



# Weighting and pruning based ensemble deep random vector functional link network for tabular data classification



Qiushi Shi<sup>a</sup>, Minghui Hu<sup>a</sup>, Ponnuthurai Nagarathnam Suganthan<sup>a,b,\*</sup>, Rakesh Katuwal<sup>a</sup>

<sup>a</sup> School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore

<sup>b</sup> KINDI Center for Computing Research, College of Engineering, Qatar University, Doha, Qatar

## ARTICLE INFO

### Article history:

Received 23 February 2022

Revised 28 April 2022

Accepted 26 June 2022

Available online 27 June 2022

### Keywords:

Ensemble deep random vector functional link (edRVFL)

Weighting methods

Pruning

UCI classification datasets

## ABSTRACT

In this paper, we first integrate normalization to the Ensemble Deep Random Vector Functional Link network (edRVFL). This re-normalization step can help the network avoid divergence of the hidden features. Then, we propose novel variants of the edRVFL network. Weighted edRVFL (WedRVFL) uses weighting methods to give training samples different weights in different layers according to how the samples were classified confidently in the previous layer thereby increasing the ensemble's diversity and accuracy. Furthermore, a pruning-based edRVFL (PedRVFL) has also been proposed. We prune some inferior neurons based on their importance for classification before generating the next hidden layer. Through this method, we ensure that the randomly generated inferior features will not propagate to deeper layers. Subsequently, the combination of weighting and pruning, called Weighting and Pruning based Ensemble Deep Random Vector Functional Link Network (WPedRVFL), is proposed. We compare their performances with other state-of-the-art classification methods on 24 tabular UCI classification datasets. The experimental results illustrate the superior performance of our proposed methods.

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## 1. Introduction

Deep learning has been extremely successful in recent years. Ranging from vision and video tasks to natural language processing, these deep neural networks have reached state-of-the-art results in multiple domains [1,2]. In conventional neural networks, back-propagation methods are used to train a large number of parameters in these models [3]. Although such a training method makes it possible to optimize the parameters, the time-consuming training process has become a severe problem in recently designed complex neural networks. Also, a BP-trained neural network may fall into a local minimum and gives a sub-optimal result [4–6]. By looking at the Kaggle competitions that have no relation with vision or sequence, we can easily find that deep learning is not always the best solution for diverse tasks [7,8].

At the same time, another kind of neural network based on randomization is attracting significant attention because of its superiority to overcome the shortcomings of the conventional models [5,9,10]. It has been successfully applied to a range of tasks from classification [11–13], regression [14,15], visual tracking [16], to forecasting [17,18]. Instead of using back-propagation to train,

this randomization-based neural network frequently uses a closed-form solution to optimize parameters in the model [19]. Unlike the BP-trained neural networks which need multiple iterations, the randomization-based neural networks only need to be trained once by feeding all the samples to the model together. Among these models, Random Vector Functional Link Network (RVFL) [20,21] is a typical representative with a single hidden layer. Its universal approximation ability has been proved in [22]. The weights and biases are randomly generated in this neural network. And its uniqueness lies in a direct link that connects the information from the input layer to the output layer. It has various applications [23–25]. However, due to different random seeds and perturbations in the training set, this randomized neural network can perform quite differently in each realization [26]. To increase the performance, stability, and robustness of this model, two improved structures named Deep Random Vector Functional Link Network (dRVFL) and Ensemble Deep Random Vector Functional Link Network (edRVFL) were proposed [27]. The dRVFL network is a deep version of RVFL network, which allows the existence of multiple hidden layers, while edRVFL network treats each hidden layer as a classifier to compose an ensemble.

However, with the edRVFL network goes deeper, the divergence of the randomized hidden features will become a serious problem. Therefore, using normalization methods to re-normalize the hidden features is extremely important for improving the performance

\* Corresponding author.

E-mail addresses: [qiushi001@e.ntu.edu.sg](mailto:qiushi001@e.ntu.edu.sg) (Q. Shi), [e200008@e.ntu.edu.sg](mailto:e200008@e.ntu.edu.sg) (M. Hu), [epnsugan@ntu.edu.sg](mailto:epnsugan@ntu.edu.sg) (P.N. Suganthan), [rakeshku001@e.ntu.edu.sg](mailto:rakeshku001@e.ntu.edu.sg) (R. Katuwal).

of the edRVFL network. In this paper, we employ the batch normalization scheme [28] to do the re-normalization work. To the best of our knowledge, this is the first time that batch normalization is introduced to the randomized neural network. After the re-normalization process, the mean and the variance of the hidden features will become 0 and 1. Then, we scale and shift these values to increase the expression capacity of the neural network.

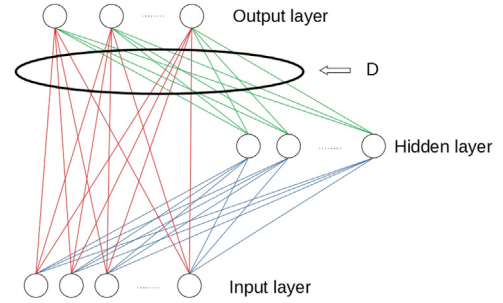
Besides, there are still some drawbacks to the edRVFL network. Firstly, for every layer (or classifier) in the edRVFL network, they share the same training samples. Meanwhile, these training samples have the same weights in the training process. Compared to ensemble methods that using differing training bags for each classifier, these ensemble frameworks which utilize similar training sets usually perform worse [29,30]. Moreover, the testing accuracy for the last few layers may slightly go down when the network becomes deeper. We believe that some inferior features can be generated since we randomly generate the weights for the hidden neurons. And these useless features will propagate to deeper layers inducing further inferior features to decrease the overall testing accuracy.

Thus, for solving the first problem, we introduce a weighting matrix. Each training sample will be allocated a particular weight when performing the closed-form solution depending on its performance in the previous layer. Our approach differs from Weighted Extreme Learning Machine [31] which gives weights to each sample for addressing the problem of imbalance learning. The main purpose is to ensure that different classifiers can have their preference for a particular portion of the training samples that were not classified with high confidence in the previous layer. We have also tried to apply the sample weighting method of Adaboost [32]. However, most of the samples will be given weights near zero while only a few can be allocated reasonable weights. Therefore, we propose a new weighting method for the edRVFL network in this paper, and this improved variant of the edRVFL network is named Weighted Ensemble Deep Random Vector Functional Link Network (WedRVFL).

Besides, pruning algorithms are widely used to reduce the heavy computational cost of deep neural networks in low-resource settings [33]. Different effective techniques have been proposed to cut off the redundant part of the neural network models [34–37]. In our case, we perform it by selecting some inferior features in the hidden layer and prune them permanently. The selection process can help to prevent the propagation of inferior features and maintain the testing accuracy for deeper layers. We named this improved variant of edRVFL network as Pruning-based Ensemble Deep Random Vector Functional Link Network (PedRVFL). Although there was previous work that applying pruning strategy to the RVFL network in [38], we would like to highlight that our work is different from theirs at the following point: They do pruning after training to shrink the size of the neural network. However, we perform pruning during the generation step so that inferior features will not propagate to deeper layers. Additionally, we integrate the advantages of WedRVFL and PedRVFL to create a combined model called Weighting and Pruning based Ensemble Deep Random Vector Functional Link Network (WPedRVFL).

The key contributions of this paper are summarized as follows:

- We introduce the batch normalization to the edRVFL network for re-normalizing the hidden features.
- We employ the weighting scheme to allocate different weights to different samples in the edRVFL network. We name it WedRVFL network. The weight matrix changes according to the samples' predictions in the previous layers. This method can make sure that each hidden layer in the network has different biases for each sample and increase the ensemble classification accuracy.



