

Research Article

An Efficient Robust Nonparametric Triple EWMA Wilcoxon Signed-Rank Control Chart for Process Location

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Received 1 July 2021; Accepted 1 October 2021; Published 26 October 2021

Academic Editor: Ishfaq Ahmad

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The nonparametric (NP) control charts are famous for detecting a shift in the process parameters (location and/or dispersion) when the underlying process characteristic does not follow the distributional assumptions. Similarly, when the cost of estimations is very high and the ranking of observational is relatively simple, the ranked set sampling (RSS) technique is preferred over the simple random sampling (SRS) technique. On the other hand, the NP triple exponentially weighted moving average (EWMA) control chart based on SRS is superior to the NP EWMA and NP double EWMA (NP DEWMA) based on the SRS technique to detect a shift in the process location. This study designed an advanced form of NP TEWMA Wilcoxon signed-rank based on RSS, denoted as TEWMA – SR_{RSS} control chart to identify a shift in the process location parameter. The Monte Carlo simulation method is used to assess the performance of the proposed TEWMA – SR_{RSS} control chart along with SRS-based NP TEWMA (TEWMA-SR), SRS-based NP TEWMA sign (TEWMA-SN), SRS-based TEWMA – \bar{X} , and RSS-based NP DEWMA-SR (DEWMA – SR_{RSS}) control charts. The study shows that the proposed TEWMA – SR_{RSS} control chart is more efficient in identifying shifts (especially in small shifts) in the process location than the existing control charts. Finally, a real-life application is also provided for the practical implementation of the proposed TEWMA – SR_{RSS} control chart.

1. Introduction

Variations are an essential part of every manufacturing and service process, and these variations can be categorized as common and special cause variations. The common cause variations are harmless, and the mechanism or process that operates under these variations is called in-control (IC). However, the special cause variations affect the product quality, and the process that runs in these variations is referred to as out-of-control (OOC). Statistical process control (SPC) tool (e.g., cause-and-effect diagram, check sheet, control charts, histogram, Pareto chart, scatter

diagram, and stratification) kit is famous for monitoring the shift (i.e., special cause variations) in the process parameters (location and/or dispersion). Compared to other SPC tools, the control charts got special attention because they are competent and easy to implement to detect the shift in the process parameters. Furthermore, the control charts can be classified into memoryless and memory-type depending on their design structures. Shewhart [1] introduced control charts known as the Shewhart control charts; these control charts are also named memoryless-type control charts. Shewhart control charts only use current information to detect a large shift quickly in the process parameters. Later

on, Page [2] and Roberts [3] proposed cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts, respectively; these control charts are also known as memory-type. Memory-type control charts are famous for identifying a small-to-moderate shift in the process parameters.

Generally, for classical parametric control charts (i.e., Shewhart, CUSUM, and EWMA), the underlying process characteristics follow a normal distribution or any other specified probability distribution to identify a shift in the process parameters. Occasionally, there may be a situation when the underlying process characteristics do not follow any specific distribution or distribution of the underlying process characteristics is in doubt. In this case, nonparametric (NP) control charts are the robust substitute for parametric control charts. These control charts are convenient because their IC run length (RL) distribution is similar for all continuous distributions. The sign (SN) and Wilcoxon signed-rank (SR) are well-known NP techniques. The SN technique only required the assumption of continuity, whereas the SR method required symmetry assumption as well [4].

Bakir and Reynolds [5] introduced an NP CUSUM control chart (NPCUSUM-SR) based on the signed-rank statistic for the process location. Similarly, Amin and Searcy [6] developed an NP EWMA-SR control chart by combining the SR technique with the EWMA control chart to monitor the process location shift effectively. In the same lines, Bakir [7] presented the SR-based Shewhart control chart. After that, Balakrishnan et al. [8] suggested an NP Shewhart control chart using runs rules to make it statistically sensitive. Correspondingly, Yang and Cheng [9] and Yang et al. [10] introduced CUSUM-SN and EWMA-SN control charts, respectively, for effective process location monitoring. In the same way, Graham et al. [11] presented the design structure of the single observation-based NP EWMA control chart. Furthermore, Chakraborty et al. [12] developed the GWMA-SR control chart to improve the EWMA-SR control chart's ability to detect small shifts. Later on, Raza et al. [13] introduced the double NP EWMA-SR based (DEWMA-SR) control chart to monitor shift and showed that it is more sensitive than the EWMA-SR control chart. Likewise, Malela-Majika [14] developed new distribution-free control charts based on the Wilcoxon rank-sum test for efficiently monitoring the process location. Also, He et al. [15] designed an NP multivariate control chart for the time between events and amplitude data. This control chart is used to track the time intervals' location shifts and the amplitudes of an event.

In SPC, sampling techniques such as simple random sampling (SRS) and perfect ranked set sampling (RSS) are well known and are commonly used with SN and SR techniques to observe the underlying process data. Furthermore, both SRS and RSS sampling techniques are widely used with parametric and NP control charts. RSS is a necessary and valuable statistical technique commonly used in statistical quality control when the precise measurement of a selected unit is either difficult or prohibitively expensive and time-consuming [16]. It is self-evident that RSS efficiency is dependent on the precision with which the randomly selected units are defined. Errors in the ranking have a

negative impact on the estimator's efficiency and result in imprecise estimates. Dell and Clutter [17] investigated the effect of error and imperfect ranking on mean estimator performance. They presented that even when imperfect rankings are used, the RSS mean estimator remains unbiased and outperforms the SRS mean estimator. However, the performance of the RSS remains superior to that of the imperfect RSS. Visual inspection of the study variable or based on the auxiliary variable may easily lead to ranking a small set of the selected units [18]. For example, hazardous waste sites with different contamination levels can be ranked by a visual inspection of soil discoloration, when the actual measurements of toxic chemicals and quantifying their environmental impact are very costly. In this regard, Salazar and Sinha [19] introduced RSS with control charts to monitor the process location shift. They demonstrated that the RSS-based control charts outperformed against SRS-based control charts. Similarly, Muttlak and Al-Sabah [20] and Abujiya and Muttlak [21] presented an RSS-based (median ranked set sampling and extreme ranked set sampling) and double RSS (DRSS) based control charts, respectively, to monitor the shift in the process location. Later, Al-Omari and Haq [22] proposed a Shewhart-type control chart based on the DRSS technique.

Haq et al. [23] suggested a maximum EWMA (Max-EWMA) control chart based on order DRSS (ODRSS) and order imperfect DRSS (OIDRSS) sampling techniques. In addition, they [24] also developed a new synthetic control chart for process location and dispersion using different RSS techniques. Furthermore, based on RSS, Haq and Khoo [25] developed synthetic EWMA and synthetic CUSUM control charts to monitor the process location. Recently, Abbas et al. [26] introduced an NP DEWMA-SR control chart using the RSS technique for process location monitoring, labelled as DEWMA – SR_{RSS} control chart.

Similarly, in the SPC, modifications and enhancements are continually practiced to enhance the performance of the memory-type control charts. For example, Shamma and Shamma [27] extended the classical EWMA control chart and suggested a double EWMA (DEWMA) control chart for the process location. The DEWMA control chart is more responsive than the classical EWMA control chart. Anwar et al. [28] and Aslam and Anwar [29] introduced modified-EWMA control charts, respectively, in the presence of auxiliary information and Bayesian methodology. Similarly, Anwar et al. [30] introduced auxiliary information-based (AIB) combined MEC for the simultaneous monitoring of process location and dispersion.

Recently, Chatterjee et al. [31] proposed a TEWMA control chart to monitor process dispersion shift. Later on, Alevizakos et al. [32] and Alevizakos et al. [4] suggested an NP TEWMA-SN (TEWMA-SN) and NP TEWMA-SR (TEWMA-SR) control charts, respectively. Furthermore, Alevizakos et al. [33] extended EWMA and DEWMA control charts and proposed a triple EWMA (TEWMA) control chart further to enhance the efficiency of the EWMA control chart. The TEWMA control chart is more responsive than the EWMA and DEWMA control charts to monitor small-to-moderate shifts in process location.

As discussed earlier, the SRS-based TEWMA-SN and TEWMA-SR control charts are well known for timely monitoring of the quality characteristics easily available. Still, when selecting quality characteristics is either difficult or expensive and time-consuming, these SRS-based control charts fail to monitor the process efficiently. To monitor such quality characteristics, this study suggested an NP triple EWMA-SR control chart with the RSS technique (TEWMA – SR_{RSS}) for effectively monitoring the process location of a continuous and symmetric distribution. Various processing environments like normal, Contaminated normal (CN), Laplace, Student’s *t*, and Logistic are used to measure the RL characteristics of the proposed control chart. The average run length (ARL), the median of the run length (MDRL), and the standard deviation of the run length (SDRL) are assessed to determine the efficiency of the proposed control chart against other control charts. The sample points before the control chart signal are referred to as RL, and the RL’s expected values are referred to as ARL. ARL₀ is the ARL of the IC process while ARL₁ is the ARL of the OOC process. A control chart is considered to be efficient when it has smaller ARL₁ values as compared to its contestant at different shifts.

The remainder of the article is as follows: Section 2 presents the design structures of existing and the proposed control charts. Similarly, Section 3 offers the implementation of the proposed control chart. Also, the comparative study is given in Section 4, while Section 5 provides an illustrative example related to the proposed control chart. Finally, Section 6 offers the summary, conclusions, and recommendations of the study.

2. Existing and Proposed Methods

This section explains the design structures of some existing and the proposed control charts. For example, Section 2.1 presents the RSS-based Wilcoxon signed-rank statistic. Similarly, methodologies of the NP EWMA SR with RSS (EWMA – SR_{RSS}) and DEWMA – SR_{RSS} control charts are given in Sections 2.2 and 2.3, respectively. Finally, the design structure of the proposed TEWMA – SR_{RSS} control chart is provided in Section 2.4.

2.1. RSS-Based Wilcoxon Signed-Rank Statistic. McIntyre [16] gives the idea of the RSS technique for data collection. Suppose RSS of size *n* is taken from a quality variable $X_{qj(h)}$, where $q = 1, 2, 3, \dots, j = 1, 2, 3, \dots, n$, and $h = 1, 2, 3, \dots, m$. *q* denotes each sample, *j* denotes the number of observations, and *h* denotes the number of cycles used in the RSS technique. $r = nm$ is the number of iterations used in the RSS technique with a sample of size *n* and *m* number of cycles. Let $R_{qj(h)}^+ = |X_{qj(h)} - \varphi_0|$, where $R_{qj(h)}^+$ denote the ranks of absolute deviations of RSS values ($X_{qj(h)}$) from median (φ_0). According to Kim and Kim [34], the statistic of RSS under SR can be written as

$$SR_{RSS_q} = \sum_{j=1}^n \sum_{h=1}^m \text{sign}(X_{qj(h)} - \varphi_0) R_{qj(h)}^+, \quad (1)$$

where

$$\text{sign}(X_{qj(h)} - \varphi_0) = \begin{cases} 1, & \text{if } X_{qj(h)} - \varphi_0 > 0, \\ 0, & \text{if } X_{qj(h)} - \varphi_0 = 0, \\ -1, & \text{if } X_{qj(h)} - \varphi_0 < 0. \end{cases} \quad (2)$$

The mean and variance of the statistic SR_{RSS} are $E(SR_{RSS}) = 0$ and $\text{var}(SR_{RSS}) = ((r(r+1)(2r+1))/6)\omega_0^2$, respectively [34]. The ω_0^2 can be defined as follows:

$$\omega_0^2 = 1 - \frac{4}{n} \sum_{j=1}^n \left(F_k(0) - \frac{1}{2} \right)^2, \quad (3)$$

where the values of $F_k(0)$ can be obtained by solving the following expression for $j = 1, 2, \dots, n$:

$$F_k(0) = \frac{r!}{(j-1)!(r-j)!} \int_{-\infty}^0 F(t)^{j-1} (1-F(t))^{r-j} f(t) dt, \quad (4)$$

whereas ω_0^2 is used to enhance the performance of the control chart. The values of ω_0^2 at different sample sizes taken from [35] are given as $(n, \omega_0^2) = (2, 0.750), (3, 0.625), (4, 0.547), (5, 0.490), (6, 0.451), (7, 0.416), (8, 0.393), (9, 0.371),$ and $(10, 0.352)$.

2.2. EWMA – SR_{RSS} Control Chart. The EWMA – SR_{RSS} control chart is more sensitive for detecting shifts in the process location/median than the NPEWMA-SR control chart. The plotting statistic of the EWMA – SR_{RSS} control chart based on equation (1) is defined as

$$E_q = \lambda SR_{RSS_q} + (1 - \lambda) E_{q-1}, \quad (5)$$

where $\lambda \in (0, 1]$ is a smoothing constant and the initial value of the E_q is equal to 0. The mean and variance of the statistic E_q are $E(E_q) = 0$ and $\text{var}(E_q) = (\lambda/(2-\lambda))(1 - (1-\lambda)^{2q})(r(r+1)(2r+1)/6)\omega_0^2$, respectively. The variance of E_q is reduced to $(\lambda/(2-\lambda))(r(r+1)(2r+1)/6)\omega_0^2$, when the term $(1 - (1-\lambda)^{2q})$ approaches to unity. The control limits of the EWMA – SR_{RSS} control chart (see Table 1) are defined below:

$$\left. \begin{aligned} UCL_{EWMA-SR_{RSS}} &= +L \sqrt{\frac{\lambda}{2-\lambda} \left(\frac{r(r+1)(2r+1)}{6} \right) \omega_0^2} \\ LCL_{EWMA-SR_{RSS}} &= -L \sqrt{\frac{\lambda}{2-\lambda} \left(\frac{r(r+1)(2r+1)}{6} \right) \omega_0^2} \end{aligned} \right\}, \quad (6)$$

where *L* is the width of the control limits, which helps to determine ARL₀. The process will be OOC if any $E_q > UCL_{EWMA-SR_{RSS}}$ or $E_q < LCL_{EWMA-SR_{RSS}}$; otherwise, it will remain in an IC state.

