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An Improved Control Chart for Monitoring Linear Profiles and Its Application in Thermal Conductivity

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ABSTRACT In most of the manufacturing processes, we encounter different quality characteristics of a product and process. These characteristics can be categorized into two kinds; study variables (variable of interest) and the supporting/explanatory variables. Sometime, a linear relationship might exist between the study and supporting variable, which is called simple linear profiles. This study focuses on the simple linear profiles under assorted control charting approach to detect the large, moderate and small disturbances in the process parameters. The evaluation of the proposed assorted method is assessed by using numerous performance measures, for instance, average run length, relative average run length, extra and sequential extra quadratic losses. A comparative analysis of the proposal is also carried out with some existing linear profile methods including Shewhart_3, Hotelling's T^2, EWMA_3, EWMA/R and CUSUM_3 charts. Finally, a real-life application of the proposed assorted chart is presented to monitor thermal management of diamond-copper composite.

INDEX TERMS Control chart, cumulative sum, exponentially weight moving average, Shewhart, thermal conductivity monitoring

I. INTRODUCTION

Control charts are magnificently applied in many industrial processes and assist the specialists in improving the performance of a process by decreasing the process variation. In some manufacturing processes, the variable of interest is associated (linearly or non-linearly) with one or more auxiliary variable(s). Monitoring the variable of interest along with the linear association of one auxiliary variable is referred to simple linear profiles. The usual practice in statistical process control (SPC) is to monitor the mean and/or variance of the process. On the contrary, in simple linear profiles, one models the slope, intercept and error deviation of the linear model.

Many monitoring structures for the simple linear profiles are developed in the literature: control chart for the monitoring of group adjusted variables was discussed in [1]–[5]. The well-known control charting structures such as multivariate Hotelling's T^2 and EWMA/R charts were suggested by Kang and Albin [6], while Gupta, *et al.* [7] suggested a Shewhart based linear profile monitoring method known as Shewhart_3 chart. Further, for simple linear profiles, Mahmoud and Woodall [8] and Yeh and Zerehsaz [9] have proposed Phase I monitoring approach while the cumulative sum (CUSUM) structure in multivariate setup was proposed by Noorossana, *et al.* [10]. Noorossana, *et al.* [11] have provided a study to resolve the issue of normality, and the change point methods were discussed by Zou, *et al.* [12] and Mahmoud, *et al.* [13]. A well-known EWMA_3 chart was proposed by Kim, *et al.* [14], and the similar CUSUM structure (CUSUM_3 chart) was introduced by Saghaei, *et al.* [15]. Noorossana and Amiri [16] proposed integrated MCUSUM and χ^2 structures while a review on linear profile methods was presented by Woodall [17]. Linear profile monitoring based on the recursive residuals was proposed

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by Zou, *et al.* [18] and based on the mixed model was studied by Jensen, *et al.* [19]. Soleimani, *et al.* [20] covered the effect of within autocorrelation, and a likelihood ratio based method was discussed by Zhang, *et al.* [21]. Most of the above-mentioned studies were examined under the fixed effect model while under a random effect model, a Phase II method was suggested by Noorossana, *et al.* [22]. The max and sum of square based linear profile monitoring methods were discussed by Mahmood, *et al.* [23], and a progressive approach for simple linear profile was suggested by Saeed, *et al.* [24]. Most of the linear profile studies were designed under simple random sampling but under different sampling environments such as ranked set and modified successive samplings were examined in [25]–[30]. The simple linear profile methods under the Bayesian approach were discussed in [31]–[34].

The Shewhart based structures are useful to detect a large amount of shift in the process parameter while for the detection of small to moderate changes, EWMA and CUSUM charts were used (cf. Faisal, *et al.* [35]). Instead of these charts, Abbas, *et al.* [36] proposed a method, which is compatible for all type of shifts (i.e., small, moderate and large) and referred to the assorted_3 chart. This study is intended to propose an assorted_3 approach under the simple linear profile setup. The rest of the article is outlined as follow: simple linear profile model is discussed in section 2; structure of the existing and proposed linear profile methods were given in section 3; a brief discussion on the performance evaluation is reported in section 4; comparative analysis of the proposed with the existing control charting methods were discussed in section 5; the implementation of proposed assorted_3 chart with real-life dataset is demonstrated in section 6, and the concluding remarks are reported in section 7.