**Fig. 1.** The structure of RVFL network. The original features have two ways to transfer to the output layer: One is going through the hidden layer (the blue lines and the green lines), and the other one is transferred through the direct link (the red lines). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

- We propose pruning based edRVFL network called PedRVFL network. Instead of pruning neurons after the training process, we cut off the inferior neurons according to their importance for classification when we are training the model. This method can prevent the propagation of detrimental features and increase the classification accuracy in deeper layers.
- The combination of weighting and pruning based edRVFL network named WPedRVFL network is also presented in the paper.
- The empirical results show the superiority of our new methods over 11 state-of-the-art methods on 24 UCI benchmark datasets.

The rest of the paper is organized as follows: [Section 2](#) outlines the basic concepts of RVFL network and illustrates the ensemble deep version of this structure. [Section 3](#) introduces the re-normalization method for the edRVFL network. Then [Section 4](#) gives details about our new proposed versions of edRVFL network. In [Section 5](#), the performance of our methods, as well as other classification methods are compared. Finally, conclusions and future research directions are presented in [Section 6](#).

## 2. Related works

In this section, we give a brief review of the structure of the standard RVFL network and ensemble deep RVFL network.

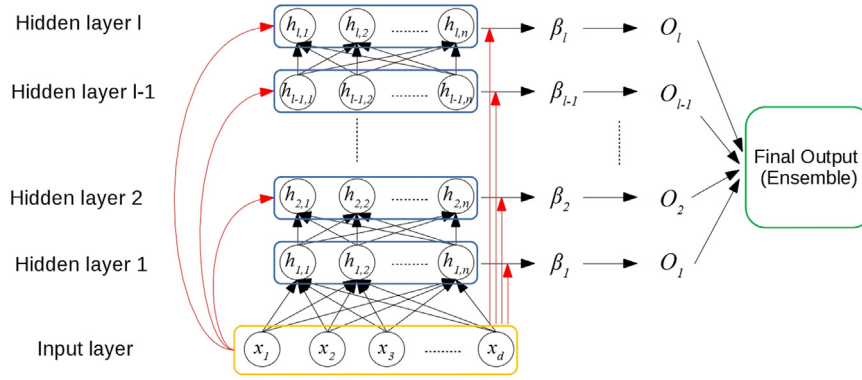
### 2.1. Random vector functional link network

As shown in [Fig. 1](#), a basic RVFL network consists of one input layer, one hidden layer, and one output layer [20]. Unlike the general neural network, the uniqueness of RVFL network is its direct link between the input layer and the output layer. This framework conveys both the linear features in the input layer and the non-linearly transformed features in the hidden layer to the output layer. Because the parameters for the hidden layer are randomly generated and kept fixed during the training process, the only thing it needs to learn is the output weights  $\beta$ . The solution of the  $\beta$  can be computed by solving the optimization problem given as follows:

$$O_{RVFL} = \min_{\beta} \|\mathbf{D}\beta - \mathbf{Y}\|_2^2 + \lambda \|\beta\|_2^2 \quad (1)$$

where  $\mathbf{D}$  represents all the input features and output features,  $\mathbf{Y}$  is the true vector we want to fit, and  $\lambda$  is a regularization parameter that controls how much the RVFL network cares about its model complexity.

Normally, this kind of optimization problem can be solved via Moore-Penrose pseudoinverse [39] and ridge regression [40]. For Moore-Penrose pseudoinverse, the algorithm does not consider the contribution of the part  $\|\beta\|_2^2$ , and  $\lambda$  is simply set to 0. So the



**Fig. 2.** The structure of edRVFL network. The whole network can be treated as several independent classifiers, and each classifier  $l$  has its own output weights  $\beta_l$  and output  $O_l$ . The final output is obtained by applying some ensemble methods to these independent outputs.

solution is given by:

$$\beta = \mathbf{D}^+ \mathbf{Y} \quad (2)$$

Moreover, for the ridge regression where  $\lambda$  is not equal to 0, the solution can be written as:

$$\text{Primal Space: } \beta = (\mathbf{D}^T \mathbf{D} + \lambda \mathbf{I})^{-1} \mathbf{D}^T \mathbf{Y} \quad (3)$$

$$\text{Dual Space: } \beta = \mathbf{D}^T (\mathbf{D} \mathbf{D}^T + \lambda \mathbf{I})^{-1} \mathbf{Y} \quad (4)$$

Depending on the number of total feature dimensions, the computational complexity for RVFL training can be reduced by using the primal or dual solution [5].

### 2.2. Ensemble deep random vector functional link network

With deep learning methods become more and more popular today, the deep version of the RVFL networks called Ensemble Deep Random Vector Functional Link Network (edRVFL) was proposed by [27]. It is generally accepted that ensemble learning performs better than a single learner. Inspired by this idea, the author separates the whole network into several independent classifiers. The structure of edRVFL can be found in Fig. 2.

For easy to understand, here we set the number of hidden neurons in each hidden layer the same to  $n$  and the total number of the hidden layers to  $l$ . Biases will also be omitted in the formula for simplicity. Let  $X$  be the input features, and the output of the first hidden layer can be represented as:

$$\mathbf{H}^{(1)} = g(\mathbf{X} \mathbf{W}^{(1)}), \quad \mathbf{W}^{(1)} \in \mathbb{R}^{d \times n} \quad (5)$$

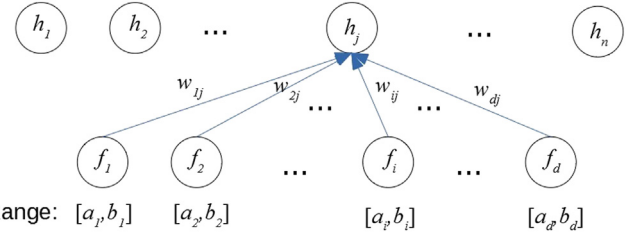
where  $d$  is the feature number of the input and  $g(\cdot)$  is the non-linear activation function used in each hidden neuron. When  $l > 1$ , this formula becomes:

$$\mathbf{H}^{(l)} = g([\mathbf{H}^{(l-1)} \mathbf{X}] \mathbf{W}^{(l)}), \quad \mathbf{W}^{(l)} \in \mathbb{R}^{(n+d) \times n} \quad (6)$$

This framework treats every hidden layer as a single classifier. For one classifier (hidden layer), the original features (features in the input layer) and all the hidden features in this layer, are serving as the input for this classifier's prediction. Either Moore-Penrose pseudoinverse [39] or ridge regression [40] can be chosen for solving the optimization problem. After getting all the hidden layers' predictions, an ensemble method such as major voting or averaging is employed to reach the final output.

### 3. Proposed re-normalization scheme for the edRVFL network

For the edRVFL network, the input samples are normalized so that the mean equals to 0 and the variance is 1 for each feature.



**Fig. 3.** The range of the input features and the generation of the first hidden layer.

Also, the hidden weights of the edRVFL network are uniform randomly generated within  $[-1, 1]$ . Suppose the feature number of the training sample is  $d$  and let the range of its  $i$ th normalized feature  $f_i$  to be  $[a_i, b_i]$ , where  $a_i \leq 0$  and  $b_i \geq 0$ . Then the input of the  $j$ th node's activation function in the first hidden layer can be written as:

$$h_j = \sum_{i=1}^d w_{ij} f_i \quad (7)$$

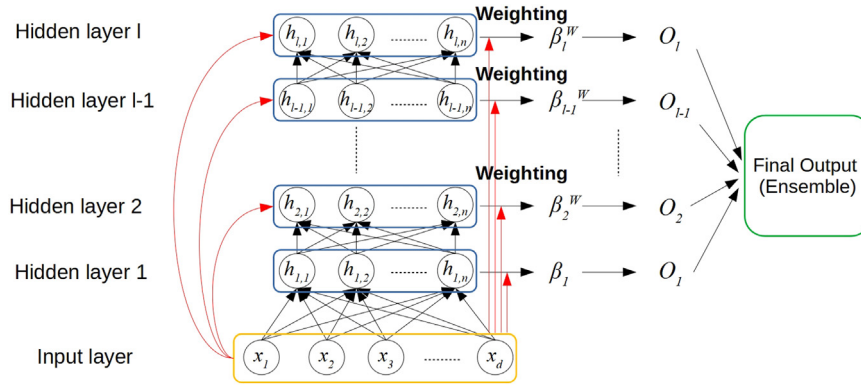
Where  $w_{ij}$  is the hidden weight between the  $i$ th normalized feature and the  $j$ th hidden node. We summarize the above operation in Fig. 3.