2.3. DEWMA – SR_{RSS} Control Chart. Abbas et al. [26] introduced the following plotting statistics of the DEWMA – SR_{RSS} control chart:

$$\left. \begin{aligned} E_q &= \lambda SR_{RSS_q} + (1 - \lambda)E_{q-1} \\ DE_q &= \lambda E_q + (1 - \lambda)DE_{q-1} \end{aligned} \right\}, \quad (7)$$

= $((r(r+1)(2r+1)/6)\omega_0^2)\lambda^4((1+\lambda^2 - (q^2+2q+1)\lambda^{2q} + (2q^2+2q-1)\lambda^{2q+2} - q^2)/(1-\lambda^2)^3)$, respectively. The control limits of the DEWMA – SR_{RSS} control chart can be designed as

The mean and variance of DE_q (DEWMA – SR_{RSS} control chart) for IC process are $E(DE_q) = 0$ and $\text{var}(DE_q)$

$$\left. \begin{aligned} LCL_{DEWMA-SR_{RSS}} &= -L \sqrt{\left(\frac{r(r+1)(2r+1)}{6}\omega_0^2\right)\lambda^4 \frac{1+\lambda^2 - (q^2+2q+1)\lambda^{2q} + (2q^2+2q-1)\lambda^{2q+2} - q^2}{(1-\lambda^2)^3}} \\ UCL_{DEWMA-SR_{RSS}} &= +L \sqrt{\left(\frac{r(r+1)(2r+1)}{6}\omega_0^2\right)\lambda^4 \frac{1+\lambda^2 - (q^2+2q+1)\lambda^{2q} + (2q^2+2q-1)\lambda^{2q+2} - q^2}{(1-\lambda^2)^3}} \end{aligned} \right\}, \quad (8)$$

where L is the control chart width and the initial value of the DE_q = 0. The process will be OOC when $LCL_{DEWMA-SR_{RSS}} > DE_q$ or $UCL_{DEWMA-SR_{RSS}} < DE_q$; otherwise, it will remain in an IC state.

of the proposed TEWMA – SR_{RSS} control chart are defined as

2.4. Proposed TEWMA – SR_{RSS} Control Chart. This section offers a proposed control chart structure. In more detail, the proposed TEWMA – SR_{RSS} control chart is designed for monitoring the process location shift. The plotting statistics

$$\left. \begin{aligned} E_q &= \lambda SR_{RSS_q} + (1 - \lambda)E_{q-1} \\ DE_q &= \lambda E_q + (1 - \lambda)DE_{q-1} \\ TE_q &= \lambda DE_q + (1 - \lambda)TE_{q-1} \end{aligned} \right\}, \quad (9)$$

The initial values of E₀, DE₀, and TE₀ are all equal to 0 (zero). The mean and variance of the statistic TE_q under IC state are $E(TE_q) = 0$ and

$$\begin{aligned} \text{var}(TE_q) &= \left[\frac{\theta^3 \lambda^6}{4} \left[\frac{q(q^2-1)(q-2)\theta^{q-3}}{1-\theta} - \frac{4q(q^2-1)\theta^{q-2}}{(1-\theta)^2} - \frac{12q(q+1)\theta^{q-1}}{(1-\theta)^3} - \frac{24(q+1)\theta^q}{(1-\theta)^4} + \frac{24(1-\theta^{q+1})}{(1-\theta)^5} \right] \right. \\ &+ 2\theta^2 \lambda^6 \left[\frac{q(q^2-1)\theta^{q-2}}{1-\theta} - \frac{3q(q+1)\theta^{q-1}}{(1-\theta)^2} - \frac{6(q+1)\theta^q}{(1-\theta)^3} + \frac{6(1-\theta^{q+1})}{(1-\theta)^4} \right] \\ &\left. + \frac{7\theta \lambda^6}{2} \left[\frac{q(q+1)\theta^{q-1}}{1-\theta} - \frac{2(q+1)\theta^q}{(1-\theta)^2} + \frac{2(1-\theta^{q+1})}{(1-\theta)^3} \right] + \lambda^6 \left[\frac{1-\theta^{q+1}}{(1-\theta)^2} - \frac{(q+1)\theta^q}{(1-\theta)} \right] \frac{n(n+1)(2n+1)}{6} \omega_0^2 \right] \end{aligned} \quad (10)$$

respectively, where $\theta = (1 - \lambda)^2$. The time-varying control limits of the proposed TEWMA – SR_{RSS} control chart are specified as

where L is the width of the control limits. For large values of q , we have $\text{var}(TE_q) = [(6(1-\lambda)^6\lambda/(2-\lambda)^5) + (12(1-\lambda)^4\lambda^2/(2-\lambda)^4) + (7(1-\lambda)^2\lambda^3/(2-\lambda)^3) + (\lambda^4/(2-\lambda)^2)](n(n+1)(2n+1)/6)\omega_0^2$ and the asymptotic control limits become

$$\left. \begin{aligned} LCL_{(TEWMA-SR_{RSS})q} &= -L\sqrt{\text{var}(TE_q)} \\ UCL_{(TEWMA-SR_{RSS})q} &= +L\sqrt{\text{var}(TE_q)} \end{aligned} \right\}, \quad (11)$$

$$\left. \begin{aligned} LCL_{TEWMA-SR_{RSS}} &= -L\sigma \sqrt{\left[\frac{6(1-\lambda)^6\lambda}{(2-\lambda)^5} + \frac{12(1-\lambda)^4\lambda^2}{(2-\lambda)^4} + \frac{7(1-\lambda)^2\lambda^3}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2} \right] \frac{n(n+1)(2n+1)}{6} \omega_0^2} \\ UCL_{TEWMA-SR_{RSS}} &= +L\sigma \sqrt{\left[\frac{6(1-\lambda)^6\lambda}{(2-\lambda)^5} + \frac{12(1-\lambda)^4\lambda^2}{(2-\lambda)^4} + \frac{7(1-\lambda)^2\lambda^3}{(2-\lambda)^3} + \frac{\lambda^4}{(2-\lambda)^2} \right] \frac{n(n+1)(2n+1)}{6} \omega_0^2} \end{aligned} \right\}, \quad (12)$$

The process goes OOC if any $TE_q > UCL_{TEWMA-SR_{RSS}}$ or $TE_q < LCL_{TEWMA-SR_{RSS}}$. However, if $LCL_{TEWMA-SR_{RSS}} < TE_q < UCL_{TEWMA-SR_{RSS}}$, the process will be in IC state.

3. Implementation of the Proposed TEWMA – SR_{RSS} Control Chart

This section provides the implementation of the proposed TEWMA – SR_{RSS} control chart. Section 3.1 describes the choice of design parameters. Similarly, Section 3.2 presents the evaluation of the proposed control chart.

3.1. Choice of Design Parameters. λ and L are the design parameters of the proposed TEWMA – SR_{RSS} control chart. The control chart width L is chosen with various values of λ , such that the prespecified ARL_0 is obtained. The best subgroup size is between 5 and 10, depending on ARL_0 and shift size [5]. Different RL properties are assessed to study the performance behavior of the proposed TEWMA – SR_{RSS} control chart with $\lambda \in (0.05, 0.10, 0.25, 0.50)$ and $n \in (5, 10)$. ARL_0 is assumed to be 370 for this study.

3.2. Evaluation. This section provides the RL characteristics of the different distributions (normal and non-normal distributions) for IC robustness and IC performance of the proposed control chart. The proposed TEWMA – SR_{RSS} control chart's RL features are evaluated using normal and non-normal distributions.

3.2.1. Evaluation of the Run-Length Distribution. The performance of the proposed control chart can be assessed in both normal and non-normal symmetric distributions. The distributions used in this analysis are as follows:

- (i) Standard normal distribution, i.e., $N(0, 1)$
- (ii) Double exponential (Laplace) distribution, i.e., $DE(0, (1/\sqrt{2}))$
- (iii) Heavy tail Student's t distribution t_{ν} , with the degree of freedom (ν) = 4 and 8
- (iv) The logistic distribution, i.e., $LOG(0, (\sqrt{3}/\pi))$
- (v) Contaminated normal (CN) distribution, which is a mixture of $N(0, \sigma_1^2)$ and $N(0, \sigma_0^2)$, i.e., $(1 - \alpha)N(0, \sigma_1^2) + \alpha N(0, \sigma_0^2)$, with α such that $(1 - \alpha)\sigma_1^2 + \alpha\sigma_0^2 = 1$, where $\alpha = 0.05$ and $(\sigma_1/\sigma_0) = 2$. For comparison purpose, all the considered distributions in this analysis were reparametrized with mean zero and unit standard deviation. The probability density functions of the distributions used in this study are shown in Table 2.

3.2.2. Monte Carlo Simulation. The Monte Carlo simulation is used to obtain the RL characteristics of the proposed control chart. The simulation algorithm is developed in R software to compute the RL characteristics. 10^4 random samples of the size $n > 1$ from any distribution covered by the study for a shift (δ) were generated. The simulation

algorithm to determine the nominal values of ARL_0 under various distributions can be explained in the following steps [36]:

- (a) A finite loop is practiced to generate samples from different distributions
- (b) Select a particular value for λ
- (c) Determine the TE_q plotting statistic offered in equation (9)
- (d) Let us assume $L = 3$
- (e) Calculate the $UCL_{TEWMA-SR_{RSS}}$ and $LCL_{TEWMA-SR_{RSS}}$ from equation (11)
- (f) Plot the TE_q statistic against control limits in step (e) over q
- (g) If the $TE_q > UCL_{TEWMA-SR_{RSS}}$ or $TE_q < LCL_{TEWMA-SR_{RSS}}$, record the sample number of TE_q statistic as an RL. For example, at $q = 255$, if $TE_q > UCL_{TEWMA-SR_{RSS}}$ or $TE_q < LCL_{TEWMA-SR_{RSS}}$ record 255 as a first RL.
- (h) Repeat steps from (a) to (g) for 10^5 times and record RLs.
- (i) Compute the average of 10^5 times noted RLs, which is called ARL_0 .
- (j) If $ARL_0 = 400$; otherwise, adjust L (i.e., $L < 3$ or $L > 3$) constant accordingly in step (d) and repeat steps from (a) to (i) to obtain $ARL_0 = 400$.
- (k) To compute the ARL_1 values, again generate samples from relevant distribution and repeat steps from (b) to (i).

3.2.3. In-Control Robustness of the Proposed Control Chart. The EWMA and DEWMA – \bar{X} control charts have the limitation that their IC RL features do not remain the same for all continuous distributions. The IC RL characteristics of the NP control chart's characteristics remain the same for all symmetric continuous distributions [4]. The simulated results of the proposed control chart with different combinations of λ and L to obtain the nominal $ARL_0 = 370$ are shown in Tables 3 and 4. The IC RL distribution of the proposed TEWMA – SR_{RSS} control chart seems to be similar for all distributions used in this study. The RL distribution is positively skewed when $ARL > MDRL$.

3.2.4. Out-of-Control Performance. OOC performance of the proposed control chart can be discussed under perfect RSS and imperfect RSS. The OOC control chart's efficiency is primarily compared, and it demonstrates how sensitive the control chart is to detect shifts in process parameters. Tables 3 and 4 show the OOC performance of the proposed TEWMA – SR_{RSS} control chart for the various distributions evaluated in this study. For small and moderate shifts, the OOC performance of the proposed TEWMA – SR_{RSS} control chart increases as n increases; i.e., ARL_1 values decrease by increasing n . The proposed scheme's RL values reduce with the rise in n and m values. The OOC RL performance of the

TABLE 1: Details about the control charts used in this study.

Control chart's name	Notation	Plotting statistics	Control limits
1 NP EWMA under RSS	$EWMA - SR_{RSS}$	$E_q = \lambda SR_{RSS_q} + (1 - \lambda)E_{q-1}$	$UCL_{EWMA-SR_{RSS}} = +L\sqrt{\frac{(\lambda/(2-\lambda))((r(r+1)(2r+1))/6)\omega_0^2}{(r(r+1)(2r+1))}}$ $LCL_{EWMA-SR_{RSS}} = -L\sqrt{\frac{(\lambda/(2-\lambda))((r(r+1)(2r+1))/6)\omega_0^2}{(r(r+1)(2r+1))}}$
2 DEWMA under RSS	$DEWMA - SR_{RSS}$	$E_q = \lambda SR_{RSS_q} + (1 - \lambda)E_{q-1}$ $DE_q = \lambda E_q + (1 - \lambda)DE_{q-1}$	$LCL_{DEWMA-SR_{RSS}} = -L\sqrt{\frac{((r(r+1)(2r+1))(6)\omega_0^2)\lambda^4((1+\lambda^2 - (q^2 + 2q + 1)\lambda^{2q} + (2q^2 + 2q - 1)\lambda^{2q+2} - q^2)/(1 - \lambda^2)^3)}{((r(r+1)(2r+1)(2r+1))/6)\omega_0^2}}$ $UCL_{DEWMA-SR_{RSS}} = +L\sqrt{\frac{((r(r+1)(2r+1)(2r+1))/6)\omega_0^2)\lambda^4((1+\lambda^2 - (q^2 + 2q + 1)\lambda^{2q} + (2q^2 + 2q - 1)\lambda^{2q+2} - q^2)/(1 - \lambda^2)^3)}{((r(r+1)(2r+1)(2r+1))/6)\omega_0^2}}$
3 TEWMA under RSS	$TEWMA - SR_{RSS}$	$E_q = \lambda SR_{RSS_q} + (1 - \lambda)E_{q-1}$ $DE_q = \lambda E_q + (1 - \lambda)DE_{q-1}$ $TE_q = \lambda DE_q + (1 - \lambda)TE_{q-1}$	$LCL_{TEWMA-SR_{RSS}} = -L\sigma\sqrt{\frac{[(6(1-\lambda)^5\lambda/(2-\lambda)^5) + (12(1-\lambda)^4\lambda^2/(2-\lambda)^4) + (7(1-\lambda)^3\lambda^3/(2-\lambda)^3) + (\lambda^4/(2-\lambda)^2)](n(n+1)(2n+1)(6)\omega_0^2)}{[(6(1-\lambda)^6\lambda/(2-\lambda)^5) + (12(1-\lambda)^4\lambda^2/(2-\lambda)^4) + (7(1-\lambda)^3\lambda^3/(2-\lambda)^3) + (\lambda^4/(2-\lambda)^2)](n(n+1)(2n+1)(6)\omega_0^2)}}$ $UCL_{TEWMA-SR_{RSS}} = +L\sigma\sqrt{\frac{[(6(1-\lambda)^5\lambda/(2-\lambda)^5) + (12(1-\lambda)^4\lambda^2/(2-\lambda)^4) + (7(1-\lambda)^3\lambda^3/(2-\lambda)^3) + (\lambda^4/(2-\lambda)^2)](n(n+1)(2n+1)(6)\omega_0^2)}{[(6(1-\lambda)^6\lambda/(2-\lambda)^5) + (12(1-\lambda)^4\lambda^2/(2-\lambda)^4) + (7(1-\lambda)^3\lambda^3/(2-\lambda)^3) + (\lambda^4/(2-\lambda)^2)](n(n+1)(2n+1)(6)\omega_0^2)}}$

TABLE 2: PDFs of the continuous distributions used for this study.

