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A Novel Simulation-Based Adaptive MEWMA Approach for Monitoring Linear and Logistic Profiles

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ABSTRACT As a common approach in the development of control charts in Statistical Process Control (SPC), an industrial process is monitored with one or more quality characteristics using their corresponding distributions. Note though, modelling the quality characteristics through a relation between some independent and dependent variables is an alternative approach which is designated as profiles monitoring. This study proposes the integration of the adaptive approach to the conventional Multivariate Exponentially Weighted Moving Average (MEWMA) control chart to improve its detection ability in phase II application. The run length characteristics of the adaptive MEWMA chart are measured with the use of Monte Carlo simulations by which better performance of the proposed method than numerous existing competitors including the conventional MEWMA chart is indicated in monitoring linear and logistic profiles. Finally, a real-life example from semiconductor manufacturing is provided to demonstrate the implementation and superiority of the proposed adaptive MEWMA chart over the conventional MEWMA chart.

INDEX TERMS Control chart, linear profiles, logistic profiles, profile monitoring, statistical process control (SPC).

I. INTRODUCTION

As a common approach in industrial processes, statistical process control (SPC) is mostly reduced assignable causes and expurgated adverse conditions to improve the final quality of the products. SPC controls and monitors the whole production's process, finds internal system problems and provides several complementary choices to optimize the production process with the seven existing tools; i.e. cause-and-effect diagram, Pareto chart, control chart, histogram, scatter diagram, check sheet and stratification; see [1], [2]. As the most popular approach among seven tools, control charts have been widespread as an effective tool for quality control in manufacturing processes since the 1920s. It is implemented either in phase I or II applications. In phase I, an initial retrospective study to find the proper estimations of reliable process parameters is conducted while phase II applications start with on-line monitoring of the process with the aim of inspecting the occurrence of the assignable causes in which the process stability conditions are no longer met (i.e., the process moves from IC (in-control) to OC (out-of-control) situation). It is measured by definition of ARL (average run length) criteria meaning the average number of charting statistics to be plotted on the control chart before an OC signal. Naturally, it is expected that a control chart has greater (lower) ARL values in IC (OC) conditions which are denoted by ARL_0 (ARL_1), respectively; see [3].

To monitor a process in SPC with control charts, two different ways have been usually applied in the previous researches. In the first manner, the process is formulated with a univariate or multivariate distribution of a single or multiple quality characteristics. On the other hand, as a secondary and state-of-the-art method, the quality characteristic of such a process could be exhibited by a regression model between the dependent or response and explanatory or independent variables. In the related SPC literature, this approach has been usually termed as profile monitoring. According to [3], profile monitoring aims to investigate whether a predefined IC relationship (or profile) is still valid or the IC model has been changed to an unknown OC model.

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Several studies have shown that monitoring functional relationship or equivalently profile monitoring could be caused to a faster, cheaper and more profitable results in industrial process in comparison with direct monitoring or monitoring quality characteristics [1], [4]. The main reason for this superiority is that some unnatural trends and assignable causes could not be found in monitoring quality characteristics while profile monitoring is able to detect them. So, it has been gaining increasing attentions in scientific researches and practical applications in recent years [5].

Profile monitoring has been employed efficiently in a wide range of industrial processes such as calibration [6], deep reactive ion etching machines from semiconductor manufacturing [7], wood composites [8], remote laser welding [9], thermal conductivity management [10], health-care monitoring [11], pharmaceutical industry [12], agriculture [13], optical imaging system [14], 3D printing [15] and so forth. In these applications, different profiles types entailing linear [16], nonlinear [17], logistic [18], Poisson [19] and non-parametric models [20] have been considered as the IC model. In these studies, it was required to establish a relationship between some independent and dependent variables in such a process. In other words, it does not matter whether the variables have been located in a specific range but it is important that the relationship between them to be controlled over time. So, as the dependent variable changes during the process, the independent variables must change according to the predefined IC model.

Since this paper focuses on phase II monitoring of linear and logistic profiles, a brief literature review about them is only provided below. Also, some descriptions about application of memory type control chart have been provided. Note though, an interested reader is referred to Maleki, *et al.* [5] for more details on other types of profile control charts.

In the linear profiles' SPC literature, the well-known Exponentially Weighted Moving Average (EWMA) charts have been developed in several researches. First, Kang and Albin [6] utilized the EWMA chart to monitor the residuals of the profile parameters, this chart is denoted by EWMAR. Next, Kim, *et al.* [21] applied a novel transformation on the independent variables and proposed three individual independent EWMA charts, denoted as EWMA3. Zou, *et al.* [7] improved the detection ability of the EWMAR and EWMA3 control charts by introducing the Multivariate EWMA (MEWMA) monitoring scheme. This approach not only generated lower ARL_1 values, but also was able to detect decreasing error variance shifts. It could be deduced that most of the existing subsequent works have been designed based on the EWMAR [22], EWMA3 [23]–[26] and MEWMA [27]–[29] control charts.

In some applications, it is also common to employ profile monitoring for discrete, categorical, and unsorted response variables; for example, categorizing the product quality level in two groups or counting the number of nonconforming outcomes. In these situations, some of the conventional assumptions considered in linear profiles entailing the responses follow a normal distribution, establishing a linear relationship between response and independent variables and so forth may be violated. Hence, in such circumstances, the generalized linear models (GLM) can be employed to describe the profiles which include a wide range of distributions, such as the Bernoulli, gamma, exponential, Poisson and logistic. As one of the fundamental researches in this field, Yeh, *et al.* [30] provided some different types of Hotelling's T^2 statistics for monitoring logistic profiles. In addition to the T^2 method, Soleymanian, *et al.* [31] extended the MEWMA and Likelihood Ratio Test (LRT) schemes for monitoring binary logistic profiles in phase II. The modified version of LRT scheme was also proposed by Qi, *et al.* [32], where the weighted effects of the previous samples were augmented by computation of the current charting statistic. Application of change point theory has been also applied in monitoring logistic profiles; for instance, Shadman, *et al.* [33] utilized the change point estimator in combination with the Rao score test to obtain a new statistic for the binary logistic profiles. Also, some other ideas were introduced in Shang, *et al.* [34], Huwang, *et al.* [35], Shang, *et al.* [19], Song, *et al.* [36] and Alevizakos, *et al.* [37] for monitoring logistic profiles.

The content analysis of related literature revealed that different memory-type control charts have been developed in profile monitoring, especially for linear profiles. For example, Riaz, *et al.* [25] and Huwang, *et al.* [38] used ranked set sampling approach for monitoring linear profiles. That is, the basic idea is that the samples are gathered as a memory and then are ranked with a predefined criterion. By this end, a sequence of sorted samples generates a new statistic about the process. As an alternative approach, Variable Sampling Interval (VSI) scheme has been employed in some papers [39]–[43]. In the VSI approach, the sampling interval is not fixed, that is, it varies according to the IC and OC occurrence probabilities considering the previously obtained charting statistics.

In a different approach, Haq [29] monitored linear profiles with an adaptive approach. In this method, whose main idea has been extracted from Haq and Khoo [44], the smoothing constant of the EWMA chart, usually denoted with λ, is assigned with regard to the unbiased estimator of the process parameters' shift.