Based on the condition that  $w_{ij} \in [-1, 1]$ , the range of  $h_j$  can be given by  $[\sum_{i=1}^d a_i, \sum_{i=1}^d b_i]$ . And this range will keep increasing with more and more hidden layers been generated. Since randomization has the potential to generate inferior features, and this situation can be worse due to this large range. Therefore, re-normalization is crucial for avoiding divergence of the hidden features in the edRVFL network.

Batch normalization is widely used in back-propagation-based deep neural networks to address the Internal Covariate Shift issue during the training process [28]. In this work, we use it to help the edRVFL network re-normalize its hidden features. To the best of our knowledge, this is the first time that batch normalization is introduced to the randomized neural network. The batch normalization generally has two steps. The first step is to make the inputs' mean and variance to 0 and 1, respectively. Suppose we have  $m$  inputs and let the  $i$ th input to be  $x_i$ , then the mean value  $\mu$  and variance  $\sigma^2$  can be obtained by:

$$\mu = \frac{1}{m} \sum_{i=1}^m x_i \quad (8)$$

$$\sigma^2 = \frac{1}{m} \sum_{i=1}^m (x_i - \mu)^2 \quad (9)$$



**Fig. 4.** The structure of WedRVFL network. From  $l \geq 2$ , it gives each training sample different importance values when calculating  $\beta$ . This method ensures that every hidden layer has its preference for a particular subset of hard-to-classify training samples.

And the normalization can be done by:

$$\hat{x}_i = \frac{x_i - \mu}{\sqrt{\sigma^2 + \epsilon}} \quad (10)$$

After we have normalized the inputs, we can do scaling and shifting to increase the approximation ability of the network:

$$y_i = \gamma \hat{x}_i + \alpha \quad (11)$$

Here  $\gamma$  and  $\alpha$  are parameters that we need to set for the network. In the back-propagation-based deep neural networks, these two parameters are learned by the back-propagation which are similar to the hidden weights. However, we treat  $\gamma$  and  $\alpha$  as hyperparameters for the edRVFL network. The validation set is used to help us find the best configuration of these two hyperparameters.

#### 4. Ensemble deep random vector functional link networks with weighting & pruning

In this section, we propose two improved variants of edRVFL network called WedRVFL and PedRVFL in Section 4.1 and Section 4.2. In Section 4.3, their combination WPedRVFL is presented.

##### 4.1. Weighted ensemble deep random vector functional link network

Weighting [41] is a widely used method in boosting ensemble learning and can make each classifier focus its preference on a particular set of samples. Therefore, we propose our own weighting methods in this paper. In our Weighted Ensemble Deep Random Vector Functional Link Network (WedRVFL), we apply weighting by treating each hidden layer as an independent classifier. The samples which are hard to predict will be given higher weights in the next classifier. The typical structure of the WedRVFL network is shown in Fig. 4.

The generation step for the WedRVFL is exactly the same as the basic edRVFL using (5)(6). Moreover, the way of calculating the output weights  $\beta_1$  of the first hidden layer is also based on (3) and (4) (Ridge regression is chosen here). Suppose there are  $m$  training samples, after finishing the training for the first layer, some of these samples are predicted correctly while others are assigned wrong labels in this layer's classification. Denote these two groups of samples as  $S_r$  and  $S_w$ , respectively. And let the size of  $S_r$  and  $S_w$  to be  $n_r$  and  $n_w$ . Then we can have:

$$n_r + n_w = m \quad (12)$$

For these samples in set  $S_w$ , which means they are predicted wrongly in the current layer, we should give them a higher importance value in the next layer. On the other hand, for these samples

which have correct classification, their importance value should be decreased in the next classifier.

In the first hidden layer, there is no weighting scheme for different samples. However, we still can treat this situation as that all the training samples are sharing the same weight 1. From the second hidden layer, we give the samples in set  $S_r$  weight  $\omega_r$  and samples in set  $S_w$  weight  $\omega_w$ . Since we know that  $\omega_r$  should be less than 1 and a positive value. The range of  $\omega_r$  is (0,1] ( $\omega_r = 1$  if all the samples are predicted correctly in the last layer). Recall that when all the samples are sharing the same weight 1, the sum of these weights is equal to the size of the training set  $m$ . For numerical stability, we should follow the same rule in our weighting scheme. Therefore, we can obtain the following equation:

$$n_r \times \omega_r + n_w \times \omega_w = m \quad (13)$$

So that the weight  $\omega_w$  for set  $S_w$  can be expressed by  $\omega_r$  as:

$$\omega_w = \frac{m - n_r \times \omega_r}{n_w} \quad (14)$$

It is worth mentioning that although we have two different weights for  $S_r$  and  $S_w$ , we only need to set one hyperparameter  $\omega_r$  during the experiments. The validation set can be used to find the best setting for this hyperparameter.

After getting the value of  $\omega_w$ , we can create the weight matrix  $\mathbf{W}_l$ , where  $\mathbf{W}_l \in \mathbb{R}^{m \times 1}$  and  $l \geq 2$  (weighting scheme starts from the second hidden layer). The  $i$ th value in  $\mathbf{W}_l$  represents the weight we give to the  $i$ th training sample (either  $\omega_r$  or  $\omega_w$ ). Thus, when  $l \geq 2$ , (3) and (4) will turn into new forms:

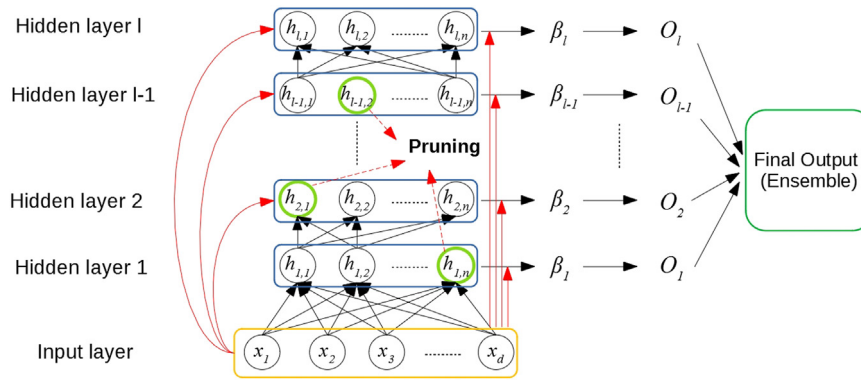
$$\text{Primal Space: } \beta_l^w = (\mathbf{D}^T \mathbf{W}_l^* \mathbf{D} + \lambda \mathbf{I})^{-1} \mathbf{D}^T \mathbf{W}_l^* \mathbf{Y} \quad (15)$$

$$\text{Dual Space: } \beta_l^w = \mathbf{D}^T (\mathbf{W}_l^* \mathbf{D} \mathbf{D}^T + \lambda \mathbf{I})^{-1} \mathbf{W}_l^* \mathbf{Y} \quad (16)$$

$$\text{where } \mathbf{W}_l^* = \begin{bmatrix} \mathbf{W}_l^{(1)} & 0 & \dots & 0 \\ 0 & \mathbf{W}_l^{(2)} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{W}_l^{(m)} \end{bmatrix}, \text{ which is the } (m \times m)$$

diagonal form of  $\mathbf{W}_l$ .