(i)	Standard normal	$f(X) = (e^{(-x^2/2)/\sqrt{2\pi}})$, where $X \in \mathfrak{R}$, $M_0 = 0$, and $\sigma^2 = 1$
(ii)	Student's t_v	$f(X) = (\Gamma((v+1)/2)/(\Gamma(v/2)\sqrt{v\pi}))(1+(X^2/v))^{-(v+1)/2}$, where $X \in \mathfrak{R}$, $M_0 = 0$, and $\sigma^2 = (v/(v-2))$ and $v = 4, 8$ are taken
(iii)	Logistic	$f(X) = e^{(-\pi X)/\sqrt{3}} / ((\sqrt{3}/\pi)(1 + e^{(-\pi X)/\sqrt{3}})^2)$, where $X \in \mathfrak{R}$, $M_0 = 0$, and $\sigma^2 = (3/\pi^2)$
(iv)	Laplace	$f(X) = (1/2)e^{- X }$, where $X \in \mathfrak{R}$, $M_0 = 0$, and $\sigma^2 = (1/2)$
(v)	Contaminated normal (CN)	$f(X) = (0.95e^{(-X^2/2)/\sqrt{2\pi}}) + (0.05e^{-(X^2/2\sigma_0^2)}/\sigma_0\sqrt{2\pi})$, where $X \in \mathfrak{R}$, $M_0 = 0$, and $\sigma^2 = 0.95 + 0.05\sigma_0^2$

TABLE 3: RL characteristics of the proposed TEWMA – SR_{RSS} control chart under different distributions with nominal ARL₀ = 370 and $n = 5$.

(λ, L)		δ															
		0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00		
(0.05, 1.585)	N(0, 1)	ARL	371.11	206.77	168.62	92.67	50.53	33.91	19.12	12.02	7.94	2.35	1.37	1.08	1.01	1.00	
		MDRL	218.00	131.00	111.00	70.00	41.00	28.00	17.00	9.00	7.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	453.10	234.01	183.70	90.33	49.06	29.15	15.72	10.54	6.62	1.77	0.75	0.31	0.08	0.00	
	CN	ARL	369.92	212.31	164.04	94.31	51.93	35.63	19.37	12.06	8.35	2.51	1.46	1.14	1.01	1.00	
		MDRL	217.50	139.00	113.00	65.00	40.00	27.50	16.00	9.00	6.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	441.07	235.19	179.28	97.35	50.15	33.35	16.16	10.77	7.50	2.08	0.87	0.43	0.13	0.04	
	Logistic	ARL	371.69	189.29	156.51	80.72	49.37	30.49	16.21	10.27	7.20	2.18	1.31	1.10	1.01	1.00	
		MDRL	230.50	118.00	112.00	62.00	41.00	25.00	13.00	8.00	6.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	429.72	216.77	159.50	83.63	44.70	26.42	14.26	8.94	6.39	1.61	0.66	0.33	0.11	0.03	
	Laplace	ARL	371.65	158.45	123.59	63.49	34.18	22.54	12.25	7.52	5.48	2.01	1.32	1.12	1.03	1.00	
		MDRL	237.00	106.50	87.00	50.00	28.00	18.00	10.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	428.92	173.36	126.83	60.33	30.92	20.29	10.72	6.89	4.85	1.50	0.66	0.38	0.17	0.06	
	t_4	ARL	369.29	215.22	187.04	111.34	62.00	41.65	23.55	15.39	10.43	3.34	1.79	1.36	1.09	1.02	
		MDRL	232.50	126.50	123.00	83.00	47.00	33.00	20.00	12.00	8.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	423.91	254.75	210.06	111.40	59.02	39.91	20.45	12.90	9.58	2.79	1.31	0.70	0.34	0.17	
	t_8	ARL	370.51	202.75	174.37	99.36	58.92	37.64	20.00	13.33	8.89	2.80	1.58	1.20	1.03	1.01	
		MDRL	231.00	128.00	118.00	70.00	45.00	33.00	15.00	10.00	7.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	455.11	247.02	189.65	102.22	55.97	32.31	17.50	11.80	7.86	2.40	1.03	0.54	0.17	0.12	
	(0.10, 1.837)	N(0, 1)	ARL	370.58	239.49	200.36	107.44	57.60	38.66	21.58	13.26	9.19	3.09	1.75	1.28	1.03	1.00
			MDRL	241.50	164.00	143.00	75.50	44.00	32.00	19.00	12.00	8.00	2.00	1.00	1.00	1.00	1.00
			SDRL	399.98	252.12	200.34	104.09	52.29	31.47	16.68	9.58	6.52	2.10	0.97	0.54	0.18	0.03
		CN	ARL	371.69	222.24	189.81	110.71	64.19	40.58	21.02	13.65	9.67	3.28	1.79	1.32	1.05	1.01
			MDRL	240.50	155.00	136.50	75.00	48.00	34.00	18.00	12.00	8.00	3.00	2.00	1.00	1.00	1.00
			SDRL	400.58	233.16	189.80	111.14	60.27	32.09	16.17	9.93	6.98	2.33	1.02	0.60	0.22	0.08
Logistic		ARL	369.89	220.35	179.15	97.85	52.17	35.77	18.26	11.25	8.24	2.73	1.67	1.25	1.04	1.01	
		MDRL	257.50	150.00	124.00	72.00	40.00	29.00	16.00	10.00	7.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	379.94	229.75	177.94	90.99	47.57	29.96	13.90	8.42	5.94	1.80	0.89	0.52	0.20	0.07	
Laplace		ARL	369.00	179.37	145.59	72.15	39.00	25.62	14.36	8.83	6.40	2.42	1.52	1.27	1.05	1.01	
		MDRL	257.50	117.50	103.00	54.00	31.00	22.00	13.00	8.00	5.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	379.94	191.51	141.81	65.46	33.78	19.39	10.66	6.28	4.77	1.58	0.83	0.56	0.23	0.10	
t_4		ARL	369.26	257.35	229.58	142.19	81.25	48.90	26.11	16.68	12.29	3.98	2.27	1.60	1.16	1.04	
		MDRL	239.00	166.50	154.00	102.00	59.50	38.00	23.00	14.00	11.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	382.67	289.42	240.47	134.47	75.25	42.73	19.56	12.46	8.78	2.78	1.43	0.90	0.43	0.22	
t_8		ARL	370.13	241.63	203.62	125.90	67.99	44.76	22.63	15.92	10.57	3.42	1.92	1.38	1.06	1.01	
		MDRL	250.00	169.50	139.00	89.50	52.00	37.00	20.00	14.00	9.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	417.20	260.53	202.72	129.48	59.38	36.44	17.13	11.11	7.52	2.42	1.14	0.62	0.27	0.08	

TABLE 3: Continued.

(λ, L)		δ															
		0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00		
(0.25, 2.204)	$N(0, 1)$	ARL	370.74	284.31	253.51	151.84	88.78	54.57	24.92	15.57	10.38	3.59	2.09	1.44	1.07	1.01	
		MDRL	267.50	199.50	167.00	108.50	64.00	38.50	19.00	13.00	9.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	369.16	291.85	267.11	151.35	88.66	50.80	20.17	11.01	6.99	2.13	1.06	0.63	0.26	0.08	
	CN	ARL	371.10	293.39	250.56	154.48	88.79	57.94	27.45	15.74	11.22	3.78	2.12	1.58	1.13	1.02	
		MDRL	273.00	214.50	172.50	109.50	64.00	42.00	21.00	13.00	10.00	3.50	2.00	1.00	1.00	1.00	
		SDRL	366.10	279.85	252.76	141.01	83.90	53.54	23.59	11.44	7.55	2.10	1.08	0.75	0.37	0.15	
	Logistic	MDRL	265.50	180.50	169.50	103.00	58.00	36.00	18.50	12.00	8.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	360.62	282.24	242.01	135.38	69.56	40.44	16.29	9.51	5.85	1.87	1.02	0.65	0.28	0.13	
		ARL	370.38	230.11	199.04	105.76	52.44	30.72	15.08	10.52	7.07	2.82	1.83	1.35	1.10	1.02	
	Laplace	MDRL	265.50	156.50	144.00	75.00	38.50	23.00	13.00	9.00	6.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	360.62	223.94	196.34	100.81	47.67	26.97	11.19	7.14	4.41	1.68	0.97	0.65	0.33	0.17	
		ARL	371.04	304.19	282.63	193.67	113.38	72.96	32.78	19.37	13.04	4.52	2.52	1.85	1.28	1.11	
	t_4	MDRL	251.00	208.50	200.00	136.50	80.00	51.00	25.00	15.00	11.00	4.00	2.00	2.00	1.00	1.00	
		SDRL	367.97	313.54	280.19	186.75	112.53	70.92	27.16	16.71	8.95	2.58	1.48	0.98	0.55	0.36	
		ARL	370.07	116.22	108.76	81.73	54.05	36.08	19.70	12.12	8.85	3.08	1.89	1.35	1.07	1.01	
	t_8	MDRL	261.50	82.50	74.00	60.50	39.00	26.00	16.00	11.00	8.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	366.37	115.99	114.08	80.49	53.32	36.90	16.95	9.09	6.20	1.92	1.04	0.59	0.26	0.11	
		ARL	371.08	311.27	311.97	210.28	137.41	87.02	38.24	22.17	12.96	3.72	2.12	1.47	1.07	1.00	
	(0.50, 2.491)	$N(0, 1)$	MDRL	252.50	207.50	204.50	147.50	97.00	63.00	28.00	16.00	10.00	3.00	2.00	1.00	1.00	1.00
			SDRL	381.21	322.22	323.87	205.59	134.81	82.86	35.61	19.95	10.58	2.13	1.09	0.67	0.26	0.04
			ARL	369.07	317.34	298.10	218.94	142.93	91.04	41.29	22.56	14.44	3.95	2.29	1.54	1.12	1.02
		CN	MDRL	254.00	223.00	210.00	150.00	99.00	65.00	30.00	17.00	11.00	4.00	2.00	1.00	1.00	1.00
			SDRL	371.98	317.49	304.17	207.28	141.42	86.41	38.00	20.71	11.92	2.38	1.17	0.74	0.35	0.13
			ARL	371.67	305.18	284.01	191.15	116.23	72.54	34.44	16.81	11.03	3.48	1.98	1.39	1.08	1.02
Logistic		MDRL	257.50	201.00	199.00	129.50	87.00	52.00	26.00	13.00	9.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	356.53	305.98	274.70	187.91	113.63	70.19	31.77	13.28	8.81	1.81	1.07	0.62	0.30	0.13	
		ARL	371.67	274.31	254.24	151.90	81.51	52.05	22.67	12.59	8.14	2.82	1.88	1.41	1.11	1.02	
Laplace		MDRL	257.50	191.50	179.00	112.00	60.00	36.00	17.00	10.00	7.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	356.53	269.48	255.88	148.13	77.52	49.70	19.76	10.15	6.24	1.72	1.01	0.66	0.35	0.13	
		ARL	371.87	322.54	311.03	239.91	161.54	109.66	52.45	28.87	18.26	4.66	2.75	1.92	1.28	1.12	
t_4		MDRL	250.00	210.50	220.50	167.00	110.00	76.50	36.00	20.00	14.00	4.00	3.00	2.00	1.00	1.00	
		SDRL	384.56	349.01	284.83	233.04	160.25	106.96	52.03	26.96	15.06	2.91	1.56	1.03	0.58	0.38	
		ARL	371.82	320.55	293.03	225.65	143.39	95.97	46.46	24.48	15.52	4.19	2.37	1.65	1.18	1.04	
t_8		MDRL	260.00	221.00	203.00	158.00	103.00	67.00	34.00	18.00	12.00	4.00	2.00	1.00	1.00	1.00	
		SDRL	367.33	335.03	302.40	221.66	139.56	96.20	42.63	20.99	12.36	2.51	1.29	0.82	0.42	0.20	
		ARL	371.08	311.27	311.97	210.28	137.41	87.02	38.24	22.17	12.96	3.72	2.12	1.47	1.07	1.00	

Laplace distribution is higher as compared to the other distributions used in this study (see Figures 1 and 2). The proposed control chart's ability to detect shifts deteriorates when it occurs later for small values of λ ($\lambda = 0.05$ or $\lambda = 0.10$), whereas this trend is noticed for large values of λ ($\lambda = 0.25$ or $\lambda = 0.50$) for large shifts (see Figures 3 and 4). The proposed control chart's OOC performance under imperfect RSS is also determined to illustrate the superiority of perfect RSS or generally RSS over imperfect RSS (see Tables 5-7). For example, under normal distribution, at $\lambda = 0.05$, $n = 5$, and $\delta = 0.025, 0.05, 0.075$, and 0.10 , the ARL values of the proposed control chart under RSS are 206.77, 92.67, 50.53, and 33.9, whereas the ARL values of the proposed control chart under imperfect RSS are 295.71, 178.64, 111.43, and 73.63 (see Tables 3 and 5). In general, the proposed control chart under RSS is very sensitive to detect small and moderate shifts in the process location.

In terms of OOC performance at a specific shift (δ), the proposed TEWMA - SR_{RSS} control chart outperforms all other control charts. For instance, at $\delta = 0.05$, $\lambda = 0.25$, and $n = 5$, the ARL₁ values of the proposed TEWMA - SR_{RSS}, TEWMA-SR, TEWMA-SN, TEWMA - \bar{X} , and DEWMA - SR_{RSS} control charts are 151.84, 251.08, 341.92, 337.05, and 167.12, respectively. The ARL is used as a metric to compare the proposed and existing control charts. Some findings are presented in the percentage decreasing in $ARL = ((ARL_0 - ARL_1)/ARL_0) \times 100\%$. Under normal setup, the TEWMA - SR_{RSS} control chart displaying a 3 percent improvement in the process median minimizes the ARL by 54.56 percent for $n = 5, \lambda = 0.05$, and $ARL_0 = 370$, whereas at the same shift (δ), the proposed control chart reduced the ARL by 80.55 percent for $n = 10, \lambda = 0.05$, and $ARL_0 = 370$ (see Tables 3 and 4).

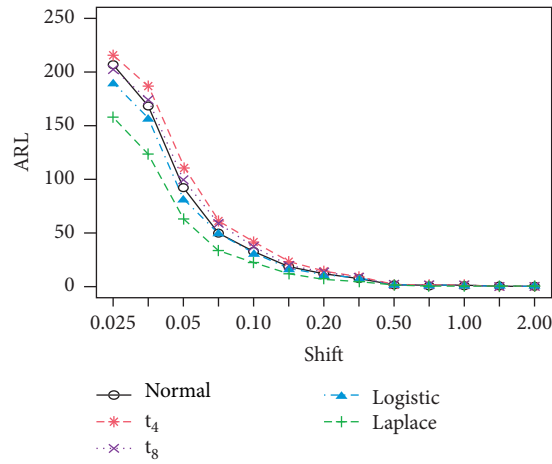


FIGURE 1: ARL behavior of various distributions using the proposed TEWMA – SR_{RSS} control chart when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

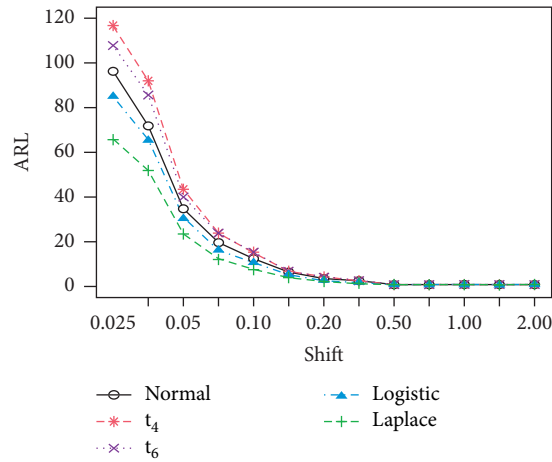


FIGURE 2: ARL behavior of various distributions using the proposed TEWMA – SR_{RSS} control chart when $n = 10$, $\lambda = 0.05$, and $ARL_0 = 370$.