Adaptive control charts which have been frequently implemented in monitoring quality characteristics, usually adjusts the sample size, sampling interval, chart statistic or other parameters based on the estimated shifted parameters, see [45] and [46]. Based on related SPC literature, it can be deduced that very little attention has been paid to adaptive control charts in profile monitoring and, more precisely, all of the existing memory-type approaches have been extended for linear profiles only; see, for instance [25], [29], [39], [42], [47]. In this paper, a novel MEWMA chart based on the adaptive approach (implemented using a completely different mechanism as compared to the older above-mentioned methods) is proposed for phase II applications of profile monitoring. The adaptive approach which considers the ration of samples as the main criteria in decision making is incorporated into the design of the MEWMA charting statistics (by Zou, *et al.* [7] and Soleymanian, *et al.* [31]) for monitoring simple linear and binary logistic profiles, respectively. The performance of the proposed scheme is evaluated through extensive Monte Carlo simulations as well as an illustrative example based on semiconductor manufacturing. To sum up, the main contributions of this paper are as follows:

- Employing of adaptive control chart in monitoring linear and logistic profiles in phase II.
- Definition a heuristic structure for the proposed method.
- Increasing the detection ability of conventional MEWMA control chart with the proposed adaptive approach.

The rest of the article is organized as follows: in **Section II**, the formulations of linear and binary logistic profiles are discussed. **Section III** illustrates the design of the proposed control chart. **Section IV** gives a comprehensive simulation study of the proposed method and the performance comparison based on the ARL criteria. **Section V** provides a practical example to show the implementation of the proposed method, and finally, in **Section VI,** concluding remarks and recommendations for future research ideas are provided.

II. MEWMA STATISTIC IN PROFILE MONITORING

A. MEWMA STATISTIC IN LINEAR PROFILES

In phase II applications, the IC general linear profile model of j^{th} sample containing n sets of $(\boldsymbol{X_j}, \boldsymbol{Y_j})$ is shown as:

$$Y_j = \mathbf{X}_j\boldsymbol{\beta}_j + \varepsilon_j. \tag{1}$$

The responses in (1), i.e., $\boldsymbol{Y}_j$, is defined as the vector of j^{th} dependent variable; while, $\boldsymbol{X}_j$ ($n \times p$ with $n > p$) is indicator of the matrix of independent variables. The vector of parameters, β_j is defined as a p-dimensional vector of IC parameters and ε_{ij} is regarded to follow a normal distribution, denoted as $N(0, \sigma_0^2)$. By this definition, the IC parameters are shown with $\beta_0 = (\beta_{01}, \beta_{02},\ldots, \beta_{0p})$ and σ_0. Also, the explanatory variables are considered as a fixed values in each profile; hence, the j index is omitted from $\boldsymbol{X}_j$ and it is denoted by $\boldsymbol{X}$ hereafter.

The estimators of coefficients and error variance in (1) (i.e., the p-dimensional vector $\hat{\boldsymbol{\beta}}_j$ and $\hat{\boldsymbol{\sigma}}_j^2$) are computed via the ordinary least square (OLS) method with the following formulations:

$$\begin{aligned}\hat{\boldsymbol{\beta}}_j &= (\boldsymbol{X}\boldsymbol{X})^{-1\prime}\boldsymbol{X}'\boldsymbol{Y}_j,\\ \hat{\boldsymbol{\sigma}}_j^2 &= \frac{1}{n-p}(\boldsymbol{Y}_j - \boldsymbol{X}\hat{\boldsymbol{\beta}}_{\boldsymbol{j}})'(\boldsymbol{Y}_j - \boldsymbol{X}\hat{\boldsymbol{\beta}}_{\boldsymbol{j}}).\end{aligned} \tag{2}$$

To monitor the IC model in (1), Zou, *et al.* [7] scaled the estimations of parameters as follows:

$$\begin{aligned}Z_j(\beta) &= \frac{\hat{\boldsymbol{\beta}}_j - \beta_0}{\sigma_0},\\ Z_j(\sigma) &= \Phi^{-1}\{F((n-p)\frac{\hat{\boldsymbol{\sigma}}_j^2}{\sigma_0^2}; n-p)\}.\end{aligned} \tag{3}$$

The above formulation transforms the probability distribution of an 'estimator' into a normal distribution, where $\Phi^{-1}(.)$ is the inverse of the standard normal cumulative distribution function. Also, the chi-square cumulative distribution function is shown with $F(., \nu)$ (ν is the indicator of the degrees of freedom).

In the above relation, $\mathbf{Z}_j$ consisting of $p+1$ elements, has a normal distribution with mean vector $\mathbf{0}$ and covariance matrix $\sum = \begin{pmatrix}(X'X)^{-1} & 0\\ 0 & 1\end{pmatrix}$. To construct the MEWMA estimator similar to EWMA statistic, Zou *et al.* [4] defined $\boldsymbol{W}_j$ as the MEWMA statistic of the vector $\mathbf{Z}_j$ from the first to the j^{th} profile, i.e.:

$$\mathbf{W}_j = \theta\mathbf{Z}_j + (1-\theta)\mathbf{W}_{j-1}, \tag{4}$$

In most of the EWMA control charts, the smoothing parameter (θ) is considered as a fixed value between 0.1 and 0.2; so, without loss of generality, it is taken as 0.2 in this paper. For ease in computations, the initial value of $\boldsymbol{W}_j$ (i.e., $\boldsymbol{W}_0$) is adjusted based on the IC model. Due to aiming to have a unique statistic, an approach similar to Hotelling's T^2 is applied for the above statistics and we have an OC signal when:

$$U_j = \mathbf{W}_j^T\boldsymbol{\Sigma}^{-1}\mathbf{W}_j > L\frac{\theta}{2-\theta}. \tag{5}$$

So, U_j is the final decision-making criteria in a way that an OC signal is observed when a single plotted point of U_j is beyond the Upper Control Limit (UCL), where UCL is equivalent to $L\frac{\theta}{2-\theta}$ (for better understanding, we show it as UCL_{MEWMA}). For a specified value of θ, UCL_{MEWMA} is calculated by adjusting the parameter L such that the IC process is able to reach the predefined value of ARL_0.

B. MEWMA STATISTIC IN LOGISTIC PROFILES

Suppose that for the j^{th} profile collected over time, there is a set of n observations $(\boldsymbol{x_i}, y_{ij}; i = 1,2,\ldots,n; j = 1,2,..)$ in which y_{ij} is the i^{th} response variable and $\boldsymbol{x_i}$ is a vector including p independent variables ($\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{ip})$), and their values are assumed to be constant in different profiles. Since we do not use the j index for the independent variable, then $\boldsymbol{X}$ is the $n \times p$ matrix of independent variables in the form of $\boldsymbol{X} = [\boldsymbol{x_1};\boldsymbol{x_2};\ldots,\boldsymbol{x_n}]$.

To establish a logistic profile, a relationship between the response and independent variables is modeled with the assumption of:

(1) y_{ij} (i.e., response variables) are binomial variables with parameters m and π_{ij} ($y_{ij} \sim binomial(m, \pi_{ij})$), where m is the total number of trials in each observation and π_{ij} is the probability of success for the i^{th} observation in the j^{th} profile.
(2) The relationship between π_{ij} and explanatory variable vectors is expressed using Logit link function:

$$\begin{aligned}g(\pi_{ij}) &= \mathbf{log}(\frac{\pi_{ij}}{1-\pi_{ij}}) = x_i\boldsymbol{\beta}_j\\ &= \beta_{1j}x_{i1} + \beta_{2j}x_{i2}+,\ldots,+\beta_{pj}x_{ip},\end{aligned} \tag{6}$$

in which $\beta_j = (\beta_{1j}, \beta_{2j}, \ldots, \beta_{pj})$ is the vector of the model's parameters and we will have $x_{i1} = 1$ so that β_{1j} is equal to the intercept.

Yeh, *et al.* [30] suggested Iterative Weighted Least Squares (IWLS) algorithm for estimating the model parameters in the j^{th} profile and the same approach has been employed in this paper (a MATLAB code was written by the authors for this aim and it can be made available upon request).

