researcher also used the Vocabulary Instruction Implications for Teacher Practice guide. The study indicated a relationship between the acquisition of vocabulary and mathematical self-efficacy.

The Frayer model was not only applied to mathematics but proved to be effective in other subjects, for examples, Arabic subject (fandi, 2005) – (Alaa 2012). Science subject (Trask 2011 – (Hussein, 2014) – (Khadeeja, 2014) – (Estacio &

Martinez, 2017). English subject (Nahampum & Sibarani 2014), (Sullivan,2014) Social Studies (Ilter, 2012). The success of the model has been shown to enhance the acquisition of concepts by students in different subjects.

### 2.2.5 Analysis of the studies

There is a common theme in the four studies highlighted in the previous section to support the use of Frayer's Model. Three studies used or incorporate Frayer's model with other teaching models. Monroe and Pendergrass (1997) use a combination of Frayer's Model and a modified concept of a definition graphic organizer to form an integrated graphics model (also referred to as a discussion model or Integrated CD-Frayer Model).

Estacio and Martinez (2017) use a Modified Frayer Model which combined Frayer's Model with the 4 Pics One Word game. Brooke (2017) uses the Collaborative Four-Square Frayer Model which combines the original Frayer Model, the 4-Square strategy, and the integrated CD-Frayer model. Three studies used the original Frayer model Alsamei (2003), Al-Wazzan (2009). Since its inception by Frayer, Fredrick, and Klausmeier in 1969, Frayer's Model has been modified and implemented in numerous different ways. Apart from the modifications made to the original Frayer model as mentioned earlier, Frayer's Model can be used in any level of study. Sullivan uses the model among university students; three studies use the model among fourth-grade students, while Estacio and Martinez use the model among high school students.

Major emphasis is placed on vocabulary development in all four studies. According to the previous studies, to acquire concepts, whether mathematical or scientific, the key is in understanding the vocabulary used by the concept. Focusing on vocabulary development is in line with the main features of Frayer's Model. Vocabulary under a concept can be easily and graphically organized by providing a

definition, characteristics, non-characteristics, examples, and non-examples of all terms and concepts. Based on the study's findings, vocabulary development guarantees the acquisition of concepts, especially mathematical concepts.

There is a key difference between previous and current studies on Frayer's Model: the Monroe and Pendergrass's study can be considered as a previous study since it was conducted in 1997 while the studies by Brooke and Estacio and Martinez can be considered as current studies since they were conducted in 2017. Current studies on Frayer's Model have incorporated technology in their models. Estacio and Martinez incorporate the 4 Pics One Word game application in their teaching model. Brooke incorporates Google Docs© in his study which allowed for both direct and indirect means of instructing students. The aim of incorporating technology in both studies is to improve the effectiveness of Frayer's Model.

### 2.2.6 Implications of the Literature Review in Designing the Present Study

Emphasis on vocabulary development is placed in previous studies. According to the studies, to acquire concepts, whether mathematical or scientific, the key is in understanding the vocabulary used in the concept. The focus on vocabulary development is in line with the main features of Frayer's Model. Vocabulary under a concept can be graphically organized by providing a definition, characteristics, non-characteristics, examples, and non-examples of the vocabulary. Based on the study's findings, vocabulary development guarantees the acquisition of concepts, especially mathematical concepts.

There is a key difference between previous and current studies on Frayer's Model. The Monroe and Pendergrass's study can be considered as a previous study since it was conducted in 1997 then the studies by Alsamei, Alwazzan. While Brooke can be considered as the current study since they were conducted in 2017.

Distinguishes this study from other studies:

1-first study conducted at the third level of the primary stage.

2- Teaching numbers and operations in mathematics and especially the multiplication unit as one of the basic skills at this stage

3-Employed technology and integrated into the lesson plans.

4-Work on teachers training by providing techniques and supportive educational resources to achieve the goal of the study.

### 2.2.7 Setback to using Frayer's Model

Despite the numerous advantages of using Frayer's Model in learning environments, it has one major setback. According to Greenwood (2002), Frayer's Model was

> "The most time consuming and labor-intensive model. Teachers must be purposeful when selecting the concept that would be developed using this model; the Frayer's Model should be reserved for only the most challenging and conceptually hard to understand concepts (p.261)."

Thus, teachers should be careful in the number of times they use Frayer's Model to explain concepts. Several factors could lead to the model being ineffective including students losing interest, difficulty managing time, and information overload. Therefore, "To be an effective tool in creating an understanding of concepts, completing the Frayer model with students should include both oral discussion and written information components" (Monroe & Pendergrass, 1997). So, as it has been previously stated, "the time is taken to complete [Frayer's model] was overshadowed by the positive retention of concept knowledge the students demonstrated after its use" (Brooke, 2017).

# CHAPTER 3: RESEARCH METHODOLOGY

## 3.1 Research design

The research design refers to the procedures and methods used in a research study to answer the research question (Priviera, 2014). For this study, a quasi-experimental approach design was used. Which is appropriate in studies when entire groups of participants are used in an experiment rather than assigning participants at random to experiments treatments (Wiersma & Jurs, 2005).

In this study, whole classes were used as they were without random assignment of participants. As highlighted by Goodwin (2005), such a study aims to evaluate the effectiveness of a proposed solution to a problem, specifically to evaluate the impact of the Frayer Model in the acquisition of mathematical concepts. A quasi-experimental research design is also appropriate when comparisons need to be made before and after the implementation of a solution in both the control and experimental groups (Privitera, 2014). The choice of this research design was also informed by other similar studies. Other researchers who have researched the effectiveness of teaching models in the acquisition of mathematical concepts through instructions have used this research design (Sanders, 2007; Wolf, 2013; Iwankovitsch, 2013). The notational paradigm of the design can be summarized as shown below:

| | | | |
|---|---|---|---|
| Experimental Group | $O_1$ | X | $O_3$ |
| Control Group | $O_2$ | (X) | $O_4$ |

**Key:** $O_1$ and $O_2$ represent the pre-test observations; X represents the solution; (X) shows no solution applied. $O_3$ and $O_4$ represent post-test observations for the experimental and control groups respectively. The dashed line separating the parallel rows indicates that the experimental and control groups have not been equated by randomization.

Source: Cohen, Manion and Morrison (2011, p. 323).

This study made an effort to include groups that were as equivalent as possible to reduce the threats posed by internal validity which also allowed for the minimization of possible effects from the reactive arrangements (Gay, Mills and Airasian, 2009).

**3.2 Research methodology:**

This research was quantitative research. The rationale behind choosing this methodology was because this study aimed to establish the relationships between a specific Mathematics teaching strategy and the students' performance in Mathematics, specifically the acquisition of mathematical concepts (Cohen, Manion & Morrison, 2011). Moreover, null hypotheses guided this study to determine relationships between variables. A quantitative methodology was chosen for this study because the study used an achievement test to measure the performance of students as a result of the instruction. The research was also deductive to generalize the findings to a larger population (Mugenda, 2003). The study used achievement tests to collect quantitative data.

The researcher followed the procedures of experimental research ,which included choosing a suitable experimental design of the mathematical concepts to reach the desired results of the. The researcher adopted a design for the experimental and control groups based on the following schedule:

Table 1. Type of Variable

| The Group | The Independent Variable | The Dependent Variable |
|---|---|---|
| Experimental | Using Frayer's Model | The Acquisition of Mathematical Concept |
| Control | Using Definition-only method | |

### 3.3 Research context / setting

#### a. Variables

Acquisition of mathematical concepts were the key dependent variables for this study.

Also, Independent variables are represented in the methods used. The Frayer model was the primary teaching strategy used for this study for the experimental group while the traditional method was used for the control group. More specifically, the following were considered the main variables for this study.

Control variables

Performance in Mathematics is as a result of various factors which include school ethos, resources such as personnel and teaching aids, the attitude of both teachers and students towards mathematics, mathematical vocabulary, Arabic Language proficiency, and mathematical language. This study aimed to determine the effectiveness of Frayer's Model in the acquisition of mathematical concepts. Therefore, certain factors were controlled including the mathematics textbooks and teacher's qualifications. To control these factors, the study only involved teachers who were qualified in mathematics education.

#### b. Participants

The study was conducted in two primary schools. The first was Omar Bin Al Khattab Primary School for boys located in Al Wa'ab area. The second was Khadija bint Khuwailid Primary School for girls located in Al Mamoura area. The selected primary schools were purposively and were based on the researcher's work as a standard specialist in the Ministry of Education. The two schools were also chosen because both schools meet the applicable requirements of the study. Both schools are government schools and administer students of the same age. Both schools also apply the same

curricula for mathematics.

### c. Sampling techniques

Sampling refers to the process of selecting several people in a study such that those selected represent a larger group of people (Mugenda & Mugenda, 2003). Sampling aims to make sure that the researcher gathers information about a population. For this study, some samples were drawn from the target population. The target population was third-grade students in primary schools in Qatar. Different sampling techniques were employed to select a sample from each.

A simple random sampling technique was used to select the four classes of third-grade students involved in the study. Two classes were randomly selected from the boys' school, and similarly, two classes were also randomly selected from the girls' school. Each of the four classes provided 25 students for the study, giving a total of 100 students between both control and experimental groups. As mentioned in Chapter 1, this research targeted third-grade students in Qatar primary students because, according to the national results of students in Mathematics for the 3$^{rd}$ grade in the study year 2017-2018, a significant percentage of the students failed. The Qatar government wants to test the students at an earlier stage so that they are better prepared and have a good mathematical foundation in the form of acquisition of mathematical concepts.

Following the above sampling techniques, a sample size of 100 third grade students from the two primary schools was chosen for the study. The current community consists of the primary schools in Qatar during the academic year 2017—2018.

The current research required choosing two schools: one boy and one girl's primary school. Whole classes were used as they were without random assignment of the sample size. This aimed at avoiding distractions in the school while conducting the study.

One hundred students of the 3rd grade were determined from four academic classes. Classes A and C were selected randomly as an experimental group to teach the mathematical concepts by using Frayer's Model. While Classes B and D were chosen as a control group to use the definition-only method. As follows in the schedule:

Table 2. Number of students

| Group | Classes | Number of Students |
|---|---|---|
| Experimental school | (A) from Boys school | 25 |
| | (C) from Girls school | 25 |
| Control school | (B) from Boys school | 25 |
| | (D) from Girls school | 25 |

### 3.4 Data generation methods

The researcher prepared the study tools which were as follows:

#### 3.4.1 Choice of methods

*a.* The Scientific Material

An academic teaching plan for the multiplication unit from the mathematics book for the first term 2018 – 2019 was developed (Appendix B). This is because many mathematical concepts are covered under the multiplication unit. Frayer's Model is considered one of the academic models that focuses on the main characteristics of the concept and providing supportive examples. A list of forms of multiplication concepts was determined, including multiplication by equivalent groups, multiplication by repeated addition, multiplication by array, multiplication using the number line, multiplication using patterns and multiplies, distribution property, multiplication properties, missing number in multiplication, and problem-solving related to

multiplication.

The rationale for choosing the multiplication unit:

The rationale for choosing the multiplication unit for this study was based on the idea that the multiplication unit promotes awareness of structure and early development in the acquisition of mathematical concepts (Mulligan & Mitchelmore, 2009). Mathematical units including multiplication unit, algebra, and spatial reasoning improve the structural development in the mathematical thinking of young students; specifically, from the second grade to the fifth grade. According to Mulligan & Mitchelmore (2009, p.305), all mathematical concepts are based on pattern and structure. According to Warren (2005), "The power of Mathematics lies in relations and transformations which give rise to patterns and generalizations. Abstracting patterns is the basis of structural knowledge, the goal of Mathematics learning." Research has been conducted to examine how young students represent different mathematical situations in structural characteristics. The findings of such studies show that students who perform poorly in Mathematics gave pictorial and iconic representations that were organized poorly and lacking in structure. However, in the case of those performed well, the structure of their mathematical representations was well developed (Mulligan & Mitchelmore, 2009). Other research based on Mathematics education has also found out that students who appreciate the structure of mathematical representations and processes are more likely to acquire a deep understanding of mathematical concepts (Pitta-Pantazzi, Gray & Christou, 2004; Gray, Pitta & Tall, 2000).

Mulligan & Mitchelmore (2009, p.34) defines a mathematical pattern as "any predictable regularity, usually involving numerical, spatial or logical relationships." In any mathematical pattern, the elements are organized regularly (Mulligan &

Mitchelmore, 2009). The organization of a mathematical pattern is referred to its structure. Multiplication involves the iteration of identical numerical units. Repeating patterns are crucial in the acquisition of mathematical concepts in a student's early stage of development from the second grade to the fifth grade.

A mathematical structure is expressed in the form of a generalization, either numerical, spatial or logical relationship. Figure 2 can be considered in illustrating mathematical pattern and structure in Mathematics education among young students. The figure shows a rectangle divided into squares. Adults can identify the pattern as a 3 × 5 square pattern; however, for young students, this pattern is difficult to identify (Outhred & Mitchelmore, 2000). The young students are unable to perceive the implicit structure of the figure which is three rows of five equally sized squares or five columns of three with their sides aligned vertically and horizontally. Repetition is a key feature in the structure of the figure. Awareness of such grid patterns and the structure can help young learners acquire mathematical concepts. For example, counting the squares as composite units leads to skipping counting (e.g., 5, 10, 15 by five and 3, 6, 9, 12, 15 by threes) and therefore to multiplication as a binary operation (e.g., three times 5).

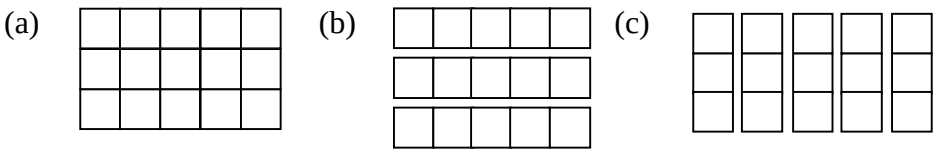



Figure 2.Rectangular grid perceived as (a) 3 x 5, (b) 3 rows of 5, (c) 5 columns of 3.

Mulligan & Mitchelmore (2009) researched grade 1 students to examine the conceptual understanding of mathematical concepts instead of procedural understanding. The study by Mulligan and Mitchelmore was focused on testing the

structural development of students' responses. Multiplicative reasoning was found to be very important for Mulligan and Mitchelmore's study. Tasks were developed such that is students would be able to identify, visualize, represent, or replicate elements of pattern and structure.

Acquisition of mathematical concepts is based on identifying patterns in mathematical tasks. The multiplication unit of the Mathematics syllabus is based on patterns which can be represented graphically. The multiplication unit was therefore perfect for this study. With using Frayer's Model's graphic organizer for this study, it is easier to incorporate the multiplication unit. Since the multiplication unit is based on patterns, mathematical concepts under this unit can be easily represented graphically.

*b. Framework for the lesson plan:*

The framework for the lesson plan (shown in Appendix B) used in this study was based on Nancy Frey and Douglas Fisher's "Gradual Release of Responsibility Instructional Framework" (2013). According to Frey and Fisher (2013, p.1), "the gradual release of responsibility model of instruction suggests that cognitive work should shift slowly and intentionally from teacher modeling to joint responsibility between teachers and students, to independent practice and application by the learner." The lesson plan for this study is therefore based on the gradual release of responsibility model.

The gradual release of responsibility model is designed such that all responsibilities of a mathematical task is shifted from the teacher to the student. This model is built on several theories:

i. Jean Piaget's work on cognitive structures and schema (1952)
ii. Lev Vygotsky's work on zones of proximal development (1962, 1978)
iii. Albert Bandura's work on attention, retention, reproduction, and motivation (1965)

iv. David Wood, Jerome Bruner, and Gail Ross's work on scaffolded instruction (1976)

Based on these theories, they all propose that interactions with others lead to learning, and when these interactions are intentional, specific learning occurs. The gradual release of responsibility instructional framework developed by Frey and Fisher has four primary components. These four components include focus lessons, guided instructions, productive group work, and independent learning.

In **focus lessons**, the teacher determines the objective of the lesson and comes up with a model based on what he/she thinks of the lesson. The objectives set for the lesson should focus on the outcomes expected from the lesson. The model developed by the teacher should aim at providing the students with examples of the language and thinking needed for active learning. This is in line with Frayer's Model that would be used in this study to teach mathematical concepts (Frey and Fisher, 2013).

Under **guided instructions**, the second key components of the framework developed by Frey and Fisher is that the teacher strategically uses questions, cues, and prompts to help the students understand the concept being taught. This can be conducted with the whole class. However, Frey and Fisher recommend using smaller groups of students to improve the effectiveness of the developed lesson plan. The small groups can be created based on the instructional needs of the students. During guided instruction, the teacher provides the students with instructional scaffolds. This enables the teacher to release responsibility to the students. The instructional scaffolds ensure the students are successful in the tasks assigned to them (Frey and Fisher, 2013).

**Underproductive group work**, the students work together in groups to come up with something related to the topic that is being taught. For the group work to be productive, the students should use academic language, and they should also be able to

account for their role in the group effort. This part of the lesson plan should allow students with a chance to put their understanding of the topic together before applying their understanding independently (Frey and Fisher, 2013).

The final part of the lesson plan focuses on independent learning. Under this part, the students are required to apply whatever they have learned in the class, together with whatever they have learned outside of the class. This can be done by assessing the students to evaluate the understanding of the students' understanding of the unit taught. The teacher then identifies the need to reteach the class based on the results of the assessment. It is important that the assessment of students' understanding does not come too soon in the instructional cycle. This is because the students must practice whatever they have learned before they can apply the knowledge acquired in new situations.

The framework for the teaching plan proposed by Frey and Fisher can be used in any order when developing a lesson plan (Frey and Fisher, 2013). However, Frey and Fisher insist that all four elements must be used. The lesson plan developed for this study, shown in Appendix B, followed the order provided by Frey and Fisher in their gradual release of responsibility instructional framework; focus lessons, guided instructions, productive group work, and independent learning.

C. Training of participating teachers

The teachers, one for each group (control and experimental), took part in a training session in their respective schools. The training sessions were conducted during the teachers' free time and after school. The sessions lasted for about an hour. The sessions emphasized the importance of Mathematics vocabulary in improving the performance of students in Mathematics. Project materials were handed out to the teachers with an explanation regarding each material. The materials included the Mathematics concepts

test, felt pens, lesson plan templates, manila papers, and consent forms. A detailed working schedule was also presented to the teachers during the training sessions. The detailed working schedule aimed to ensure uniformity in time in teaching the concepts and giving the tests. The teachers also received information on how to use the lesson plan prepared by the researcher, and how to use graphic organizers in teaching mathematical concepts.

### 3.4.2 Methods

***a.*** **Multiplication concept test:**

Examinations are the most used and the most convenient method in the assessment of the results of learning. However, using multiple choice exams makes the student get used to choosing the correct answer (Carson and Ruth, 1991, p.370). To prevent this, the exam design used here consists of essay questions, one question for each of the multiplication concepts that were taught (shown in Appendix C), and the exam includes all multiplication concepts, which were taught in the academic unit. The mathematical concept test that was used to assess the students was adopted and modified from other international tests that have been undertaken by other third grade students. The four international tests that were sampled include the Math Mammoth Grade 3- A Worktext South African Version by Maria Miller, the Math Mammoth End of the Year Test-Grade 3, the California Standard Test for Grade 3, and the Conceptual Understanding Mini-Assessment by Students Achievement Partners.

To confirm the veracity of the exam, a list of mathematical concepts to be covered and including the educational goals, was submitted to a group of judges who are specialists in the Mathematics Department in the Ministry of Education and Higher Education with the aim of getting their approval regarding the extent of the exam to cover the concepts. The exam was applied to the experimental and control group that

consisted of 100 students.