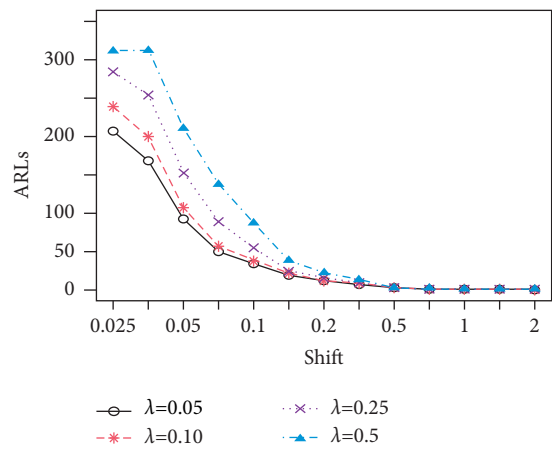


FIGURE 3: ARL comparisons of the proposed TEWMA – SR_{RSS} control chart for different values of λ when $n = 5$ and $ARL_0 = 370$.

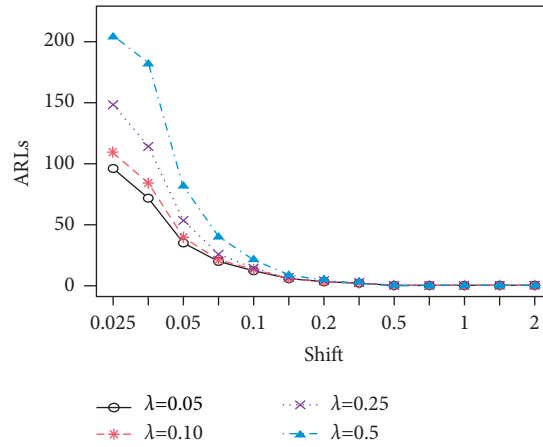


FIGURE 4: ARL comparisons of the proposed TEWMA – SR_{RSS} control chart for different values of λ when $n = 10$ and $ARL_0 = 370$.

TABLE 4: RL characteristics of the proposed TEWMA – SR_{RSS} control chart under different distributions with nominal $ARL_0 = 370$ and $n = 10$.

(λ, L)		δ														
		0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00	
(0.05, 1.3635)	$N(0, 1)$	ARL	369.61	96.01	71.86	34.99	19.89	12.62	6.24	3.75	2.47	1.06	1.00	1.00	1.00	1.00
		MDRL	240.0	67.0	54.0	29.0	16.0	10.0	5.0	3.0	2.0	1.0	1.0	1.0	1.0	1.0
		SDRL	418.41	101.64	70.49	30.60	16.96	10.99	5.50	3.19	1.95	0.26	0.00	0.00	0.00	0.00
	CN	ARL	369.54	98.54	77.99	37.52	20.21	13.22	6.20	3.85	2.65	1.11	1.01	1.00	1.00	1.00
		MDRL	244.00	69.00	61.00	32.00	16.00	11.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	429.63	102.05	74.36	31.93	17.22	11.36	5.24	3.25	2.18	0.37	0.07	0.00	0.00	0.00
	Logistic	ARL	370.04	85.10	65.60	30.63	16.53	10.77	5.08	3.09	2.19	1.06	1.00	1.00	1.00	1.00
		MDRL	215.50	64.00	53.00	25.00	13.00	8.00	4.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	451.55	83.95	61.17	27.07	14.16	9.51	4.44	2.54	1.64	0.26	0.04	0.00	0.00	0.00
	Laplace	ARL	371.04	65.82	51.95	23.68	12.32	7.75	4.03	2.36	1.74	1.04	1.00	1.00	1.00	1.00
		MDRL	218.00	55.00	42.00	19.00	9.00	6.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	452.29	61.73	46.09	20.99	11.01	6.81	3.50	1.90	1.19	0.21	0.00	0.00	0.00	0.00
t_4	ARL	370.73	116.79	91.87	43.49	24.36	15.90	7.42	4.85	3.29	1.23	1.02	1.00	1.00	1.00	
	MDRL	228.50	84.00	69.00	35.00	20.00	13.00	5.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	421.19	125.85	92.98	40.01	20.16	13.58	6.69	4.04	2.71	0.54	0.14	0.00	0.00	0.00	
t_8	ARL	369.67	107.75	85.82	40.15	23.71	14.78	7.08	4.29	2.87	1.11	1.01	1.00	1.00	1.00	
	MDRL	225.00	77.00	65.00	33.00	20.00	12.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	441.35	113.15	83.92	36.90	18.63	12.45	6.15	3.58	2.35	0.39	0.09	0.00	0.00	0.00	
(0.10, 1.57)	$N(0, 1)$	ARL	369.43	109.52	84.48	39.80	21.08	13.55	6.82	4.30	2.87	1.13	1.00	1.00	1.00	1.00
		MDRL	242.50	76.00	61.00	33.00	18.00	12.00	6.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	401.47	109.19	79.78	32.99	16.75	10.21	5.30	3.26	2.14	0.39	0.04	0.00	0.00	0.00
	CN	ARL	369.18	117.33	91.87	41.32	23.19	14.61	7.15	4.58	3.14	1.17	1.01	1.00	1.00	1.00
		MDRL	244.50	84.00	68.00	32.50	21.00	13.00	6.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	401.76	115.00	87.66	36.27	16.79	10.75	5.59	3.48	2.30	0.46	0.07	0.00	0.00	0.00
	Logistic	ARL	369.92	104.66	73.89	35.34	19.33	12.30	6.06	3.73	2.46	1.09	1.00	1.00	1.00	1.00
		MDRL	243.00	77.00	54.00	29.50	17.00	10.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	419.02	99.93	71.94	29.90	14.51	9.21	4.56	2.78	1.76	0.33	0.06	0.00	0.00	0.00
	Laplace	ARL	369.92	80.30	57.70	25.08	13.95	8.93	4.65	2.88	2.01	1.08	1.00	1.00	1.00	1.00
		MDRL	243.00	60.00	45.00	22.00	12.00	7.50	4.00	2.00	1.00	1.00	1.00	1.00	1.00	1.00
		SDRL	419.02	73.03	52.34	19.71	10.63	6.76	3.66	2.16	1.38	0.29	0.07	0.00	0.00	0.00
t_4	ARL	370.86	137.04	109.45	50.21	25.91	17.07	9.22	5.67	3.94	1.33	1.04	1.00	1.00	1.00	
	MDRL	251.00	98.00	80.00	39.00	23.00	15.00	8.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	388.38	138.72	108.92	43.31	19.84	12.63	6.93	4.30	3.07	0.66	0.20	0.05	0.00	0.00	
t_8	ARL	370.56	126.76	101.54	48.45	23.01	15.84	8.16	5.04	3.33	1.21	1.01	1.00	1.00	1.00	
	MDRL	257.50	87.00	68.50	38.00	20.00	14.00	7.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	387.66	130.74	104.01	41.84	17.88	11.16	6.14	3.68	2.39	0.53	0.10	0.00	0.00	0.00	

TABLE 4: Continued.

(λ, L)		δ															
		0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00		
(0.25, 1.867)	$N(0, 1)$	ARL	370.25	148.30	114.50	53.87	26.46	15.29	7.57	4.94	3.56	1.25	1.02	1.00	1.00	1.00	
		MDRL	271.50	103.00	78.00	40.00	20.00	13.00	7.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	353.98	143.90	113.04	49.08	22.94	11.38	5.06	2.97	2.09	0.52	0.13	0.00	0.00	0.00	
	CN	ARL	371.05	154.57	129.10	59.23	26.81	16.47	7.95	5.34	3.66	1.30	1.02	1.00	1.00	1.00	
		MDRL	255.50	105.50	90.00	44.00	21.00	14.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	383.00	152.56	124.95	56.42	22.54	12.56	4.99	3.36	2.09	0.56	0.14	0.00	0.00	0.00	
	Logistic	ARL	369.80	142.12	110.97	45.00	22.01	13.10	6.86	4.23	3.03	1.20	1.01	1.00	1.00	1.00	
		MDRL	249.00	97.00	78.00	32.00	18.00	11.00	6.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	402.95	147.38	102.81	41.45	16.83	9.21	4.22	2.72	1.85	0.46	0.09	0.03	0.00	0.00	
	Laplace	ARL	369.80	111.92	78.38	33.46	16.67	9.76	5.28	3.30	2.50	1.14	1.02	1.00	1.00	1.00	
		MDRL	249.00	81.00	54.00	26.00	14.00	9.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	402.95	105.95	75.98	28.81	12.94	6.80	3.23	2.04	1.50	0.37	0.14	0.00	0.00	0.00	
	t_4	ARL	368.97	186.35	140.23	71.38	34.86	19.43	9.78	6.19	4.22	1.56	1.09	1.01	1.00	1.00	
		MDRL	262.00	131.00	97.50	50.00	26.00	16.00	9.00	6.00	4.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	359.24	181.23	140.44	67.16	29.67	14.58	6.51	4.05	2.63	0.79	0.30	0.11	0.00	0.00	
	t_8	ARL	371.13	177.20	148.49	62.09	30.50	17.80	9.01	5.81	3.75	1.41	1.04	1.00	1.00	1.00	
		MDRL	259.00	125.00	101.50	44.50	23.00	15.00	8.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	367.57	173.08	155.73	54.46	25.64	13.55	5.73	3.45	2.21	0.69	0.20	0.05	0.00	0.00	
	(0.50, 2.1185)	$N(0, 1)$	ARL	370.25	203.92	181.53	81.69	39.73	21.85	8.99	5.16	3.64	1.35	1.02	1.00	1.00	1.00
			MDRL	261.00	144.00	131.50	55.00	29.00	16.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00
			SDRL	361.13	203.76	174.27	81.56	36.79	19.27	6.49	3.35	2.03	0.56	0.15	0.00	0.00	0.00
		CN	ARL	371.05	220.20	176.01	89.51	45.28	23.66	9.17	5.40	3.82	1.40	1.05	1.00	1.00	1.00
			MDRL	266.50	144.00	129.50	63.00	33.00	17.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00
			SDRL	355.54	224.64	167.97	86.11	44.09	20.66	6.81	3.40	2.19	0.59	0.21	0.04	0.00	0.00
Logistic		ARL	369.71	207.36	144.94	72.68	33.57	18.75	7.77	4.62	3.28	1.26	1.01	1.00	1.00	1.00	
		MDRL	262.00	140.50	107.50	53.00	25.00	15.00	7.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	389.82	220.88	139.79	69.42	31.08	15.69	5.48	2.89	1.81	0.49	0.12	0.04	0.00	0.00	
Laplace		ARL	369.71	165.97	118.11	47.01	21.84	12.14	5.65	3.50	2.55	1.22	1.02	1.00	1.00	1.00	
		MDRL	262.00	112.00	81.50	33.50	16.00	9.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	389.82	177.25	114.36	42.64	19.12	9.66	3.58	2.00	1.38	0.45	0.14	0.05	0.00	0.00	
t_4		ARL	369.17	242.08	202.82	104.20	55.12	29.44	12.56	7.09	4.64	1.64	1.12	1.03	1.00	1.00	
		MDRL	261.50	172.00	141.00	76.00	38.00	22.00	9.00	6.00	4.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	359.03	241.26	199.81	101.71	54.08	26.61	10.82	4.73	2.73	0.79	0.34	0.16	0.04	0.00	
t_8		ARL	369.74	240.61	203.22	99.49	48.13	26.55	10.96	5.74	4.13	1.56	1.06	1.01	1.00	1.00	
		MDRL	253.00	173.00	138.00	67.00	34.50	20.00	9.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	
		SDRL	366.34	229.90	209.30	100.88	44.50	24.28	8.71	3.80	2.32	0.74	0.24	0.08	0.00	0.00	

4. Comparative Study

This section provides the proposed control chart’s comparisons to several existing control charts, which includes DEWMA – SR_{RSS} [26], TEWMA – \bar{X} [33], TEWMA-SN [32], and TEWMA-SR [4] control charts. More detail is provided in the following sections.

4.1. Proposed versus TEWMA-SR Control Chart. The proposed TEWMA – SR_{RSS} control chart outperforms the TEWMA-SR control chart. For instance, at $n = 5$ and $\lambda = 0.05$ with $\delta = 5$ percent shift in the Logistic distribution, the proposed TEWMA – SR_{RSS} control chart reduces the ARL₁ value by almost 78.28 percent, whereas the TEWMA-SR control chart reduces the ARL₁ value by 58.74 percent (see Table 8). Similarly, under $t_{(4)}$ distribution, when $n = 10$ and $\lambda = 0.10$ along with $\delta = 5$ percent, the ARL₁ values of the proposed TEWMA – SR_{RSS} control chart are reduced by 86.46 percent, while the TEWMA-SR control chart reduces

the values by 72.30 percent (see Tables 4 and 9). Figures 5 and 6 highlight the proposed TEWMA – SR_{RSS} control chart superiority over the TEWMA-SR control chart. It is worth mentioning that, for small shifts, the proposed TEWMA – SR_{RSS} control chart outperforms the TEWMA-SR control chart for different combinations of λ and n .

4.2. Proposed versus TEWMA-SN Control Chart. The RL features of the TEWMA-SN control chart for the process location are listed in Table 10. The proposed TEWMA – SR_{RSS} control chart is more efficient than the TEWMA-SN control chart. For example, under Laplace distribution, the proposed TEWMA – SR_{RSS} control chart with $n = 10$ and $\lambda = 0.05$ with $\delta = 10$ percent shows OOC situation almost after 7.75 observations, whereas the TEWMA-SN control chart gives OOC state nearly after 25.67 observations (see Tables 4 and 10). Similarly, in the $t_{(8)}$ distribution scenario, at $n = 10$ and $\lambda = 0.05$ with a shift of 5 percent, the TEWMA-SN control chart decreases the ARL₁

TABLE 8: ARL values of the proposed TEWMA – SR_{RSS}, TEWMA-SR, TEWMA-SN, TEWMA- \bar{X} , and DEWMA – SR_{RSS} control charts for $\lambda = 0.05$ under various distributions when $ARL_0 = 370.$.