Soleymanian, *et al.* [31] and Shadman, *et al.* [33] developed MEWMA statistic for logistic profiles. In this scheme, the chart statistic is defined as:

$$U_j = (\mathbf{Z}_j - \boldsymbol{\beta}_0)' \sum\nolimits_z^{-1} (\mathbf{Z}_j - \boldsymbol{\beta}_0), \tag{7}$$

where $\mathbf{Z}_j$ is established with an EWMA form as $\mathbf{Z}_j = \theta \hat{\boldsymbol{\beta}}_j + (1-\theta)\mathbf{Z}_{j-1}$, and $\sum_z = \frac{\theta}{2-\theta} \sum_0$ is the variance-covariance matrix ($\sum_0$ is obtained from phase I) and $\mathbf{Z}_0$ is equal to the IC parameters (i.e., $\boldsymbol{\beta}_0$). The chart triggers an OC signal when U_j is greater than UCL_{MEWMA} which is obtained using simulation to achieve a specific ARL_0.

III. THE PROPOSED METHOD

The main idea of this paper is to provide an adaptive MEWMA control chart that is based on the ratio of samples in three specific regions from IC range entailing Lower Control Limit (LCL) (which is equal to 0 for the MEWMA) to UCL ([0-UCL_{MEWMA}]). Naturally, such an OC situation will allow more data to be generated far from (close to) the LCL (UCL), respectively. More discussions about this phenomenon could be found in Yeganeh and Shadman [48] from which it could be inferred that the average of generated MEWMA statistics (U_j) is increased when there are more plotted points near the UCL, whereas it is decreased in the condition of having more samples in the closeness of the LCL (for both the linear and logistic profiles).

From the abovementioned regions, three specific states with three fixed coefficients ($c_1 \leq 1 \leq c_2 \leq c_3$) are defined for the MEWMA control chart as follows:

- State 1 (S1): It considers the samples plotted in the range [0-$\frac{UCL_{MEWMA}}{3}$] with the coefficient c_1.
- State 2 (S2): It considers the samples plotted in the range [$\frac{UCL_{MEWMA}}{3}$-$\frac{2UCL_{MEWMA}}{3}$] with the coefficient c_2.
- State 3 (S3): It considers the samples plotted in the range [$\frac{2UCL_{MEWMA}}{3}$-UCL_{MEWMA}] with the coefficient c_3.

Suppose we are in the j^{th} sample over time. The number of samples in each state is denoted as d_{1j}, d_{2j} and d_{3j}. Then, the ratios of samples are computed as $r_{1j} = \frac{d_{1j}}{j}$, $r_{2j} = \frac{d_{2j}}{j}$ and $r_{3j} = \frac{d_{3j}}{j}$ (with $r_{1j} + r_{2j} + r_{3j} = 1$). Considering these ratios, the Adaptive Rate (AR_j) is obtained as:

$$AR_j = c_1 \times r_{1j} + c_2 \times r_{2j} + c_3 \times r_{3j}. \tag{8}$$

In this formula c_1, c_2 and c_3 are the control chart's parameters all of which are designed to reach a predefined ARL_0. By the above definition, it is expected that AR_j to be lower than 1 when there are more samples in the first state (since r_{1j} has the most impact) and it could be greater than 1 when there are more samples in the second and third states.

In the j^{th} generated sample, the MEWMA statistic is updated with the following equation:

$$U_j^* = U_j \times AR_j. \tag{9}$$

The OC signal is triggered when U_j^* or U_j are greater than UCL_{MEWMA}. By the above procedure, U_j^* are updated regarding to the process condition in a way that it is expected to have $U_j^* > U_j$ in OC situations and vice versa.

Let the values of c_1, c_2, c_3 and ARL_0 be the determined values (the manner by which they determined are discussed later). Figure 1 illustrates the signaling procedure and the computation of ARL_1 (in *MaxIt* iterations) for the proposed method.

The procedure outlined in Figure 1 can be used either for linear or logistic profiles. Also, the standard deviation of run length (SdRL) is computed in a similar manner to ARL; hence, it is denoted by SdRL_0 and SdRL_1 for the IC and OC profiles, respectively.

Designing of the proposed method entails choosing a proper value for c_1, c_2, c_3 and UCL_{MEWMA}. Similar to Hafez Darbani and Shadman [39], Haq, *et al.* [42] and Yeganeh, *et al.* [22], simulation-based approach is utilized in this paper for control limit adjustment.

The designing idea is based on the three main directions. First, the value of UCL_{MEWMA} is only related to the ARL_0, or equivalently, it is the same with the common MEWMA approach. Considering the ration of sample in IC data generation for the coefficients of states 1 and 2, i.e., c_1 and c_2 is the second one, and the logical relation between coefficients ($c_1 \leq 1 \leq c_2 \leq c_3$) is the last. To this end, 10000 IC profiles are generated from 1 and the averages of ratios in each state are computed for estimation of the initial values of c_1 and c_2. Then, c_3 is obtained corresponding to the sum of the obtained differences. Equation (10) summarizes the formulas for obtaining the initial values of c_1, c_2 and c_3.

$$\begin{aligned} c_1 &= \bar{r}_1, \\ c_2 &= 1 + \bar{r}_2, \\ c_3 &= c_2 + 1 - \bar{r}_1 + \bar{r}_2 = 2c_2 - c_1. \end{aligned} \tag{10}$$

In the above formula, $\bar{r}_1$ and $\bar{r}_2$ are the average of 10000 obtained ratios in states 1 and 2 from the IC data generation. Considering the initial values and UCL_{MEWMA}, the ARL_0 is calculated. Considering multiplying the relative differences of the obtained ARL_0 and the desired one in $\bar{r}_1$, the c_1 is changed (if the obtained ARL_0 is lower (greater) than the desired one, c_1 is decreased (increased), respectively). Then, ARL_0 is computed with the new value of c_1. The above procedure should be iterated with c_2 if the desired ARL_0 is not reached with the new c_1 (by multiplying relative difference in $\bar{r}_2$). Similarly, if it is not possible to reach the desired value, the updating procedure could be done with c_3 (with multiplying relative difference in $\bar{r}_1 + \bar{r}_2$). For better understanding of the designing procedure, the following steps are provided.