Based on the suggestions provided by the specialists in the Mathematics Department in the Ministry of Education and Higher Education, the exam consisted of only essay questions. Essay questions provide a better opportunity for the teacher to evaluate the students' understanding of the content taught. Multiple choice questions only test the ability of the students to recall facts or information, which is disadvantageous.

In assessing the acquisition of mathematical concepts, essay questions are more effective. According to Barbara Davis (1993, p.272), "Essay tests let students display their overall understanding of a topic and demonstrate their ability to think critically, organize their thoughts, and be creative and original." While essay and short answer questions are easier to design than multiple choice tests, they are more difficult and time-consuming to score. Moreover, essay tests can suffer from unreliable grading; that is, grades on the same response may vary from reader to reader or from time to time by the same reader. For this reason, some faculty prefers short-answer items to essay tests.

On the other hand, essay tests are the best measure of students' skills in higher-order thinking and written expression." Essay tests are appropriate when aiming to analyze, synthesize, or evaluate students' acquisition of concepts that have been taught. In the design of essay questions, there is a need to be specific, and the words used in formulating the questions should give the student hints on what the test or examiner requires. Time is also a crucial aspect of assessments with essay questions.

The essay questions in the mathematics concept test were based on multiplication concepts that were taught in the class. Appendix C also shows the answers to the questions that were attempted by the students. Since it was a mathematics test, the answers were straight-forward and grading the exams was not challenging.

The data was prepared before it was analyzed using the Statistical Package for Social Sciences (SPSS) computer software. The data were first edited and coded for use in the software. A computer code sheet was developed from a codebook. The computer code sheet was later used in the synthesis of the data. After entry of the data, it was cleaned with the aim of detecting and removing any errors that may have taken place during data entry. The data was cleaned by running simple frequency analysis on the variables and through random cross-tabulation.

The data was primarily analyzed using T- TESTS in order to compare means between the groups. In this study, a chi-square test has also been conducted to evaluate the relationship between the students' performance in the test (which is an indication of acquisition of the mathematical concepts taught) and the way used by the students to solve the problem. A chi-square test is also conducted to evaluate the relationship between the students' performance in the test and the way used by the students to write distribution. In SPSS, the chi-square option is used on the statistics subcommand of the crosstabs command to obtain the test statistic and its associated p-value. The two relationships have also been evaluated using the Kruskal-Wallis H test.

### 3.4.3 Procedure

Step 1: Preparation of educational materials for research

It was necessary first to analyze the unit plan that would guide the teachers in teaching the multiplication unit. The theoretical framework was used to analyze the contents of the curriculum. The unit plan was then prepared by the researcher and reviewed by specialists from the Department of Early Childhood Development. The lesson plan included different aspects of the multiplication unit such as facts and ideas, terminologies and vocabulary, multiplication concepts, values and trends, skills, drawings, pictures and illustrations, and questions. The prepared unit plan has been

attached in Appendix B.

Second, the lesson plan was developed. The lesson plan was designed based on the gradual release of responsibility instructional framework developed by Frey and Fisher and is shown in Appendix D. The lesson plan was then presented to a group of arbitrators to express their opinion on a few elements of the lesson plan. The arbitrators checked on the suitability of the content and the relevance of the objectives set in the lesson plan. The arbitrators also checked on the suitability of the lesson plan and its relevance to the curriculum in the State of Qatar. The lesson plans were presented to a group of specialists from the Ministry of Early Education to adopt the plan in terms of (Fit the lesson plan to the age stage- Contains all required items, and the possibility of adding what they find appropriate. to modify plans in line with the Frayer model. The opinions of the arbitrators were also sought on the suitability of the evaluation techniques to determine the achievement of goals of the lesson. Another element that was advised upon by the arbitrators was the adaptation of teaching techniques such that they are in line with the content and goals of the lesson. The researcher implemented the proposals made by the arbitrators in the lesson plan. Educational related activities were also developed based on the students' interests. A diverse amount of activities was developed including both individual and group work activities.

Equivalent Groups

The researcher was careful to adjust the variables that may affect the experiment and affect the accuracy of the results. The equivalence between the two groups of research was carried out in a variable (Previous information on mathematical concepts). Where the researcher used the results of the students in the pretest of the multiplication unit within the Mathematics sourcebook for grade third (Appendix).

The T-Test was used for two independent samples, to compare the mean scores of the

two groups (experimental and control). The equivalence process showed that the two groups were equal in the mean scores of the pretest.

The following is an explanation of statistical equivalence in the previous variables between the two research groups:

Table 3. Statistical equivalence in the previous variables between the research groups

| Group | Mean | N | Std. Deviation | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|
| Experimental | 25.4700 | 50 | 3.83148 | -0.428 | 49 | 0.671 |
| Control | 25.8300 | 50 | 3.89218 | | | |

Table 3.1 reflected that there was no significant difference in the mean scores between the experimental and the control groups that were considered for the study (25.4 versus 25.8, $p>0.05$). Hence, it could be concluded that the academic knowledge and cognitive abilities of the study participants regarding the mathematical concepts were comparable for the participants belonging to the experimental and control groups. Therefore, the present study ensured that the findings of it were reliable and reproducible.

Step 2: Preparation of research tools

This involved the planning and preparation of the Mathematics concept test. First, the aim of the test was determined. The Mathematics concept test aimed to access the effectiveness of the Frayer model in the acquisition of mathematical concepts. The next activity was based on the specification table on the dimensions of selected unit subjects as dimensions of the test. The researcher then decided on the type of test and the formulation of the questions for the test. The researcher decided to use essay

questions instead of the multiple-choice questions. This was aimed at measuring the students had truly acquired the multiplication concepts that had been taught during the study. It was aimed at avoiding situations whereby a student chooses the right answer (in multiple choice questions), but he/she has not truly acquired the mathematical concept being assessed. In the formulation of the test questions, the examiner/researcher considered that all questions asked in the Mathematics test concept were covered under the multiplication unit that had been taught. The researcher ensured the language was clear and easy for the students to comprehend. The researcher also ensured that the nature of the questions was diverse. The test instructions were also formulated. The test instructions were formulated using a clear language and determining the target of the test. The time and duration of the test were also determined at this stage. The correction key for the test was determined which targeted providing the teachers and assisters with help to know the mechanism of correction and the difference in grades for each student.

The next phase of this second stage involved verifying the validity of the test that had been developed and to find the stability coefficient of the test. In verifying the stability of the test, the mathematics concept test was viewed by a group of arbitrators and specialists in the field of mathematics. Some of the specialists were mathematics teachers in primary schools. The stability of the test was verified to ensure that the questions of the test were based on third-grade mathematical concepts as indicated in the curriculum content. Also, it was to ensure the adequacy of the questions and the clarity of the instructions. The group of arbitrators was to suggest addition or removal of appropriate and necessary changes. This phase also involved finding the stability coefficient of the test. This aimed to find the time for the test. The test time was determined by finding the average between the time taken by the first student and the last student to complete the test questions. The appropriate time for the test was 45

minutes. The researcher used the SPSS program to find the stability of the test.

Step 3: Selection of the research group and the identification of variables

Teaching strategies for the acquisition of mathematical concepts were the key independent variables for this study. This involved the use of the lesson plans based on the Frayer Model in the teaching of the multiplication unit for the grade three students. Mathematical test scores constituted the dependent variables for this study. The scores gauged the acquisition of mathematical concepts taught. The Frayer model was the primary teaching strategy used for this study for the experimental group while the definition-only strategy was used for the control group.

During the teaching of the multiplication unit to the students, two teachers were chosen to teach for the experimental and control groups. The teachers were considered to have the same qualifications and the same years of experience. The teachers taught the first class using the definition-only method while the second class was taught using the Frayer model.

Step 4: Conducting a research experiment

The Mathematics teachers from both schools first underwent training on how to employ the models in enhancing the acquisition of the mathematical concepts among the students, including the Frayer's teaching model. The training program was conducted on 25$^{th}$ September 2018. The coordinators of the training workshop were instructed to measure the impact of the training workshop for teachers. Presentation of the workshop was done until the 9$^{th\ of}$ October 2018. Before using Frayer's Model, all participating students in both control and experimental groups were given a pretest to gauge their mean scores for comparison with the posttest after using the new technique.

The second phase of this step involved implantation of the lesson plan. The lesson plan for this study was implemented during the first semester of the academic

year 2018-2019. The teachers also committed themselves to the study by implementing the lesson plans handed to them and being under the supervision of the researcher. The implementation of the lesson plan took around two weeks, from 11th November 2018 to 23rd November 2018.

Finally, the posttest, like a measuring instrument, was administered to the students. The mathematical concept test was undertaken by both the control and experimental groups of third-grade students. The correlation model was used to prepare for the statistical processing of the results, analysis, and interpretation. This was carried out on 29th November 2018.

**3.5 Data analysis**

a. Validity

Content and construct validity of the research tools were assessed during the design stage. Some of the items used in the study were adapted from previous studies. The mathematical concept test that was used to assess the students was adopted and modified from other international tests that have been undertaken by other third grade students. The four international tests that were sampled include the Math Mammoth Grade 3- A Worktext South African Version by Maria Miller, the Math Mammoth End of the Year Test- Grade 3, the California Standard Test for Grade 3, and the Conceptual Understanding Mini-Assessment by Students Achievement Partners. This strengthened the content and validity of this study.

Also, the mathematical concept test was also reviewed by arbitrators, specialists, and teachers in the field of Mathematics. This was to ensure the test was based on mathematical concepts for third-grade students and that the instructions were clear. This group of people offered supportive proposals, as well as appropriate and necessary amendments for the test that was later issued to the students.

b. Instrument Reliability:

Kuder-Richardson Formula 20

To determine the stability of the test was measured through the computation of Kuder-Richardson Formula 20 (KR20) (Brunning & Kintz, 1996)

According to Cortina (1993),

$KR20 = \left(\frac{N}{N-1}\right)\left(\frac{\partial 2 - \sum p(1-p)}{\partial 2X}\right)$, Where N =Total Number of students;

P = Difficulty, index; $\partial$ Stander deviation and X= Scores mean.

Table 4. Reliability Test

| | marks | Question | x% | Sd | Sd^2 (variance) | Difficulty indix (p) | 1-p | p*(1-p) | KR20 |
|---|---|---|---|---|---|---|---|---|---|
| Equivalent group | 4 | Q1 | 0.915 | 0.557326 | 0.310612 | 0.92 | 0.09 | 0.08 | 0.780 |
| repeated Addition | 4 | Q2 | 0.935 | 0.527218 | 0.277959 | 0.94 | 0.06 | 0.06 | 0.763 |
| Array | 4 | Q3 | 0.955 | 0.437526 | 0.191429 | 0.96 | 0.05 | 0.04 | 0.747 |
| Number line | 2 | Q4 | 0.88 | 0.476381 | 0.226939 | 0.88 | 0.12 | 0.11 | 0.811 |
| patterns and factors | 4 | Q5 | 0.89 | 0.611455 | 0.373878 | 0.89 | 0.11 | 0.10 | 0.802 |
| Distribution property | 2 | Q6 | 0.86 | 0.496518 | 0.246531 | 0.86 | 0.14 | 0.12 | 0.830 |
| Multiplication properties | 4 | Q7 | 0.91 | 0.597956 | 0.357551 | 0.91 | 0.09 | 0.08 | 0.784 |
| Missing number | 2 | Q8 | 0.96 | 0.274048 | 0.075102 | 0.96 | 0.04 | 0.04 | 0.743 |
| Problem Solving | 4 | Q9 | 0.92 | 0.62073 | 0.385306 | 0.92 | 0.08 | 0.07 | 0.776 |
| | | | | | | | | 0.699325 | **0.782** |

When measuring stability, the result was positive with the stability coefficient (0.782). KR-20 associated with the items. This result be acceptable and reassuring to a partial sample, as explained by Clark and Watson (1995), the alpha value is acceptable if it is greater than (0.60). Considering The scale in the sample results has the internal consistency of its terms. It shows a strong level of stability and reliability of the measuring instrument.

### 3.6 Logistical and ethical considerations:

Permission was sought to research the Qatar University Institutional Review Board. Approval to conduct the research was given by the board. The heads of the two schools were also informed of the research, and their permission sought before involving the teachers and students. The researcher also sought consent from the respondents who were taking part in the study. The participants in the study were informed that the information they gave in the study would be treated as confidential information and would only be used for the study.

Measures were put in place to make sure that both the control and experimental groups were not disadvantaged. The study was scheduled such that it did not interfere with the school's normal programs. The test was also carried out after the classes, and the teachers in the two schools agreed to expose the control group to the solution (the Frayer Model used to teach the experimental group) in a later time after the study was completed. Thus, this was aimed at not disadvantaging the students who were part of the control group.

### 3.7 Limitations of research

There was some resistance from the heads of the school as they were unwilling to let their students take part in this study. However, the researcher was able to explain to them the purpose and importance of the study. This helped to overcome their initial hesitation. Also, the head of schools was assured that the findings of the study would not be used to evaluate their schools but rather the results would be used only for the study.

Limited resources were also another limitation in this study. The heads of the schools were not compensated for their participation in this study which limited the researcher's access among the student population. However, the researcher made them

understand that the study will benefit their students and play a part in the teaching of Mathematics.

# CHAPTER 4: RESULTS

The results were expressed in terms of the research questions that were considered in this study.

## 4.1 Revisit research questions

*1st Research Question: Outputs and Analysis*

i. 4A) Is there a statistically significant difference in the mean score between the experimental and control groups in the Multiplication Concept Test?

Using the previously stated hypotheses:

$$H_0: \bar{x}_e = \bar{x}_c$$

$$H_1: \bar{x}_e \neq \bar{x}_c$$

Where $\bar{x}_e$ is the mean score of the experimental group, and $\bar{x}_c$ is the mean score of the control group , Two-tailed unpaired t-tests were conducted at a 95% confidence level with $\alpha = 0.05$ to evaluate the research question whether there are any statistically significant differences in the mean scores related to effect of the teaching method between the experimental group (which were taught according to the Frayer's model) and their peers belonging to the control group (who were taught through tradition method ).

Two-tailed unpaired t-tests were conducted to evaluate the research question whether there are any statistically significant differences in the mean scores related to the effect of method between the experimental group and their peers belonging to the control group. The descriptive statistics and the significance values for the referred t-tests are presented in Table 4A.1 on the following page.

Table 5 (4A.1) Statistical Significance of the Independent Samples t-Test (unpaired t-test) for comparing the various functions and total scores for the mathematics multiplication test between the experimental and control groups

| Math concepts | Groups | Mean | St. Deviation | t | df | sig (2-tailed) | Effect Size |
|---|---|---|---|---|---|---|---|
| Equivalent Group | Control | 3.58 | 0.60911 | -0.805 | 49 | 0.425 | 0.13 |
| | Experimental | 3.66 | 0.55733 | | | | |
| Repeated addition | Control | 3.5 | 0.67763 | -1.476 | 48.47 | 0.146 | 0.4 |
| | Experimental | 3.74 | 0.52722 | | | | |
| Array | Control | 3.2 | 1.04978 | -3.362 | 30.99 | 0.002 | 0.77 |
| | Experimental | 3.82 | 0.43753 | | | | |
| Number line | Control | 1.4 | 0.67006 | -2.572 | 42.461 | 0.014 | 0.62 |
| | Experimental | 1.76 | 0.47638 | | | | |
| Patterns & Factor | Control | 3.1 | 0.8391 | -1.534 | 49 | 0.132 | 0.63 |
| | Experimental | 3.56 | 0.61146 | | | | |
| Distributive Property | Control | 1.26 | 0.72309 | -2.523 | 46.241 | 0.015 | 0.74 |
| | Experimental | 1.72 | 0.49652 | | | | |
| Multiplication Properties | Control | 3.34 | 0.84781 | -0.889 | 49 | 0.378 | 0.41 |
| | Experimental | 3.64 | 0.59796 | | | | |
| Missing Number | Control | 1.72 | 0.70102 | -0.25 | 49 | 0.804 | 0.37 |
| | Experimental | 1.92 | 0.27405 | | | | |
| Problem Solving | Control | 2.74 | 1.15723 | -4.495 | 41.114 | 0 | 1 |
| | Experimental | 3.68 | 0.62073 | | | | |
| TOTAL | Control | 23.84 | 3.94016 | -3.736 | 43.997 | 0.001 | 1.08 |
| | Experimental | 27.5 | 2.66688 | | | | |

By examining this table, there were noticeable differences in the means of the various forms of multiplication concepts related to the effective teaching method as well as between the total scores that were evident between the participants belonging to the experimental and control groups.

The results also showed statistically significant differences in the following mathematical forms of multiplication concepts: Multiplication using (Array - Number line - Patterns & Factors - Distributive property– Problem-Solving). These concepts formed clear differences between the experimental and control groups

($p \leq 0.05$). The results indicated that there are no statistically significant differences in the following forms of multiplication: Multiplication using (Equivalent groups, Repeated Addition, and Missing numbers).

According to the hypotheses of the research and T-Test analysis on the experimental and control groups, it was found that there were statistically significant differences at the level of function 0.05 in the mean scores of the experimental and control groups. The experimental group and the table (4A.1) show this. Overall, the results indicated that there were statistically significant differences in mathematical concepts test for the experimental group, with a mean of 27.5 while the control groups obtained 23.8.

An effect size is a quantitative measure of the magnitude of a phenomenon; it was used in the analysis to find the relationship between the methods used by the research groups. For most types of effect size, a larger absolute value always indicates a stronger effect. Where the effect size in this study plays an important role in the analysis process, In addition, the prominent role in the determination of the effective methods by students during the testing of mathematical concepts.

Effect Size analysis, which shows the extent of the practical difference in the teaching of mathematical concepts. The table shows that there is a significant rate in problem-solving (1.0). I Array and distribution property that gets a rate close to large 0.77 and 0.74, respectively. The number line has an average rate of 0.62.

The following forms of multiplication concepts (repeated addition/multiplication property and missing numbers) indicated chose to -average rate (less than 0.5), while equivalent groups obtained a small rate of 0.13.

*2rd Research Question: Outputs and Analysis*

4B) How might the use of Frayer's Model affect the student's ways to solve problems related to multiplication concept?

The analysis of the data using T-Test on the experimental and control groups showed that there were significant differences between experimental and control groups

students in the mathematical concepts test, fig 4B.1 and table (4B.2) show this. The Frayer's model was most effective (as evident from the student's approach in solving the multiplication function). A chi-square test was undertaken to compare the frequency of the students between the experimental and control in applying different ways for solving the multiplication function.