δ	Control charts	L	Normal	t_4	t_8	Logistic	Laplace	CN
$n = 5$								
0.00	TEWMA – SR _{RSS}	1.585	371.11	369.03	370.51	371.69	371.65	369.92
	TEWMA-SR	1.761	370.21	370.70	370.50	369.93	369.93	370.41
	TEWMA-SN	1.755	370.12	371.34	369.35	370.63	370.63	371.45
	TEWMA – \bar{X}	1.761	370.42	384.87	375.61	369.18	368.30	420.27
	DEWMA – SR _{RSS}	1.742	370.75	369.11	37.065	369.58	368.53	371.09
0.05	TEWMA – SR _{RSS}	1.585	92.67	111.34	99.36	80.72	63.49	94.31
	TEWMA-SR	1.761	170.21	133.43	154.45	152.63	117.81	164.23
	TEWMA-SN	1.755	215.69	147.36	177.50	173.25	107.59	190.75
	TEWMA – \bar{X}	1.761	157.04	159.27	155.59	156.45	156.49	164.35
	DEWMA – SR _{RSS}	1.742	95.77	114.90	101.40	84.81	66.13	97.51
0.10	TEWMA – SR _{RSS}	1.585	33.91	41.65	37.64	30.49	22.54	35.63
	TEWMA-SR	1.761	71.18	52.78	63.28	63.83	47.12	68.51
	TEWMA-SN	1.755	102.07	59.59	75.47	74.30	43.97	84.11
	TEWMA – \bar{X}	1.761	63.92	64.94	63.34	64.59	64.94	65.32
	DEWMA – SR _{RSS}	1.742	36.12	45.08	40.27	33.16	25.23	37.70
0.25	TEWMA – SR _{RSS}	1.585	7.94	10.43	8.89	7.20	5.48	8.35
	TEWMA-SR	1.761	18.95	13.75	16.66	16.93	12.66	18.06
	TEWMA-SN	1.755	28.46	15.85	20.09	20.07	12.44	22.93
	TEWMA – \bar{X}	1.761	16.57	16.59	16.24	16.59	16.59	16.57
	DEWMA – SR _{RSS}	1.742	9.03	12.07	11.74	9.62	7.32	10.56
0.50	TEWMA – SR _{RSS}	1.585	2.35	3.34	2.80	2.18	2.01	2.51
	TEWMA-SR	1.761	6.16	4.50	5.48	5.45	4.46	5.83
	TEWMA-SN	1.755	9.62	5.20	6.70	6.62	4.71	7.60
	TEWMA – \bar{X}	1.761	5.07	4.99	5.02	5.10	5.07	5.00
	DEWMA – SR _{RSS}	1.742	2.88	3.91	2.99	2.78	2.09	2.84
0.75	TEWMA – SR _{RSS}	1.585	1.37	1.79	1.58	1.31	1.32	1.46
	TEWMA-SR	1.761	3.15	2.49	2.86	2.88	2.57	2.98
	TEWMA-SN	1.755	4.88	2.81	3.47	3.45	2.80	3.83
	TEWMA – \bar{X}	1.761	2.44	2.38	2.42	2.44	2.42	2.40
	DEWMA – SR _{RSS}	1.742	1.85	1.99	1.89	1.73	1.79	1.72
1.00	TEWMA – SR _{RSS}	1.585	1.08	1.36	1.20	1.10	1.12	1.14
	TEWMA-SR	1.761	2.06	1.77	1.94	1.95	1.85	1.97
	TEWMA-SN	1.755	3.02	1.93	2.25	2.26	2.01	2.41
	TEWMA – \bar{X}	1.761	1.54	1.50	1.53	1.53	1.52	1.52
	DEWMA – SR _{RSS}	1.742	1.44	1.67	1.46	1.33	1.32	1.31
1.25	TEWMA – SR _{RSS}	1.585	1.01	1.09	1.03	1.01	1.03	1.01
	TEWMA-SR	1.761	1.58	1.44	1.53	1.54	1.51	1.52
	TEWMA-SN	1.755	2.14	1.52	1.69	1.70	1.61	1.74
	TEWMA – \bar{X}	1.761	1.19	1.18	1.19	1.19	1.19	1.18
	DEWMA – SR _{RSS}	1.742	1.03	1.19	1.09	1.04	1.07	1.05
1.50	TEWMA – SR _{RSS}	1.585	1.00	1.02	1.01	1.00	1.00	1.00
	TEWMA-SR	1.761	1.33	1.28	1.31	1.32	1.33	1.30
	TEWMA-SN	1.755	1.66	1.32	1.39	1.40	1.38	1.38
	TEWMA – \bar{X}	1.761	1.06	1.06	1.07	1.06	1.06	1.05
	DEWMA – SR _{RSS}	1.742	1.00	1.06	1.02	1.00	1.00	1.00
$n = 10$								
0.00	TEWMA – SR _{RSS}	1.3635	369.61	370.73	369.67	371.04	371.04	369.54
	TEWMA-SR	1.773	370.71	370.32	371.50	371.75	371.75	370.57
	TEWMA-SN	1.774	370.63	369.08	368.98	370.48	370.48	371.49
	TEWMA – \bar{X}	1.761	370.31	378.00	365.57	366.34	368.59	425.74
	DEWMA – SR _{RSS}	1.503	369.55	369.84	371.06	371.38	370.20	370.52
0.05	TEWMA – SR _{RSS}	1.3635	34.99	43.49	40.15	30.63	23.68	37.52
	TEWMA-SR	1.773	111.13	85.42	102.17	101.11	77.70	106.18
	TEWMA-SN	1.774	135.28	94.83	118.69	115.90	66.23	129.65
	TEWMA – \bar{X}	1.761	102.68	105.01	103.54	102.70	102.90	105.26
	DEWMA – SR _{RSS}	1.503	37.77	47.80	43.22	33.16	26.02	39.68

TABLE 8: Continued.

δ	Control charts	L	Normal	t_4	t_8	Logistic	Laplace	CN
0.10	TEWMA – SR _{RSS}	1.3635	12.62	15.90	14.78	10.77	7.75	13.22
	TEWMA-SR	1.773	42.69	31.60	38.05	38.30	29.06	40.28
	TEWMA-SN	1.774	53.12	35.75	45.53	44.56	25.67	50.43
	TEWMA – \bar{X}	1.761	39.00	39.58	39.03	39.29	39.27	39.33
	DEWMA – SR _{RSS}	1.503	14.58	17.68	15.62	12.68	9.50	15.23
0.25	TEWMA – SR _{RSS}	1.3635	2.47	3.29	2.87	2.19	1.74	2.65
	TEWMA-SR	1.773	10.40	7.46	9.29	9.26	7.09	9.84
	TEWMA-SN	1.774	13.53	8.61	11.39	11.15	6.65	12.78
	TEWMA – \bar{X}	1.761	9.34	9.34	9.45	9.38	9.38	9.31
	DEWMA – SR _{RSS}	1.503	3.54	4.17	3.71	2.82	1.98	2.99
0.50	TEWMA – SR _{RSS}	1.3635	1.06	1.23	1.11	1.26	1.04	1.11
	TEWMA-SR	1.773	3.12	2.35	2.82	2.78	2.35	2.95
	TEWMA-SN	1.774	4.13	2.60	3.43	3.36	2.34	3.89
	TEWMA – \bar{X}	1.761	2.72	2.69	2.72	2.69	2.69	2.67
	DEWMA – SR _{RSS}	1.503	1.33	1.38	1.27	1.46	1.09	1.18
0.75	TEWMA – SR _{RSS}	1.3635	1.00	1.02	1.01	1.00	1.00	1.01
	TEWMA-SR	1.773	1.62	1.39	1.53	1.52	1.43	1.55
	TEWMA-SN	1.774	2.02	1.45	1.75	1.73	1.44	1.92
	TEWMA – \bar{X}	1.761	1.43	1.41	1.43	1.42	1.41	1.41
	DEWMA – SR _{RSS}	1.503	1.00	1.05	1.04	1.01	1.00	1.01
1.00	TEWMA – SR _{RSS}	1.3635	1.00	1.00	1.00	1.00	1.00	1.00
	TEWMA-SR	1.773	1.18	1.12	1.16	1.16	1.15	1.15
	TEWMA-SN	1.774	1.34	1.14	1.24	1.24	1.16	1.29
	TEWMA – \bar{X}	1.761	1.10	1.09	1.10	1.09	1.09	1.09
	DEWMA – SR _{RSS}	1.503	1.01	1.03	1.02	1.02	1.01	1.00
1.25	TEWMA – SR _{RSS}	1.3635	1.00	1.00	1.00	1.00	1.00	1.00
	TEWMA-SR	1.773	1.04	1.04	1.05	1.04	1.05	1.03
	TEWMA-SN	1.774	1.11	1.04	1.08	1.08	1.06	1.09
	TEWMA – \bar{X}	1.761	1.01	1.02	1.02	1.02	1.02	1.01
	DEWMA – SR _{RSS}	1.503	1.00	1.01	1.00	1.00	1.01	1.00
1.50	TEWMA – SR _{RSS}	1.3635	1.00	1.00	1.00	1.00	1.00	1.00
	TEWMA-SR	1.773	1.01	1.01	1.01	1.01	1.02	1.01
	TEWMA-SN	1.774	1.03	1.01	1.02	1.02	1.02	1.02
	TEWMA – \bar{X}	1.761	1.00	1.00	1.00	1.00	1.00	1.00
	DEWMA – SR _{RSS}	1.503	1.00	1.00	1.00	1.00	1.00	1.00

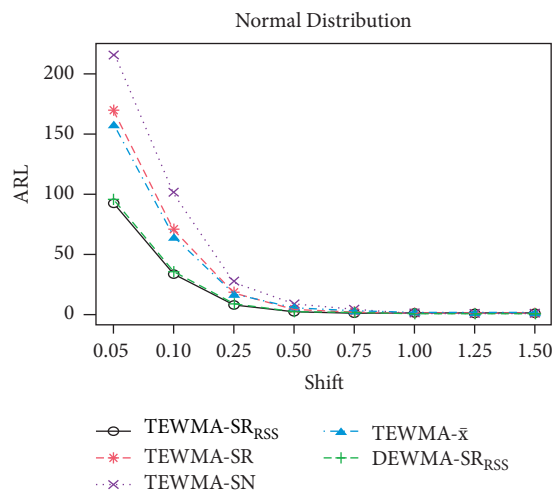


FIGURE 5: ARL_s of the proposed TEWMA – SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA – \bar{X} , and DEWMA – SR_{RSS} control charts for normal distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

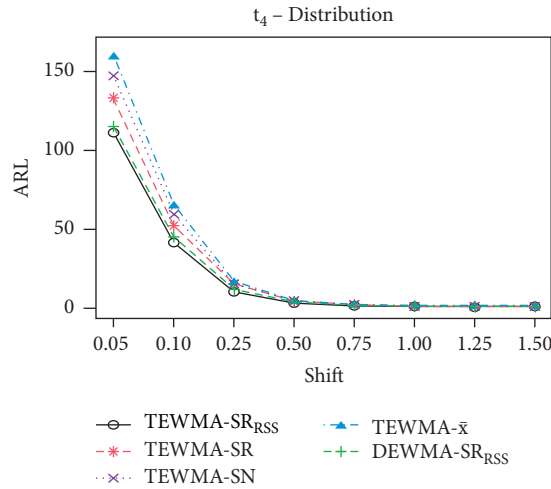


FIGURE 6: ARL_s of the proposed TEWMA – SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA – \bar{X} , and DEWMA – SR_{RSS} control charts under t_4 distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

TABLE 9: RL characteristics of the TEWMA-SR control chart under different distribution when $ARL_0 = 370$.

(λ, L)	Distr.	δ									
		0.00	0.05	0.10	0.25	0.50	0.75	1.00	1.25	1.50	
		$n = 5$									
(0.10, 2.04)	$N(0, 1)$	ARL	370.40	202.43	87.08	21.41	7.41	4.03	2.79	2.30	2.10
		MDRL	249.00	139.00	64.00	19.00	6.00	3.00	2.00	2.00	2.00
		SDRL	387.42	207.07	81.89	15.83	5.07	2.47	1.31	0.71	0.39
	t_4	ARL	371.00	160.93	62.47	15.69	5.56	3.29	2.53	2.23	2.11
		MDRL	252.00	112.00	49.00	14.00	5.00	2.00	2.00	2.00	2.00
		SDRL	389.09	161.68	54.78	11.01	3.70	1.84	1.04	0.64	0.42
	t_8	ARL	370.56	185.17	76.89	18.89	6.65	3.71	2.68	2.28	2.11
		MDRL	247.00	128.00	58.00	17.00	6.00	3.00	2.00	2.00	1.00
		SDRL	397.41	189.51	71.05	13.66	4.54	2.19	1.19	0.69	0.41
	Logistic	ARL	369.90	183.20	76.96	19.07	6.63	3.72	2.69	2.28	2.12
		MDRL	249.00	125.00	57.00	17.00	6.00	3.00	2.00	2.00	2.00
		SDRL	389.84	185.38	70.88	13.76	4.51	2.20	1.21	0.70	0.42
Laplace	ARL	369.90	142.71	55.49	14.53	5.52	3.39	2.61	2.28	2.13	
	MDRL	249.00	100.00	43.00	13.00	5.00	2.00	2.00	2.00	2.00	
	SDRL	389.84	141.45	48.47	10.35	3.71	1.93	1.13	0.71	0.46	
CN	ARL	369.40	195.44	83.32	20.41	7.06	3.83	2.70	2.25	2.08	
	MDRL	250.00	134.00	62.00	18.00	6.00	3.00	2.00	2.00	2.00	
	SDRL	386.73	199.04	77.80	14.96	4.77	2.29	1.22	0.66	0.36	
(0.25, 2.435)	$N(0, 1)$	ARL	370.04	251.08	123.74	26.12	8.21	4.71	3.36	2.72	2.39
		MDRL	255.00	173.00	87.00	20.00	7.00	4.00	3.00	3.00	2.00
		SDRL	371.26	248.36	120.31	20.92	4.68	2.28	1.35	0.88	0.61
	t_4	ARL	369.94	208.62	88.84	17.89	6.29	3.89	2.98	2.56	2.34
		MDRL	255.00	145.00	64.00	15.00	6.00	3.00	3.00	2.00	2.00
		SDRL	369.71	207.56	83.88	12.73	3.36	1.78	1.14	0.81	0.62
	t_8	ARL	370.11	234.16	110.11	22.47	7.42	4.36	3.20	2.66	2.38
		MDRL	254.00	164.00	78.00	18.00	7.00	4.00	3.00	2.00	2.00
		SDRL	370.95	233.40	105.28	17.24	4.15	2.07	1.25	0.86	0.62
	Logistic	ARL	370.34	232.92	109.14	22.91	7.42	4.36	3.21	2.67	2.38
		MDRL	254.00	162.00	77.00	18.00	7.00	4.00	3.00	2.00	2.00
		SDRL	374.21	229.98	104.65	17.78	4.13	2.06	1.28	0.86	0.62
Laplace	ARL	370.34	191.59	77.88	16.52	6.23	3.98	3.09	2.64	2.40	
	MDRL	254.00	134.00	56.00	14.00	6.00	4.00	3.00	2.00	2.00	
	SDRL	374.21	188.39	73.25	11.80	3.37	1.87	1.23	0.87	0.66	
CN	ARL	370.82	244.07	117.97	24.67	7.86	4.52	3.25	2.65	2.35	
	MDRL	255.00	168.00	84.00	19.00	7.00	4.00	3.00	2.00	2.00	
	SDRL	369.23	243.23	112.20	19.61	4.41	2.14	1.28	0.83	0.57	

TABLE 9: Continued.