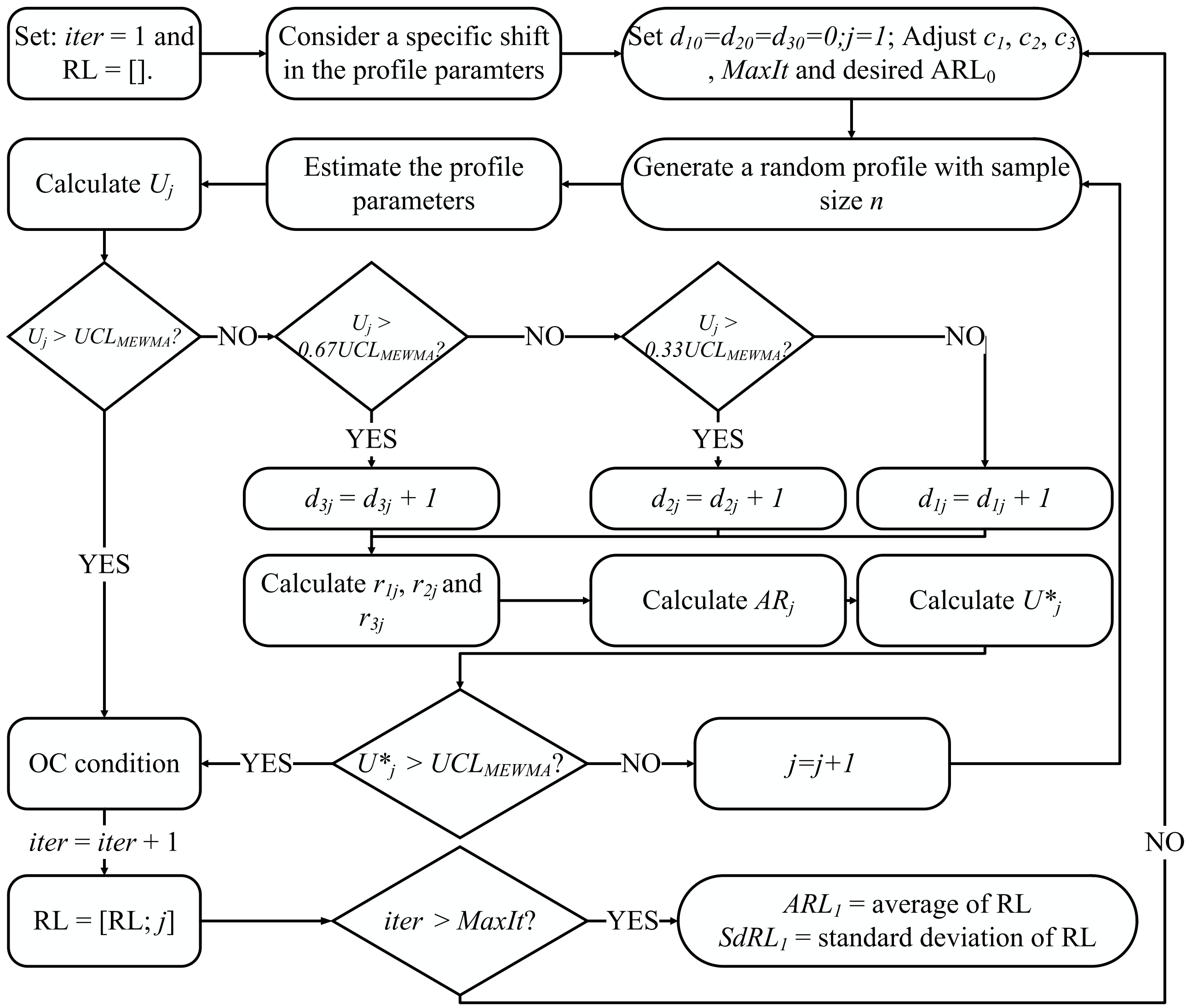

FIGURE 1. The signaling and ARL_1 computing procedure in the proposed method.

- **Step 1**: Adjust UCL_{MEWMA} to reach desired ARL_0.
- **Step 2**: Generate 10000 IC profiles and compute $\bar{r}_1$ and $\bar{r}_2$.
- **Step 3**: Calculate initial values of coefficients by using (10).
- **Step 4**: Calculate ARL_0 with regard to the obtained coefficients.
- **Step 5**: Calculate relative difference of obtained and desired ARL_0.
- **Step 6**: Update only one coefficient, respectively, by multiplying relative difference in $\bar{r}_1$ (for c_1), $\bar{r}_2$ (for c_2) and $\bar{r}_1 + \bar{r}_2$ (for c_3).
- **Step 7**: Iterate Steps 4, 5 and 6 to reach the desired ARL_0.

Note that in Step 7, we iterate the previous three steps by updating one coefficient at a time.

IV. SIMULATION RESULTS

In this section, the performance of the proposed method for simple linear and binary logistic profiles in phase II is investigated through extensive Monte Carlo simulations. The IC models have been gathered from Zou, *et al.* [7] and Shadman, *et al.* [33] respectively.

A. PERFORMANCE COMPARISON IN SIMPLE LINEAR PROFILES

Following Zou, *et al.* [7], the IC model is defined as:

$$y_{ij} = 3 + 2x_i + \varepsilon_{ij},$$
$$i = 1, 2, 3, 4; j = 1, 2, \ldots,$$
$$\varepsilon_{ij} \sim N(0, 1). \tag{11}$$

To adjust ARL_0 at 200 in phase II, UCL_{MEWMA} is obtained as 1.3189 ($L = 11.867$) based on Table 1 in

TABLE 1. A step-by-step procedure for designing of the coefficients in simple linear profile.

Step	c_1	c_2	c_3	ARL_0	Relative difference	Updated coefficient	Change magnitude
Initial	0.77	1.23	1.69	188.3	-0.058	c_1	-0.045
1	0.74	1.23	1.69	190.4	-0.048	c_2	-0.011
2	0.74	1.22	1.69	200.6	0.003	-	-

TABLE 2. ARL_1 comparisons under positive shift in simple linear profile.

Method	λ									
	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMAR	66.50	17.70	8.40	5.40	3.90	3.20	2.70	2.30	2.10	1.90
EWMA3	59.10	16.20	7.90	5.10	3.80	3.10	2.60	2.30	2.10	1.90
MEWMA	59.90	17.20	8.50	5.80	4.10	3.30	2.80	2.40	2.20	2.00
AMEWMA	**53.75**	**15.53**	**7.34**	**4.67**	**3.45**	**2.98**	**2.45**	**2.12**	**1.93**	**1.73**
Improvement%	0.10	0.10	0.14	0.19	0.16	0.10	0.13	0.12	0.12	0.14
Method	η									
	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMAR	119.00	43.90	19.80	11.30	7.70	5.80	4.70	3.90	3.40	3.00
EWMA3	101.60	36.50	17.00	10.30	7.20	5.50	4.50	3.80	3.30	2.90
MEWMA	99.00	35.00	16.40	9.80	6.90	5.30	4.30	3.70	3.20	2.90
AMEWMA	**88.70**	**27.13**	**14.14**	**8.40**	**5.95**	**4.70**	**3.70**	**3.24**	**2.77**	**2.53**
Improvement%	0.10	0.22	0.14	0.14	0.14	0.11	0.14	0.12	0.13	0.13
Method	γ									
	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMAR	34.3	12	6.1	**3.9**	**2.9**	**2.3**	**1.9**	**1.7**	**1.7**	**1.4**
EWMA3	33.5	12.7	7.2	5.1	3.9	3.2	2.8	2.5	2.3	2.1
MEWMA	32.9	12.1	7	4.9	3.8	3.1	2.69	2.3	2.08	1.9
AMEWMA	**27.98**	**10.14**	**5.81**	4.10	3.10	2.51	2.18	1.88	1.72	1.58
Improvement%	0.15	0.16	0.17	0.16	0.18	0.19	0.19	0.18	0.17	0.17

Huwang, *et al.* [27]. The designing procedure in simple linear profile is illustrated here and the same approach has been carried out for the logistic model.

To this end, 10000 IC profiles were generated with the above model and then the average of ratio of statistics in the states 1 [0-0.4395], 2 (0.4395-0.879] and 3 [0.879-1.3186] were obtained as 0.77 ($\bar{r}_1$), 0.23 ($\bar{r}_2$) and 0, respectively. So, from Equation (10), the initial values of c_1, c_2 and c_3 are 0.77, 1.23 and 1.69 (1.23+1-0.77+0.23), respectively. By this adjustment, ARL_0 and relative difference were obtained as 188.3 and -0.058 ($\frac{188.3-200}{200}$), respectively (so the change magnitude is $0.77 \times (-0.058) = -0.045$). The procedure was iterated with a new value of c_1 ($0.77 - 0.045 \times 0.77 = 0.74$) and ARL_0 became 190.4 in step 1 (the change magnitude is $|1\text{-}1.23| \times (-0.048) = \text{-}0.011$). By updating c_2 as 1.22 ($1.23 - 0.011 \times 1.23 = 1.22$), we were able to reach ARL_0 nearly equal to 200 at the second step. For better clarification, the details of the designing procedure for the above example are provided in Table 1.