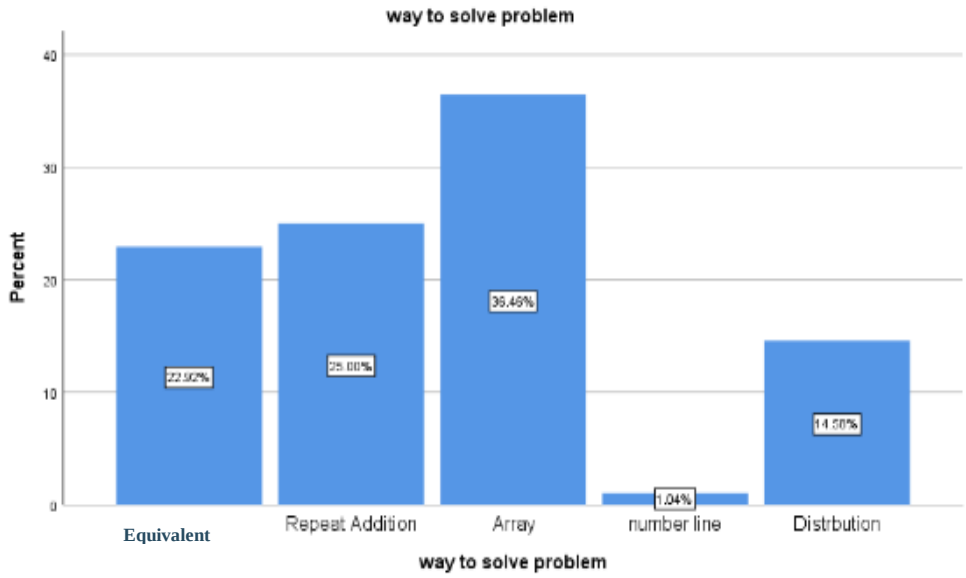



Figure 3 (4B.1) Reflects the ways in solving a multiplication function

Table 6. Frequency Table to solve multiplication problems Q9

| | | Frequency | Percent |
|---|---|---|---|
| Valid | | 22 | 22.00 |
| | Repeat Addition | 24 | 24.00 |
| | Array | 35 | 35.00 |
| | Number Line | 1 | 1.00 |
| | Distribution | 14 | 14.00 |
| | Total | 96 | 96.00 |

Fig 4B.3 and tables 4B.4 reflected that the ways to solve a multiplication problem significantly differed between the experimental and control groups. The experimental group used more forms of multiplication concept based on Frayer's model compared to the control group in solving a multiplication problem, and the results were statistically significant ($X^2$=17.04, p=0.002). Hence, it can be interpreted that the Frayer's teaching model helped the third-grade students to develop competence in solving a multiplication problem based on the forms of the number line and distribution properties. The solutions of the experimental group showed different percentages in solving the questions related multiplication concept while the control group was limited to three solutions

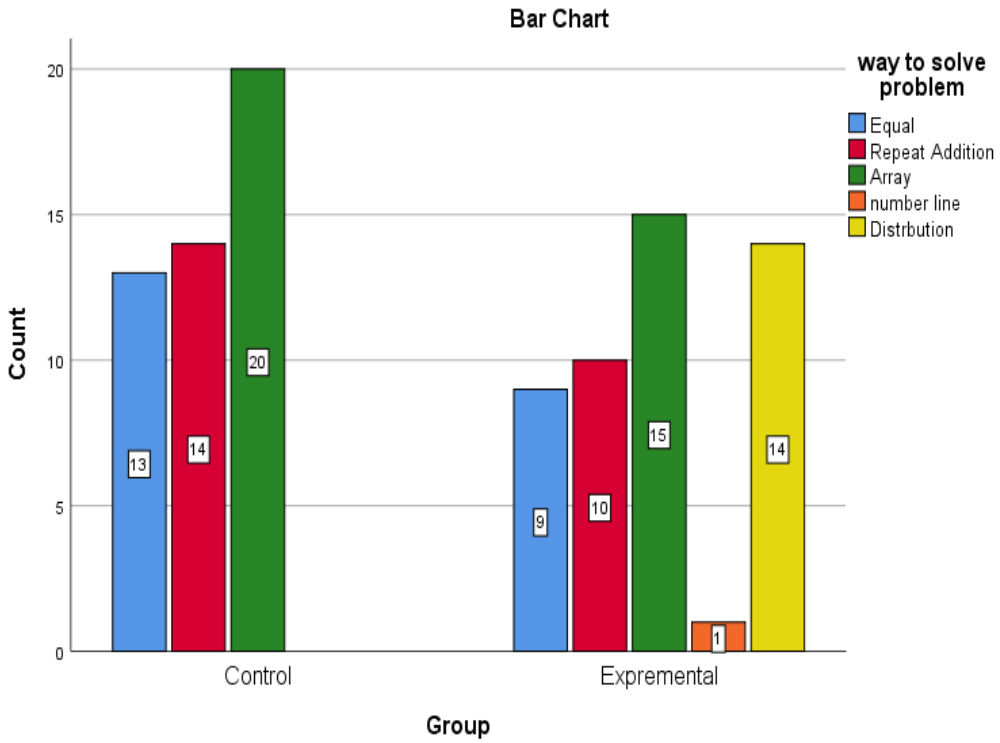



Figure 4 (4B.3). Comparison between the experimental and control groups in solving the multiplication problems Q9

Table 7: Way to solve problem Crosstabulation Q9

| | | Equal | Repeat Addition | Array | number line | Distribution | Total |
|---|---|---|---|---|---|---|---|
| Control | Count | 13 | 14 | 20 | 0 | 0 | 47 |
| | % within Group | 27.7% | 29.8% | 42.6% | 0.0% | 0.0% | 100.0% |
| | % within way to solve problem | 59.1% | 58.3% | 57.1% | 0.0% | 0.0% | 49.0% |
| | Adjusted Residual | 1.1 | 1.1 | 1.2 | -1.0 | -4.0 | |
| Experimental | Count | 9 | 10 | 15 | 1 | 14 | 49 |
| | % within Group | 18.4% | 20.4% | 30.6% | 2.0% | 28.6% | 100.0% |
| | % within way to solve problem | 40.9% | 41.7% | 42.9% | 100.0% | 100.0% | 51.0% |
| | Adjusted Residual | -1.1 | -1.1 | -1.2 | 1.0 | 4.0 | |
| Total | Count | 22 | 24 | 35 | 1 | 14 | 96 |
| | % within Group | 22.9% | 25.0% | 36.5% | 1.0% | 14.6% | 100.0% |
| | % within way to solve problem | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |

Table 8: Chi-Square Tests Q9

| | Value | df | Asymptotic Significance (2-sided) |
|---|---|---|---|
| Pearson Chi-Square | 17.074[a] | 4 | 0.002 |
| Likelihood Ratio | 22.871 | 4 | 0.000 |
| Fisher test | 11.301 | 1 | 0.001 |
| N of Valid Cases | 96 | | |

a. 2 cells (20.0%) have expected count less than 5. The minimum expected count is .49.

The Chi-Square test was used and adopted for several reasons, first because the data were not systematically distributed secondly the data analysis adopted by specialized consultants. The analysis of the Fisher test shows that a range of methods (less than 5). This indicates a statistical significance between the methods used to

answer the ninth question in mathematical test and the groups of study (experimental and control). This analysis focuses on identifying the most effective methods for students.

The table indicates that the adjustment of the first three methods (Equivalent groups - repeated addition – Array) does not indicate that there are statistically significant differences through the Adjusted Residual analysis as the percentage of students within the group is higher in control group more than the experimental group.

For example, the number of students who used the Array method within the groups was 42.6% of the control group compared to the experimental group show 30.6% for the following reasons:

1 - Students in the control group focused on the first three forms as previous concepts and were emphasized during the teaching unit.

2 - Repeated methods for more than once for students and solving many activities depending on the three methods.

Method 4: The number line method There are no statistically significant differences and the table showed that only one student in the experimental group used this method, while no student used this method in the control group

Method 5 indicates that there is a statistically significant difference (greater than 2) by an adjusted residual analysis for the experimental group. The results indicate that 28 students used the distribution method during the test. On the other side, no students used this method in the control group.

We found that the experimental group used all methods in varying percentages, where the distribution method obtained the largest percentage of 28.6% followed by array method by 30.6%, while the method of repeated addition and equivalent groups obtained similar ratios 20.4% and 18.4% respectively finally 2% Only for the number

line method.

Question 6 was analyzed in the test, which refers to whom students wrote the multiplication fact using the distributive property. The results indicated that the experimental group used various numerical sentences to express the multiplication fact, for example;

Solution 1: (5x3) + (5x3)

Solution 2: (4x6) + (1x6)

Solution 3: (5x1) + (5x5)

Solution 4: (2x6) + (3x6)

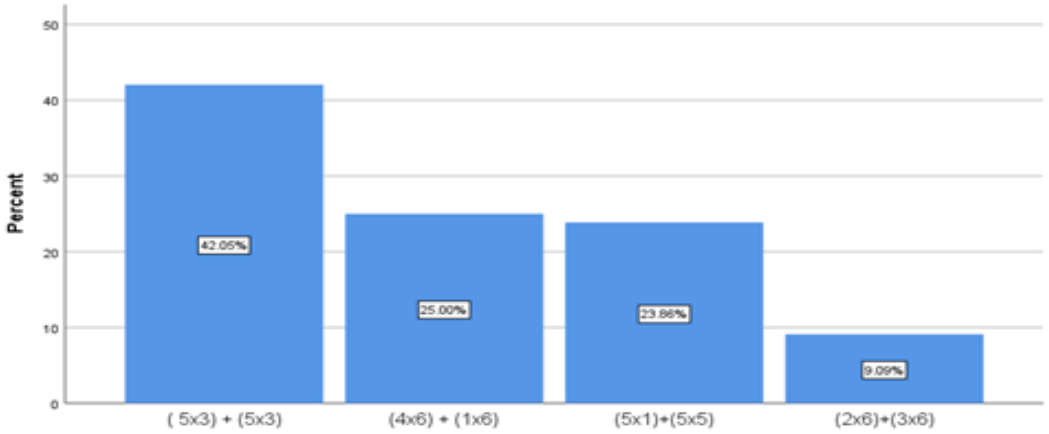



Figure 5 (4B.6) Ways to write Distributive Property Q6

Table 9 (4B.7) Frequency Table way to write Distribution property Q6

| | | Frequency | Percent |
|---|---|---|---|
| Valid | ( 5x3) + (5x3) | 37 | 37 |
| | (4x6) + (1x6) | 22 | 22 |
| | (5x1)+(5x5) | 21 | 21 |
| | (2x6)+(3x6) | 8 | 8 |
| | Total | 88 | 88 |

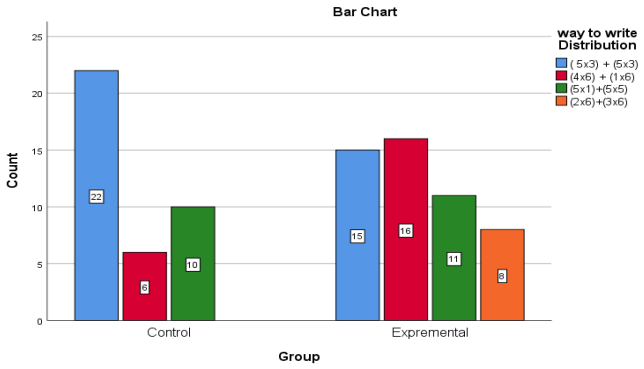



Figure 6 (4B.8) Comparison between the groups in write the distributive Property.Q6

Table 10 (4B.9) Ways to write distribution property Q6

| | | ( 5x3) + (5x3) | (4x6) + (1x6) | (5x1)+(5x5) | (2x6)+(3x6) | Total |
|---|---|---|---|---|---|---|
| Control | Count | 22 | 6 | 10 | 0 | 38 |
| | % within Group | 57.9% | 15.8% | 26.3% | 0.0% | 100.0% |
| | % within way to write Distribution | 59.5% | 27.3% | 47.6% | 0.0% | 43.2% |
| | Adjusted Residual | 2.6 | -1.7 | 0.5 | -2.6 | |
| Expremental | Count | 15 | 16 | 11 | 8 | 50 |
| | % within Group | 30.0% | 32.0% | 22.0% | 16.0% | 100.0% |
| | % within way to write Distribution | 40.5% | 72.7% | 52.4% | 100.0% | 56.8% |
| | Adjusted Residual | -2.6 | 1.7 | -0.5 | 2.6 | |
| Total | Count | 37 | 22 | 21 | 8 | 88 |
| | % within Group | 42.0% | 25.0% | 23.9% | 9.1% | 100.0% |
| | % within way to write Distribution | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |

Table 11: Chi-Square Tests Q6

| | Value | df | Asymptotic Significance (2-sided) |
|---|---|---|---|
| Pearson Chi-Square | 12.514[a] | 3 | 0.006 |
| Likelihood Ratio | 15.545 | 3 | 0.001 |
| Fisher's test | 6.447 | 1 | 0.011 |
| N of Valid Cases | 88 | | |

a. 2 cells (25.0%) have expected count less than 5. The minimum expected count is 3.45.

Tables 4C.8 and 4C.9 reflected that the ways to apply the distributive property to solve a multiplication problem (question 6) significantly differed between the experimental and control groups. The experimental group used more forms based on Frayer's model compared to the control group in solving a multiplication problem. The results were statistically significant ($X^2$=17.04, p=0.002). The experimental group used the (4*6) + (1*6) and (2*6) + (3*6) distribution properties significantly higher than the control group for the same properties. Hence, it can be interpreted that the Frayer's teaching model helped the third-grade students to develop competence on novel distribution properties to solve a multiplication function.

The following table shows the different ways of writing multiplication sentences using distribution property. Fisher test indicates that there was a statistical difference between the various methods of writing the multiplication sentences in question 6 and between the study groups (experimental and control)

The results of the Crosstabulation table indicate the percentage of students 'achievement in the methods used. The students' answers were limited to four methods as shown in the table below. The first method shows a statistically significant relationship (2.6> 2) in favor of the control group. The percentage of students within the group was 57.9%, while the percentage of students in the experimental group was 30%. The students of the control group were able to apply this method as one of the easiest ways to write the multiplication clause where the grid is divided equally. The second method indicates that there is no statistically significant relationship (1.7 <2) for the experimental group. They outperform the control group with 17 students. In addition to dividing the grid horizontally and wrote the multiplication sentence in the following form (4x6) + (1x6). The answer in the third method was divided vertically, with no statistically significant relationship (0.5 <2) Between bath groups. Where the

control group used this method 26.3% of their peers in the experimental group by 22%

Finally, method 4 showed a statistically significant relationship (2.6> 2) for the experimental group they Divide the grid horizontally and divide the five rows into 2 and 3 and write the multiplication sentence as follows (2x6) +(3x6). This method is unusual since no student has used this method in the control group and demonstrates the students' understanding of the reality of multiplication and writing different ways to find the product.

Therefore, the traditional way of writing the sentence is the first method which has most commonly used by students in the control groups, while the experimental group used more diverse methods, where it was the second most common method in this group.

### 4.2. Present results and findings

Based on the analysis of all the previous statistical tests, unpaired and paired t-tests, and the chi-square test for independence, between the experimental and control groups, there is a significant difference in the mean scores for the mathematical forms of Array, Number Line, Distributive Property, and Problem Solving, for the groups that used Frayer's Model. However, there were no significant differences in the mean scores for the mathematical forms of the equivalent group, repeated addition, patterns & factors, multiplication properties, and missing numbers for those groups that used Frayer's Model. Also, the overall result for the unpaired sample t-test, there were significantly higher scores for the experimental group over the control group. So, while there were more subjects that showed no significant difference between the two groups than those that showed a significant difference, the overall result implies that Frayer's model is more effective in teaching students' the multiplication concepts

It should also be noted that even when the results were not significant, the mean

scores of the experimental groups are always higher than those of the control groups. Thus, every group that learned via the new model either performed similarly or better than the groups using the old method. The Chi-square results also indicate that there is a significant difference between the expected results of the control and experimental groups.

# CHAPTER 5: DISCUSSION AND CONCLUSION

## 5.1 Discussion of the results

**The first hypothesis** that was considered for this study contended that there is a significant difference between the mean score of the experimental group and the control group in testing the multiplication concepts. According to the results, the experimental group performed better compared to the control group in five out of the ten multiplication forms that were evaluated in the study participants ($p<0.05$). Since previous forms of multiplication were taught to students at the second-grade level, students were able to understand these concepts very well in both the experimental and control groups. The forms where the experimental group outperformed their counterparts in the control group were on the array, number line, patterns and factor, distributive functions, and problem-solving. In the overall test, the experimental group students achieved a mean score of 27.5 (S. D= 2.67)

It could thus be concluded that Frayer's Model is more effective than the traditional method in the acquisition of mathematical concepts across the third-grade students. Such findings complement the philosophies that underpin acquisition and learning of mathematical concepts

Smith (2006) and Mulligan & Mitchelmore (2009) highlighted the importance of understanding patterns and structures in developing multiplication concepts in elementary school-going students. The findings of the present study also help to conclude that Frayer's model might encourage "Constructivism Learning Model" across the students. Various authors have supported that the constructivism model of learning mathematical concepts is more suitable across primary school-going children.

The findings of the present study are also in line with the findings of Monroe and Pendergrass (1997). Monroe and Pendergrass conducted a study on fourth-grade

students to evaluate the effect of two vocabulary instructional approaches- the integrated CD-Frayer's model and the definition only model. The experimental group taught using the CD-Frayer's model achieved a better mean of 12.857 (S. D= 10.543) compared to the control group who were taught using the definition only model who obtained a mean of 8.444 (5.989), and $p< 0.041$. Indicated that, like this study, there is a difference in students' performance in Mathematics while comparing the effects of the two teaching models. The integrated CD-Frayer's model is more effective in giving vocabulary instructions to students compared to the definition only model. This means students taught using the Frayer's Model can perform better in Mathematics tests since they have a better and deeper of the mathematical language, vocabulary, and concepts. Also, Moore and Radiance (1984) noted that "teachers who use the graphical organizer to teach the findings of the present study are also in line with the findings of Monroe and Pendergrass (1997).

Sanders (2007) carried out a similar study to investigate the difference in the performance of students who had been taught using two different teaching strategies in mathematical vocabulary instructions- direct instruction and keyword mnemonics. However, the group that was taught using the keyword mnemonics performed better than the group taught using the direct instruction method. The keyword mnemonics group obtained a mean score of 33.65, compared to a mean of 30.53 for the direct instruction group. These were measured from the posttest and follow up a test of the mathematical vocabulary assessment, $F (1, 206) = 13.196$, $p< 0.0005$. Roe and Smith (2012) saw that due to the structure and thought processes involved in the Frayer's Model teaching strategy provides students with an opportunity to develop a deep understanding of mathematical vocabulary and ultimately acquisition of the mathematical concepts they have been taught. Therefore, the study concludes that the

Frayer's Model is a good teaching approach for mathematical vocabulary instruction and ultimately acquisition of mathematical concepts as indicated by improved performance in mathematical tests. The findings of the present study and the previous studies reflect that the Frayer's model may be integrated with other teaching methodologies or modified to develop mathematical concepts in the target population. However, the design of such teaching models should emphasize on the philosophy of improving decision-making and innovation-based concepts that would promote constructivism learning approaches.

**The second hypothesis** that was considered in this study contended that *Frayer's teaching model significantly help third-grade students in solving a multiplication problem by using different forms of the multiplication concept. ($p>0.05$).* The present study showed that the experimental group engaged a mix of different strategies in solving problems compared to their control counterparts ($p=0.002$).

The present study reflected that the ways to solve a multiplication problem significantly differed between the experimental and control groups. The experimental group used more forms based on Frayer's model compared to the control group in solving a multiplication problem, and the results were statistically significant ($X^2=17.04$, $p=0.002$). Hence, it can be interpreted that the Frayer's teaching model helped the third-grade students to develop competence in solving a multiplication problem based on the forms of multiplication of the number line and distribution properties. According to the results, the experimental group acquired more problem-solving techniques compared to the control group. There were four different ways (equal groups, repeated addition, array, number line, and distribution) that the students could use to solve Question 9 of the test. In the experimental group, each of the four ways was used to solve the problems by the students whereas, in the control group, only

three ways were used.