II. PRELIMINARIES TO SIMPLE LINEAR PROFILES

Let a study variable Y with the explanatory variable X is observed in a paired form such as (Y_{ij}, X_i) for the i^{th} random sample, collected with respect to time j, then the simple linear regression model, is described as follows:

$$Y_{ij} = \beta_0 + \beta_1 X_i + \epsilon_{ij}; \quad i = 1, 2, \ldots, n; \; j = 1, 2, \ldots \quad (1)$$

where β_0, β_1 and ϵ_{ij} represents intercept, slope and error term, respectively. It is assumed that the ϵ_{ij} follows a normal distribution with mean ($\mu = 0$) and variance ($\sigma^2 = 1$). The ordinary least square (OLS) estimates of the parameters are described as follows:

$$\hat{\beta}_{1j} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})Y_{ij}}{\sum_{i=1}^{n}(X_i - \bar{X})^2} = \frac{S_{XY(j)}}{S_{XX}},$$
$$\hat{\beta}_{0j} = \bar{Y}_j - \hat{\beta}_{1j}\bar{X},$$

where $\bar{Y}_j = \sum_{i=1}^{n} Y_{ij}/n$, $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S_{XX} = \sum_{i=1}^{n}(X_i - \bar{X})^2$. The expected values, variances and co-variance term of $\hat{\beta}_{0j}$ and $\hat{\beta}_{1j}$ are defined as follows:

$$E\left(\hat{\beta}_{1j}|X\right) = \beta_1; \quad E\left(\hat{\beta}_{0j}|X\right) = \beta_0,$$
$$Var\left(\hat{\beta}_{1j}|X\right) = \frac{\sigma^2}{S_{XX}}; \quad Var\left(\hat{\beta}_{0j}|X\right) = \sigma^2\left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right],$$
$$Cov\left(\hat{\beta}_{1j}, \hat{\beta}_{0j}|X\right) = -\frac{\sigma^2\bar{X}}{S_{XX}}.$$

In most of the studies, mean square error (MSE) is used to provides an unbiased estimate of the error variance σ^2 and computed by

$$MSE_j = \frac{\sum_{i=1}^{n}(Y_{ij} - \hat{Y}_{ij})^2}{n-2} = \frac{\sum_{i=1}^{n} e_{ij}^2}{n-2},$$

where $\hat{Y}_{ij}$ is the j^{th} predicted value for the i^{th} random sample. Generally, when we are interested in the monitoring of two or more process parameters than it is necessary to make them independent from each other. In simple linear profiles, slope and intercept have a covariance, and in order to meet zero covariance, the coded model is a productive approach. To obtain the coded model, we transformed the X_i values such as $X_i' = X_i - \bar{X}$ and the obtained model named by the transformed model can be represented as follows:

$$Y_{ij} = B_0 + B_1 X_i' + \epsilon_{ij}; \quad i = 1, 2, \ldots, n; \; j = 1, 2, \ldots \quad (2)$$

where intercept of the transformed model (B_0) is equals to $\beta_0 + \beta_1\bar{X} + \beta\sigma\bar{X}$, and slope of the transformed model (B_0) is equals to $(\beta_1 + \beta\sigma)X_i'$. It is noted that β represents a shift in terms of σ, in the slope of the original model (given in Eq. (1)). Similarly, one may obtain OLS estimates (i.e., b_{0j}, b_{1j} and mse_j) and other properties for the parameters of transformed model. Several studies on monitoring linear profile parameters are accessible in the latest literature, some of which are shortly outlined below.

III. METHODS OF SIMPLE LINEAR PROFILES

This section is designed to formulate the monitoring methods based on the preliminaries reported in section 2. Further, the section is divided into existing and the proposed simple linear profile methods, which are discussed into following subsections.

A. EXISTING SIMPLE LINEAR PROFILE METHODS

In this subsection, we will provide the structure of all existing simple linear profile methods, which will further be used as the counterparts of the stated proposal.