Hence, the final coefficients are 0.74, 1.22 and 1.69. With this adjustment, next, we compare our proposed adaptive method, denoted as AMEWMA, with the existing MEWMA by [7], EWMAR by [6] and EWMA3 by [21]. Table 2 provides the ARL_1 values for increasing shifts from the IC model parameters. The magnitude of shifts in intercept, slope and standard deviation are shown with λ, η and γ, respectively. For example, when the OC profiles are generated with the intercept equal to 3.2, the AMEWMA chart has the best ARL_1 performance which is equal to 53.75, and yields a 10% improvement in comparison with MEWMA chart (i.e., $\frac{|59.9-53.75|}{59.9} = 0.1$). It can be seen that the detection ability of MEWMA chart is improved when the adaptive approach is used, with the percentage improvement in the range of 10 to 22%. It is worth mentioning that for large

TABLE 3. ARL_1 comparisons under negative shift in simple linear profile.

Method	λ									
	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.8	-1	-1.5	-2
EWMAR	-	68.70	-	17.60	-	8.30	5.30	3.90	-	1.90
EWMA3	132.20	59.50	28.50	16.40	10.80	7.90	5.10	3.80	2.40	1.90
MEWMA	131.00	58.60	29.50	17.20	11.50	8.50	5.50	4.10	2.60	2.00
AMEWMA	**124.39**	**56.78**	**25.46**	**14.75**	**10.56**	**7.53**	**4.8**	**3.55**	**2.25**	**1.69**
Improvement%	0.05	0.03	0.14	0.14	0.08	0.11	0.13	0.13	0.13	0.16
Method	η									
	-0.025	-0.0375	-0.05	-0.0625	-0.075	-0.10	-0.125	-0.15	-0.2	-0.25
EWMAR	118.7	-	44.2	-	20	11.4	7.8	5.9	3.9	3.3
EWMA3	102.90	59.10	36.50	24.00	17	10.30	7.20	5.50	3.80	2.90
MEWMA	99.6	56.8	35.2	22.8	16.5	9.8	6.9	5.3	3.7	2.9
AMEWMA	**90.78**	**52.16**	**30.47**	**20.17**	**14.35**	**8.68**	**6.08**	**4.67**	**3.17**	**2.52**
Improvement%	0.09	0.08	0.13	0.12	0.13	0.11	0.12	0.12	0.14	0.13
Method	γ									
	0.95	0.9	0.85	0.8	0.75	0.7	0.65	0.55	0.45	0.35
EWMA3	283.4	320.3	287.2	208.7	138.2	91.3	60.2	27.7	14.8	8.6
MEWMA	**280.5**	290.5	**235.6**	170.0	113.5	75.2	49.3	22.8	12.4	7.7
AMEWMA	288.43	**270.39**	237.64	**167.52**	**106.6**	**64.26**	**40.52**	**18.62**	**10.08**	**6.63**
Improvement%	-0.03	0.07	-0.01	0.01	0.06	0.15	0.18	0.18	0.19	0.14

shifts in the standard deviation, the EWMAR chart tends to outperform the other competing charts in Table 2; however, the AMEWMA chart still has a slight advantage over the MEWMA chart.

Similar to increasing shifts, Table 3 summarizes the ARL_1 values for decreasing shifts. We can see that there are scant improvements in small shifts while AMEWMA has more superiority with respect to increasing the intercept and slope shifts magnitude. In decreasing error variance shift, occurrence of the bias effect, i.e., having $\text{ARL}_1 > \text{ARL}_0$, is seen. More discussion about this situation has been provided in Huwang, *et al.* [27] and Yeganeh, *et al.* [28]. Except these limited cases, AMEWMA chart has the best method in other simulations.

Similar to SdRL_0 and SdRL_1, median of run lengths denoted by MRL_0 and MRL_1 are defined in phase II and they are two another supplementary criterion for phase II comparisons, see for instance the discussions in [16, 23, 26, 49]. It is important for such a proper approach to reduce SdRL_1 and MRL_1 in addition to ARL_1. Table 4 illustrates the values of SdRL_1 and MRL_1 in addition to SdRL_0 and MRL_0 values for EWMA3, AMEWMA and MEWMA schemes for positive shifts of simple linear profile. The results of EWMA3 have been gathered from Abbas, *et al.* [49] and other values were based on our simulations.

It is obvious that the proposed method is able to decrease SdRL_1 and MRL_1 values for AMEWMA chart in comparison with MEWMA and EWMA3 charts in most of the shifts especially in small and moderate shifts.

For more illustration of improvements in small and moderate shifts, Figure 2 compares the SdRL_1 values in some specific shift of intercept and standard deviation. As can be seen, improvements in standard deviation shifts are more tangible then intercept and slope (for brevity they have been neglected).

B. PERFORMANCE COMPARISON IN BINARY LOGISTIC PROFILES

The IC binary logistic model is described in Shadman, *et al.* [33] as follows:

$$\begin{aligned} g(\pi_{ij}) &= \log\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) = -2.8 + x_i, \\ &\qquad i = 1, 2, \ldots, 10, n = 10, \; m = 30, \\ x &= 0.1 : 0.1 : 1, \\ \sigma_1 &= 0.4676, \sigma_2 = 0.6907, \\ \sum_0 &= \begin{pmatrix} 0.2186 & -0.2936 \\ -0.2936 & 0.4771 \end{pmatrix}. \end{aligned} \tag{12}$$

To reach ARL_0 equal to 200, the contol chart limits entailing UCL_{MEWMA}, c_1, c_2 and c_3 are obtained as 12.9, 0.9, 1.4 and 1.9. The details are similar to Table 1 and they are neglected for brevity. The shift magnitudes ($\Delta = [\delta_1\sigma_1 \;\; \delta_2\sigma_2]$) are reported with δ_1 and δ_2 for the shifts in intercept and slope. Due to investigation of joint shifts, the Non

TABLE 4. SdRL and MRL (top and bottom) comparisons under shifts in simple linear profile.

Method	λ										
	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
EWMA3	201.4	54.09	11.52	4.17	2.20	**1.34**	0.97	0.73	0.56	**0.46**	**0.43**
	145	44	13	**7**	**5**	4	**3**	**2**	**2**	**2**	**2**
MEWMA	232.4	54.54	11.81	4.64	2.52	1.43	1.00	**0.66**	**0.58**	0.48	**0.43**
	157	40	14	8	**5**	4	**3**	3	**2**	**2**	**2**
AMEWMA	**197.4**	**47.64**	**10.04**	**3.32**	**2.10**	**1.34**	**0.95**	0.78	0.60	0.51	0.58
	128	**39**	**12**	**7**	**5**	**3**	**3**	**2**	**2**	**2**	**2**
Improvement%	0.15	0.13	0.15	0.28	0.17	0.06	0.05	-0.18	-0.03	-0.06	-0.35
	0.18	0.03	0.14	0.13	0.00	0.25	0.00	0.33	0.00	0.00	0.00

Method	η										
	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
EWMA3	201.4	98.46	31.29	12.25	10.17	3.43	2.33	1.71	1.31	1.03	0.85
	145	74	27	14	9	6	5	**4**	4	**3**	3
MEWMA	232.4	93.54	29.75	10.89	6.07	**2.91**	2.26	**1.41**	**1.10**	**0.93**	**0.79**
	157	71	28	13	9	6	5	**4**	4	**3**	3
AMEWMA	**197.4**	**85.93**	**28.53**	**9.94**	**5.10**	2.95	**1.97**	1.44	1.11	0.94	**0.79**
	128	**64**	**21**	**11**	**7**	**5**	**4**	**4**	**3**	**3**	**2**
Improvement%	0.15	0.08	0.04	0.09	0.16	-0.01	0.13	-0.02	-0.01	-0.01	0.00
	0.18	0.10	0.25	0.15	0.22	0.17	0.20	0.00	0.25	0.00	0.33