According to the results, there is a statistically significant relationship between the number of ways used by a student in solving a mathematical problem and the teaching model used ($p<0.05$). The experimental group taught using the Frayer's Model, acquired more ways to solve a mathematical problem. Question 6 of the test examined how students in both groups distributed a multiplication problem. Again, students in the experimental group used more ways to distribute the multiplication problem. Also, more students ($n= 12$) in the control group did not attempt the question whereas, in the experimental group, all the students attempted the question. The results also show that there is a statistically significant relationship between the performance of the students and the way to write distribution of mathematical problem ($p<0.05$). Based on such findings the present study reflected that the *Frayer's teaching model significantly helped third-grade students in solving a multiplication function based on the multiplication concepts.*

This finding supports that teaching using the Frayer's Model enables students to become strategic problem solvers. Visuals used in Frayer's Model enhance the thinking process of the students. Through the graphic organizer, the students can organize their thoughts and process the information in the question. Students can separate what is essential in the question of what is not essential. Processing the information also allows the students to think of various ways they could use to solve mathematical problems. Since the teaching model used to teach the students is organized, during tests, it becomes easier for the students to recall whatever they were taught. Moreover, in lessons based on Frayer's Model, the students are at the center of the teaching as they are heavily involved in the learning process. This enhances the acquisition of the mathematical concepts taught by the teacher. In the Frayer's graphical

organizer under examples or characteristics of a mathematical concept, the student establishes relationships between a mathematical concept and other concepts. Students taught using the Frayer's Model can relate different mathematical problems and problem-solving techniques. The students are therefore able to solve a single problem using different techniques.

**5.2 Summary of the findings**

This study was conducted with the aim of determining the impact of Frayer's Model, as a teaching model, on the acquisition of mathematical concepts and effect on the performance of third-grade students in Mathematics in Qatar primary schools. The strength of this study was that it aimed to identify the strategies that influence students' acquisition of mathematical concepts and how the Frayer's Model and the definition only method compare in developing such concepts across the concerned stakeholders.

The study focused on five practices involving Mathematics vocabulary that differentiated the two teaching models. The five practices included: determining the mathematical vocabulary proficiency of the students before the beginning of teaching, teaching of mathematical vocabulary by definition only method, teaching mathematical vocabulary by both direct teaching and meaningful context methods, consideration of the mathematical vocabulary proficiency of the students during the setting of Mathematics items, and consideration of mathematical vocabulary proficiency of students by authors of Mathematics textbooks during writing of the books.

Focus on teaching method during the time ten teaching strategies were developed. Their impact was also noted, and they all had a positive impact on the students' acquisition of mathematical concepts hence improvement in the performance of students in mathematical tests. The ten teaching strategies include:

i. Using synonyms for simple words

ii. Integrating the four modes of language during Mathematics lessons which include listening, speaking, reading, and writing
iii. Breaking complex and difficult words into simpler segments or words that the students can easily understand
iv. Regular teaching of mathematical language structure and vocabulary
v. Teaching students how to learn and study new mathematical vocabulary
vi. Incorporate the use of technology in teaching and learning of mathematical vocabulary
vii. The teacher talking loudly while explaining mathematical concepts and solving problems on the chalkboard
viii. Using simplified speech while teaching mathematical concepts, especially for young students
ix. Using different ways to demonstrate and supplement written or spoken instructions
x. Using graphic organizers based on Frayer's Model

The fourth objective of the study was to develop a general framework for employing Frayer's model in the development of the mathematical concept. In developing the general framework, Frayer's Model was used. The teachers involved in the study, together with the researcher and the arbitrators from the Ministry of Education agreed on a general framework for employing the Frayer's Model in the acquisition of mathematical concepts. They agreed that the lesson plan should have three parts: Introduction, Development, and Conclusion. The lesson must integrate the use of graphic organizers which are based on the Frayer's Model. From the framework, a sample lesson plan for the multiplication unit was developed. The same framework had been employed in developing lesson plans that were used to teach the experimental

group. Based on the performance of the group in the posttest, it shows that the framework, based on the Framework model, is useful in the acquisition of mathematical concepts among students.

In general, the following is a summary of the main findings of the study:

i. The students exposed to the mathematical forms of multiplication concept using the Frayer's Model performed better compared to those taught using the traditional method hence an indication of improved acquisition of mathematical concepts

ii. The effective strategy for acquisition of mathematical concepts was established to be the use of graphical organizers based on the Frayer's Model since the model is centered on the student

iii. Students taught using the Frayer's Model acquire different and more problem-solving techniques which generally enhances their acquisition of mathematical concepts.

**5.3 Conclusions**

These conclusions are essentially according to a context, based on the results of previous studies,

and testing their veracity at different stages private schools and school contexts beyond Qatar.

Following the above findings, the study made five logical conclusions. First, the study concluded that a teaching approach that is well developed and focused on frayer model instructions could improve the students' acquisition of mathematical concepts and hence their performance in Mathematics for primary third graders. Secondly, the use of graphical organizers based on Frayer's Model can be an effective method for Mathematics concept instruction that helps the students to develop a deeper

understanding of the mathematical concepts taught and hence improve the students' ability to acquire the mathematical concepts.

The third conclusion was that the lesson plan developed from a general framework using Frayer's Model for vocabulary instruction could be used to enhance the acquisition of mathematical concepts among students. This is because the lesson plan is centered on the student and there is a more profound understanding of the mathematical concepts hence the acquisition of the concepts. The fourth conclusion was that, apart from improving the mathematical vocabulary of students, Frayer's Model enhances the acquisition of mathematical concepts by availing numerous different techniques to solve mathematical problems. The conclusion was that students who are proficient in mathematical vocabulary face many problems while studying Mathematics. These problems include the inability to comprehend mathematical word problems, not able to verbally express mathematical concepts, and difficulty in understanding whatever the teacher is teaching. Also, if the students do not acquire proficiency in Mathematics, then they would not be able to read Mathematics textbooks, they would be unable to understand mathematical problems during tests hence poor performance in the subject, and most importantly they would not be able to acquire mathematical concepts.

Finally, Various elements of this study are new and different. First, it involved third-grade students. Previous studies on Mathematics and the Frayer's Model have involved older age groups. This study has shifted the age group. This can be attributed to the fact that the third grade in Qatar will soon begin taking the TIMSS exams. It is, therefore, essential to try and improve their performance in the exams early since Qatar has performed poorly in the sixth and eighth TIMSS exams. Second, this study is among the new studies that apply Frayer's Model in the State of Qatar. Third, this study is

based on the new curricula developed in Qatar which is based on the main competencies: creative and critical thinking, linguistic competence, numerical efficiency, communication, cooperation and participation, research, and problem-solving. Fourth, most studies conducted in Mathematics use the Geometry unit. This study has used the Multiplication unit. The rationale for choosing this unit has been discussed in Chapter 3.

The thesis of the study was that the Frayer's Model is an effective teaching model in the acquisition of mathematical concepts among students, more specifically, third-grade students in Qatar primary schools. The poor performance in Mathematics in various grades in Qatar primary schools is due to the inability of the students to acquire the mathematical concepts that they are taught which stems from difficulty understanding mathematical vocabulary. Through this study, it was shown that the Frayer's Model is an effective model in helping third-grade students in Qatar primary schools in acquiring mathematical concepts through enhancement of their mathematical vocabulary.

## 5.4 Recommendations

Based on the conclusions of this study, policy recommendations were made to various primary stakeholders in Qatar's education sector. Suggestions for further areas were also provided.

### Policy recommendations

Recommendations were made to various stakeholders in Mathematics education based on the conclusions of the current study.

For Mathematics teachers in primary schools, it is crucial to emphasize to them the role of mathematical concept on how students' performance in Mathematics or whether they acquire the mathematical concepts they are taught. According to the

findings of the current study, Teachers should, therefore, use simple and appropriate in teaching, assessing, and during learning of Mathematics.

In Qatar, the Ministry of Education and Higher Education is charged with curriculum development. The ministry should, therefore, design the Mathematics materials for the curriculum such that it enhances the students' readability. Simple and appropriate Mathematics language should be used in the materials. This would improve the students' ability to acquire mathematical concepts hence improving their performance in Mathematics. The Ministry of Education and Higher Education can pilot the prototype lesson plan developed in this study in some primary schools within Qatar. The pilot prototype lesson plan was based on mathematical vocabulary instruction in the acquisition of mathematical concepts. From the prototype, the lesson could then be adopted in the schools to teach Mathematics.

### 5.5 Recommendations for future research

Although the present study showed that the effectiveness of Frayer's model in teaching mathematical concepts across third-grade students. Future studies should explore the effectiveness of teaching multiplication concepts through Frayer's model by incorporating a large sample size and cross-sectional and multi-centric study designs. Such study designs would increase the reliability and reproducibility of the effectiveness of Frayer's model irrespective of the demographic and cultural background of the study participants. Moreover, the learning theories employed by the respective study participants could have also confounded the findings of the present study. Future studies should try to minimize the confounding effects of learning theories applied or practiced by the study participants at baseline. Such study design help to develop conclusive evidence regarding the effectiveness of the Frayer's Model as a teaching model for developing mathematical concepts across the concerned

stakeholders irrespective of the learning theories that they apply while acquiring concepts. Hence, the present study recommends further research by adding second dependent variable such as “spatial abilities” and investigate the effect of using the Frayer’s Model on the spatial abilities of students.

## REFERENCES


Abu El-Ela 2013 Effectiveness of a proposed program based on some of the teachings approaches to the development of mathematical concepts and problem-solving skills and the trend towards learning mathematics for students in the first-grade secondary, Ph.D. thesis, Fayoum University, Egypt

New Jersey Department Of Education - Report Card Narratives. (n.d.). Retrieved from http://www.nj.gov/education/pr/1213/narrative/13/1750/13-1750-070.html

Acharya, B. (2017). Factors Affecting Difficulties in Learning Mathematics by Mathematics Learners. International Journal of Elementary Education, 6(2), 8. doi: 10.11648/j.ijeedu.20170602.11

Al-Jazzar 2002 The effectiveness of the use of multimedia-assisted learning in acquiring some levels of learning of scientific concepts by the FRAIB model for evaluating concepts. Journal of Education for Educational, Psychological and Social Research. Faculty of Education, University, 47-73

Zhang, Jie, Niu, C., Munawar, S., & Anderson, R. (2016). What Makes a More Proficient Discussion Group in English Language Learners' Classrooms? Influence of Teacher Talk and Student Backgrounds.

African Centered Education Research Papers - Academia.edu. (n.d.). Retrieved from http://www.academia.edu/Documents/in/African_Centered_Education

Alliance, E. (2006). Closing the achievement gap: Best practices in teaching mathematics. Retrieved March 8, 2010

African Centered Education Research Papers - Academia.edu. (n.d.). Retrieved from http://www.academia.edu/Documents/in/African_Centered_Education

Barton, M. L. (1997). Addressing the literacy crisis: Teaching reading in the content areas. NASSP Bulletin, 81(587), 22-30. doi:10.1177/01963659708158705

Brewer, D., Goldman, C., Augustine, C., Zellman, G., Ryan, G., & Stasz, C. (2006). An Introduction to Qatar's Primary and Secondary Education Reform. Supreme Education Council.

Brooke, P. (2017). Improving the self-efficacy of math learners using a direct and focused approach to vocabulary clarification. University of Lethbridge Research Repository.

Brunn, M. (2002). The four-square strategy. The Reading Teacher, 55(6), 522-525.

Battista, M. C. (1999). Spatial structuring in geometric reasoning. Teaching Children Mathematics, 6, 171-177.

Blanton, M., & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. Journal for Research in Mathematics Education, 36, 412-446.

Bruner, J.S. &et. (1956) A study of thinking, Gohnwilers sons, NewYork.

Clark, R. (2007). Writing strategies for social studies. USA: Shell Education

Cohen, V. & Cowen, J. (2008). Literacy for children in an information age: Teaching reading, writing, and thinking USA: Thompson Wadsworth

Collier, P. (2007) Bottom Billion. The Blackwell Encyclopedia of Sociology: 1-3.

Carraher, D. W., Schliemann, A. D., Brizuela, B. M., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. Journal for Research in Mathematics Education, 37, 87-115

Doty, J., Camron, G. & Barton, M. (2003). Teaching Reading in Social Studies. USA: ASCD.

Dunston, P. J., & Tyminski, A. M. (2013). What's the Big Deal about Vocabulary?. Mathematics Teaching in the Middle school, 19(1), 38-45.

Daro, P. (2006, February 15). Math Warriors, Lay Down Your Weapons. Education Week, 33, 35

Estacio, R., & Martinez, D. (2017). The use of modified Frayer model in developing science vocabulary of senior high school students. New Trends and Issues Proceedings on Humanities and Social Sciences, 4(1), 36-42. doi: 10.18844/prosoc.v4i1.2049

English, L. D. (1999). Assessing for structural understanding in children's combinatorial problem-solving. Focus on Learning Problems in Mathematics, 21(4), 63-82.

Fishman, B., Marx, R., Best, S., & Tal, R. (2003). Linking teacher and student is learning to improve professional development in systemic reform. Teaching and Teacher Education, 19(6), 643-658.

Frayer, D.A., & Klausmeier, H.G. (1969). A schema for testing the level of concept mastery, Technical Report #16. The University of Wisconsin

Frayer, L. E., Fredrick, W. C., & Klausmeier, H. J. (1969). A Schema for testing the level of concept mastery, Working Paper No. 16, Madison, Wisconsin: Wisconsin research and development center for cognitive learning, University of Wisconsin.

Fisher, Douglas, and Nancy Frey. 2008. Word Wise and Content Rich, Grades7□12: Five Essential Steps to Teaching Academic Vocabulary. Portsmouth, NH: Heinemann.

Frayer, Dorothy A., Wayne C. Frederick, and Herbert J. Klausmeier. 1969. A Schema for Testing the Level of Cognitive

Mastery. Working Paper No. 16 Wisconsin Research and Development Center. Madison: University of Wisconsin

Fitzgerald, Jill, and Michael F. Graves. 2005. "Reading Supports for All." Educational Leadership 62 (4): 68.71.

Gafoor, A., & Sarabi, M. (2015). Relating Difficulty in School Mathematics to Nature of Mathematics: Perception of High School Students from Kerala. Regional Institute of Education (NCERT), Mysuru.

Gay L. R., Mills, G.E., & Airasian P.W. (2009). Educational Research: Competencies for analysis and applications. Merrill/Pearson.

Gillis, V. R., & MacDougall, G. (2007). Reading to learn science as an active process. The Science Teacher, 74(5), 45-50.

Gray, E., Pitta, D., & Tall, D. (2000). Objects, actions, and images: A perspective on early, number development. The Journal of Mathematical Behavior 18(4): 401-413.

Greenwood, S. (2010). The Power of Words: Learning Vocabulary in Grades 4-9. London: Rowman & Littlefield Publishers Inc.

Grehan, M., O'Shea, A., & Bhaird, C. (2010). How do students deal with difficulties in mathematics? CETL-MSOR Conference 2010, 34-38.

Godino, J. D. & Batanero, C. (1994). Significado personal e institucional de los objetos matemáticos. Recherches en Didactique des Mathématiques, 14 (3): 325-355.

Godino, J. D. & Batanero, C. (1996). Clarifying the meaning of mathematical objects as a priority area of research in mathematics education. In: A. Sierpinska (Ed.), What is Research in Mathematics Education, and What Are its Results? Dordrecht: Kluwer A. P. (in press). https://oapub.org/edu/index.php/ejes/article/view/517

Hejaze, R. (2018). Qatar's Performance in International Tests: PIRLS, TIMSS, PISA, and TOEFL. Retrieved from https://sites.northwestern.edu/qtaaleam/qatars-performance-in-international-tests/#.XAySjmgzZPY

Hunting, R. (2003). Part-whole number knowledge in preschool children. Journal of Mathematical Behavior, 22, 217-235.

Iwankovitsch, D.L. & Stranderford, N.S. (2013) Effective instructional strategies to enhance vocabulary development in elementary classrooms.

ILter, I. (2015). The investigation of the effects of Frayer model on vocabulary knowledge in social studies. Elementary Education Online, 14(3), 1106-1129.

Koc, M., & Fadlelmula, F. (2016). Overall Review of Education System in Qatar. Lambert Academic Publishing.

Karjala, L. (2010). Math vocabulary instruction in an inclusive classroom 1 direct instruction and the Frayer model: Effects on mathematics achievement in an inclusive classroom. (Unpublished master thesis). South West State University Marshall Minnesota

Macceca, S. (2007). Reading strategies for social studies. Washington: Shell education

Marzano, R.J. (2004). Building background knowledge for academic achievement: Research on what works in schools. Ascd.

Marzano, R.J. & Simms, J.A. (2013). Vocabulary for the common core. Alexandria: Marzano research laboratory.

Monroe, E. E. (1997). Using graphic organizers to teach vocabulary: Does available research inform mathematics instruction? (ERIC Document Reproduction Service No. ED414256)

Monroe, E. E., & Pendergrass, M. R. (1997). Effects on mathematical vocabulary instruction on fourth-grade students. Reading Improvement, 34(3), 120-143.

Mugenda, A.G. (2008). Social science research: Theory and principles. Nairobi: Applied.

Mugenda, O.M, & Mugenda, A. (2003). Research methods quantitative and qualitative approaches.

Mullign, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. Mathematics Education Research Journal 21(2): 33-49.

Mundia, L. (2010). Problems in Learning Mathematics: Comparison of Brunei Junior High School Students in Classes with and without Repeaters. Journal of Mathematics Research, 2(3), 150-160. doi: 10.5539/jmr.v2n3p150.

Mulligan J & Mitchelmore M (2009) Awareness of Pattern and Structure in Early

Mathematical Development, Mathematics Education Research Journal 21(2), 33-49.

Mehryar, N. (2003)." The use of innovative teaching methods for-maximizing the enjoyment from learning mathematical concepts," University of Southern Queensland, Toowoomba, Australia. , Columbus.

Mousley, J&Peery, B. (2009). Developing mathematical Concepts- Australian Pre-school Setting, The Background Proceeding ofthe32ndannual conference of the Mathematics Education Research Group of Australasia, Vol (1), pp104-12

Macceca, S. (2007). Reading strategies for social studies. Washington: Shell education.

Monroe, E.E., & Pendergrass, M.R. (1997). Effects of mathematical vocabulary instruction on fourth-grade students. Reading improvement, 34(3), 120- 132.

Nahampun, E.E., & Sibarani, B. (2014). The effect of using the Frayer model on students' vocabulary mastery. General Journal of Applied Linguistics of FBS Unimed, 3(1), 1-8

Neyland, J. (Ed) (1994). Mathematics Education: A Hand Book for Teachers. New Zealand, Wellington College of Education.

Novriani, M., & Surya, E. (2010). Analysis of Student Difficulties in Mathematics Problem Solving Ability at MTs SWASTA IRA Medan. International Journal of Sciences: Basic and Applied Research, 33(3), 63-75.

Novriani, M. R., & Suyra, E. (2017). Analysis of student difficulties in mathematics problem-solving ability at MTs SWASTA IRA Medan. International Journal of Sciences: Basic and applied research 33(3): 63-75.

Nelson, J & Michael, J. (1980). Secondary Social Studies Reflective Teacher. A Research on Teacher Reflective Thinking Vol (48), No. (6), pp. 23–54.

Nahampun, E.E., & Sibarani, B. (2014). The effect of using Frayer model on students'

vocabulary mastery. General Journal of Applied Linguistics of FBS Unimed, 3(1), 1-8.

Nagy, William E. 1988. Teaching Vocabulary to Improve Reading Comprehension. Newark, DE: International Reading Association.

Outhred, L.N. & Mitchelmore, M.C. (2000). Young children's intuitive understanding of rectangular area measurement. Journal for research in mathematics education: 144-167.