From the above, we know that the training samples which are difficult to train will be given higher importance values when calculating the loss in the next layer. This method ensures that for each training sample, whether it is hard to train or not, there will exist some corresponding layers that are good at predicting such samples. Hence, every sample may have been predicted correctly in some layers. Intuitively, the wrong prediction of WedRVFL can be less than the normal edRVFL after ensemble aggregation. This claim is supported by our empirical simulation results in the experimental part.



**Fig. 5.** The structure of PedRVFL network. It chooses to cut off some inferior neurons starting from the first hidden layer. Here  $h_{1,n}$ ,  $h_{2,1}$ , and  $h_{l-1,2}$  are identified as inferior ones and will be removed.

#### 4.2. Pruning based ensemble deep random vector functional link network

The pruning method is widely used in today's neural networks. It was proposed to improve the efficiency of the neural network models when facing a limited computational budget. However, we apply the pruning method in our model for a completely different reason. Since the weights between hidden layers in our neural network are randomly generated and kept fixed, we believe some inferior features will be created and propagated to deeper layers. With the layer number increasing, the accuracy of a single layer's prediction will slightly go down. Hence, we decide to cut off some inferior hidden neurons to prevent them from participating in the generation of the deeper hidden layers. The structure of PedRVFL is shown in Fig. 5.

For deciding which neurons should be cut off, [34] proposed a pruning method based on the sensitivity calculation. Besides, Penalty-term methods are also widely used by researchers [42,43]. In recent days, some new pruning schemes targeted on deep neural networks have been investigated [36,37]. In this paper, we set a criterion based on the output weights which also belongs to the sensitivity methods. For each hidden neuron  $h_{l,n}$  in layer  $l$ , it has  $k$  corresponding weights in the  $\beta_l$  where  $k$  refers to the number of classes. Let these weights be  $a_{l,n,1}, a_{l,n,2} \dots a_{l,n,k}$ , and the absolute value for them represent the importance for this neuron to different classes. Therefore, we should use the sum of these values to evaluate how much is this hidden neuron contributed to the whole prediction:

$$\theta_{l,n} = \sum_{i=1}^k |a_{l,n,i}| \quad (17)$$

Depending on the pruning rate we set, hidden neurons with the lowest values of  $\theta$  are pruned. It is worth mentioning that by applying the pruning method, our neural network will have fewer hidden neurons. That means the complexity of the framework is also reduced. However, according to the experimental results, this model turns out to have better performance on multiple tasks.

#### 4.3. Weighting and pruning based ensemble deep random vector functional link network

Weighting and Pruning based Ensemble Deep Random Vector Functional Link Network (WPedRVFL) is a combination of the above two models. It has both advantages of WedRVFL and PedRVFL. The structure of this neural network is shown in Fig. 6.

To better illustrate the training procedure of the WPedRVFL, we summarize the steps in Algorithm 1.

#### Algorithm 1: WPedRVFL.

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**Input:** A set of training samples  $\mathbf{X} \in \mathbb{R}^{m \times d}$ .

- 1 **for** Every Hidden Layer  $l$  **do**
- 2     Initialize the hidden weights.
- 3     Do re-normalization and generate the hidden features using (11), (5) and (6).
- 4     **Weighting:** Get the optimal output weights  $\beta_l^W$  by (15) and (16). When  $l = 1$ , all the samples share the same weight 1.
- 5     Predict the labels of the training samples.
- 6     **Weighting:** Calculate the weight  $\omega_w$  for the wrong predicted samples based on the  $\omega_r$  we set by (14).
- 7     **Pruning:** Obtain the importance value  $\theta$  for each hidden nodes using (17).
- 8     **Pruning:** Cut off a number of the inferior neurons according to the pruning rate  $p$  we set.
- 9 **end**
- 10 Combine all the predictions from every hidden layer using ensemble methods.

**Output:** Hidden weights, the final prediction of the training samples, and the output weights  $\beta_l^W$  of every hidden layer.

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It is also worth highlighting that a BP-trained network needs to re-train all the parameters in the previously trained hidden layers every time if we set different number of the hidden layers. However, in our edRVFL based neural networks, we only need to train the model once with the maximum permitted hidden layer number. Since there is no need for training the hidden weights and biases in edRVFL based neural networks, results of different numbers of hidden layers can be obtained by eliminating the last few layers' predictions in the final output.

#### 4.4. Justification for choosing tabular datasets

Deep neural networks show great performance in diverse areas such as image classification, object detection, and natural language processing. However, the concept of the neural network is not new. The shallow neural networks also reach good performance on UCI tabular datasets. On the contrary, most deep networks may have poor results on the tabular data because of over-fitting problems [44]. We believe the advantages of deep neural networks are brought by their specific structures, such as the Convolutional Neural Network (CNN) block for image-related tasks and the Long Short-Term Memory (LSTM) for text-related datasets. Since our proposed models are randomized neural networks that do not have

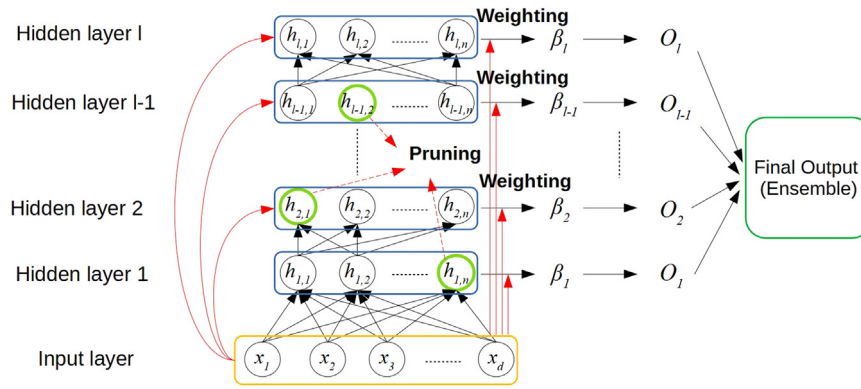


Fig. 6. The structure of WPedRVFL network. It combines the weighting and the pruning methods to achieve better performance.

**Table 1**  
Datasets used in this work.

Dataset	#Patterns	#Features	#Classes
abalone	4177	9	3
adult	48,842	15	2
arrhythmia	452	263	13
bank	4521	17	2
cardio-10	2126	22	10
cardio-3	2126	22	3
chess-krvkp	3196	37	2
congressional-voting	435	17	2
contrac	1473	10	3
glass	214	10	6
letter	20,000	17	26
molec-biol-splice	3190	61	3
monks-3	554	7	2
musk-2	6598	167	2
oocytes_trisopterus_states_5b	912	26	2
pendigits	10,992	17	10
spambase	4601	58	2
statlog-image	2310	19	7
statlog-landsat	6435	37	6
statlog-shuttle	58,000	10	7
wall-following	5456	25	4
waveform	5000	22	3
waveform-noise	5000	41	3
wine-quality-white	4898	12	7

\* We name these datasets the same as in [44].

these specific feature extraction modules, we only investigate their performance on UCI tabular datasets. Therefore, we set the limitation of tabular data classification in this research. However, once features are extracted by CNN or LSTM, edRVFL can be employed for the classification of the extracted tabular features [45].

## 5. Experiments

### 5.1. Datasets

In this paper, we use 24 classification datasets from the UCI machine learning repository [46] to compare the performance of our methods with other state-of-the-art networks. These datasets are from diverse application areas like physics, geology, and biology and usually serve as the benchmarks for the classification task [44,46]. We follow the same data pre-processing and partitions as used in [44]. The details of these datasets are shown in Table 1.