Method	γ										
	0	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
EWMA3	201.4	29.72	9.59	4.80	3.08	2.16	1.72	1.40	1.21	1.05	0.93
	145	24	9	5	4	**3**	**2**	**2**	**2**	**2**	**1**
MEWMA	232.4	29.40	8.90	4.29	2.71	2.04	1.65	1.34	**1.02**	0.99	0.99
	157	28	10	6	5	**3**	3	**2**	**2**	**2**	2
AMEWMA	**197.4**	**22.75**	**6.95**	**4.02**	**2.63**	**1.93**	**1.53**	**1.21**	**1.02**	**0.89**	**0.85**
	128	**21**	**8**	**5**	**3**	**3**	**2**	**2**	**2**	**2**	**1**
Improvement%	0.15	0.23	0.22	0.06	0.03	0.05	0.07	0.10	0.00	0.10	0.14
	0.18	0.25	0.20	0.17	0.40	0.00	0.33	0.00	0.00	0.00	0.50

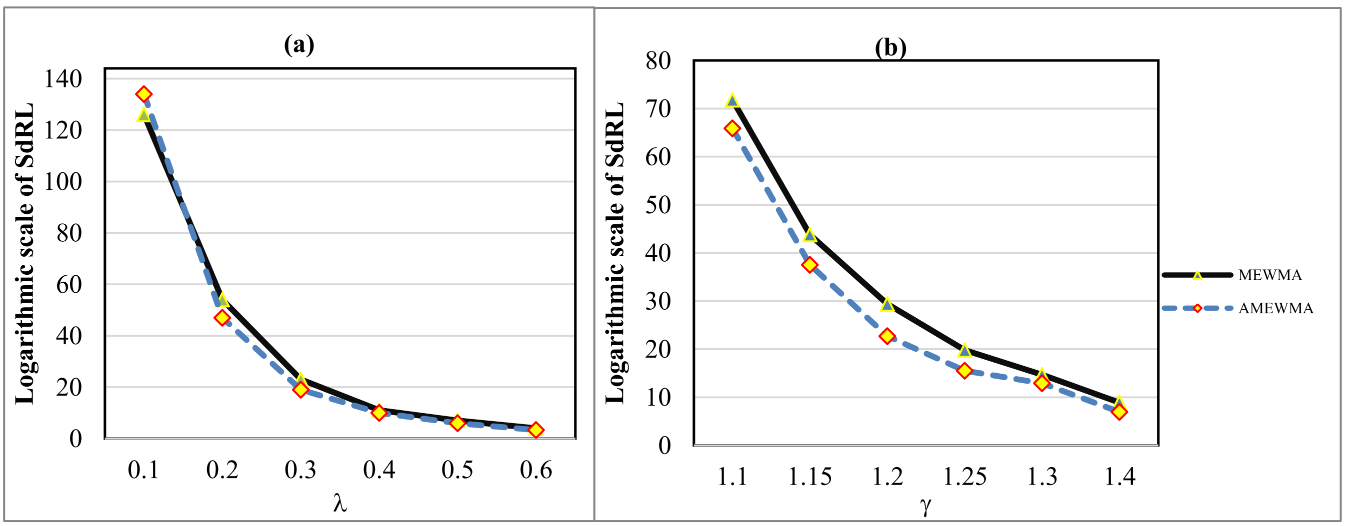

FIGURE 2. The $SdRL_1$ comparisons between AMEWMA and MEWMA when there are shifts in the intercept (left panel) and standard deviation (right panel).

Centrality Parameter (*NCP*) index, defined with $\Delta' \sum_0^{-1} \Delta$, is calculated for each shift magnitude. The competitors are MEWMA by [33], LRT by [31] and Hotelling's T^2 by [31] approaches.

TABLE 5. ARL$_1$ comparisons under intercept shift in binary logistic profile.

NCP	δ_1	T^2	LRT	MEWMA	AMEWMA	Improvement%
0.057	0.1	333.33	**177.11**	505	557.54	-0.10
0.23	0.2	529.11	**119.85**	260.15	204.62	0.21
0.51	0.3	850	70.58	67.62	**50.01**	0.26
0.92	0.4	1184.19	40.75	25.42	**18.70**	0.26
1.44	0.5	1141.16	24.13	13.72	**10.45**	0.24
2.07	0.6	753.37	14.6	9.12	**7.37**	0.19
3.68	0.8	163	5.82	5.16	**4.33**	0.16
5.76	1	37.49	**2.4**	3.98	3.15	0.21
12.97	1.5	1.21	**1.17**	2.48	2.07	0.17
23.06	2	1.13	**1**	1.91	1.5	0.21
51.89	3	**1**	**1**	1.25	**1**	0.20

TABLE 6. ARL$_1$ comparisons under slope shift in binary logistic profile.

NCP	δ_2	T^2	LRT	MEWMA	AMEWMA	Improvement%
0.06	0.1	288.52	**176.39**	333.98	340.32	-0.02
0.23	0.2	378.62	**118.78**	164.74	130.41	0.21
0.51	0.3	464.44	71.74	52.98	**38.63**	0.27
0.92	0.4	505.86	40.32	22.09	**17.14**	0.22
1.44	0.5	471.29	23.69	12.62	**9.94**	0.21
2.07	0.6	352.25	13.82	8.6	**7.08**	0.18
3.68	0.8	114.93	5.62	5.25	**4.33**	0.18
5.76	1	31.78	**2.75**	3.84	3.11	0.19
13	1.5	2.82	**1.19**	2.41	2.07	0.14
23	2	1.13	**1**	1.87	1.52	0.19
51.8	3	**1**	**1**	1.26	**1**	0.21

The simulation results for the shift in the intercept ($\delta_2 = 0$) are shown in Table 5 for AMEWMA chart as well as the other competitors. Similar to linear profiles, the bias effect is also observed for small shifts of the MEWMA, AMEWMA and T^2 schemes. Except for some limited shifts in which the bias effect is seen, the AMEWMA chart has a better detection ability than the MEWMA chart. The improvement range is between 16 to 26%. Comparing with LRT chart, the AMEWMA chart has superior ability in the moderate shifts, while the LRT chart has better small and large shifts detection ability.

The simulation results for the shift in the slope ($\delta_1 = 0$) are shown in Table 6 for AMEWMA chart as well as the other competitors. The conclusions are similar to the shift in intercept. The best improvement percentage occurred in $NCP = 0.51$, i.e., 27%.

Table 7 provides the results of the joint shifts for the AMEWMA chart and other competitors when a step shift takes place in the intercept. Although the bias effect has occurred in MEWMA scheme for $NCP = 0.22$, AMEWMA scheme did not suffer from it. The improvement range is between 10 to 33%. The competition between the AMEWMA and LRT charts is the same as in the previous shifts.

From all the results, it could be concluded that AMEWMA chart had a superior performance compared with MEWMA in most of the shifts. Also, it could be said that the maximum improvement had been observed in the medium shift sizes and that is why the AMEWMA chart outperforms the LRT chart for medium shifts.

Superiority of AMEWMA over other competitors is obvious in simple linear profiles but one may want to investigate precisely the overall performance of our proposed method over other competitors. For this aim, relative mean index (RMI) was implemented to show the overall detection ability of MEWMA, AMEWMA and LRT approaches (T^2 was omitted due to significant weakness against other methods). The formula of RMI has been provided in several references (see for example [50], [51]) and the lower the RMI the better overall performance would be. The RMI values for the shifts in intercept (Table 5), slope (Table 6) and joint shifts (Table 7) are gathered in Figure 3.

TABLE 7. ARL_1 comparisons under joint shift in binary logistic profile.