Olson, L. (2005, October 26). "NAEP Gains are elusive in Key Areas." Education Week

Piaget, J. & Cook, M. (1952). The origins of intelligence in children. New York: International Universities Press: 8(5).

Piaget, J. (1976). Piaget's theory. In Piaget and his school (pp. 11-23). Springer, Berlin, Heidelberg.

Pitta-Pantazzi, D., Gray, E., & Christou, C. (2004). Elementary school students' mental representations of fractions. Proceedings of the 28th annual conference of the international group for the psychology of mathematics education 4: 41-48.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing a conceptual understanding and procedural skill in mathematics: An iterative process. Journal of educational psychology, 93(2), 346.

Roe, B. &Smith, S. (2012). Teaching reading in today's elementary schools. Canada: Wadsworth: Cengage Learning.

Rukangu, S.M. (2000). Pupils development of spatial ability in mathematics: An issue of learning environment in selected secondary schools in Kenya. Unpublished Ph.D. thesis, Kenyatta University.

Russell, W., Waters, S. and Turner, T. (2013). Essentials of Elementary Social Studies. London: Routledge.

Sanders, J.A., Verhulst, F. & Murdock, J.A. (2007). Averaging methods in nonlinear dynamical systems. New York: Springer: 59.

Sullivan, M. T. (2014). Using adapted Frayer model as a graphic organizer for graph vocabulary. In N. Sonda & A. Krause (Eds.), JALT2013 Conference Proceedings. Tokyo: JALT.

Tambychik, T., & Meerah, T. (2010). Students' Difficulties in Mathematics Problem-Solving: What do they Say? Procedia - Social and Behavioral Sciences, 8, 142-151. doi: 10.1016/j.sbspro.2010.12.020.

Teacher Resource Guide. (2006). Florida center for reading the research. Retrieved on 8 May 2015 from the website: http://www.fcrr.org/curriculum/PDF/G 2-3/2-3TRG_2.pdf. TNR Font.

Toh, T. (2007). An Algebra Content Upgrading Course for In-service Mathematics Teachers. A Singapore Experience International Journal of Mathematics Education in Science and Technology. 38 (4), 489-500.

Trask, C.P. (2011). Raising the achievement of special education students through vocabulary instruction. Lagrange University, Georgia.

Trask, C.P. (2011). Raising achievement of special education students through vocabulary instruction. Lagrange University, Georgia.

Urquhart, V. & Frazee, D. (2012). Teaching Reading in the Content Areas. USA: ASCD.

Usiskin, Z. (2001). A Collection of Content Deserving to be a Field. The Mathematics Educator. 6 (1), 86-98.

Vygotsky, L.S. (1962). Thought and language (E. Hanfmann &G. Vakar, trans.).

Vygotsky L. (1978). Interaction between learning and development. Readings on the development of children. 23(3): 34-41.

Wiersma, W., & Jurs, S.G. (2005). Research methods in education (Eighth Edition).

Wilkins, J. (2002). The impact of teachers' content knowledge and attitudes on instructional beliefs and practices. Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.

Wood, D., Bruner, J.S., Ross, G. (1976). The role of tutoring in problem solving. Journal of child psychology and psychiatry. 17(2): 89-100.

Wanjiru, B & O-Connor, M (2015) Effects of Mathematical Vocabulary Instruction on Students' Achievement in Mathematics in Secondary Schools of Murang' a County, Kenya, Journal of Education and Practice, 6(18), 1-8

Vrugt, A., Oort, F. J., & Zeeberg, C. (2002). Goal orientations perceived self-efficacy and study results amongst beginners and advanced students. British Journal of Educational Psychology, 72(3), 385-397. doi:10.1348/000709902320634285

Lesson Seeds ~ State Curriculum 5.c.2.b ~ Social Studies (2009). A Study on the Problems and Improvement Directions of Needs Analysis in Social Studies Curriculum Development. Journal of Research in Curriculum Instruction, 13(1), 171-195. doi:10.24231/rici.2009.13.1.171

Frayer, J. B. Jules. (2011). Benezit Dictionary of Artists. doi: 10.1093/benz/9780199773787.article.b00067805

Etoom And Abushreekh Pdf Journals from which the literature was retrieved. (1988). Journal of Molecular Structure, 191, ix-xi. doi:10.1016/0022-2860(88)80032-2

Wihlborg, A. (2012). Difference in mortality and fracture risk among participants and non-participants in a population-based study. Bone, 50, S135. doi: 10.1016/j.bone.2012.02.414

Tam, K. K. (2007). Ohio Mental Health Consumer Outcomes System: Report 15. PsycEXTRA Dataset. doi:10.1037/e527942011-001

TECHNICAL SESSION PARTICIPANTS. (1979). Naval Engineers Journal, 91(3), 52-52.

doi:10.1111/j.1559-3584. 1979.tb04041.x

Hampton, M. A. (1983). Geotechnical framework study of the Kodiak Shelf, Alaska. Open-File Report. doi:10.3133/ofr83171

Cohen, F. (2007). Solve Performance Problems with FastSOA Patterns. FastSOA, 103-118. doi:10.1016/b978-012369513-0/50006-9

WOOTTON, C. (2007). Extracting Text from PDF Files. Developing Quality Metadata, 415-416. doi:10.1016/b978-0-240-80869-7.50069-9

Decker, L. C. (2008). Pipeline Integrity Management: Estimating and Prioritizing the Risk Resulting from Pipeline Facility Operations. 2008 7th International Pipeline Conference, Volume 1. doi:10.1115/ipc2008-64364

Bruyning, C. F. (1985). Epidemiology of Gastrointestinal Helminths in Human Populations. Chemotherapy of Gastrointestinal Helminths, 7-65. doi:10.1007/978-3-642-69527-8_2

Watson, A., & Bills, L. (2011). Working Mathematically on Teaching Mathematics: Preparing Graduates to Teach Secondary Mathematics. Constructing Knowledge for Teaching Secondary Mathematics, 89-101. doi:10.1007/978-0-387-09812-8_6

International Financial Reporting Standards (IFRS) for Smes. (2015). Understanding IFRS Fundamentals, 383-393. doi:10.1002/9781119197690.ch41

Ko, H., & Youn, C. (2011). Effects of laughter therapy on depression, cognition, and sleep among the community-dwelling elderly. Geriatrics & Gerontology International, 11(3), 267-274. doi:10.1111/j.1447-0594.2010. 00680.x

Making Language Learning Strategies Research Useful: Insights from China. (n.d.). Language Learning Strategies and Individual Learner Characteristics: Situating Strategy Use in Diverse Contexts. doi:10.5040/9781350005075.ch-007

المراجع العربية

الوزان ، بلسم وليد مجيد .(2009) أثر استخدام نموذج فراير في اكتساب المفاهيم الرياضية لدى تلاميذ الصف الخامس الأبتدائي .رسالة ماجستير غير منشورة ، الجامعة المستنصرية : العراق .

حسين، خديجة. ( 2014 (. أثر استعمال نموذج فراير في تصحيح الأخطاء الشائعة لدى طالبات الصف الثاني المتوسط في المفاهيم الكيميائية. مجلة جامعة بابل، العلوم الانسانية ،22 (1) ،196-218

السامعي، قائد محمد. ) 2003 (. أثر نموذج اكتساب المفاهيم والتعميمات في تحصيل الرياضيات. رسالة ماجستير غير منشورة، العراق: جامعة بغداد.

فندي، أسماء وغيدان، سهام. ( 2011 (. اثر نموذجي الانتقاء وفراير في اكتساب المفاهيم النحوية لدى طالبات الصف الاول المتوسط81-33. . ، مجلة الفتح، 1

# Appendices

## Appendix (A): List of arbitrators for study tools.

| N | Name | Occupation | Workplace |
|---|---|---|---|
| 1 | Ferrous Rahmouni | English Specialist | Ministry of education |
| 2 | Maha Aqle | Mathematics Specialist | Ministry of education |
| 3 | Lolwa Mohamed Al-Kubaisi | Mathematics Specialist | Ministry of education |
| 4 | Zaalit Izouaouene | Mathematics Specialist | Ministry of education |
| 5 | Nessrin Shousha | Mathematics Specialist | Ministry of education |
| 6 | Sumaya Alyafei | Mathematics Specialist | Ministry of education |
| 7 | Ruba Tahaineh | Mathematics Specialist | Ministry of education |
| 8 | Noor Al-Ansari | Mathematics Specialist | Ministry of education |
| 9 | Eman Maki | Mathematics Specialist | Ministry of education |
| 10 | Dr. Rania Albakoor | Mathematics Specialist | Ministry of education |
| 11 | Abdel Fattah Khalil Zaghloul | Mathematics assessment specialist | Ministry of education |
| 12 | Afrah Johar | Mathematics Coordinator | Omar Bin Al Khattab Primary |
| 13 | Raeeda alsalah | Mathematics Coordinator | Khadija bint Khuwailid Primary |
| 14 | shahd hannun | Mathematics Teacher | Omar Bin Al Khattab Primary |
| 15 | Ghada Garar | Mathematics Teacher | Khadija bint Khuwailid Primary |
| 16 | Dr. Mahmood Ahmed Ahmed | Senior Researcher | Qatar University |
| 17 | Mohammad Mollazehi | MASTER STUDENT | College of Arts and Sciences |
| 18 | Dr. Areej Isam Barham | Associate Professor of Mathematics Education | College of Education |

Appendix (B): Unit Plan

| Number of Classes (10) | Pages (193 – 246) | Unit 5 "Multiplication properties: Basic multiplication facts of numbers 3,4,6,7,8." | Grade three First Semester |
|---|---|---|---|
| Apply Multiplication properties in 0-1 / Distributed properties / Apply properties Number 3 as factor / apply properties Number 4 as factor / apply Properties Number 6 and 7 as factors / Apply properties Number 8 as a factor / Practice multiplication fact - Repeated logical justification. Vocabulary included in life matters (bowls - socks - brick molds - octopus - ceramic pieces - fig fruit - boats - outer space - stickers - lamps - apples - dragon snails - sensors) | | The main and sub-headings in the module or lesson and new words | Vocabulary |
| Counting - one property in multiplication – zero property in multiplication - substitution property … - distribution property - factors - multiplication - product - double - the numerical sentence | | Words or terms have verbal and mental connotations | Concepts and terminology |
| *The distribution properties states that a fact can be divided into two multiplied facts by the same number, equal to the result of combining the fact of the original multiplication. *Use multiplication facts for number 1 and facts multiplication for number 2 to find the result of multiplication in number 3. Use double multiplication fact to number 2 to find the product in number 4 -Use multiplication facts for number 5 and multiplication facts for number 1 to find the product in number 6. Use multiplication facts for number 5 and multiplication facts for number 2 to find the product in number 7. Double the multiplication facts for number 2 to find the product in number 8. Use double multiplication fact to number 4 to find the product in number 8- | | Facts are data, events or phenomena that prove their validity. Ideas Set general facts that explain phenomena or relationships. | Facts and ideas |
| The multiplication symbols x … Equalization= .°°°°°°°Addition + ….. Line numbers - Cubes – Array - Images of the environment during the presentation of life issues | | Forms, drawings or images expressing scientific ideas or schemes containing knowledge. New icons that support scientific content | Shapes, images, and symbols |

| | | |
|---|---|---|
| Used distribution property to find the missing factor in multiplication fact.<br>Use distribution property to find the product./ Using Array to Represent Multiplication Facts in number 3 / Using Double in finding the product of number 4. / Using multiplication facts in number 2 to find the product in number 4.<br>Using modeling to represent and solve multiplication problems.<br>Using facts multiply the number 2-4 in finding the multiplication result of the number 8. / Use known facts and strategies to find product | The mental and scientific practices of the students and the exposure of the students to educational experiences intended and planned | Skills |
| Objects are used as integer (cubes / counter / grid / line numbers / objects) to present multiplication facts in numbers 3-4-6-7-8<br>Activities using storytelling of some problem solving.<br>To attract attention. / Activities in which the student uses modeling and tools to represent multiplication facts. / Activities related to the use of strategies and use tools to solve issues including multiplication in numbers 3-4-6-7-8.<br>Activities include exploration using different methods to find the product. | All student activities are supported by the teacher and serve the scientific content | Activities |

| | | |
|---|---|---|
| Value of tender / Cleanliness / The Preservation of the environment.<br>Maintain health.<br>The Importance of Sports P 208 (Soccer - Jumping the Rope)<br>Healthy Eating (Fruits ... p. 214-212)<br>Module project: inherited traits | Values represent the criteria by which attitudes or behavior are judged.<br>The trend is an individual concept that determines human tendencies | Values and trend |
| ➢ Numerical (mathematical skills).<br>➢ Linguistic (Vocabulary and terminology).<br>➢ Problems Solving.<br>➢ Cooperation and participation (in various activities).<br>➢ Communication.<br>➢ Creativity and critical thinking (higher thinking skills).<br>➢ Research and investigation (unit project). | A combination of knowledge, skills and trends that are appropriately used or applied in a context | Competencies |

Appendix (C): Mathematical Concepts Test

**Mathematical Concepts Test**

**Pre-test**

**School year 2018 - 2019**

## Question 1: vocabulary

Select the appropriate term from the box and type it in the blank space:

| Arrays ..... One properties ..... Product ..... Commutative property ..... Factors |
|---|

a) Numbers you multiply together to get another number called ----------

b) The result you get when you multiply numbers together called ----------.

c) "property that the factors can be multiplied in any order, and the result of the multiplication remains the same called ----------

---

## Question 2 : Multiplication as Repeated Addition :

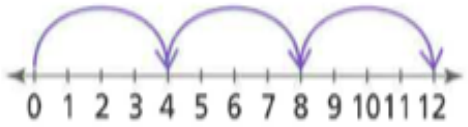
Complete each numeral sentence:

| | |
|---|---|
| b) ..... 2 + 2 + 2 + 2 = 4 X ------ | a) ... 2 x 6 .... = ..... -------- ... + ..... --------- |

---

## Question 3: Multiplication using number line:

A) Abdullah drew the line of numbers shown, what is the multiplication facts shown by the number line?




| | | | |
|---|---|---|---|
| A) ……3 x 5 = 15 | B) ……3 x 4 = 12 | C) ……3 x 3 = 9 | D) ……3 x 6 = 18 |

B) Explain the multiplication facts on number line, then find the product?

| | |
|---|---|
|   | ……3 x 2 = ……. |

---

## Question 4: Properties of Multiplication:

| | |
|---|---|
| A) ……10 x 1 = ……. | B) ……0 x 5 = ……. |

---

**Question 5: Multiplication Using Arrays::**

**How can I represent 6 x 3 using an Array? Draw a grid and explain how to use it to find the product?**

Scoring Sheet (pre-test)

| Questions | Mathematical Concept | Question mark | Student Marks |
|---|---|---|---|
| 1 | Vocabulary of multiplication | 3 | |
| 2 | Multiplication as repeated addition | 2 | |
| 3 | Multiplication using a number line | 2 | |
| 4 | Properties of Multiplication | 2 | |
| 5 | Multiplication Using Arrays | 1 | |
| Total | | 10 | |

# Mathematical Concepts Test

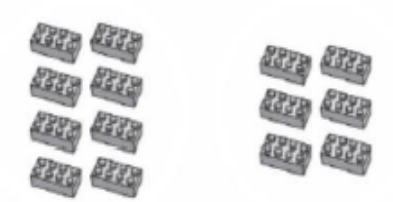
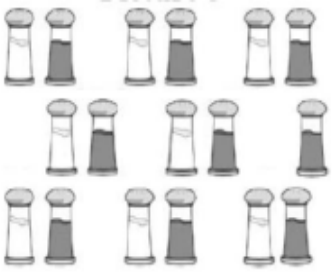
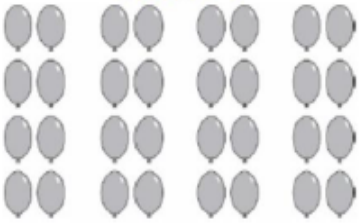
## School year 2018 - 2019

| Name | | Grade | Third | |
|---|---|---|---|---|

| | Question number | Evaluation Items | Marks | Student Mark |
|---|---|---|---|---|
| 1 | A<br>B | Multiplication with equal groups | 4 | |
| 2 | - | Multiplication using repeated addition | 4 | |
| 3 | A<br>B | Multiplication using Array. | 4 | |
| 4 | A<br>B | Multiplication using Number line. | 2 | |
| 5 | A<br>B | Multiplication using patterns & factors. | 4 | |
| 6 | A<br>B | Multiplication using Distributive property. | 2 | |
| 7 | - | Multiplication properties. | 4 | |
| 8 | - | Missing number. | 2 | |
| 9 | A<br>B | Problem Solving multiplication | 4 | |
| | Final Marks | | 30 | |

2

Question 1: a) Select Images that have Equal Groups ?

| | |
|---|---|
| Picture 1  | Picture 2  |
| Picture 3  | Picture 4  |

B) Find the Product by Drawing Equal groups ?

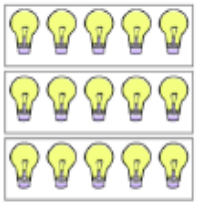
2

4 × 6 = _____

4

**Question 2 :** Write the Sentences of the Addition and the Multiplication for all of the following.

a.



_____+ _____ + _____

-------- x --------- = -------- Lamps

b.

Two groups each of them has 6 balls.

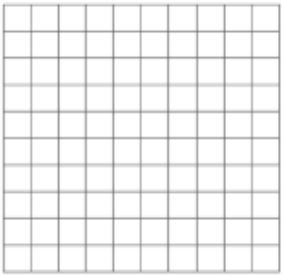
_____+ _____

-------- x --------- = -------- Balls

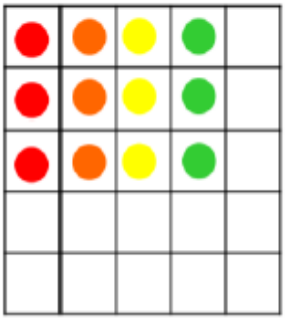
**Question 3:** a) Draw a grid of squares to represent the fact of the multiplication, then write the product?

2

2 X 5 = --------



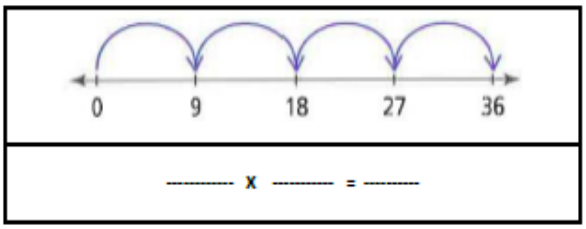
B) Write the multiplication fact that represents the following grid?



___ X ___ = ____

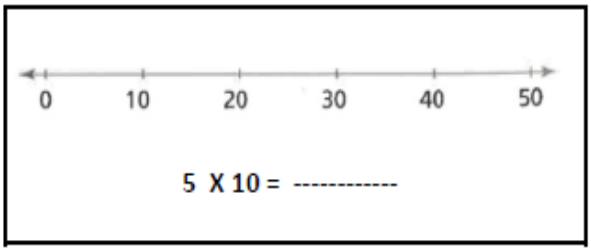
2

Question 4: a) What the product shown by the following number line?




2

B) Explain the multiplication fact by using number line, then write the product ?




1

Question 5 : a) Choose the Multiples of number 7 of the following numbers:

| | | | | |
|---|---|---|---|---|
| 24 | 35 | 50 | 56 | 62 |

2

B) Which of the following numbers is a factors of number 18 ?