### 5.2. Other methods used for comparison

For evaluating the performance of our proposed edRVFL variants, we compare them with other 4 deep learning models, 4 randomized neural networks, 1 random forest, and the original

**Table 2**  
Hyperparameters considered for edRVFL based methods.

Hyperparameter	Considered values
Regularization parameter $\lambda$	$\lambda$ belongs to $2^x$ , $x \in [-12, 12]$
Number of hidden neurons $n$	[20,1000]
Maximum number of hidden layers $l_{\max}$	10
$\gamma$ in batch normalization	[0.5,2]
$\alpha$ in batch normalization	[-2,2]
Weight $\omega_r$ for the correctly predicted samples	(0,1), 1 means there is no weighting in the network
Pruning Rate $p$	[0,1], 0 means there is no pruning in the network

edRVFL network without re-normalization in this work. These methods (1–10), as well as our new proposed methods (11–14), are listed as follows:

- 1) Self Normalizing Neural Network (SNN): Self normalizing networks with SELUs activation function and ranks the best among the FNNs [44].
- 2) MSRAinit (MS): The deep neural network with Microsoft weight initialization [47].
- 3) Highway (HW): Highway networks [48].
- 4) ResNet (ResNet): Residual networks adapted to FNNs using residual blocks with 2 or 3 layers [49].
- 5) Stochastic configuration networks (SCN): The randomized neural network generated incrementally by stochastic configuration[50].
- 6) Broad learning system (BLS): The RVFL based model with a complete paradigm shift in discriminative learning [51].
- 7) Type-2 fuzzy BLS (FBLS): The fuzzy BLS learning algorithm [52].
- 8) Heterogeneous oblique random forest (obRaF(H)): The heterogeneous version of the oblique random forest [53].
- 9) Hierarchical ELM (H-ELM): Hierarchical Extreme Learning Machine for multi-layer perception [54].
- 10) edRVFL\_O: The original ensemble deep random vector functional link neural network without re-normalization [27].
- 11) edRVFL\_N: The new edRVFL network with re-normalization that proposed in this work.
- 12) PedRVFL: Pruning based edRVFL with re-normalization proposed in this work.
- 13) WedRVFL: Weighting based edRVFL with re-normalization proposed in this work.
- 14) WPedRVFL: The combination of PedRVFL and WedRVFL with re-normalization proposed in this work.

**Table 3**  
Comparison of Accuracy(%) on 24 UCI Datasets.

Dataset	SNN[44]	MS[47]	HW[48]	Resnet[49]	SCN[50]	BLS[51]	FBLs[52]	obRaF(H)[53]	H-ELM[54]	edRVFL_O[27]	edRVFL_N <sup>†</sup>	WedRVFL <sup>†</sup>	PedRVFL <sup>†</sup>	WPedRVFL <sup>†</sup>
abalone	66.57	62.84	64.27	64.66	64.00	60.06	64.17	65.93	63.77	65.83±0.36	66.13±0.32	66.87±0.22	<b>67.05±0.01</b>	66.89±0.15
adult	84.76	84.87	84.53	84.84	85.01	85.15	<b>85.61</b>	85.21	85.05	85.21±0.15	85.26±0.15	85.30±0.13	85.42±0.14	85.46±0.14
arrhythmia	65.49	63.72	62.83	64.60	44.91	62.23	68.40	72.33	72.12	69.06±0.69	72.43±0.66	73.22±0.32	73.66±0.39	<b>73.88±0.36</b>
bank	89.03	88.76	88.85	87.96	88.83	88.19	88.69	89.07	89.20	89.77±0.11	90.13±0.11	90.92±0.11	90.19±0.11	<b>91.14±0.01</b>
cardio-10	83.99	84.18	84.56	81.73	81.26	83.47	82.15	82.37	82.39	82.37±0.47	83.24±0.42	84.56±0.52	84.08±0.26	<b>85.30±0.26</b>
cardio-3	91.53	89.64	91.71	90.21	91.57	91.33	92.87	93.47	90.68	92.71±0.25	93.42±0.25	93.47±0.21	93.55±0.01	<b>94.20±0.18</b>
chess-krvkp	98.37	99.00	99.00	99.12	97.77	98.75	98.68	99.15	99.00	99.08±0.15	99.21±0.17	99.36±0.01	99.33±0.01	<b>99.47±0.01</b>
congressional-voting	61.47	60.55	58.72	59.63	60.09	59.40	57.79	61.19	61.24	61.01±0.28	61.98±0.29	61.91±0.23	61.93±0.27	<b>62.16±0.25</b>
contrac	51.90	51.36	50.54	51.36	47.75	41.78	49.38	51.97	54.08	51.33±0.50	54.04±0.49	55.53±0.51	54.94±0.37	<b>55.68±0.19</b>
glass	<b>73.58</b>	60.38	64.15	64.15	66.79	61.23	65.66	73.95	68.87	65.13±0.88	70.73±0.83	71.75±0.67	71.19±0.47	72.35±0.87
letter	97.26	97.12	89.84	97.62	86.11	93.99	92.75	97.39	93.15	97.43±0.23	97.54±0.22	<b>97.77±0.15</b>	97.66±0.07	97.73±0.11
molec-biol-ssplice	83.72	84.82	<b>88.33</b>	85.57	75.75	74.84	77.24	86.76	82.40	84.01±0.38	84.31±0.39	84.07±0.50	85.22±0.22	85.69±0.57
monks-3	60.42	74.54	58.80	58.33	69.42	52.37	59.95	55.84	78.70	55.02±2.02	75.48±2.04	80.29±1.93	81.98±1.12	<b>82.35±1.57</b>
musk-2	98.03	99.45	99.15	99.64	96.71	98.77	97.12	98.03	98.32	98.54±0.25	99.33±0.26	99.54±0.19	99.28±0.01	<b>99.74±0.01</b>
oocytes_trisopterus_states_5b	93.42	94.30	93.42	89.47	89.91	57.46	83.33	93.75	92.06	93.91±0.20	93.97±0.20	94.19±0.18	95.20±0.23	<b>95.21±0.30</b>
pendigits	97.06	97.14	96.71	97.08	97.05	97.45	98.44	97.14	97.41	97.49±0.12	97.97±0.13	98.05±0.15	97.89±0.11	<b>98.20±0.11</b>
spambase	93.00	94.61	94.35	94.61	91.71	92.15	90.60	94.87	92.67	93.83±0.16	94.08±0.17	94.11±0.23	94.18±0.01	<b>94.72±0.14</b>
statlog_image	95.49	<b>97.57</b>	95.84	95.84	94.97	89.90	94.73	97.38	95.28	96.82±0.21	97.05±0.17	97.56±0.14	97.44±0.18	97.40±0.13
statlog_landsat	91.00	90.75	91.10	90.55	90.25	83.47	86.95	91.79	91.22	91.15±0.51	91.64±0.51	<b>92.19±0.25</b>	92.15±0.17	91.85±0.12
statlog_shuttle	99.90	99.83	99.77	99.92	99.79	96.82	99.15	99.90	99.88	99.91±0.02	99.92±0.02	99.93±0.01	99.93±0.01	<b>99.94±0.01</b>
wall-following	90.98	90.76	<b>92.30</b>	90.12	85.41	89.53	83.84	90.64	89.46	90.28±0.43	90.79±0.45	91.04±0.55	91.37±0.31	92.20±0.33
waveform	84.80	83.12	83.20	83.60	84.76	83.48	80.86	86.28	86.16	85.95±0.11	85.97±0.11	86.59±0.01	86.83±0.01	<b>86.97±0.12</b>
waveform-noise	86.08	83.28	86.96	85.84	83.70	82.44	84.66	86.76	86.08	85.68±0.14	86.17±0.11	86.92±0.01	86.98±0.15	<b>87.13±0.13</b>
wine-quality-white	63.73	64.79	55.64	63.07	55.96	55.15	51.47	63.21	55.49	63.29±0.40	63.70±0.39	64.76±0.38	64.22±0.41	<b>65.66±0.31</b>
<b>Mean Accuracy</b>	83.40	83.22	82.27	82.48	80.40	78.31	80.60	83.93	83.53	83.12±0.38	84.77±0.37	85.41±0.32	85.49±0.21	<b>85.89±0.27</b>
<b>Ave. Rank</b>	8.23	8.42	8.88	9.06	11.54	11.92	10.92	6.27	8.67	7.75	5.31	3.31	3.15	<b>1.58</b>

\* Methods with <sup>†</sup> are proposed in this paper with re-normalization.