(λ, L)	Distr.	δ									
		0.00	0.05	0.10	0.25	0.50	0.75	1.00	1.25	1.50	
		$n = 10$									
(0.10, 2.058)	$N(0, 1)$	ARL	370.56	135.11	49.19	11.81	3.79	1.96	1.34	1.11	1.03
		MDRL	247.00	94.00	39.00	10.00	3.00	2.00	1.00	1.00	1.00
		SDRL	398.14	136.85	42.90	8.61	2.76	1.25	0.65	0.34	0.17
	t_4	ARL	371.81	102.96	35.83	8.65	2.86	1.65	1.26	1.11	1.05
		MDRL	248.00	74.00	30.00	7.00	2.00	1.00	1.00	1.00	1.00
		SDRL	399.99	100.54	29.33	6.38	2.04	0.96	0.54	0.34	0.23
	t_8	ARL	370.70	123.16	43.53	10.62	3.42	1.84	1.31	1.12	1.04
		MDRL	248.00	87.00	35.00	9.00	3.00	1.00	1.00	1.00	1.00
		SDRL	400.75	122.75	36.85	7.71	2.46	1.14	0.62	0.36	0.21
	Logistic	ARL	370.85	122.50	43.60	10.57	3.40	1.83	1.31	1.12	1.04
		MDRL	86.00	35.00	9.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	401.74	123.07	37.52	7.79	2.44	1.12	0.61	0.35	0.21
	Laplace	ARL	370.85	93.19	32.58	8.25	2.87	1.70	1.30	1.14	1.06
		MDRL	247.00	68.00	27.00	7.00	2.00	1.00	1.00	1.00	1.00
		SDRL	401.74	90.83	26.50	6.12	2.03	1.01	0.60	0.38	0.25
	CN	ARL	370.25	128.82	46.63	11.22	3.58	1.87	1.30	1.09	1.02
		MDRL	248.00	91.00	37.00	10.00	3.00	1.00	1.00	1.00	1.00
		SDRL	400.04	129.39	40.58	8.26	2.61	1.17	0.60	0.31	0.15
(0.25, 2.448)	$N(0, 1)$	ARL	370.18	180.52	68.40	13.21	4.44	2.45	1.69	1.33	1.14
		MDRL	254.00	126.00	50.00	11.00	4.00	2.00	2.00	1.00	1.00
		SDRL	375.22	180.15	63.64	9.23	2.53	1.28	0.78	0.53	0.36
	t_4	ARL	369.69	143.29	47.66	9.55	3.41	2.04	1.51	1.26	1.14
		MDRL	255.00	101.00	35.00	9.00	3.00	2.00	1.00	1.00	1.00
		SDRL	375.73	142.46	42.57	6.15	1.95	1.06	0.69	0.49	0.36
	t_8	ARL	369.78	168.23	60.52	11.79	4.05	2.30	1.63	1.31	1.15
		MDRL	254.00	117.00	44.00	10.00	4.00	2.00	1.00	1.00	1.00
		SDRL	374.36	166.36	55.56	7.86	2.30	1.19	0.76	0.53	0.37
	Logistic	ARL	370.66	168.70	60.21	11.69	4.01	2.28	1.62	1.31	1.15
		MDRL	255.00	116.00	43.00	10.00	4.00	2.00	1.00	1.00	1.00
		SDRL	376.79	168.98	55.64	7.93	2.29	1.18	0.75	0.53	0.37
	Laplace	ARL	370.66	131.82	43.16	9.07	3.43	2.10	1.57	1.31	1.16
		MDRL	255.00	92.00	32.00	8.00	3.00	2.00	1.00	1.00	1.00
		SDRL	376.79	130.99	38.69	5.86	1.97	1.10	0.74	0.53	0.39
	CN	ARL	371.72	175.24	65.05	12.48	4.22	2.35	1.63	1.29	1.12
		MDRL	256.00	123.00	47.00	11.00	4.00	2.00	1.00	1.00	1.00
		SDRL	376.29	173.67	60.76	8.60	2.42	1.22	0.74	0.50	0.33

TABLE 10: RL characteristics of the TEWMA-SN control chart when $ARL_0 = 370$.

λ	L	δ									
		0.00	0.05	0.10	0.25	0.50	0.75	1.00	1.25	1.50	
		$n = 5$									
0.05	1.755	ARL	370.16	300.16	127.36	65.39	23.63	12.29	7.37	3.34	1.76
		MDRL	226.00	189.00	90.00	52.00	19.00	10.00	6.00	3.00	1.00
		SDRL	433.64	343.98	132.82	61.43	20.07	10.43	6.22	2.64	1.15
0.10	2.057	ARL	370.66	317.07	150.59	75.51	25.58	13.51	8.32	3.90	2.07
		MDRL	244.00	209.00	104.00	56.00	22.00	12.00	7.00	3.00	1.00
		SDRL	403.07	343.76	154.96	71.44	20.33	10.12	6.20	2.87	1.43
0.25	2.444	ARL	370.71	341.92	201.57	108.16	32.90	15.61	9.40	4.71	2.86
		MDRL	255.00	236.00	140.00	77.00	25.00	13.00	8.00	4.00	2.00
		SDRL	376.22	344.07	198.79	103.51	27.87	11.13	5.81	2.49	1.16
0.50	2.744	ARL	370.00	354.05	254.57	160.73	52.56	21.94	11.65	5.15	3.10
		MDRL	256.00	244.00	176.00	112.00	37.00	16.00	9.00	5.00	3.00
		SDRL	373.11	356.39	252.52	158.19	49.29	18.87	8.65	2.69	1.10
0.75	2.807	ARL	369.67	360.66	294.33	215.10	88.43	38.79	19.31	6.74	3.34
		MDRL	260.00	249.00	205.00	149.00	63.00	27.00	14.00	5.00	3.00
		SDRL	367.82	367.82	358.70	291.34	213.85	86.50	36.72	17.46	4.80

TABLE 10: Continued.

λ	L	δ									
		0.00	0.05	0.10	0.25	0.50	0.75	1.00	1.25	1.50	
$n = 10$											
0.05	1.774	ARL	370.65	254.20	80.02	38.63	13.34	6.56	3.77	1.68	1.09
		MDRL	224.00	157.00	61.00	31.00	10.00	5.00	3.00	1.00	1.00
		SDRL	441.63	297.47	80.63	35.43	11.91	5.93	3.37	1.24	0.36
0.10	2.047	ARL	370.66	280.36	97.51	45.03	15.25	7.84	4.72	2.19	1.30
		MDRL	247.00	189.00	72.00	36.00	13.00	7.00	4.00	2.00	1.00
		SDRL	395.10	293.48	93.13	38.35	11.10	5.68	3.38	1.37	0.55
0.25	2.454	ARL	370.39	316.88	137.44	61.44	17.26	8.43	5.16	2.44	1.35
		MDRL	252.00	219.00	96.00	44.00	14.00	8.00	5.00	2.00	1.00
		SDRL	379.72	320.57	135.97	57.89	13.23	5.50	3.25	1.52	0.66
0.50	2.761	ARL	370.52	337.38	189.41	97.16	24.83	10.23	5.77	2.84	1.77
		MDRL	254.00	233.00	133.00	69.00	18.00	8.00	5.00	3.00	2.00
		SDRL	375.53	337.71	186.49	94.36	22.07	7.67	3.56	1.30	0.66
0.75	2.891	ARL	370.35	351.34	244.79	148.51	42.51	15.95	7.76	3.11	1.79
		MDRL	255.00	243.00	169.00	103.00	30.00	12.00	6.00	3.00	2.00
		SDRL	369.68	351.31	243.05	148.06	40.84	14.16	6.06	1.70	0.70

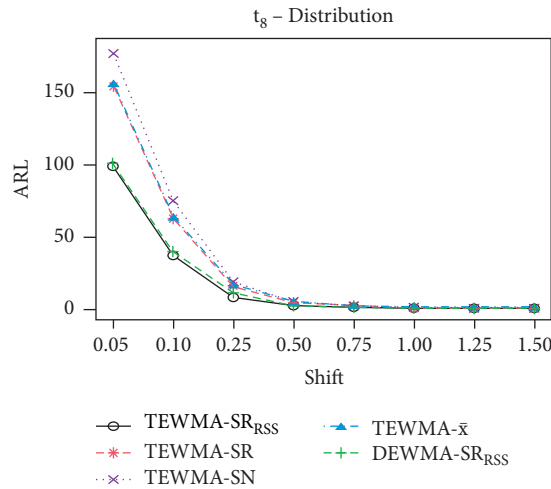


FIGURE 7: ARL_s of the proposed TEWMA - SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA - \bar{X} , and DEWMA - SR_{RSS} control charts under t_8 distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

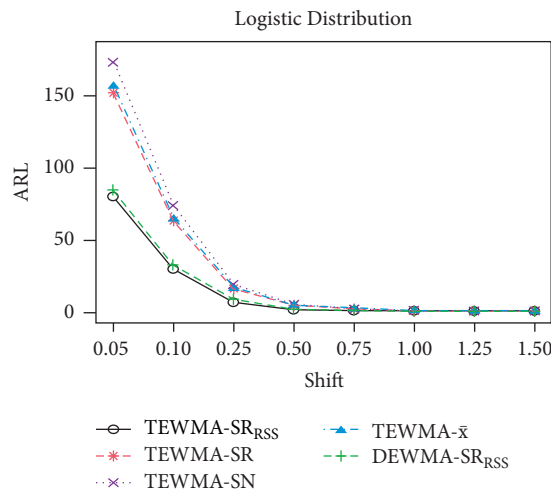


FIGURE 8: ARL_s of the proposed TEWMA - SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA - \bar{X} , and DEWMA - SR_{RSS} control charts under logistic distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

TABLE 11: RL characteristics of the TEWMA- \bar{X} control chart under different distributions when $ARL_0 = 370$.

(λ, L)		δ										
		0.00	0.05	0.10	0.25	0.50	0.75	1.00	1.25	1.50		
		$n = 5$										
(0.10, 2.052)	$N(0, 1)$	ARL	369.03	307.06	200.72	65.87	21.83	11.54	7.17	4.79	3.55	
		MDRL	250.00	202.00	136.00	49.00	19.00	10.00	6.00	4.00	3.00	
		SDRL	395.95	328.82	211.07	60.94	16.99	8.57	5.37	3.60	2.72	
	t_4	ARL	370.29	186.99	77.45	18.30	5.80	2.86	1.75	1.26	1.10	
		MDRL	242.00	124.00	56.00	16.00	5.00	2.00	1.00	1.00	1.00	
		SDRL	400.55	194.09	74.04	13.24	4.36	2.15	1.18	0.60	0.35	
	t_8	ARL	369.13	187.84	75.73	18.58	6.07	2.91	1.78	1.31	1.10	
		MDRL	241.50	129.50	56.00	17.00	5.00	2.00	1.00	1.00	1.00	
		SDRL	401.65	197.86	71.48	13.83	4.56	2.18	1.16	0.65	0.34	
	Logistic	ARL	369.25	182.82	76.01	18.40	5.95	2.94	1.80	1.28	1.10	
		MDRL	251.00	129.00	57.00	16.00	5.00	2.00	1.00	1.00	1.00	
		SDRL	395.31	190.55	71.15	14.08	4.51	2.17	1.20	0.63	0.35	
	Laplace	ARL	369.16	181.96	76.63	18.55	5.95	2.85	1.78	1.31	1.10	
		MDRL	253.00	130.00	57.00	16.00	5.00	2.00	1.00	1.00	1.00	
		SDRL	396.30	191.07	72.13	14.25	4.47	2.07	1.17	0.64	0.33	
	CN	ARL	370.86	184.21	75.84	18.45	5.88	2.84	1.76	1.30	1.10	
		MDRL	238.00	127.50	57.00	16.00	5.00	2.00	1.00	1.00	1.00	
		SDRL	406.48	187.56	70.40	13.78	4.48	2.11	1.14	0.65	0.35	
			$n = 10$									
	(0.25, 2.456)	$N(0, 1)$	ARL	369.58	337.05	253.31	92.03	26.45	12.90	8.04	5.49	4.12
			MDRL	255.00	237.00	180.00	63.00	21.00	11.00	7.00	5.00	4.00
			SDRL	371.62	344.14	249.89	90.27	21.86	9.25	5.23	3.44	2.57
		t_4	ARL	370.01	244.15	112.68	22.79	6.56	3.35	2.09	1.50	1.20
			MDRL	259.00	163.00	80.00	18.00	6.00	3.00	2.00	1.00	1.00
SDRL			374.25	244.05	106.57	18.02	4.02	2.03	1.24	0.79	0.47	
t_8		ARL	369.15	245.69	113.57	22.10	6.60	3.42	2.12	1.50	1.22	
		MDRL	252.00	170.00	79.00	18.00	6.00	3.00	2.00	1.00	1.00	
		SDRL	378.00	244.08	113.25	17.96	4.07	2.08	1.26	0.77	0.48	
Logistic		ARL	370.81	235.39	109.97	22.42	6.72	3.44	2.10	1.50	1.22	
		MDRL	259.50	165.00	76.00	18.00	6.00	3.00	2.00	1.00	1.00	
		SDRL	379.06	236.63	106.03	17.58	4.19	2.15	1.21	0.77	0.48	
Laplace		ARL	370.35	238.73	113.35	22.41	6.52	3.39	2.11	1.50	1.21	
		MDRL	259.00	165.00	79.00	18.00	6.00	3.00	2.00	1.00	1.00	
		SDRL	385.77	241.88	109.29	17.63	3.99	2.11	1.24	0.77	0.49	
CN		ARL	370.73	240.89	111.72	21.68	6.62	3.29	2.11	1.52	1.21	
		MDRL	246.50	171.00	79.00	17.00	6.00	3.00	2.00	1.00	1.00	
		SDRL	382.48	237.76	108.46	17.44	4.19	2.04	1.25	0.79	0.48	
		$n = 10$										
(0.10, 2.052)		$N(0, 1)$	ARL	371.96	311.91	204.02	64.05	21.87	11.43	6.99	4.93	3.50
			MDRL	249.00	206.00	139.50	48.00	19.00	10.00	6.00	4.00	3.00
			SDRL	403.38	328.93	212.02	60.24	16.61	8.44	5.35	3.77	2.64
		t_4	ARL	370.15	128.10	45.77	10.67	3.17	1.60	1.15	1.04	1.01
			MDRL	246.00	91.00	37.00	9.00	3.00	1.00	1.00	1.00	1.00
	SDRL		397.13	125.50	39.69	7.95	2.35	0.99	0.44	0.22	0.11	
	t_8	ARL	369.95	124.79	44.48	10.74	3.24	1.63	1.15	1.03	1.01	
		MDRL	257.00	91.00	36.00	9.00	3.00	1.00	1.00	1.00	1.00	
		SDRL	381.78	120.87	38.66	7.79	2.44	1.06	0.43	0.18	0.08	
	Logistic	ARL	370.40	123.96	44.63	10.63	3.20	1.62	1.16	1.03	1.00	
		MDRL	242.00	91.00	37.00	9.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	402.56	121.81	37.00	7.85	2.42	1.00	0.46	0.18	0.07	
	Laplace	ARL	370.40	124.11	44.62	10.64	3.18	1.61	1.16	1.03	1.00	
		MDRL	239.00	91.00	36.00	9.00	2.00	1.00	1.00	1.00	1.00	
		SDRL	399.49	121.43	38.26	7.94	2.42	1.00	0.45	0.19	0.06	
	CN	ARL	368.84	123.63	43.39	10.44	3.18	1.64	1.16	1.03	1.00	
		MDRL	240.00	85.50	35.00	9.00	3.00	1.00	1.00	1.00	1.00	
		SDRL	397.07	123.35	36.98	7.95	2.32	1.02	0.44	0.17	0.06	

TABLE 12: Continued.