Number Factors 18

---------- X -------- = 18

- ◯ 2
- ◯ 3
- ◯ 6
- ◯ 8
- ◯ 9

2

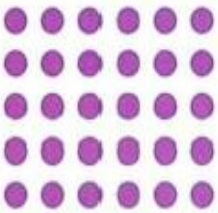
1

Question 6: a) Choose the correct segmentation represented by the following grid :

| | |
|---|---|
|  | ☐ (2 X 7) + (5 X 7)<br>☐ (4 X 7) + (1 X 7)<br>☐ (2 X 5) + (5 X 7) |

B) Divide the grid into two partial grids and write the multiplication fact that represents each partial grid?

1



5 X 6 = ( ---- X -----) + ( ---- X -----)

4

Question 7: Compare , Write < or > or = in the following sentences?

| | |
|---|---|
| A) 0 X 6 ◯ 0 X 0 | C) 1 X 7 ◯ 5 X 1 |
| B) 3 X 0 ◯ 1 X 3 | D) 2 X 4 ◯ 4 X 2 |

**Question 8:** Find the missing number in the following sentences?

2

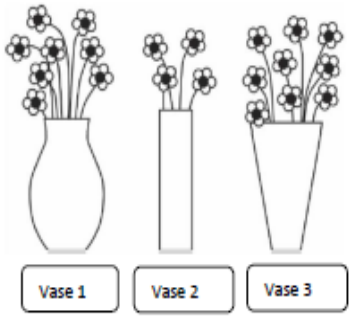
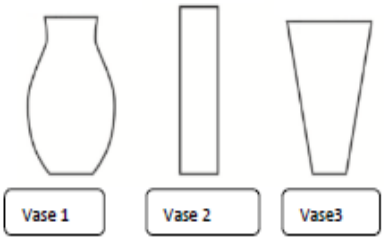
| | |
|---|---|
| A) 5 X --------- = 25 | B ) 9 X 3 = --------- |

---

**Question 9:** A) Jassim wants to re-distribute the flowers so that each vase has the same number of flowers.

1- Draw the flowers with the new distribution?

2

| Jassim Flowers | Distribution of new flowers |
|---|---|
|   |   |

1 - write the multiplication fact , which represents the arrangement of new flowers?

_____ × _____ = _____

B) Saeed has some tomatoes arrange it in 3 rows, and 8 columns.

| Part A | Part B |
|---|---|
| A) Find the total number of the tomatoes? | B) Draw the Problem any way you choose. |

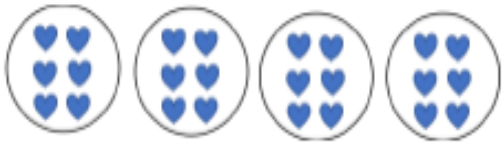
| |
|---|
| 2 |

**Questions Ended**

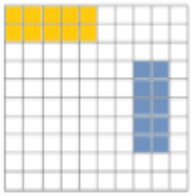
## Mathematical Concepts Test

## Mathematics (Model Answer)

## School year 2018 - 2019

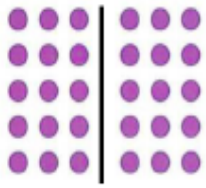
| Question 1 | Question Marks: four . |
|---|---|
| A) Main item: Select the image that contains an equal group (Picture 1)<br>Main item: Select the image that contains an equal group (Picture 4)<br>B) Main item: Draw four groups each of them contains six or draw six groups each of them contains four correctly or in any other way considered correct<br>  <br>Main item: Write the product 24 | |
| 4 Marks | Four main items are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items are true. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

| Question 2 | Question Marks: four |
|---|---|
| A) Main item: Write the addition sentence correctly 5 + 5 + 5.<br>Main item: Write the product correctly 5 x 3 = 15 lamps.<br>B) Main item: Write the addition sentence correctly 6 + 6.<br>Main item: write the product correctly 6 x 2 = 12 balls. | |
| 4 Marks | Four main items are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

| Question 3 | Question Marks: four |
|---|---|
| A) Main item: Draw the grid correctly.<br>Main item: Write the product 10.<br><br>B) Main item: write the multiplication fact correctly 3X4 or 4X3<br>Main item: write the product correctly 12.  | |
| 4 Marks | Four main items are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| Zero | There are no correct items. |

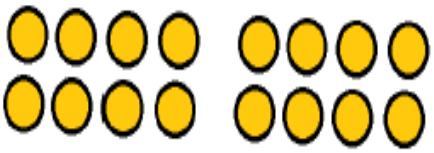
| Question 4 | Question Marks: Two |
|---|---|
| A) Main item: Write the multiplication sentence correctly 9 X 4 = 34<br><br>B) Main item: Write the product using the number line 50   | |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

| Question 5 | Question Marks: Four Marks |
|---|---|
| A) Main item: Choose the first multiples correctly 35.<br>Main item: Choose the second multiples correctly 56.<br>B) Main item: Write the first factor of number 18 correctly (2).<br>Main item: Write the second factor of number 18 correctly (9).<br>Or choose factors 3 x 6 | |
| 4 Marks | Four main items are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

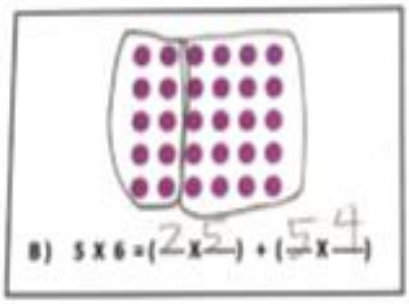
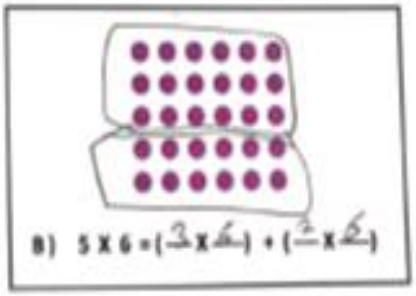
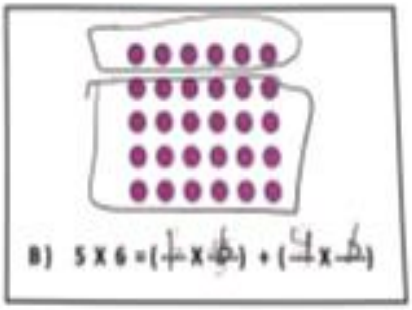
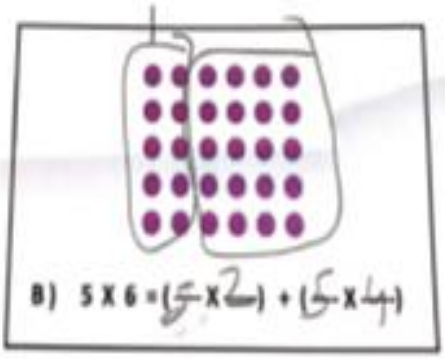
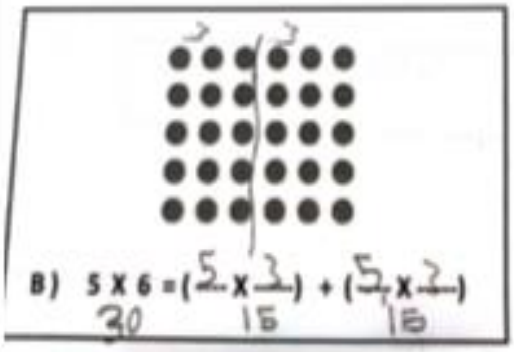
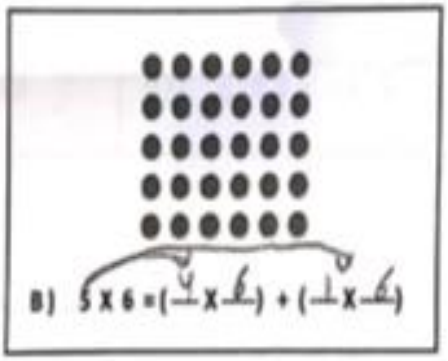
| Question 6 | Question Marks: 2 |
|---|---|
| A) Main item: Select the correct segmentation (4 X 7) + (1 X 7)<br><br>B) Main item: Write the correct segmentation for the shown grid.<br>(5 x 3) + (5 x 3) or (1 x 6) + (4 x 6)<br>Or any other method considered correct  | |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

| Question 7 | Question Marks: four |
|---|---|
| Main item: 0 = 0<br>Main item: 7 > 5<br>Main item: 0 < 3<br>Main item: 8 = 8 | |
| 4 Marks | Four main items are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

| Question 8 | Question Marks: Two |
|---|---|
| A) Main item: Write the missing number correctly 5.<br><br>B) Main item: Write the product correctly 27. | |
| 2 Marks | Two main items are correct. |
| 1 Marks | One main item is correct. |
| zero | There are no correct items. |

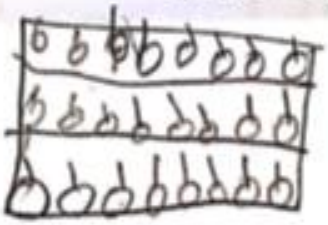
| Question | Question Marks: four |
|---|---|
| A) Main item: Arrange the new flowers so that there are seven flowers in each vase.<br>Main item: Write the multiplication sentence after arrangement 7 X 3 = 21<br>B) Main item: Write the multiplication sentence which represented by problem 3 X 8 = 24<br>Or in any other manner considered correct.<br>Main item: Represent the problem through drawing correctly .<br>  <br>Or in any other form considered correct (by pitchers - grid - number line ) | |
| 4 Marks | Four main item are correct. |
| 3 Marks | Three main items are correct. |
| 2 Marks | Two main items correct. |
| 1 Marks | One key item is correct. |
| zero | There are no correct items. |

Good luck....

| Samples of student answer Question 6 | |
|---|---|
|   |   |
|   |   |
|   |   |

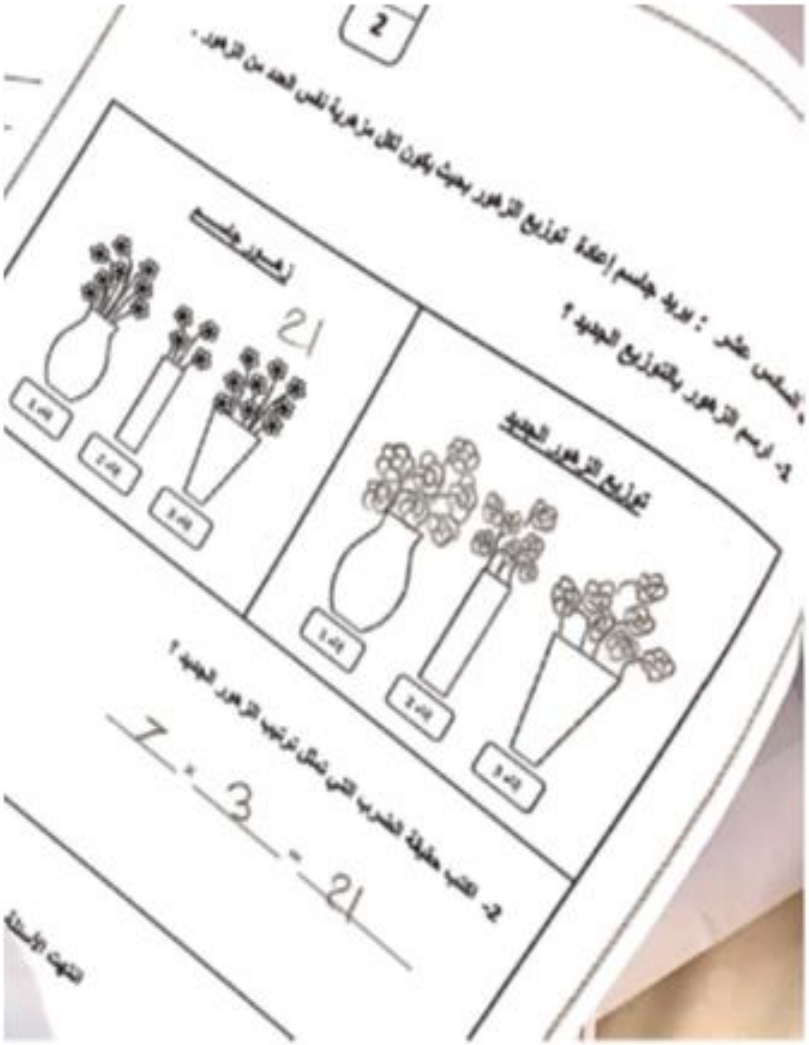
## Samples of student answer Question 9

| الجزء B | الجزء A |
| --- | --- |
| أمثل المسألة بأي طريقة تختارها | أوجد العدد الكلي لحبات الطماطم ؟ |
| $8 \times 3 = (4 \times 3) + (4 \times 3)$ | 24 |

| الجزء B | الجزء A |
| --- | --- |
| أمثل المسألة بأي طريقة تختارها | أوجد العدد الكلي لحبات الطماطم ؟ |
|  | $3 \times 8 = 24$<br>$8+8+8=24$ |

| الجزء B | الجزء A |
| --- | --- |
| أمثل المسألة بأي طريقة تختارها | أوجد العدد الكلي لحبات الطماطم ؟ |
| $3 \times 8 = 24$<br>$(1 \times 8) + (2 \times 8) = 24$<br>$8 + 16 = 8$ | $3 \times 8 = 24$ |

**Samples·of·student·answer·in·problem-solving·**



Appendix (D): Test Feedback

**Notes regarding the both tests for measuring the**

**mathematics concepts - for the third elementary grade**

Firstly, I agree with the point of view that suggested that the test of measuring mathematics concepts is conducted through essay questions, for measuring accurately, because the student may choose to answer in multiple-choice questions correctly but does not express that he has the concept.

**The first test (consists of 20 multiple choice paragraphs) :**

- Modifying the number of objective questions from 10 to 20 in the instructions of starting questions, since the test consists of 20 questions.
- Please note that the current educational source, which is available for the students, does not have bilingual terms, so please modify this in questions Q2, Q19, Q20.
- Please note that the number of choices in math tests prepared by the Student Assessment Department for the first three grades, consist of three choices.
- Please note that multiple-choice questions are in an imperative form or question form rather than a completion form. When the question is in an imperative form, a point is be placed at the end of the question (.). In addition, when the question is in a question form, a question mark is be placed at the end of the question (?).
- Please note the standardization of the formatting of the choices codes A, B, C, D so that the same alignment will be in all questions.
- Please correct answer of Q11and Q8 in the answer form.
- It is preferable to avoid as much as possible the theoretical questions that measure the understanding of terms, but it is replaced by a numerical fact, then the student is requested to write the number that expresses the term. Therefore, many students do not have enough ability to read and write correctly.

- Please note that the currency symbol precedes the number such as (QR 15).
- Please note that the text of the question is in (Bold). The first word in the question is two or three spaces ahead of paragraph numbering in questions 10 to 20.
- Please note that the choices in the objective questions are derived from the common mistakes of the students.
- Please note that the pictures are clear in Q17, Q14.

**The second test (consists of 16 essay questions)**

- Please note that the pictures are clear in Q7, Q5, and Q1.
- Please note that when the question is in an imperative form, a point is placed at the end of the question (.), whereas the question is in a question form, a question mark is placed at the end of the question .(?)
- Please note that the mathematical relationships in Q3 are modified, Where the mathematical relationships are always written from the left not from the right.
- Please note that the grade estimation guide is written as follows in the following form:

| **Question Number 3** | **Question degree: 4 points** |
|---|---|
| **Main element:** Writing the Addition operation correctly<br>5 + 5 + 5<br>**Main element:** Writing the multiplication fact correctly<br>3 x 5 = 15<br>**Main element**: Writing the Addition operation correctly<br>6 + 6<br>**Main element**: Writing the multiplication fact correctly<br>6 × 2 = 12<br>Other solution methods will be observed. | |
| 4 points | Four correct main elements |
| 3 points | Three correct main elements |
| 2 points | Two correct main elements |
| 1 point | One correct main element |
| Zero | There are no correct elements |

- Please modify Q6 in the answer form and write (Groups) instead of (Group) in Q1, write (Factors) instead of (Multiples) in Q8, and the currency symbol shall precede the number such as (QR 32) in Q15.
- Please note referring to the other solution methods in the questions that have more than one way to answer them in the grade estimation guide (as in Q2, for example, there are other methods of drawing and solving).
- Please consider the distribution of scores in questions require drawing with the writing of the mathematical sentence or the result, so that part of the score is allocated to the drawing.
- Please note that it is expedient to include Q12 before Q15 as it is a real-life question and the student may use the grid (Q4) or the numbers line (6), but the multiplication through the distribution method (multiplication using segmentation method) is not used.
- Emphasis on reviewing and minimizing the questions in the test So that it takes time to conduct the test and focus on the basic concepts of the multiplication process.

**Mathematics assessment specialist**

**Abdel Fattah Khalil Abdel Fattah Zaghloul**

**Monday, 2018-12-3**

Appendix (E): Samples lesson plan

جامعة قطر
QATAR UNIVERSITY
كلية التربية
قسم العلوم التربوية

Mathemtics Model Lesson Plan

Name: Muneera Jassim Al dehneem Date: 22-11-2018

The assistant teacher:----------

Number of students: 27 students Grade: grade three

Time: 50 mins Topic: Apply the properties of number 4 As a factor.

| The Goals: | The Qatari Standards: |
|---|---|
| By the end of this lesson, most students should be able to:<br>1.Use his/her knowledge about the multiplication by 2 to find the multiplication by 4.<br>2. Use the properties of multiplication by 4. | 3.3.1 Can build knowledge about the multiplication facts up to 10X10 and memorize them very well. |
| The Materials:<br>Data show-presentation- flashcards-mini boards- multiplication cards. | Learning Resources:<br>Student's book- teacher's guide- educational sites (supporting resources). |
| Keyword/ vocabulary:<br>عوامل: Factor<br>multiply : الضرب<br>الشبكة: Array<br>The Distributive property: خاصة التوزيع | |

Starter: learning by games (the competency is: communication)/ 10 mins

The objective: to revise the multiplication by 2,3 and 5.

-Teacher's role: Teacher displays an interactive game about multiplication and chooses some students from each group to participate. The teacher explains the activity to students.

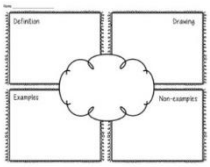
Student's role: students listen to the teacher's explanation.

http://webcdn.abcya.com/games/math_bingo.htm

-Teacher says: "we are going to summarize the previous lesson by doing this activity in groups".

-Teachers give instructions and explain the activity to students. She displays the Frayer model on the board (reference 1 (R1) and explains that the purpose of the Frayer Model is to identify and define unfamiliar concepts and vocabulary. Students define a concept/word/term, describe its essential characteristics, provide examples of the idea and suggest non examples of the idea (knowing what a concept isn't helps define what it is).

This information is placed on a chart that is divided into four sections to provide a visual representation for students.
The four sections/ parts are:



1. The definition of multiplication by 3
2. The characteristics of the multiplication process (facts)
3. Write an example of the multiplication by 3.
4. Write an example which is not applicable to the concept of multiplication by 3.

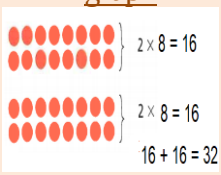
-Then, teacher distributes the Frayer Model papers and asks students to stick the papers in the right place on the Fryer model (each group will have a worksheet and small papers).

- Teacher assesses the Ss' using the checklist to check their understanding and clarify more activities during the lesson for low achievers.