1) THE HOTELLING'S T^2 CHART

In simple linear profiles literature, the Hotelling's T^2 chart for the monitoring of intercept and slope was proposed by Kang and Albin [6]. The j^{th} statistic of the Hotelling's T^2 chart is expressed as follows:

$$T_j^2 = \left(V_j - U\right)^T \Sigma^{-1}\left(V_j - U\right),$$

where $V_j = \left(\hat{\beta}_{0j}, \hat{\beta}_{1j}\right)^T$, $U = (\beta_0, \beta_1)^T$, and

$$\Sigma = \begin{bmatrix} \sigma^2\left[\frac{1}{n} + \frac{\bar{X}^2}{S_{XX}}\right] & -\frac{\sigma^2\bar{X}}{S_{XX}} \\ -\frac{\sigma^2\bar{X}}{S_{XX}} & \frac{\sigma^2}{S_{XX}} \end{bmatrix}.$$

The upper control limit of the Hotelling's T^2 statistic is obtained as $UCL_H = \chi^2_{2,\alpha}$. When the process is stable, the Hotelling's T^2 statistic follows a non-central χ^2 distribution with non-centrality parameter (τ), which is equals to $n(\varphi\sigma + \beta\sigma\bar{X})^2 + (\beta\sigma)^2 S_{XX}$. Where φ and β are the shifts in the intercept and slope of the original model given in Eq. (1).

2) THE EWMA/R CHART

For the linear profile monitoring, a combined structure centered on the EWMA and R charts was suggested by Kang and Albin [6]. Basically, the combination of these charts was used to serve the purposes such as (i) detecting changes in the error variance of the model given in Eq. (1), and (ii) addressing the unusual state of the error variance. The j^{th} EWMA chart statistic is measured as follows:

$$Z_j = \lambda\bar{e}_j + (1-\lambda)Z_{j-1},$$

where $\lambda\epsilon(0, 1]$ is the weighting parameter, $\bar{e}_j = \sum_{i=1}^{n} e_{ij}/n$ and initial EWMA value is considered as zero ($Z_0 = 0$). The process is declared as out-of-control (OOC) when $Z_j < LCL_E$ or $Z_j > UCL_E$. Where

$$LCL_E = -L_1\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}; \quad UCL_E = L_1\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}.$$

Further, the j^{th} statistic of the R chart and control limits can be obtained as follows:

$$R_j = max_i\left(e_{ij}\right) - min_i\left(e_{ij}\right),$$
$$LCL_R = \sigma(d_2 - L_2 d_3); \quad UCL_R = \sigma\,(d_2 + L_2 d_3),$$

where d_2 and d_3 are the unbiased constants which are reported in [37].

3) THE SHEWHART_3 CHART

A Shewhart based simple linear profile method, which is referred to the Shewhart_3 chart was suggested by Gupta, *et al.* [7]. The intercept, slope and mean square error were used as the plotting statistics, which were plotted against the following control limits.

$$\text{For the intercept:} \begin{cases} LCL_{SI} = B_0 - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}} \\ UCL_{SI} = B_0 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{n}}, \end{cases}$$

$$\text{For the slope:} \begin{cases} LCL_{SS} = B_1 - Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{XX}}} \\ UCL_{SS} = B_1 + Z_{\alpha/2}\sqrt{\frac{\sigma^2}{S_{XX}}}, \end{cases}$$

$$\text{For the error variance:} \begin{cases} LCL_{SE} = \frac{\sigma^2}{n-2}\chi^2_{(1-\alpha/2),(n-2)} \\ UCL_{SE} = \frac{\sigma^2}{n-2}\chi^2_{(\alpha/2),(n-2)}, \end{cases}$$

where $Z_{\alpha/2}$ is the $(\alpha/2)^{th}$ quantile point of the standard normal distribution while $\chi^2_{(1-\alpha/2),(n-2)}$ and $\chi^2_{(\alpha/2),(n-2)}$ are the lower and upper quantile points of the χ^2 distribution with $(n-2)$ degrees of freedom, respectively.