(λ, L)		δ															
		0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00		
(0.25, 2.362)	N(0, 1)	ARL	370.58	301.13	273.03	167.12	102.33	58.70	30.54	17.37	10.93	3.47	2.04	1.47	1.08	1.01	
		MDRL	258.50	199.50	189.00	110.00	75.00	43.00	23.00	14.00	9.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	362.34	306.15	284.31	169.26	99.93	54.43	26.51	13.30	7.40	1.99	0.99	0.65	0.27	0.08	
	CN	ARL	371.19	294.66	257.26	177.09	104.49	63.79	29.44	17.46	11.15	3.60	2.14	1.51	1.10	1.02	
		MDRL	274.50	202.00	176.50	128.00	72.00	47.00	21.00	14.00	9.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	353.65	299.44	256.86	167.69	105.05	58.35	26.81	14.12	8.05	2.05	1.07	0.67	0.31	0.15	
	Logistic	ARL	371.17	277.17	257.67	149.26	83.13	52.45	23.74	13.67	9.30	3.11	1.91	1.39	1.08	1.02	
		MDRL	275.00	188.00	176.00	101.50	60.00	40.00	18.00	11.00	8.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	348.81	281.41	258.33	150.34	81.19	43.81	20.29	9.88	6.42	1.75	0.99	0.62	0.29	0.14	
	Laplace	ARL	371.11	247.40	222.46	116.11	59.40	36.86	16.14	9.95	7.10	2.72	1.68	1.36	1.11	1.02	
		MDRL	275.00	162.50	165.00	85.00	43.00	27.00	13.00	8.00	6.00	3.00	1.00	1.00	1.00	1.00	
		SDRL	348.81	260.58	212.18	110.40	55.16	32.49	12.30	7.19	4.63	1.48	0.86	0.62	0.33	0.15	
	t_4	ARL	369.52	307.12	297.61	207.84	132.84	77.75	37.64	22.42	14.38	4.45	2.56	1.79	1.28	1.10	
		MDRL	243.00	211.00	208.50	150.00	89.50	57.00	29.00	17.00	12.00	4.00	2.00	2.00	1.00	1.00	
		SDRL	376.56	315.27	279.56	200.11	139.05	72.20	31.43	18.34	10.67	2.66	1.44	0.89	0.55	0.34	
	t_8	ARL	369.32	285.23	271.09	184.52	109.21	69.61	31.69	18.25	11.77	3.82	2.25	1.62	1.15	1.05	
		MDRL	255.50	196.50	196.50	130.00	81.00	52.00	24.00	15.00	10.00	3.00	2.00	1.00	1.00	1.00	
		SDRL	354.63	280.49	266.28	186.00	102.36	65.63	27.83	14.36	8.30	2.23	1.18	0.77	0.39	0.22	
	(0.50, 2.601)	N(0, 1)	ARL	371.14	336.08	330.95	232.29	171.68	101.87	50.11	26.02	16.06	4.19	2.38	1.79	1.31	1.11
			MDRL	258.00	240.00	221.00	164.50	120.50	73.00	34.00	18.00	12.00	4.00	2.00	2.00	1.00	1.00
			SDRL	380.72	326.29	339.71	222.02	164.63	95.45	49.06	24.46	13.48	2.44	1.02	0.65	0.47	0.31
		CN	ARL	370.72	316.65	306.03	239.43	164.08	103.67	53.53	28.89	17.49	4.27	2.55	1.83	1.37	1.15
			MDRL	251.00	219.00	209.00	168.50	115.00	72.00	39.00	22.00	13.00	4.00	2.00	2.00	1.00	1.00
			SDRL	361.63	317.66	294.81	225.10	163.16	101.67	52.28	26.52	14.78	2.47	1.13	0.67	0.51	0.36
Logistic		ARL	369.36	316.07	300.44	220.68	133.29	85.18	40.43	19.95	13.48	3.67	2.20	1.72	1.33	1.12	
		MDRL	251.50	217.50	218.00	148.00	94.00	56.00	30.00	14.00	10.00	3.00	2.00	2.00	1.00	1.00	
		SDRL	357.96	306.79	298.34	222.27	128.58	87.07	36.99	17.98	11.63	2.05	0.96	0.68	0.47	0.33	
Laplace		ARL	371.17	291.10	261.49	170.05	102.20	61.00	26.56	13.83	9.45	3.13	2.05	1.64	1.31	1.15	
		MDRL	252.50	208.00	190.50	120.00	76.00	42.00	19.00	10.00	7.00	3.00	2.00	2.00	1.00	1.00	
		SDRL	363.24	282.51	261.00	168.35	97.94	57.95	24.30	11.94	7.29	1.71	0.94	0.65	0.48	0.36	
t_4		ARL	369.99	341.67	332.01	268.76	178.68	128.88	63.64	36.72	22.19	5.23	2.91	2.12	1.57	1.29	
		MDRL	256.50	224.00	226.00	189.00	125.00	89.00	48.00	27.00	16.00	4.00	3.00	2.00	2.00	1.00	
		SDRL	375.89	347.85	335.27	261.40	177.72	130.43	59.68	34.08	20.12	3.49	1.46	0.92	0.62	0.47	
t_8		ARL	370.07	326.33	319.56	233.66	169.06	118.75	56.05	31.40	19.15	4.50	2.66	1.95	1.39	1.19	
		MDRL	234.50	223.00	222.00	164.00	123.00	88.00	41.00	22.00	14.00	4.00	2.00	2.00	1.00	1.00	
		SDRL	383.44	327.84	325.64	228.44	162.83	113.12	53.16	29.25	16.76	2.69	1.24	0.76	0.51	0.40	

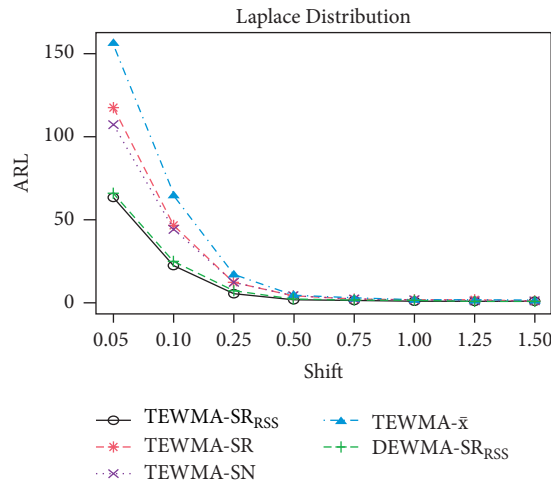


FIGURE 9: ARL_s of the proposed TEWMA – SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA – \bar{X} , and DEWMA – SR_{RSS} control charts under Laplace distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

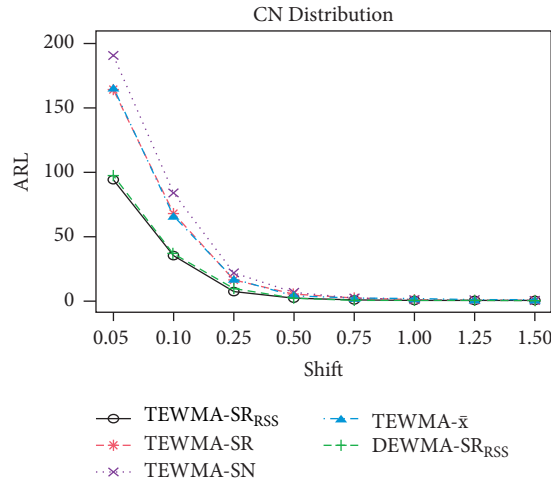


FIGURE 10: ARL_s of the proposed TEWMA – SR_{RSS} versus TEWMA-SR, TEWMA-SN, TEWMA – \bar{X} , and DEWMA – SR_{RSS} control charts under CN distribution when $n = 5$, $\lambda = 0.05$, and $ARL_0 = 370$.

TABLE 13: RL characteristics of the DEWMA – SR_{RSS} control chart under different distributions with nominal $ARL_0 = 370$ and $n = 10$.

(λ, L)		Distr.	δ													
			0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00
(0.05, 1.503)	N(0, 1)	ARL	369.55	95.01	77.78	34.77	19.00	11.58	5.85	3.56	2.54	1.13	1.00	1.00	1.00	1.00
		MDRL	236.00	69.00	59.00	29.00	16.00	10.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	408.60	97.32	74.53	30.63	15.45	9.08	4.41	2.54	1.82	0.37	0.03	0.00	0.00	0.00
	CN	ARL	370.52	105.62	83.63	36.68	19.39	12.23	6.14	3.88	2.62	1.15	1.01	1.00	1.00	1.00
		MDRL	243.50	77.00	62.00	28.00	16.00	10.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	413.79	106.34	83.56	32.13	15.93	9.82	4.74	2.80	1.77	0.38	0.08	0.00	0.00	0.00
	Logistic	ARL	371.38	85.92	66.99	31.16	15.76	9.68	5.13	3.10	2.22	1.08	1.01	1.00	1.00	1.00
		MDRL	236.00	65.00	48.00	26.00	13.00	8.00	4.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	443.42	82.79	67.57	26.84	12.95	7.64	3.92	2.28	1.42	0.28	0.07	0.00	0.00	0.00
	Laplace	ARL	370.20	68.96	51.92	24.02	12.46	7.50	3.87	2.52	1.85	1.06	1.00	1.00	1.00	1.00
		MDRL	261.00	52.00	39.00	20.00	10.00	6.00	3.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	467.24	65.38	50.61	19.88	9.92	5.84	2.87	1.70	1.10	0.24	0.06	0.03	0.00	0.00
	t_4	ARL	369.84	121.67	100.83	44.80	22.07	14.68	7.14	4.70	3.17	1.28	1.04	1.00	1.00	1.00
		MDRL	263.50	88.00	72.00	37.00	18.00	12.00	6.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	427.48	121.96	99.10	39.73	18.40	12.18	5.71	3.49	2.34	0.54	0.21	0.05	0.00	0.00
	t_8	ARL	371.06	109.28	83.63	40.22	20.97	12.62	6.74	4.07	2.71	1.17	1.02	1.00	1.00	1.00
		MDRL	236.50	73.00	59.00	32.00	18.00	10.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	416.69	118.78	84.24	36.85	17.32	10.10	5.40	3.10	1.93	0.41	0.12	0.03	0.00	0.00
(0.10, 1.709)	N(0, 1)	ARL	368.73	118.72	92.48	42.78	20.97	13.35	6.56	4.18	2.83	1.17	1.02	1.00	1.00	1.00
		MDRL	247.50	82.00	67.00	32.00	17.50	12.00	6.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	366.84	118.50	90.41	37.88	16.37	9.30	4.53	2.74	1.82	0.42	0.12	0.00	0.00	0.00
	CN	ARL	369.38	124.70	100.42	45.84	22.50	13.72	6.88	4.38	2.96	1.21	1.01	1.00	1.00	1.00
		MDRL	251.50	86.00	69.50	36.00	19.00	12.00	6.00	4.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	382.14	120.47	99.07	39.73	16.95	9.63	4.83	3.04	2.00	0.46	0.12	0.00	0.00	0.00
	Logistic	ARL	370.94	106.29	83.50	35.06	17.89	11.39	5.97	3.59	2.53	1.13	1.00	1.00	1.00	1.00
		MDRL	245.50	77.00	59.00	28.00	15.00	10.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	405.08	104.58	81.39	28.29	13.93	8.14	3.97	2.44	1.61	0.35	0.04	0.00	0.00	0.00
	Laplace	ARL	369.30	84.66	61.28	26.85	13.35	8.47	4.44	2.87	2.15	1.10	1.01	1.00	1.00	1.00
		MDRL	257.00	62.00	46.00	22.00	11.50	8.00	4.00	2.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	448.58	83.63	57.53	21.01	9.81	5.86	3.20	1.82	1.29	0.33	0.08	0.00	0.00	0.00
	t_4	ARL	370.49	145.27	120.54	53.29	26.23	16.73	8.48	5.35	3.60	1.37	1.05	1.01	1.00	1.00
		MDRL	251.00	104.00	82.00	40.00	22.00	15.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	396.26	140.85	118.90	50.69	20.54	12.50	6.07	3.61	2.52	0.60	0.22	0.08	0.00	0.00
	t_8	ARL	371.30	136.63	106.14	47.20	23.88	14.28	7.29	4.68	3.32	1.25	1.01	1.00	1.00	1.00
		MDRL	258.00	93.00	77.00	36.00	20.00	12.00	6.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	383.52	136.48	105.01	42.24	19.12	11.38	5.14	3.15	2.15	0.51	0.10	0.03	0.00	0.00

TABLE 13: Continued.