NCP	(δ_1,δ_2)	T^2	LRT	MEWMA	AMEWMA	Improvement%
0.22	(0.1,0.1)	458.8	**121.35**	224.15	182.09	0.19
0.34	(0.2,0.05)	616.28	94.83	132.71	**89.49**	0.33
0.5	(0.2,0.1)	718.52	73.53	65.84	**51.05**	0.22
0.89	(0.3,0.1)	1035	42.92	26.35	**20.24**	0.23
1.39	(0.4,0.1)	1011	24.8	14.14	**11.12**	0.21
2.69	(0.3,0.4)	316.9	9.49	6.9	**5.5**	0.20
5.67	(0.9,0.1)	39.1	**2.94**	4.01	3.16	0.21
6.66	(0.5,0.6)	21.74	**2.34**	3.57	2.88	0.19
8.01	(0.3,0.9)	11.62	**1.81**	3.19	2.22	0.30
9.32	(0.8,0.5)	7.51	**1.54**	2.95	2.41	0.18
14.1	(1,0.6)	2.38	**1.11**	2.36	2.02	0.14
17.8	(0.8,1)	1.53	**1.03**	2.09	1.89	0.10

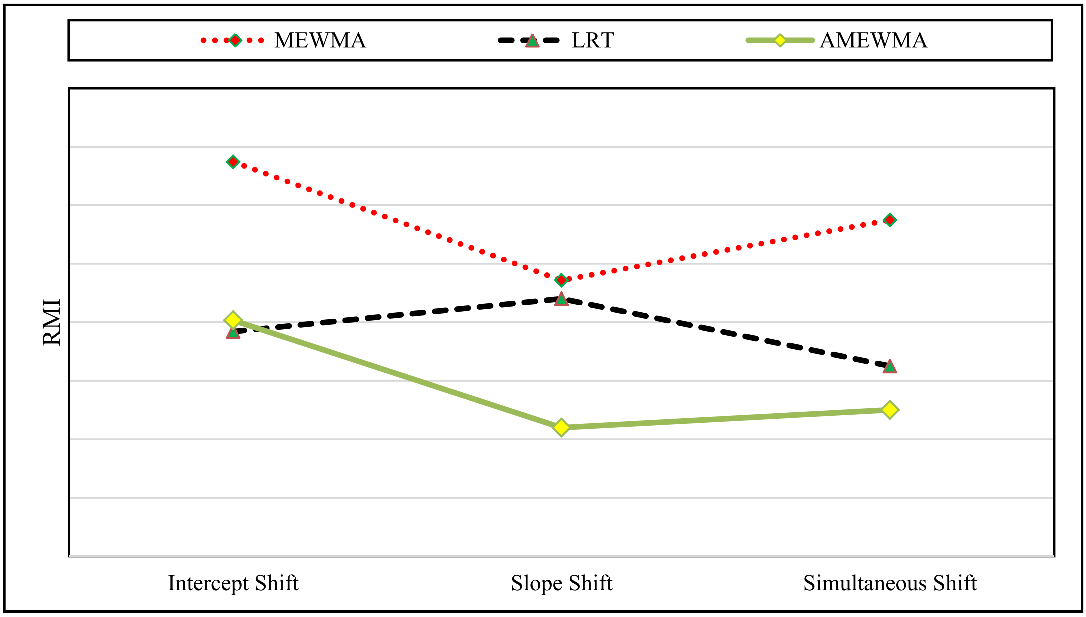

FIGURE 3. The RMI comparisons for the unique (intercept and slope) and simultaneous shifts in the binary logistic model parameters.

As can be seen, the overall superiority of AMEWMA over MEWMA is obvious for logistic profiles. Also, AMEWMA outperformed LRT in slope and simultaneous shifts while LRT achieved a slightly better performance in intercept shifts as compared to the AMEWMA.

V. ILLUSTRATIVE EXAMPLE

To illustrate the practical application of our proposed method, a well-known deep reactive ion-etching process from semiconductor device fabrication has been selected to clarify implementation of the AMEWMA approach; see also [7], [20], [27], [28] who used the same dataset. During this process, an electron microscopy scanner investigates an etched wafer in research labs. This wafer is utilised in some intricate chemical–mechanical reactions on a complicated automative system, denoted as inductively coupled plasma silicon etcher, produced by a supplier of plasma company.

Several adjustment entailing deterministic iterative etching and deposition phases are required in the machine process chamber to have an accurate final outcome. Another important machine's factor to be taken into consideration is the profile of the trench. It does not only significantly have a massive impact on the downstream operations, but also adjusts the quality of the final product. To monitor this profile of a trench, a novel approach was proposed by Zou, *et al.* [7]. they used the Scannig Electronic Microscope (SEM) date instead of the measurement angles of the sidewalls.

The IC model of this process is a polynominal form and the IC data could be obtained from Appendix C in Zou, *et al.* [7]. The following equation shows the details of the IC model, that

TABLE 8. The responses and estimation of parameters for 19 OC generated profiles.

j	y_{ij}											Estimation of parameters			
1	2.98	2.19	1.88	0.55	0.34	0.34	-0.37	0.25	1.29	2.61	4.40	1.50	0.10	0.60	0.20
2	4.19	2.18	1.91	2.05	-0.30	-0.56	1.09	0.97	1.38	2.49	4.16	1.78	-0.02	0.61	0.44
3	4.24	1.88	1.32	0.73	0.36	0.28	1.11	0.73	1.07	2.80	4.13	1.70	0.06	0.59	0.18
4	3.72	2.50	1.65	0.69	0.37	0.76	0.12	0.52	0.99	2.76	3.34	1.58	-0.06	0.54	0.10
5	4.37	2.86	1.95	0.71	0.54	-0.01	0.62	0.46	1.75	2.94	3.65	1.80	-0.08	0.62	0.07
6	3.70	3.13	1.39	0.53	-0.19	0.10	0.59	1.02	1.28	2.18	3.33	1.55	-0.08	0.56	0.13
7	3.76	1.65	1.73	1.07	-0.41	-0.06	0.31	0.58	0.87	2.31	4.01	1.44	0.02	0.60	0.19
8	4.37	3.06	1.37	0.74	0.63	0.29	0.04	0.26	0.97	2.86	4.71	1.75	-0.03	0.72	0.13
9	4.05	2.61	1.06	0.47	0.71	-0.42	0.66	0.92	0.67	3.35	3.99	1.64	0.04	0.64	0.27
10	4.76	2.87	1.15	-0.06	-0.05	-0.61	0.46	0.38	1.88	3.08	4.47	1.67	0.05	0.80	0.13
11	3.85	3.09	0.92	1.62	0.10	0.08	-0.25	0.61	0.52	2.37	3.42	1.49	-0.16	0.59	0.25
12	3.44	2.56	0.92	0.65	-0.40	0.11	-0.65	0.73	0.89	2.86	4.27	1.40	0.09	0.68	0.16
13	4.44	2.58	1.17	1.25	-0.62	-0.46	0.32	0.57	1.92	2.00	3.57	1.52	-0.09	0.66	0.24
14	3.71	3.10	1.29	0.28	-0.25	-0.06	0.73	1.09	1.93	2.84	3.34	1.64	0.03	0.59	0.23
15	3.57	2.42	1.07	1.39	0.27	-0.01	-0.13	1.07	1.66	2.08	4.20	1.60	0.05	0.58	0.16
16	4.64	2.55	1.71	0.89	-0.72	-0.74	0.27	0.05	2.50	2.47	3.42	1.55	-0.09	0.71	0.40
17	2.92	2.94	0.76	0.37	0.44	0.00	-0.05	0.31	0.73	2.01	3.31	1.25	-0.05	0.53	0.16
18	4.36	2.26	1.12	1.42	0.79	-0.65	0.04	0.58	1.82	2.66	3.72	1.65	-0.04	0.63	0.24
19	3.53	2.10	2.62	1.26	-0.05	-0.76	0.07	0.00	1.67	3.38	3.48	1.57	-0.01	0.62	0.52