**Table 4**  
Statistical comparison between WPedRVFL and each of the other networks.

Methods	Ave. Rank	p-value
WPedRVFL	1.58	
PedRVFL	3.15	4.2e-4
WedRVFL*	3.31	1.6e-4
edRVFL_N*	5.31	1.8e-5
obRaF(H)[53]	6.27	1.1e-3
edRVFL_O* [27]	7.75	1.8e-5
SNN* [44]	8.23	7.1e-5
MS* [47]	8.42	2.6e-5
H-ELM* [54]	8.67	1.8e-5
HW* [48]	8.88	1.3e-4
Resnet* [49]	9.06	1.8e-5
FBS[52]	10.92	2.6e-5
SCN* [50]	11.54	1.8e-5
BLS* [51]	11.92	1.8e-5

\* Lower rank reflects better performance. The p-value is obtained from the paired Wilcoxon test. Methods that are significantly worse than the best method are marked with “\*”.

5.3. Hyperparameter settings

For these FNN methods, we use the same hyperparameter settings as in [44]. For the other RVFL based randomized neural networks, We use the official codes provided by their authors and set the ranges of the hyperparameters as suggested in their original papers. Moreover, for the edRVFL based methods, the regularization parameter  $\lambda$  is chosen from range  $2^x$ , where  $x$  belongs to  $[-12, 12]$ . The hidden neuron number  $n$  of these methods can be tuned within  $[20,1000]$ . The maximum number of the hidden layers  $l_{max}$  of the edRVFL based methods is set to 10. The two key hyperparameters  $\gamma$  and  $\alpha$  of the re-normalization are tuned among  $[0.5,2]$  and  $[-2, 2]$ , respectively. The WedRVFL can choose the best weight  $\omega_r$  in  $(0,1)$  based on the validation accuracy. Also, for pruning-based methods, the pruning rate  $p$  is tuned among  $[0,1)$ . The details about these settings are summarized in Table 2.

During the tuning process, we do 4-fold cross-validation to find the best parameter settings. We separate the whole dataset into training and testing sets 4 times. In each fold, 25% of the training data are used as the validation set, and we select the hyperparameter configuration with the best average validation accuracy. Then, we use the whole training data to re-train the models before feeding the test data into them. The testing accuracy is obtained based on the correct predictions of the networks for the test data. At last, we report the mean value of the 4 testing accuracy as the final classification result for the current dataset.

In order to test the robustness of our new methods, for each edRVFL based network, we run the above experiment 10 times with different randomized hidden features. Then, we report the mean value and the standard deviation of these 10 outcomes in Table 3.

5.4. Experimental results

The performance of all the 14 methods on 24 UCI datasets is shown in Table 3.

We take the results of SNN, MS, HW, and Resnet directly from [44]. The rankings of each classifier are used to compare their performance on all the datasets. For the ranking method, the best classifier based on the classification accuracy of one dataset is ranked 1, the second is ranked 2, and so on. Then we use the average rank of all 24 datasets to show the performance of the classifiers. The best results for each dataset, as well as the best global rank and accuracy, are given in bold.

Wilcoxon signed-rank test is a non-parametric statistical hypothesis test used to compare the performance of two related mea-

**Table 5**  
Statistical comparison of all the methods.

	WPedRVFL†	PedRVFL†	WedRVFL†	edRVFL_N†	obRaF(H)[53]	edRVFL_O[27]	SNN[44]	MS[47]	H-ELM[54]	HW[48]	Resnet[49]	FBS[52]	SCN[50]	BLS[51]
WPedRVFL†	-													
PedRVFL†		-												
WedRVFL†			-											
edRVFL_N†				-										
obRaF(H)[53]					-									
edRVFL_O[27]						-								
SNN[44]							-							
MS[47]								-						
H-ELM[54]									-					
HW[48]										-				
Resnet[49]											-			
FBS[52]												-		
SCN[50]													-	
BLS[51]														-

\* The empty cell in the table means the corresponding method in the row and column have no significant statistical difference. The symbol ‘+’ indicates that the method in the row is statistically better than the method in the column. On the other hand, the symbol ‘-’ means the method in the row is statistically worse than that in the column. \* Methods with † are proposed in this paper.



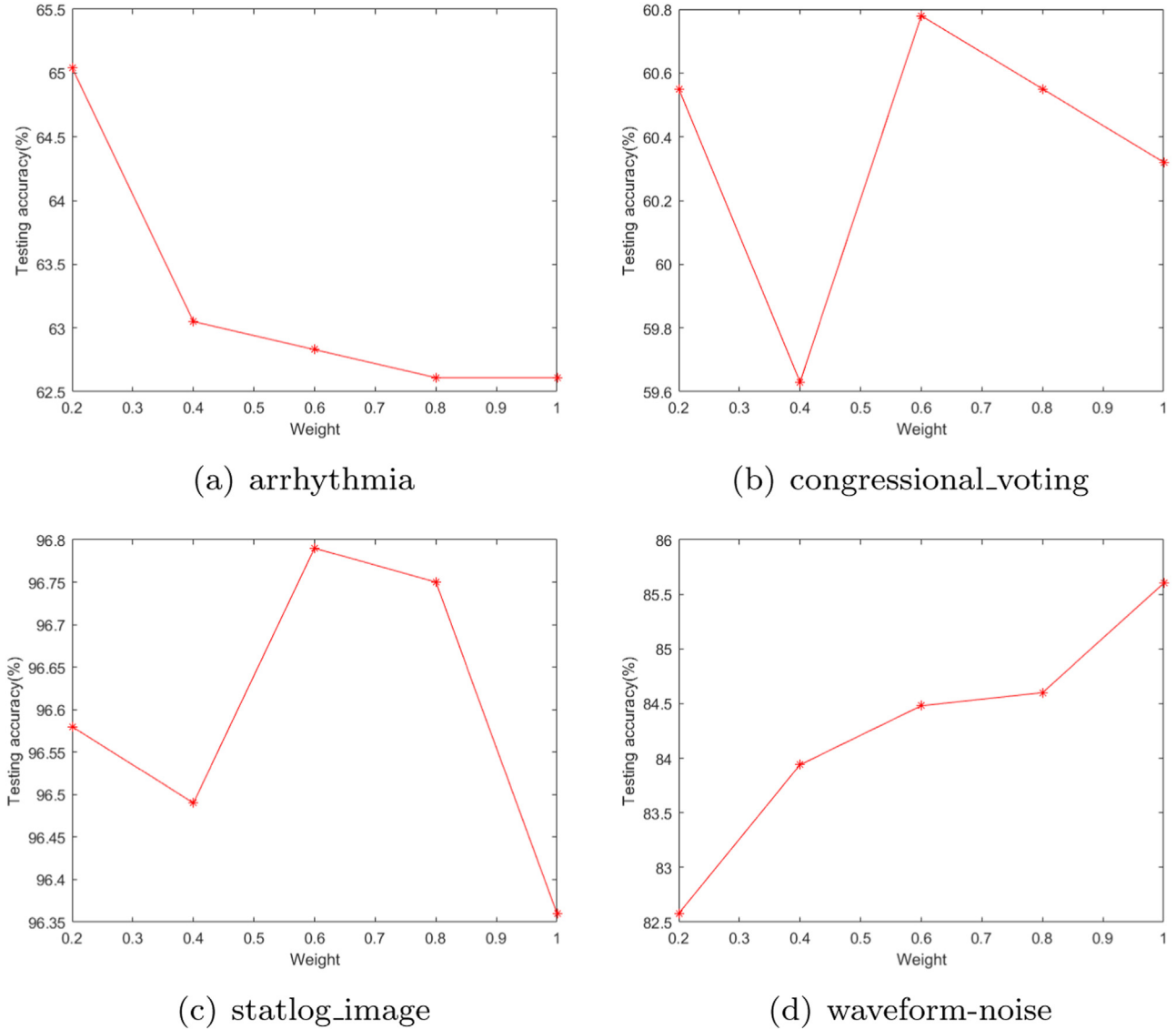


Fig. 7. The testing accuracy of WPedRVFL network with different weight  $\omega_w$  on 4 tabular datasets.