(λ, L)		Distr.	δ													
			0.00	0.025	0.03	0.05	0.075	0.10	0.15	0.20	0.25	0.50	0.75	1.00	1.50	2.00
(0.25, 1.995)	$N(0, 1)$	ARL	368.94	163.89	125.88	61.02	28.97	15.79	7.67	4.75	3.41	1.36	1.03	1.00	1.00	1.00
		MDRL	261.50	110.00	86.00	45.00	20.00	13.00	7.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	361.42	161.56	121.85	57.69	26.80	11.94	5.13	2.78	1.90	0.57	0.16	0.00	0.00	0.00
	CN	ARL	369.14	179.78	142.14	60.23	30.03	16.78	8.48	5.10	3.62	1.39	1.05	1.00	1.00	1.00
		MDRL	267.00	121.50	99.50	44.00	23.00	13.00	7.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	370.83	176.44	139.84	55.70	26.50	13.80	5.81	3.13	2.13	0.56	0.21	0.04	0.00	0.00
	Logistic	ARL	371.19	151.80	119.49	52.95	22.92	13.92	6.63	4.08	2.94	1.26	1.02	1.00	1.00	1.00
		MDRL	248.00	105.00	85.00	37.00	17.50	11.00	6.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	383.41	152.05	116.71	51.97	18.61	10.78	4.24	2.37	1.60	0.47	0.14	0.00	0.00	0.00
	Laplace	ARL	370.19	121.18	91.21	36.84	16.40	9.61	5.06	3.38	2.52	1.18	1.02	1.00	1.00	1.00
		MDRL	248.00	88.00	62.00	27.00	13.00	8.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	383.41	116.57	91.76	30.96	12.75	6.64	2.99	1.82	1.27	0.41	0.13	0.04	0.00	0.00
	t_4	ARL	369.05	196.86	156.35	78.67	36.05	20.40	9.98	5.87	4.22	1.60	1.13	1.02	1.00	1.00
		MDRL	247.50	140.00	108.00	59.00	26.00	15.00	9.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00
		SDRL	376.25	193.47	162.10	70.86	30.50	17.24	6.79	3.74	2.44	0.72	0.34	0.15	0.03	0.00
t_8	ARL	370.83	197.61	155.40	76.39	32.26	19.25	9.26	5.47	3.89	1.50	1.06	1.00	1.00	1.00	
	MDRL	273.00	134.50	108.00	55.00	23.00	15.00	8.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	361.65	203.48	149.05	73.48	29.81	15.24	6.87	3.44	2.16	0.63	0.24	0.05	0.00	0.00	
(0.50, 2.205)	$N(0, 1)$	ARL	370.24	222.16	196.77	98.15	47.68	26.30	10.28	5.80	3.80	1.47	1.04	1.00	1.00	1.00
		MDRL	261.00	164.00	138.50	69.00	33.00	19.00	8.00	5.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	362.63	215.71	194.23	97.34	47.38	23.66	8.33	3.79	2.16	0.60	0.20	0.03	0.00	0.00
	CN	ARL	370.38	228.32	202.24	102.61	50.84	28.10	10.92	6.13	4.10	1.51	1.08	1.00	1.00	1.00
		MDRL	270.00	159.00	147.50	72.50	36.00	20.00	8.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00
		SDRL	362.46	225.70	192.80	98.27	44.75	25.79	8.48	4.19	2.37	0.63	0.26	0.04	0.00	0.00
	Logistic	ARL	371.47	226.11	167.01	89.50	40.36	22.69	8.90	4.97	3.40	1.37	1.05	1.00	1.00	1.00
		MDRL	263.50	147.00	121.00	67.00	28.00	16.50	7.00	4.00	3.00	1.00	1.00	1.00	1.00	1.00
		SDRL	381.73	255.35	168.83	83.19	40.27	20.29	7.19	3.10	1.83	0.54	0.22	0.00	0.00	0.00
	Laplace	ARL	371.27	192.89	133.97	60.97	27.58	14.45	6.25	3.76	2.70	1.27	1.04	1.00	1.00	1.00
		MDRL	263.50	125.50	95.00	40.00	19.50	11.00	5.00	3.00	2.00	1.00	1.00	1.00	1.00	1.00
		SDRL	381.73	222.77	130.95	60.00	25.77	12.07	4.27	2.11	1.42	0.47	0.19	0.06	0.00	0.00
	t_4	ARL	370.71	267.22	213.56	128.25	64.61	35.50	14.42	7.45	5.03	1.75	1.19	1.04	1.00	1.00
		MDRL	255.00	186.00	148.50	92.00	46.00	25.00	10.00	6.00	4.00	2.00	1.00	1.00	1.00	1.00
		SDRL	386.36	261.54	218.84	123.88	61.63	33.52	12.57	5.34	3.18	0.78	0.39	0.18	0.03	0.00
t_8	ARL	370.13	246.85	218.05	118.30	58.28	30.54	12.45	6.78	4.40	1.59	1.10	1.01	1.00	1.00	
	MDRL	266.00	169.00	152.50	85.00	41.00	22.00	9.00	5.00	4.00	1.00	1.00	1.00	1.00	1.00	
	SDRL	353.53	244.73	216.05	116.51	55.86	28.00	10.30	4.86	2.72	0.70	0.30	0.09	0.00	0.00	

chart decreases the ARL_1 by 30.57 percent (see Tables 3 and 12). Furthermore, under a CN environment, at $n = 10$ and $\lambda = 0.50$ with $\delta = 7.5$ percent, the proposed TEWMA – SR_{RSS} control chart gives OOC signal after 45.28 observations, whereas the TEWMA – SR_{RSS} control chart does after 50.84 observations (see Tables 4 and 13). The comprehensive RL features of the DEWMA – SR_{RSS} control chart for process location shift under selected distributions are shown in Tables 12 and 13. The TEWMA – SR_{RSS} control chart outperforms the DEWMA – SR_{RSS} control chart, as shown in Figures 9 and 10. Results indicate that the proposed structure works effectively in all environments relative to the DEWMA – SR_{RSS} control chart.

5. Illustrative Example

This section demonstrates practical application to implement the proposed TEWMA – SR_{RSS} control chart in practice. For this purpose, the proposed TEWMA – SR_{RSS} with TEWMA-SR and DEWMA – SR_{RSS} control charts are considered to monitor the process location shift. For the practical implementation of the proposed TEWMA – SR_{RSS} control chart, we assumed the data set used by Muttlak and Al-Sabah [20]. These data were also used by Abid et al. [37]

for the execution of the EWMA – SR_{RSS} control chart. The collected data were about the Pepsi Cola Company in Al-Khobar, Saudi Arabia. The company uses different production lines, like a line used for filling soft drinks in bottles is referred to as an interest production and the volume of drink in the bottle is referred to as a variable of interest. They used the RSS method to hold the appropriate volume of soft drinks in bottles during the filling process.

The RSS technique is used to collect 27 ranked set samples by repeating the two cycles four times, with each cycle having a size of $n = 3$. To compare the proposed TEWMA – SR_{RSS} control chart with TEWMA-SR and DEWMA – SR_{RSS} control charts, 27 samples with $r = 12$ are collected using RSS. The design parameters (L, λ) of the DEWMA – SR_{RSS} , TEWMA-SR, and TEWMA – SR_{RSS} are $(1.578, 0.05)$, $(1.75, 0.05)$, and $(1.305, 0.05)$, respectively, at $ARL_0 = 370$. Comparison of the proposed TEWMA – SR_{RSS} control chart with the TEWMA-SR and DEWMA – SR_{RSS} control charts can be visualized in Figures 11–13. The DEWMA – SR_{RSS} control chart shows 4 OOC signals (at sample numbers 24–27), whereas the TEWMA-SR control chart shows no OOC signals. In contrast, the proposed TEWMA – SR_{RSS} control chart shows 7 OOC signals (at sample numbers 21–27) in the process location parameter.

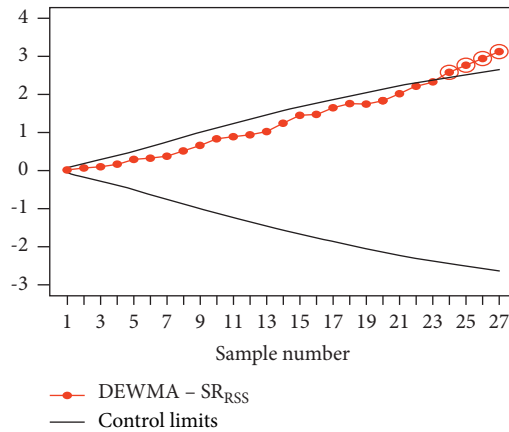


FIGURE 11: Application for the DEWMA - SR_{RSS} control chart when $L = 1.578, \lambda = 0.05,$ and $ARL_0 = 370.$

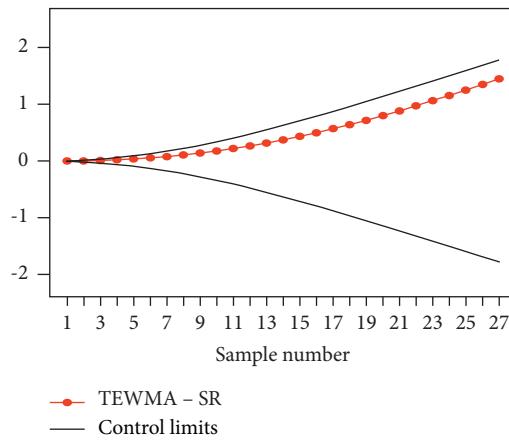


FIGURE 12: Application for the TEWMA-SR control chart when $L = 1.75, \lambda = 0.05,$ and $ARL_0 = 370.$

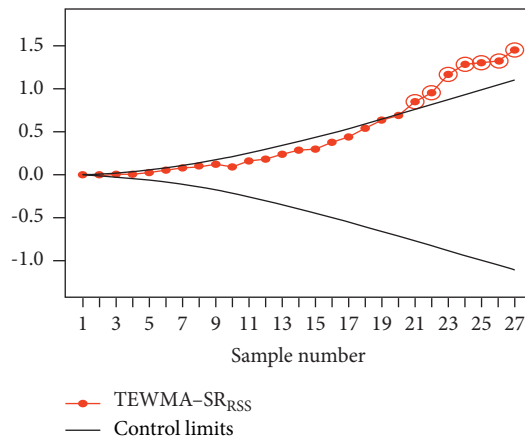


FIGURE 13: Application for the proposed TEWMA - SR_{RSS} control chart when $L = 1.305, \lambda = 0.05,$ and $ARL_0 = 370.$

These findings indicate that the proposed TEWMA - SR_{RSS} has improved performance for the monitoring of the process location as compared to DEWMA - SR_{RSS} and TEWMA-SR control charts. Hence, the proposed TEWMA - SR_{RSS} control chart under RSS has a better ability to detect shifts in process location.

6. Summary, Conclusions, and Recommendations

The ranked set sampling (RSS) technique is preferred over the simple random sampling (SRS) for the processes when the estimations are destructive or costly, and the ranking of

observational data is comparatively simple. Similarly, the nonparametric (NP) control charts are very useful to monitor shifts in the process parameters when the distribution of the underlying process is questionable or unknown. So, this study proposes an NP triple exponentially weighted moving average (TEWMA) Wilcoxon signed-rank (SR) control chart under the RSS technique (represented as TEWMA – SR_{RSS} control chart) using different continuous symmetric distributions. The Monte Carlo simulation method is used to obtain the numerical results of the proposed TEWMA – SR_{RSS} control chart along with other existing control charts. The performance of TEWMA – SR_{RSS} control chart is substantially better than the SRS-based NP TEWMA-SR, TEWMA sign, TEWMA – \bar{X} , and double EWMA-SR (DEWMA – SR_{RSS}) control charts. A real-life example is also provided to demonstrate how the proposed control chart can be used in practice. Therefore, to adopt a robust and efficient control chart, the proposed control chart gives an alternate choice to quality practitioners. The proposed work can be extended to the multivariate scenario.

Abbreviations

ARL:	Average run length
ARL ₀ :	In-control average run length
ARL ₁ :	Out-of-control average run length
CUSUM:	Cumulative sum
CUSUM-SR:	CUSUM Wilcoxon signed-rank
CUSUM-SN:	CUSUM sign
CN:	Contaminated normal
DEWMA-SR:	Double EWMA-SR
DEWMA:	Double EWMA
DEWMA – SR _{RSS} :	Double EWMA-SR with RSS
DEWMA _{TBE} :	One-sided DEWMA for time between events
DEWMA-SR:	Double NP EWMA-SR
DEWMA – \bar{X} :	Double EWMA mean
DRSS:	Double RSS
EWMA:	Exponentially weighted moving average
EWMA – SR _{RSS} :	NP EWMA-SR with RSS
EWMA _{TBE} :	One-sided EWMA for time between events
EWMA-SR:	EWMA Wilcoxon signed-rank
EWMA-SN:	EWMA sign
GEWMA-SR:	Generalised EWMA-SR
IC:	In-control
LOG:	Logistic distribution
LCL _{EWMA-SR_{RSS}} :	Lower control limit of EWMA – SR _{RSS}
LCL _{(TEWMA-SR_{RSS})<i>q</i>} :	Lower control limit of TEWMA – SR _{RSS}
LCL _{DEWMA-SR_{RSS}} :	Lower control limit of DEWMA – SR _{RSS}
MEC:	Mixed EWMA-CUSUM
MCE:	Mixed CUSUM-EWMA

MxMEC:	Auxiliary information based MEC
MxMCE:	Auxiliary information based MCE
NP:	Nonparametric
OOB:	Out-of-control
ODRSS:	Order DRSS
OIDRSS:	Order imperfect DRSS
RSS:	Ranked set sampling
RL:	Run length
SDRL:	Standard deviation of the run length
SRS:	Simple random sampling
SPC:	Statistical process control
SR:	Wilcoxon signed-rank
TEWMA:	Triple TEWMA
TEWMA – SR _{RSS} :	Triple EWMA with SR under RSS
TEWMA-SR:	TEWMA with SR
TEWMA-SN:	TEWMA signed-rank test
TEWMA _{TBE} :	One-sided TEWMA for time between events
TEWMA – \bar{X} :	TEWMA mean
UCL _{EWMA-SR_{RSS}} :	Upper control limit of EWMA – SR _{RSS}
UCL _{DEWMA-SR_{RSS}} :	Upper control limit of DEWMA – SR _{RSS}
UCL _{(TEWMA-SR_{RSS})<i>q</i>} :	Upper control limit of TEWMA – SR _{RSS}

Symbols

δ :	Amount of shift in process location
σ :	Standard deviation
n :	Sample size
λ :	Smoothing constant
\in :	Belong to
$t(\nu)$:	Student's t distribution with degree of freedom (ν)
ω_0^2 :	Variance
E_0 :	Initial value of EWMA – SR _{RSS}
L :	Control limit coefficient
E_q :	Plotting statistic of EWMA – SR _{RSS}
DE_q :	Plotting statistic of DEWMA – SR _{RSS}
TE_q :	Plotting statistic of TWMA – SR _{RSS}
φ_0 :	Median of ranked values
$R_{qj}^+(h)$:	Rank of absolute deviations
DE_0 :	Initial value of DEWMA – SR _{RSS}
TE_0 :	Initial value of TEWMA – SR _{RSS}
m :	Number of cycles
r :	Number of iterations.

Data Availability

The real-life data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The author Saddam Akber Abbasi acknowledges Qatar University for providing excellent research facilities.

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