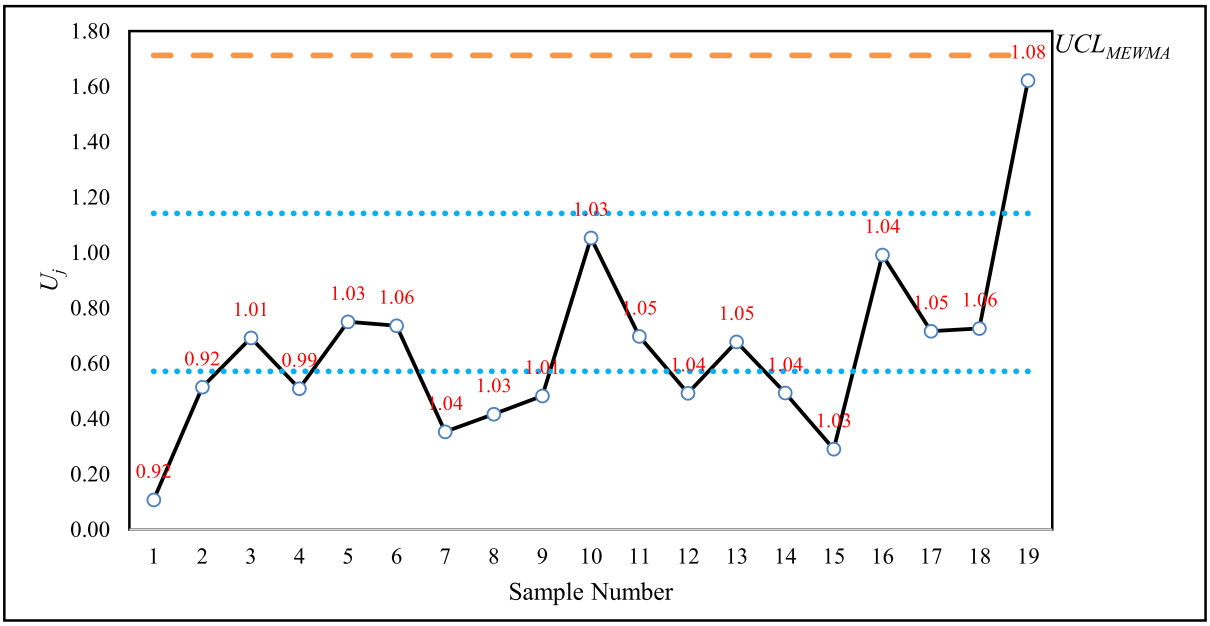

FIGURE 4. The values of the plotted statistics for the simulated OC generated profiles (AR_j values are on the top of each sample).

is, a transformation on the explanatory variables was suggeted in a way that $z_i = x_i$ and $z_i^2 = x_i^2 - \sum_{i=1}^{11} \frac{x_i^2}{11}$.

$$\begin{aligned} y_{ij} &= 1.55 + 0z_i + 0.62z_i^2 + \varepsilon_{ij}, \\ &\quad i = 1, 2, \ldots, 11; j = 1, 2, \ldots, \\ x_i &= -2.5 : 0.5 : 2.5, \\ \varepsilon_{ij} &\sim N(0, 0.16). \end{aligned} \tag{13}$$

The response and explanatory variables were defined as the shape of the profiles. For phase II analysis, it is a common approach to use simulation data instead of collecting real samples.

To reach ARL_0 equal to 370, UCL_{MEWMA} was adjusted at 1.7122 ($L = 15.41$). Based on the proposed designing approach, the states (as described in Section III) are 1 [0-0.571], 2 (0.571-1.142] and 3 (1.142-1.712] (see Figure 4).

TABLE 9. The signaling procedure for the 19 OC generated profiles.

j	$[r_{1j}\, r_{2j}\, r_{3j}]$			U_j	AR_j	U^*_j	Profile status
1	1.00	0.00	0.00	0.107	0.920	0.099	IC
2	1.00	0.00	0.00	0.514	0.920	0.473	IC
3	0.67	0.33	0.00	0.692	1.013	0.701	IC
4	0.75	0.25	0.00	0.509	0.990	0.504	IC
5	0.60	0.40	0.00	0.750	1.032	0.774	IC
6	0.50	0.50	0.00	0.736	1.060	0.780	IC
7	0.57	0.43	0.00	0.352	1.040	0.366	IC
8	0.63	0.38	0.00	0.417	1.025	0.427	IC
9	0.67	0.33	0.00	0.483	1.013	0.489	IC
10	0.60	0.40	0.00	1.053	1.032	1.087	IC
11	0.55	0.45	0.00	0.697	1.047	0.730	IC
12	0.58	0.42	0.00	0.492	1.037	0.510	IC
13	0.54	0.46	0.00	0.678	1.049	0.711	IC
14	0.57	0.43	0.00	0.493	1.040	0.513	IC
15	0.60	0.40	0.00	0.290	1.032	0.299	IC
16	0.56	0.44	0.00	0.992	1.043	1.034	IC
17	0.53	0.47	0.00	0.716	1.052	0.754	IC
18	0.50	0.50	0.00	0.726	1.060	0.770	IC
19	0.47	0.47	0.05	1.622	1.078	1.749	OC

The coefficients were obtained as 0.92, 1.2 and 1.41 by the proposed designing procedure.

To generate OC profiles, we changed the standard deviation to 0.48 from 0.4 (this is equivalent to considering a shift with magnitude 1.2 in IC standard deviation). Table 8 shows the details of the 19 OC generated profiles. The last four colunms are the estimated parameters obtained with the OLS method. For better understanding, the values of U_j s and AR_js are depicted in Figure 4.

With these data, our proposed AMEWMA chart triggers an OC signal at the 19$^{\text{th}}$ profile. The details of coefficients and obtained statistics are shown in Table 9. In the last sample, the ratios are 0.474 ($\frac{8}{19}$), 0.474 ($\frac{8}{19}$) and 0.052 ($\frac{1}{19}$) so we have $AR_{19} = 0.92 \times 0.474 + 1.2 \times 0.474 + 1.41 \times 0.052 = 1.078$. Because all the values in the fifth column are lower than 1.7122, (unlike the AMEWMA chart) the conventional MEWMA chart do not issue an OC signal in this example.

VI. CONCLUSION AND EXTENSIONS

This paper provides an effective adaptive approach for improvement of the MEWMA control chart in monitoring linear and logistic profiles for Phase II applications. In the proposed method, termed as AMEWMA, the magnitude of the MEWMA statistic, denoted by U_j in the j^{th} sample, is adjusted with regard to the ratio of generated statistics in three predefined states. A simple and useful simulation-based designing procedure was proposed to obtain the states and control limits of the AMEWMA approach. On the basis of performance evaluations in terms of ARL criterion, AMEWMA not only improved the detection ability of the MEWMA approach in phase II of monitroing linear and logistic profiles; but also outperformed some of the existing control charts (i.e., EWMA3, EWMAR, T^2, and LRT) for most shifts.

Due to suitable performance in linear, logistic and polynomial profiles, as mentioned in the illustrative example, the proposed adaptive method could be developed to other profile models (such as non-parametric, nonlinear, Poisson, geometric, etc.) in future researches. Finally, the combination of other adaptive methods such as VSI with our proposed method may result in an improved detection ability of the MEWMA control chart in phase II. Hence, it is suggested for future study to apply VSI in simple, logistic and other profile types monitoring.

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