-Teacher allocates the time required for the activity and then provides the feedback by listening to the students' answers.

| Student's role: | (Teaching and learning strategies): 10 mins<br>Teacher's role: |
|---|---|
| | Objective 1: Student uses what he/she knows about multiplication by number 2 to find the multiplication by 4<br>Activity 1: guided/ group work<br>Solve and participate (the competency is: problem solving). |
| -Use the Frayer Model. | -Teacher uses the Frayer model and use it during the lesson. (She might draw the form on the right side of the board (or print it on A3 paper) and stick it on one side of the board. |
| -Listen to the teacher's explanation. | -Teacher displays the activity on page 209:<br>  <br>Khaled made 8 hangers every week for 4 weeks. How many hangers did Khaled make? I solve this activity in any way I choose. |
| - Students read the activity silently. | -Teacher asks students. to read the activity silently. |
| -One student read the activity loudly. | -Teacher asks one student to read it loudly. |
| -Students answer the comprehension questions: | Before the problem: |

| | |
|---|---|
| 1. 8 hangers.<br>2. 4 weeks.<br>3. Find the number of the hangers made by Khaled.<br>4. The multiplication.<br>-Students use the mini boards to write<br>the multiplication required to find the solution:<br>4 × 8<br>While solving the problem/ issue:<br><br>-Students read the information in the cloud to identify the appropriate tools for the solution. | -Teacher asks students some comprehension questions:<br>1- How many hangers does Khaled make in a week?<br>2- How many weeks did Khaled work?<br>3- What is required to be done?<br>4- Which process do you need to resolve the issue?<br>5- Write the multiplication fact.<br>Objective 2: use the properties of multiplication by 4.<br><br>While solving the problem/ issue:<br><br>-Teacher displays the cloud The cloud to identify the appropriate tools for the solution.<br>  <br>I can use the structure in the solution. What are the relationships I notice when I multiply by 4? I show my work in the space below. |
| -Students use the distribution to find the output.<br><br>-Students think about distributing number 4 into 2, 2. | -Teacher asks students:<br>1- How does the distribution help me to find the output?<br>2- The teacher takes the answers from the students and writes them in the Frayer Model inside the definition box.<br>3.What are the multiplication facts you know? And how they can help me to solve the issue.<br>- The teacher takes the answers from the students and writes them in the Frayer Model inside the properties box to enable students to understand the model and the sequence of using the information to solve the problem/ the issue.<br>- Then, teacher distributes the Denis cubes to count. She gives the instructions and explains that these cubes can help students to represent the solution. (students can draw the |

| Properties (facts) | Definition |
|---|---|
| 4 is the double of 2<br>The output of 4x8 is the double of the output of 2 x8 | The distribution property enables me to break the fact of multiplication into the sum of the two multiplication facts for the same number.<br>So, to multiply by 4 I need to think of the facts of number 2 and then reduplicate it. |
| No example:<br>4+ 8 =12 | Example: I can create/ use the graph<br>  |

Collaborative: Pair work: (student's book page 209)

-Students work in pairs to solve the question (Look again) with a suitable time limit. After that, one of the students solve the question and explain the reason.

Enrichment activity/plan:

-If students didn't understand well, teacher might present another example using their books. (the source book)

graph)

- After that, teacher askes students to work in groups to solve and find the answer and then write it using the mini boards.

After solving the problem/ issue

-Teacher gives two groups of students the opportunity to present their works.

- Teacher displays the correct solution/answer inside the example box.

-Teacher writes the incorrect solutions/

> The teacher instructs the students to employ all possible methods during presentation of the illustration such as ( Equivalent Group – Number Line – Repeated Addition – Distributive property – Array )
> During the application of the Frayer model

answers in the field of no example in the Frayer Model.

Collaborative: Pair work: (student's book page 209)

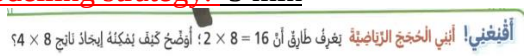
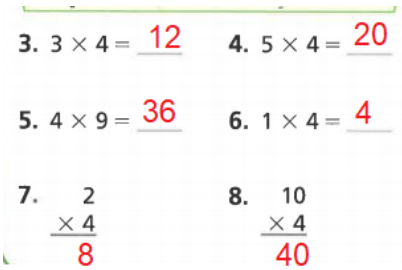
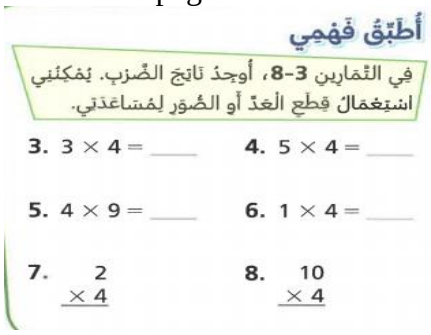
-Teacher asks students to work in pairs to solve the question (Look again) with a suitable time limit. After that, she asks one of the students to solve the question and explain the reason, while asking the rest of the students about their agreement with the given answer.

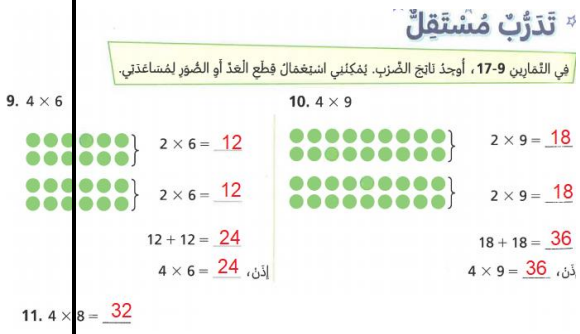
Low achievers:

-Teacher provides the Denis cubes to count to facilitate the multiplication process.

-Teacher reminds students of the multiplication facts of number 2. It can be written and placed on the board to use it/ refer back to it during the lesson.

| | |
|---|---|
| -Double the result of multiplication facts of 2 to find the corresponding facts for 4.<br>For example, double the output of multiplying 4 × 6 الى 2 × 6<br><br>-Students answer the main/ basic question based on their understanding. | -Teacher asks the main question for this lesson : 10 mins<br>How you can use the double to multiply by 4?<br>-Teacher listens to several answers from the students and says that: "we will learn more about this by solving the following example Page 210".<br><br>Activity 2: think, pair and share strategy<br>The competencies are: cooperation and ) (participation/ problem solving).<br><br>-Teacher displays the example on the board and asks one student to read the question loudly. Then, she reads the question again. |
| - One student read the question loudly. |  <br>In the arts class, Noura painted some plastic dishes. If she painted one dish every day for four weeks, how many dishes were colored by Noura? |
| -Students answer the questions:<br><br>1.One dish every day.<br>2. 7 dishes.<br>3. Find the number of dishes painted during the week.<br>4. Multiply 7x4<br>-Students listen to the teacher's instructions.<br><br>Think: students think for one minute to solve.<br>Pair: students discuss the solution in pairs for two minutes.<br>Share: each group should discuss the solution and present one answer/ one model. Then, each group presents their answers on the board for discussion. | -Teacher asks students some comprehension questions:<br><br>1-How many dishes does Nora color in a day?<br>2- How many dishes does Nora color per a week?<br>3-What is required?<br>4- How we can find the solution?<br>-The graph consists of 4 rows and in each row 7 elements, that means 2 of 7 plus 2 of 7.<br>-Teacher explains to students that number 4 is the double of number 2 and therefore we can use the multiplication facts of 2 to find the multiplication facts of 4.<br><br>-Teacher gives the instructions before starting to distribute the Frayer model and students use it to solve. |

<table>
<tr>
<td>
<table>
<tr><td>Properties/ facts<br><br>4 is the double of 2<br>The output of 4x7 is the double of the output of 2 x7</td><td>Definition<br><br>To multiply by 4, I need to think of the facts of number 2 and then reduplicate it.</td></tr>
<tr><td>No example:<br><br>4+ 7<br>or<br>14x14</td><td>Example: I can create/ use the graph<br><br> 
</td></tr>
</table>
</td>
<td>- Teacher uses the timer for the individual reflection (one minute) and then asks students to discuss the solution in pairs for two minutes. After that, each group should discuss the solution and present one answer/ one model (for one minute).<br><br>-After they have finished, teacher asks the groups to present their answers on the board for discussion.</td>
</tr>
<tr>
<td>Individual activity: convinced me/ the modelling strategy. 5 min<br>-Students read the question (convinced me) on page 210<br>4 x 8<br>2 x 8 (is the double of the fact)<br>2 x 8 = 16<br>Then:<br>16 + 16 = 32<br>-</td>
<td>Individual activity: convinced me/ the modelling strategy. 5 min<br>  <br>أَقْنِعْنِي! أَبْنِي الْحُجَجَ الرِّيَاضِيَّةَ. يَعْرِفُ طَارِقٌ أَنَّ 8 × 2 = 16؛ أُوَضِّحُ كَيْفَ يُمْكِنُهُ إِيجَادُ نَاتِجِ 8 × 4؟<br>Convince me! I build mathematical arguments. Tariq knows that: 8X2=16 Explain how he can find the output of: 4X8?<br>-Teacher asks students to solve the question (convinced me) on page 210 using the concept of paired (double) to find the output of the multiplication by drawing a Frayer Model and filling it.</td>
</tr>
<tr>
<td>Collaborative: Pair work: Active the source of learning: 5 mins<br>-Students work in pairs to answer the questions from 8-3 on page 211.<br>-The model answer:<br> 
<br><br>-Students discuss the answers on</td>
<td>Collaborative: Pair work: Active the source of learning: 5 mins<br>-Teacher asks the questions from 8-3 on page 211.<br>  <br>أُطَبِّقُ فَهْمِي<br>في التَّمارينِ 3-8، أوجدُ ناتجَ الضَّربِ. يُمْكِنُنِي اسْتِخْدامُ قِطَعِ العَدِّ أو الصُّوَرِ لِمُساعَدَتِي.<br>3. 3 × 4 = ____ 4. 5 × 4 = ____<br>5. 4 × 9 = ____ 6. 1 × 4 = ____<br>7. 2 × 4 8. 10 × 4<br><br>In the exercises from 3-8, I find the result of the multiplication. I can use the count pieces or the images to help me.</td>
</tr>
</table>

| | |
|---|---|
| the board and check their answers. | - Teacher monitors the class with no involvement to see their production and assess their learning. |
| Independent practice:<br>Individual activity: (activities with different levels)<br>1. High achievers: will answer question 9 and 10.<br>2. Mid achievers: will answer question 11.<br>3. Low achievers: will answer question 18.<br>-The model answer:<br>  <br><br>In the exercises from 9-17, I find the result of the multiplication. I can use the count pieces or the images to help me.<br><br>To answer question 18:<br><br>-Abdullah needs 4 boxes and each box has 6 pieces.<br>4x6=24 Pieces<br>-Students discuss the answers on the board.<br>-Students check their answers. | Independent practice: 10 mins<br>Individual activity: (activities with different levels)<br>-Teacher says, "Now it's time to work independently and do the activities on page 211, 212 (question 11,18,10,9)"<br>-Teacher assesses the students using the checklist to check their understanding and their achievement to the goal.<br>-Teacher allocates the time required for the activity and then provides the feedback by listening to the students' answers. Then, she discusses the answers and writes them on the board.<br><br>Note: teacher reminds students to use the substitutive feature to answer question 13.<br><br>Note for the teacher:<br>If the question is resolved by most of the students, the teacher simply asks one student to write the answer/ solution on the board.<br><br>Low achievers:<br><br>-Revise the multiplication facts represented by the graph.<br>-Revise the multiplication facts of number 2 |
| Accommodation and differentiation:<br>-Mental and oral tests<br>(Final exercises):<br>There are different levels of learning:<br>-<br>-Teachers forms different questions according to their students' needs. Students answer the given questions using the colored cards based on their levels. | Enrichment activities: (booklet)<br>-Teacher gives students the instructions and tells them that each student should open the booklet and write the following:<br>1. Goals learned today.<br>2. New vocabulary/ terms in this lesson.<br>- Teacher encourages students to choose more exercises from their books or from the high thinking questions and write them down in their booklets. |

Closure: 5 mins

-Teacher asks students:

-What do we learn today?

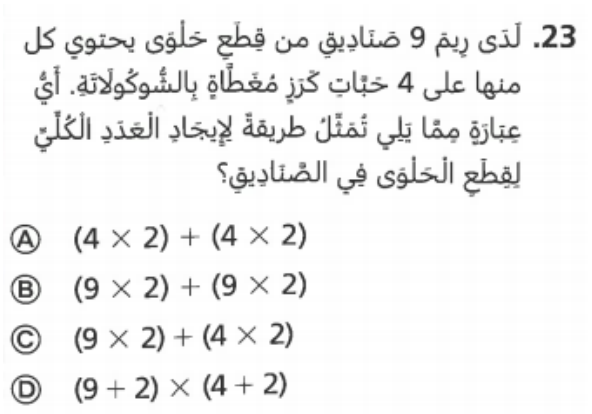
-Teacher goes through the objectives with the students and asks them to put 'thumb up/ down' to indicate whether or not they have achieved the objectives.

Evaluation/ Formative assessment

The right or wrong strategy:

-Teacher displays question 23:

**Reem has 9 boxes of candies each has 4 chocolates-covered with cherries, which of following phrases represents the way to find the total number of candies inside the boxes:**




-Teacher displays the paragraphs and asks students to read it and then raise the Yes card if the answer is correct or the No card if the answer is wrong.

-Teacher refers to the correct answer (B).

Homework:

page 213 from question 1 to 6.

Self-reflection:

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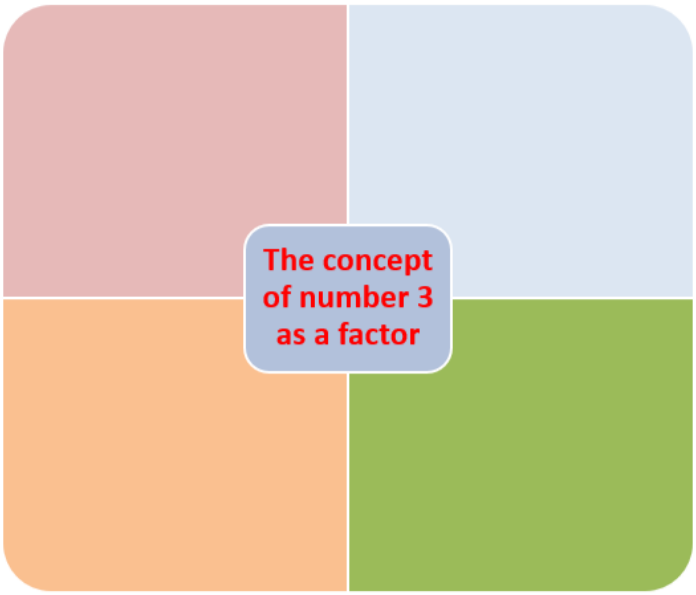
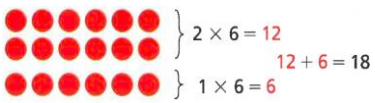
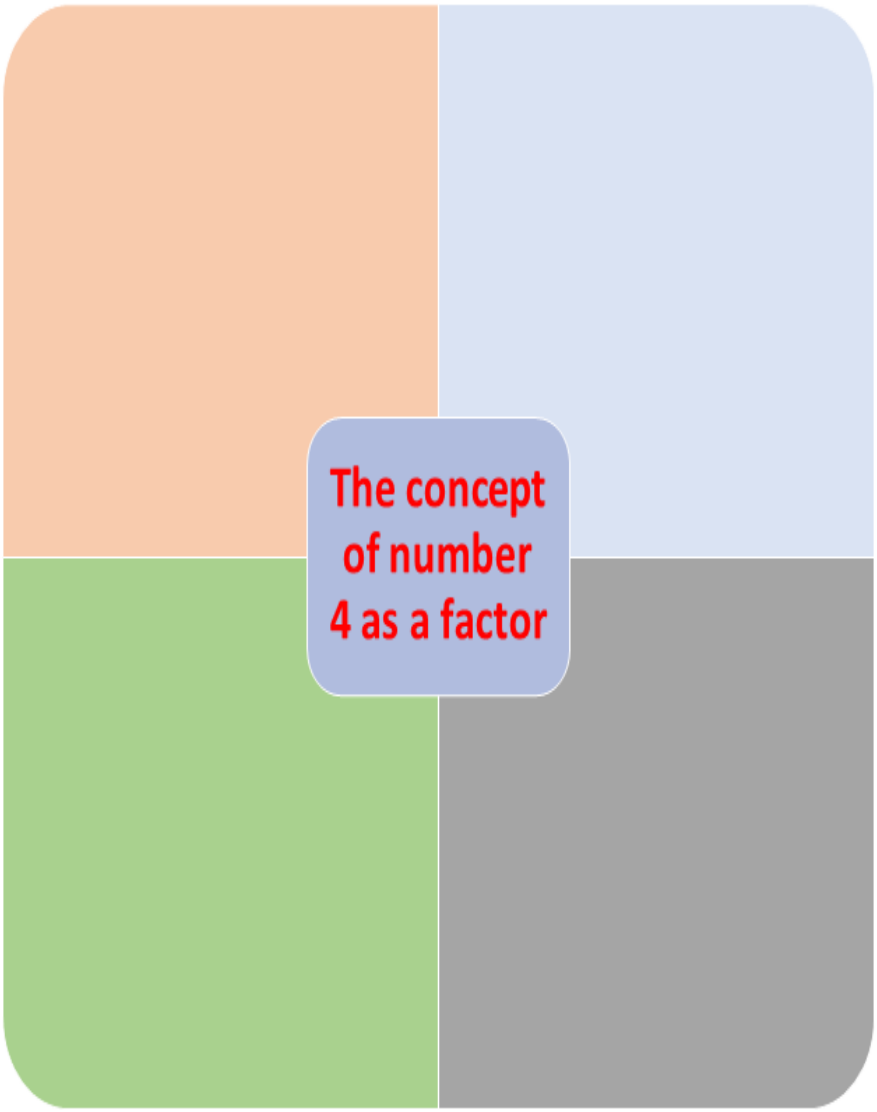
# Attached activities

Starter:




| | |
|---|---|
| -The multiplication facts of number 1<br>-The multiplication facts of number 2 |  |
| Multiplication means the process or fact of increasing the number or amount. (to sum the numbers more than once). | 6 + 3 =12 |
| The graph consists of three rows each one has six elements.<br>6x3 | |

**Activity 2: think, pair and share strategy**



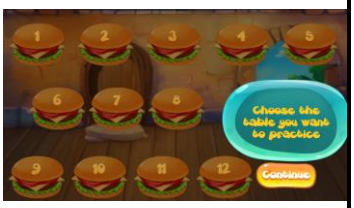

# Mathemtics Model Lesson ( 2 )

Name: Muneera Jassim Aldehneem Date: 22-11-2018

The assistant teacher: ----------

Number of students: 27 students Grade: grade three

Time: 50 mins Topic: Apply the properties of 8 as a factor.