Table 6 Standard deviation comparison between edRVFL based methods.

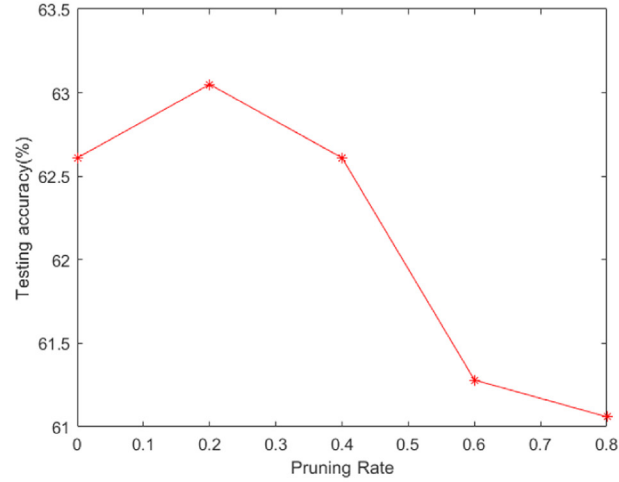
Methods	Ave. std
PedRVFL	0.21
WPedRVFL	0.27
WedRVFL*	0.32
edRVFL_N*	0.37
edRVFL_O* [27]	0.38

surements on multiple tasks [55]. In this paper, we employ it to do the pairwise comparison on the selected two methods to investigate the statistical difference between them. We first compare all the other 11 methods' performance with WPedRVFL. And the results are shown in Table 4. From the table, we can see that all leading positions are occupied by edRVFL based methods (except heterogeneous oblique random forest). The top-ranked method is WPedRVFL and followed by WedRVFL. PedRVFL takes the third place, and edRVFL\_N, obRaF(H), edRVFL\_O are following behind. By performing the statistical comparison, we know that the WPedRVFL significantly outperforms all the other competitors.

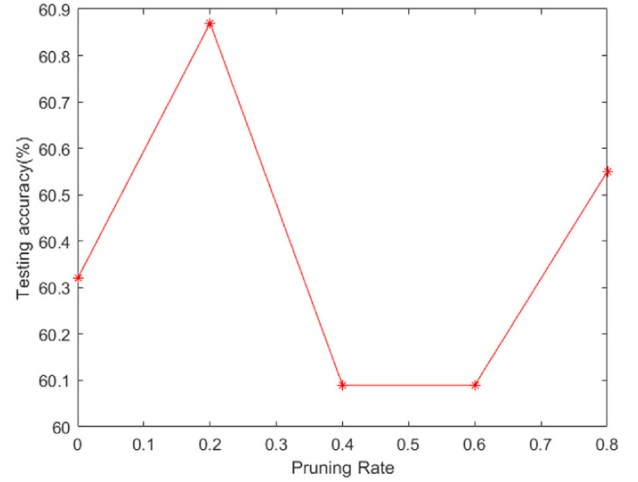
We also use the Wilcoxon test to do the pairwise comparison between all the 14 classifiers in this paper. The results are shown in Table 5. If there is no statistically significant difference between the methods in the corresponding row and column, the cells where they intersect will be empty. If the symbol in the cell is '+', that means the method in the corresponding row is statistically better than that in the column. On the contrary, the symbol '-' indicates that the method in the corresponding row is statistically worse.

Then, we pay attention to the standard deviation of these edRVFL based methods. We summarize their performance in Table 6. From it, we can learn that our new methods WedRVFL, PedRVFL, and WPedRVFL are more robust than the basic edRVFL\_O and edRVFL\_N. Among them, PedRVFL has the smallest std value. Since WPedRVFL is the combination of WedRVFL and PedRVFL, so we believe this is the reason why its std value is between WedRVFL and PedRVFL.

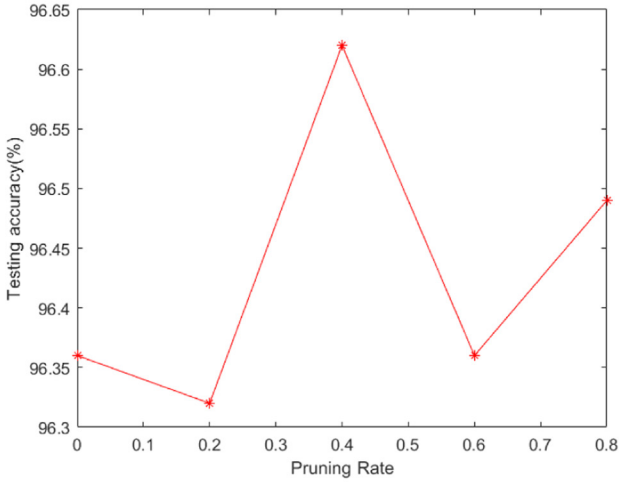
From all the Tables above, we know that our new proposed methods, edRVFL\_N, WedRVFL, PedRVFL, and WPedRVFL, show great performance on 24 UCI classification tasks. Among them, WPedRVFL takes great advantage both in the average rank and mean accuracy compared to all other methods. It significantly outperforms other FNNs like Resnet, HW, MS, and SNN. And it also has



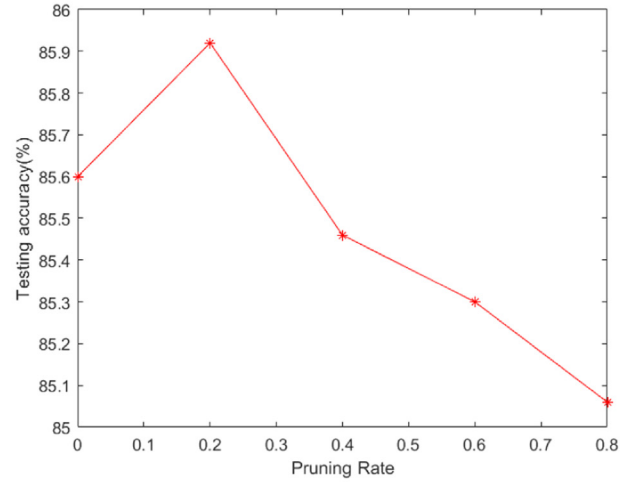
(a) arrhythmia



(b) congressional\_voting



(c) statlog\_image



(d) waveform-noise

**Fig. 8.** The testing accuracy of WPedRVFL network with different pruning rate  $p$  on 4 tabular datasets.

a significant statistical difference from other randomized neural networks. Moreover, the standard deviation of our new methods are smaller than the original edRVFL networks. Therefore, WPedRVFL can be considered as a highly competitive classifier on tabular classification tasks.

### 5.5. Effects of the key hyperparameters: Weight $\omega_r$ and pruning rate $p$

In this part, we conduct more experiments to discover the effects of the key hyperparameters. Here we select 4 datasets: arrhythmia, congressional\_voting, statlog\_image, and waveform-noise from the previous section to show how the testing accuracy changes with different settings of  $\omega_r$  and  $p$ .

We use the WPedRVFL model to perform these tests. During the experiments, except for  $\omega_r$  and  $p$ , all the other hyperparameters are fixed to control variables. Moreover, when we are testing the influence of weight  $\omega_r$ , the pruning rate  $p$  is also set to 0 (no pruning) to complete the controlled experiment. On the other hand, the weight  $\omega_r$  is set to 1 (no weighting) when the pruning rate  $p$  is investigated. We give the details of these configurations in Table 7.

**Table 7**

Hyperparameters considered in the controlled experiments.