<table>
<tr><td>The Goals:<br>By the end of this lesson, most students should be able to:<br><br>1. Use the known facts and properties to multiply by 8.<br>2. Use the multiplication facts of number 4 to find the multiplication of number 8<br>3. Use the multiplication facts for number 2 to find the multiplication of number 8</td><td>The Qatari Standards:<br><br>3.3.1 Can build knowledge about the multiplication facts up to 10X10 and memorize them very well.</td></tr>
<tr><td>The Materials:<br><br>Data show-presentation- flashcards- mini boards- multiplication cards.</td><td>Learning Resources:<br><br>Student’s book- teacher’s guide- educational sites (supporting resources).</td></tr>
<tr><td colspan="2">Keyword/ vocabulary:<br><br>-The factor: a number multiplied by another number.<br>-Multiplication: is the answer to the question of multiplication.<br>-The property of the neutral element (one) in multiplication: is the result of multiplying any number in 1 is the same number.</td></tr>
<tr><td colspan="2">  <br>Starter: the educational software strategy (the competency is: communication)/ 10 mins<br><br>-Teacher displays an electronic/ online game to revise what was taken in the multiplication table.<br><br>•https://www.timestables.com/happy-burger.html<br><br>Students will be randomly selected based on their -levels.<br><br>  </td></tr>
<tr><td>Student’s role:<br>-Use the Frayer Model.</td><td>(Teaching and learning strategies): 10 mins<br>Teacher’s role:<br>Objective 1: Use the known facts and properties to multiply by 8.<br><br>Activity 1: group work<br>Solve and participate (the competency is: problem solving).<br>-Teacher uses the Frayer model and uses it</td></tr>
</table>

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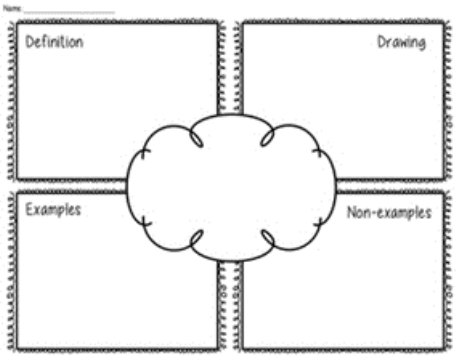
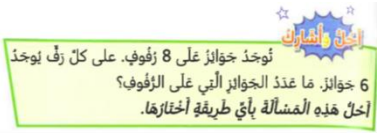
|  <br>Listen to the teacher's explanation. | during the lesson. (She might draw the form on the right side of the board (or print it on A3 paper) and stick it on one side of the board.<br>-Teacher displays the activity on page 221:<br>  <br>Solve and participate: There are prizes on eight shelves, on each shelf there are six awards/ prizes, how many prizes on the shelves? I can solve this activity in any way I choose.<br><br>Teacher asks students. to read the activity silently to identify the appropriate method for solving and thinking about solving the question on their own.<br>-Teacher asks one student to read it loudly.<br>Before the problem:<br>-Teacher asks students some comprehension questions:<br><br>- How many shelves?<br>- How many prizes per shelf?<br>- What is required to be done?<br>- Which process do you need to solve the issue/activity?<br><br>- Write the truth of multiplication.<br><br>-Teacher asks students write the answers using the small boards. |
| - Students read the activity silently.<br>-One student read the activity loudly.<br>-Students answer the comprehension questions:<br><br>- 8 shelves<br>- 6 awards<br>- Find the number of prizes on all shelves.<br>- Multiplication.<br>- Students write on the small boards<br>- Students write the fact of multiplication needed to find the solution:<br><br>8 × 6 | Objective 2: Use the multiplication facts of number 4 to find the multiplication of number 8.<br><br>While solving the problem/ issue:<br>-Teacher gives students an opportunity to work without guidance. Then she makes hints and encourages the test ideas.<br>-Teacher displays the cloud to identify the appropriate tools for the solution. |

While solving the problem/ issue:
-Students read information in the cloud.


I can understand the issues by using the known facts to solve the unknown facts. I show my work!

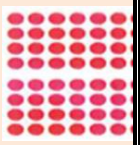
The teacher asks students some questions:
- How did you answer this issue?
- How can the graph be used for a solution?
- How does the distribution help me to find the solution?
- The teacher takes the answers from the students and writes them in the Frayer Model inside the definition box.
-What are the multiplication facts you know? And how it can help us to solve this issue.
- The teacher takes the answers from the students and writes them in the Fryer Model inside the properties box to enable students to understand the model and the sequence in using the information to solve the issue.
- Teacher distributes the worksheet containing the multiplication fact 6 × 8 and gives the count pieces.
-Teacher gives the instructions that it can help the students to represent the solution and use the graph and then divide it to use the distribution property in the solution.
- Then teacher asks students to work in group and write the solution on the small boards.

-Students discuss in groups and answer the questions:
-The graph or the distribution of 8 into 4 and 4.
- Build/ make a graph of 8 rows and 6 columns
-Split the graph into two parts and find the partial multiplication facts then collect/ sum the multiplication parts.

-Students learn the ways of solving the question.

-Students use the distribution feature to find a solution.

The teacher instructs the students to employ all possible methods during presentation of the illustration such as ( Equivalent Group – Number Line – Repeated Addition – Distributive property – Array )
During the application of the Frayer model

| Properties (facts) | Definition |
|---|---|
| Number 8 is the double of number 4<br>Double the multiplication fact of 4 to find the output of 8 x 6 | The distribution feature enables me to break/divide the fact of the multiplication into a sum of multiplication facts for the same number. The result equals to the sum of the combination of the original multiplication fact. Then multiply by 8 I should think about the facts of number 4 and then double it. |
| No example:<br>8 + 6 = 14<br>or<br>4 x 6 = 24 | Example: I can create/ use the graph<br>  <br>= ( 4 × 6 ) + ( 4 × 6 )<br>8 × 6<br>24 + 24 = 48<br>8 × 6 =48 |

After solving the problem/ issue

-Participate and discuss the answers by offering some answers from groups.

- Teacher displays the correct solution in the example box.

- Then she writes the correct answer in the definition box.

--Teacher takes one of the incorrect solutions/ answers and shows that it is written in the field of no example in the Frayer Model.

-Teacher displays the paragraph (look again):


I say/apply the general rule, explain how I can use the multiplication facts for number 2, 3 or 4 to solve the issue/ activity.

The card strategy:
-Teacher presents the following statements and asks students to complete them with the appropriate

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| -Students read the paragraph.<br><br>-Students present their ideas<br><br>-Students complete the blanks as follows:<br><br>-The multiplication facts for Number 2 are used when breaking the multiplication facts of numbers 6 and 8<br>- The multiplication facts for number 3 are used when breaking the multiplication facts for number --6--<br>- Multiplication facts for number 4 are used when breaking the multiplication facts for the number--8—<br><br>Enrichment activity/plan:<br><br>-If students didn't understand well, teacher might present another example using their books. (the source book page 242/ group D) | card:<br><br>- The multiplication facts for number 2 are used when breaking the multiplication facts of numbers---- a<br>- The multiplication facts for number 3 are used when breaking the multiplication facts for number ----<br>- Multiplication facts for number 4 are used when breaking the multiplication facts for number----<br>-The cards are:<br>8 ,7 ,6 ,8 ,7 ,6<br><br>Low achievers:<br><br>- Teacher asks students:<br>What are the complications of number 2? And what are the complications of number 3?<br><br>-Students answer:<br>-The complications of 2 are: 2, 4, 6, 8<br>-The complications of 3 are: 3, 6, 9<br>It can be written and placed on the board to use it/ refer back to it during the lesson.<br><br>  |
| Activity 1: dialogue and discussion strategy<br>The competencies are: ) | -Objective 3: Use the multiplication facts for number 2 to find the multiplication of number 8: 10 mins<br>Activity 1: dialogue and discussion strategy |

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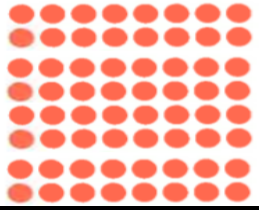
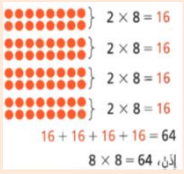
| cooperation and participation/ (problem solving).<br><br>–<br><br>– Different answers.<br><br>-Student reads the example clearly.<br><br>-Students discuss the example with the teacher.<br><br>- Students answer:<br><br>- The data are 8 rows and in each row 8 bowls.<br><br>- Required: How many bowls are there?<br><br>-Students answer:<br>The graph can be divided into two parts each with 4 rows and 8 | The competencies are: cooperation ) (and participation/ problem solving).<br>-Link between the students' thinking in (the solving and the sharing) and the concepts in the lesson.<br>-Teacher asks the basic question of the lesson on page 222:<br>-Teacher asks: How can I use the multiples (the doubles) to multiply by 8?<br><br>-Teacher listens to several answers from students and then explains that we will learn more about this by solving activity 1 on page (222):<br><br>-Teacher displays the issue using the data show and asks student to read it clearly:<br>  <br><br><br>**Student tried to throw a table tennis ball in a bowl at a school entertainment party. There were eight rows of bowls, each row had eight bowls, how many bowls did they have?**<br>**What are the multiplication facts of number 2, and the multiplication facts of number 4, which I can find in the bowl's graph?**<br><br>-The teacher asks the following questions:<br>- What are the given data?<br>- What is required?<br>- Teacher shows the graph:<br><br>  |

| Properties/ facts: | Definition: |
| --- | --- |
| 8 X8<br>Equal to 4 sets of<br>2 X 8 | In order multiply by 8, I think about the multiplication facts for number 2 and the multiplication facts of number 4. |
| No example:<br>8 =10 +2<br>Or<br>16X 2= 32 | Example: I can create/ use the graph:<br>  |

columns.

Or 4 graphs each with 2 rows and 4 columns.

-Students discover that there are two ways to solve the activity.

-Students listen to the teacher's instructions/ explanation.

-Students discuss the solution in groups.

-Students' solutions are presented and discussed:

-Teacher asks students how to divide the graph into equal graphs?

- Teacher gives the instructions before starting to distribute the Frayer model (attachment 1) to use it while solving the issue/ activity.

- Teacher uses the timer and asks students to discuss the answers in groups. Each group should choose only one method of presentation.

- After the specified time, teacher asks the groups to exchange forms between groups and each group evaluates the other group's performance (peer/colleague evaluation/assessment strategy).

- After that, teacher asks the groups to present their solutions on the board for discussion.

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| Properties/ facts:<br><br>Number 8 is the double of number 4<br>The double of the output of 4x8 is the output of 8 x 8 | Definition:<br>In order multiply by 8, I think about the multiplication facts for number 2 and the multiplication facts of number 4. | |
| No example:<br><br>12 =8 +4<br>Or<br>4X8 =32 | Example: I can create/ use the graph: | |
| Convinced me/ the think, pair and share strategy. 5 min<br><br>-A student reads the paragraph clearly<br>Think: Students think individually about the solution for half a minute.<br>Paired: Students discuss the answers in pairs.<br>Share: Students present their solutions/ answers.<br><br>Students answer:<br><br>$(8 \times 5) + (8 \times 3) = 8 \times 8$<br>40+ 24 = 64 | | Convinced me/ the think, pair and share strategy. 5 min<br><br>  <br><br><br>- Teacher displays the "convince me" paragraph from the student book p222:<br><br>- Teacher displays the issue using the data show.<br><br>- Teacher asks a student to read the activity clearly.<br><br>كيف تساعدني معرفة $8 \times 5 = 40$ على إيجاد ناتج $8 \times 8$؟ في الحل البنية<br><br>**I use the structure in the solution, how does knowledge of 5x8=40, helps me to find output of 8x8?** |

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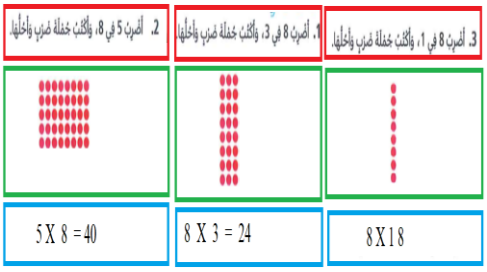
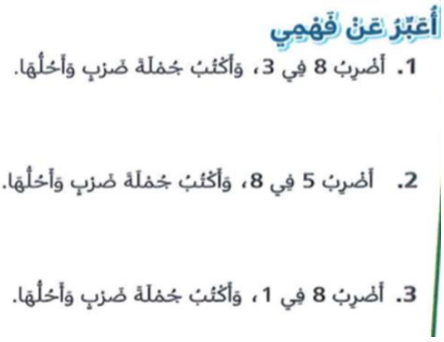
| Pair activity (activate the learning source): 5 min<br><br>- A student reads the paragraph clearly.<br><br>-Students work in pairs to answer the question.<br><br>- Students discuss the solution with the teacher.<br><br>  <br><br><br>**1. I multiply 8 by 3 and then write the multiplication phrase and solve it. 8x3=24**<br>**2. I multiply 8 by 5 and then write the multiplication phrase and solve it. 8x5=40**<br>**3. I multiply 8 by 1 and then write the multiplication phrase and solve it. 8x1=8**<br><br>-Students discuss the answers on the board.<br><br>-Students correct their wrong answers and write them in their books. | Pair activity (activate the learning source): 5 min<br><br>-Teacher displays the questions 1,2,3 on page 223:<br><br>-Teacher asks a student to read the question/activity clearly.<br><br>  <br><br><br>**1. I multiply 8 by 3 and then write the multiplication phrase and solve it.**<br>**2. I multiply 8 by 5 and then write the multiplication phrase and solve it.**<br>**3. I multiply 8 by 1 and then write the multiplication phrase and solve it.**<br><br>Teacher discusses the students and listens to various solutions and ideas.<br><br>-Teacher gives each pair the cards containing the above sentences, the multiplication graphs, the multiplication clauses and the Denis cubes (segments)<br>- Teacher asks students to use the Frayer Model to compose each |

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| | sentence with the graph and the appropriate multiplication.<br>-Teacher passes between the students to check their answers. |
| Independent practice:<br><br>Individual activity: (activities with different levels)<br><br>-Students will individually do the activities starting from level 1 exercises and gradually moving to Level 2 and Level 3.<br><br>1. High achievers: will answer question 18.<br>2. Mid achievers: will answer question 4 and 7.<br>3. Low achievers: will answer question 5 and 9.<br><br>-Students read the questions first.<br><br>- Students watch the tutorial/ educational video about the multiplication facts by 8.<br><br>- Students record/ write the answers.<br>-Students present the solution on the board.<br><br>-Students solve as the following (the model answer):<br><br>4. $8 \times 7 = 56$<br>5. $8 \times 4 = 32$<br>6. $6 \times 8 = 48$<br>7. $10 \times 8 = 80$<br>8. $9 \times 8 = 72$ 9. $8 \times 3 = 24$ | Independent practice: 10 mins<br><br>Individual activity: (activities with different levels)<br><br>-Teacher says, “Now it’s time to work independently and do the activities on page 212 and 223, from question 4 to 9.<br><br>-Teacher assesses the students using the checklist to check their understanding and their achievement to the goal.<br><br>- Teacher provides guidance when needed with monitoring observations on the achievement of the goal and reinforces them.<br><br>Note for the teacher:<br>If the question is resolved by most of the students, the teacher simply asks one student to write the answer/ solution on the board. (The small teacher strategy).<br><br>Low achievers:<br><br>-Revise the way of writing the multiplication fact represented by the graph.<br><br>-Revise the multiplication facts for number 2 and the multiplication facts for number 4. |

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|---|---|
| -Students will check their answers and correct them if there is any mistake.<br><br>-Students write the correct answers.<br><br>-Students read the question.<br><br>-Students discuss the answers in groups.<br><br>-Students raise the card Yes.<br><br>To clarify the answer:<br>Because it used the distribution function and divided the factor 8 into<br>4 + 4<br>You will get 4 x 8=32<br>32 + 32=64<br>Which is equivalent to 8 x 8 | Yes or No strategy:<br><br>-Teacher asks question 29:<br><br>29. مَهارَاتُ التَّفْكِيرِ الْعُلْيَا تَقُولُ صَبَاحُ: "لإِيجَادِ 8 × 8، يُمْكِنُنِي إِيجَادُ (4 + 4) × 8"، هَلْ أَتَّفِقُ مَعَ ذَلِكَ؟ أُوَضِّحُ إِجَابَتِي.<br><br>**High Thinking Skills: Sabah says: to Find 8x8, I can find 8x(4+4),**<br>**Do you agree with that?**<br>**Explained your answer.** |
| Accommodation and differentiation:<br><br>-Mental and oral tests<br>(Final exercises):<br><br>There are different levels of learning: -<br><br>-Teachers forms different questions according to their students' needs. Students answer the given questions using the colored cards based on their levels. | Enrichment activities: (booklet)<br><br>-Teacher gives students the instructions and tells them that each student should open the booklet and write the following:<br>1. Goals learned today.<br>2. New vocabulary/ terms in this lesson.<br><br>- Teacher encourages students to choose more exercises from their books or from the high thinking questions and write them down in their booklets. |
| Closure: 5 mins<br>Teacher's role:<br>Teacher asks students:<br>-What did we learn today?<br>-Teacher summarizes the goals learned during the lesson collectively. (with her students). | |
| Evaluation/ Formative assessment<br>-Teacher displays question 30 on page: | |


**Evaluation: A teacher has boxes of coloring pens inside the classroom cupboard, and each box contains eight colors. Draw lines showing the number of colors in each set of boxes.**

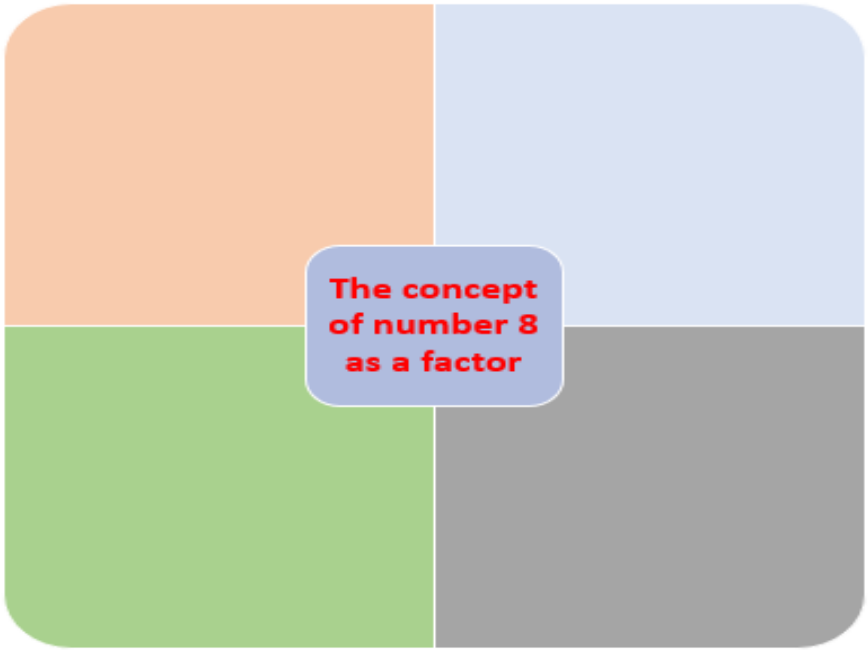
-Students answer: (the model answer)


Homework:

-Student does the homework at home. Page 225 from activity/ question 1 to 9.

# Attached activities

Activity 1:




Pair·activity:

| | |
|---|---|
| I·multiply·8·by·3·and·then·write·the·multiplication·phrase·and·solve·it. | I·multiply·8·by·1·and·then·write·the·multiplication·phrase·and·solve·it. |
| I·multiply·8·by·5·and·then·write·the·multiplication·phrase·and·solve·it. | I·multiply·8·by·1·and·then·write·the·multiplication·phrase·and·solve·it. |
| I·multiply·8·by·3·and·then·write·the·multiplication·phrase·and·solve·it. | I·multiply·8·by·1·and·then·write·the·multiplication·phrase·and·solve·it. |
| I·multiply·8·by·5·and·then·write·the·multiplication·phrase·and·solve·it.<br>I·multiply·8·by·5·and·then·write·the·multiplication·phrase·and·solve·it. | I·multiply·8·by·1·and·then·write·the·multiplication·phrase·and·solve·it. |

**Samples of Teachers Training Workshop for Mathematical Concepts**