Hyperparameter	Considered values
Regularization parameter $\lambda$	1
Number of hidden neurons $n$	500
Number of hidden layers $l$	3
$\gamma$ in batch normalization	1
$\alpha$ in batch normalization	0
Weight $\omega_r$ for the correctly predicted samples	0.2, 0.4, 0.6, 0.8, 1 or 1 (when testing different pruning rates)
Pruning Rate $p$	0, 0.2, 0.4, 0.6, 0.8, or 0 (when testing different weights)

We first give WPedRVFL's performance with different weights  $\omega_r$  on 4 datasets in Fig. 7. As we can see from Fig. 7. The weighting scheme is useful in most cases. For arrhythmia, congressional\_voting, and statlog\_image datasets, setting the weight  $\omega_r$  for the wrongly predicted samples generally increases the testing accuracy of the model. But which weight  $\omega_r$  is the most suitable one varies from case to case. Therefore, we need to use the validation data to find the best configuration of the specific dataset. On the other hand, there is an exception that the weighting scheme is

**Table 8**

Training Time comparisons between SNN and WPedRVFL on spambase dataset (s).

Dataset	SNN	WPedRVFL
arrhythmia	4.4731	0.6504
contrac	2.8705	0.4898
musk_2	7.0369	0.9814
spambase	8.6586	1.3752
waveform	6.7168	0.8336

\* Experiment environment: Intel(R) Xeon(R) CPU E5-2620; nVIDIA GeForce GTX-1080.

**Table 9**

Performance with different network settings (%).

Dataset	20 Best only	20 Worst only	All	After pruning the worst 20
abalone	65.90	64.37	67.05	67.34
bank	89.03	88.76	88.85	88.85
glass	73.58	58.49	73.58	73.61
monks-3	61.11	59.49	58.80	62.96
oocytes_trisopterus_states_5b	91.23	88.60	92.54	93.34

harmful to the classification. For the waveform-noise dataset, the testing accuracy keeps decreasing when we set lower and lower values for  $\omega_r$ .

In addition to that, we present WPedRVFL's performance of changing the pruning rate  $p$  in Fig. 8. For all 4 tabular datasets, the pruning method can always help us get better accuracy. The four line charts show a similar pattern: At the beginning, the testing accuracy increases with the pruning rate. After reaching a peak value, the accuracy drops and becomes even worse than the initial value. So same as the weight  $\omega_r$ , we need to set different pruning rates  $p$  for different cases using the validation set.

### 5.6. Training time comparison

Another advantage of randomized neural networks is their training time. Since they do not use back-propagation to train the hidden parameters, the training time of these RVFL variants can be much less than the deep learning models.

In this section, we select two representatives from two groups of the classifiers. Among the back-propagation based deep neural networks, we choose SNN since it has the highest accuracy on the benchmark datasets. Meanwhile, the most competitive classification model WPedRVFL is selected from the randomized neural networks. We present their training time (without hyperparameter tuning) on 5 selected datasets in Table 8.

### 5.7. Inferior features

Randomization has the potential to generate inferior features, and the claim can be treated as a general one. To demonstrate this claim, we conducted the following experiments. Here we use the single-hidden-layer RVFL network (without direct-link) as the backbone (No re-normalization). All the hyperparameters are fixed during these experiments (Number of the hidden neurons: 100,  $\lambda:1$ , Pruning rate: 0.2 (cutting off 20 worst hidden neurons)). We report the classification accuracy on 5 datasets (1-fold) with four different settings -20 Best (accuracy with only 20 best hidden neurons), 20 Worst (accuracy with only 20 worst hidden neurons), Original (network without pruning), and Pruning (network after removing the 20 worst neurons). We present the ACC (%) in the Table 9.

## 6. Conclusion

In this paper, we first introduce batch normalization to the edRVFL network for re-normalizing the hidden features. Then, we propose a weighted version for edRVFL network. A weight matrix is used to allocate different weights to different samples. The weight matrix changes according to the samples' predictions in the previous layers. This method can make sure that each hidden layer in the network has different biases for each sample and increase the ensemble classification accuracy. Moreover, we propose another new variant of edRVFL with the pruning method. Instead of pruning neurons after the training process, we cut off the inferior neurons according to their importance for classification when we are training the model. This method can prevent the propagation of detrimental features and increase the classification accuracy in deeper layers. Then, the combination of these two methods called Weighting and Pruning based Ensemble Deep Random Vector Functional Link Network is proposed. It takes advantage of both WedRVFL and PedRVFL and performs better overall. For evaluating the performance of our new proposed methods, we compare them, with other 10 classifiers on 24 UCI benchmark datasets. The experimental results show the superiority of our new methods on tabular classification tasks. In particular, WPedRVFL is the most competitive one among all the 14 classifiers. In addition to that, we investigate the effects of setting different weight and pruning rate values. These results illustrate how the weighting and pruning schemes can help to improve the classification results. At last, we compare the training time between our proposed model with a back-propagation based deep neural network. In our future work, we will develop methods to select only a few output layers with the highest classification accuracy to perform the final classification.

### Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Qiushi Shi** received his B.Eng. degree from Hohai University, China in 2019. Currently, he is a Ph.D. student in the School of Electrical and Electronic Engineering in Nanyang Technological University, Singapore. His research interest includes machine learning, computer vision and neural networks.

**Minghui Hu** received his B.Eng. degree from Dalian Maritime University, China in 2018 and his MSc degree from Nanyang Technological University, Singapore. Currently, he is a PhD student at the School of Electrical and Electronic Engineering at Nanyang Technological University, Singapore. His research interests include randomized neural networks, deep learning, and computer vision.

**Ponnuthurai Nagaratnam Suganthan** (or P N Suganthan) received the B.A degree, Postgraduate Certificate and M.A degree in Electrical and Information Engineering from the University of Cambridge, UK in 1990, 1992 and 1994, respectively. He received an honorary doctorate (i.e. Doctor Honoris Causa) in 2020 from University of Maribor, Slovenia. After completing his PhD research in 1995, he served as a predoctoral Research Assistant in the Dept of Electrical Engineering, University of Sydney in 1995–96 and a lecturer in the Dept of Computer Science and Electrical Engineering, University of Queensland in 1996–99. He was an Editorial Board Member of the *Evolutionary Computation Journal*, MIT Press (2013–2018), *IEEE Trans on Cybernetics* (2012 - 2018), *IEEE Trans on Evolutionary Computation* (2005 -2021). He is an associate editor of *Engineering Applications of Artificial Intelligence* (Elsevier, 2022 - ). *Applied Soft Computing* (Elsevier, 2018-), *Neurocomputing* (Elsevier, 2018- ), *Information Sciences* (Elsevier, 2009 - ), *Pattern Recognition* (Elsevier, 2001 - ) and *IEEE Trans on SMC: Systems* (2020 - ) Journals. He is a founding co-editor-in-chief of *Swarm and Evolutionary Computation* (2010 - ), an SCI Indexed Elsevier Journal. His co-authored SaDE paper (published in April 2009) won the “IEEE Trans. on Evolutionary Computation outstanding paper award” in 2012. His research interests include randomization-based learning methods, swarm and evolutionary algorithms, pattern recognition, deep learning and applications of swarm, evolutionary & machine learning algorithms. He was selected as one of the highly cited researchers by Thomson Reuters Science Citations yearly from 2015 to 2022 in computer science. He served as the General Chair of the IEEE SSCI 2013. He has been a member of the IEEE (S’90, M’92, SM’00, Fellow 2015) since 1991 and an elected AdCom member of the IEEE Computational Intelligence Society (CIS) in 2014–2016. He is an IEEE CIS distinguished lecturer (DLP) in 2018–2021.

**Rakesh Katuwal** received his B. Eng. degree from Kathmandu University, Nepal in 2014. He recently completed Ph.D. in the School of Electrical and Electronic Engineering in Nanyang Technological University, Singapore. His research interest includes various ensemble learning methods, deep learning, and computer vision.