4) THE EWMA_3 CHART

Kim, *et al.* [14] developed a memory type structure centered on the EWMA control chart and referred to the EWMA_3 chart. The EWMA_3 chart has the ability to detect small to moderate amount of shifts in the linear profile parameters. The structure of EWMA_3 chart relies on the transformed model provided in Eq. (2) and the three separate EWMA statistics based on intercept, slope and error variance are described as follows:

$$EWMA_{(I)j} = \lambda b_{0j} + (1-\lambda)\,EWMA_{(I)j-1},$$
$$EWMA_{(S)j} = \lambda b_{1j} + (1-\lambda)\,EWM_{(S)j-1},$$
$$EWMA_{(E)j} = \max\left[\lambda\ln\left(mse_j\right) + (1-\lambda)\,EWMA_{(E)j-1}, \ln\left(\sigma_0^2\right)\right],$$

where $\lambda \in (0, 1]$ is the weighting parameter, and the control limits for each statistic are described below:

$$\text{For intercept:} \begin{cases} LCL_{EI} = B_0 - L_{EI}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]} \\ UCL_{EI} = B_0 + L_{EI}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\left[\frac{1}{n}\right]}, \end{cases}$$

$$\text{For slope:} \begin{cases} LCL_{ES} = B_1 - L_{ES}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\frac{\sigma^2}{S_{XX}}} \\ UCL_{ES} = B_1 + L_{ES}\sigma\sqrt{\frac{\lambda}{(2-\lambda)}\frac{\sigma^2}{S_{XX}}}, \end{cases}$$

$$\text{For error variance:} \begin{cases} LCL_{EE} = 0 \\ UCL_{EE} = L_{EE}\sqrt{\frac{\lambda}{(2-\lambda)}Var\left[\ln\left(MSE_j\right)\right]}, \end{cases}$$

where L_{EI}, L_{ES}, L_{EE} are the charting constants, which are fixed against the IC average run length and the asymptotic variance of the $\ln\left(MSE_j\right)$ (cf. Crowder and Hamilton [38]) can be obtained as follows:

$$Var\left[\ln\left(MSE_j\right)\right] \approx \frac{2}{n-2} + \frac{2}{(n-2)^2} + \frac{2}{3\,(n-2)^3} - \frac{16}{15(n-2)^5}$$

5) THE CUSUM_3 CHART

Saghaei, *et al.* [15] proposed a cumulative sum (CUSUM) control charting structure based on three distinct CUSUM statistics (referred as CUSUM_3 control chart) to monitor simple linear profile parameters. The statistics of

TABLE 1. Control charting constant and limits for existing methods at fixed $ARL_0 = 200$.

Parameter	Hotelling's T^2	EWMA/R	Shewhat_3	EWMA_3	CUSUM_3
Intercept	$UCL_H = 10.60$	L_1=3	$Z_{\alpha/2}$= 3.14	L_{EI}=3.0156	$H_I^- = 1.198$ $H_I^+ = 1.195$
Slope				L_{ES}=3.0109	$H_S^- = 1.80$ $H_S^+ = 1.83$
Error variance	-	L_2=3.1151	$LCL_{SE} = 0.005$, $UCL_{SE} = 5.298$	L_{EE}=1.3723	$H_E^- = 0.261$ $H_E^+ = 2.510$
Design	-	λ=0.2	-	λ=0.2	$K_I^{(+,-)} = 0.50$ $K_S^{(+,-)} = 0.05$ $K_E^+ = 2.00$ $K_E^- = 0.25$

CUSUM_3 control chart was given as below:
For intercept:

$$\begin{cases} CUSUM_{(I)j}^+ = \max[0, b_{0j} - \left(B_0 + K_I^+\right) + CUSUM_{(I)j-1}^+] \\ CUSUM_{(I)j}^- = \max[0, \left(B_0 + K_I^-\right) - b_{0j} + CUSUM_{(I)j-1}^-] \end{cases}$$

For slope:

$$\begin{cases} CUSUM_{(S)j}^+ = \max[0, b_{1j} - \left(B_1 + K_S^+\right) + CUSUM_{(S)j-1}^+] \\ CUSUM_{(S)j}^- = \max[0, \left(B_1 + K_S^-\right) - b_{1j} + CUSUM_{(S)j-1}^-], \end{cases}$$

For error variance:

$$\begin{cases} CUSUM_{(E)j}^+ = \max[0, mse_j - K_E^+ + CUSUM_{(E)j-1}^+] \\ CUSUM_{(E)j}^- = \min[0, mse_j - K_E^- + CUSUM_{(E)j-1}^-], \end{cases}$$

where the initial value of each CUSUM statistic is considered as zero while the K_I, K_S and K_E are the reference values, which are equal to $\Delta/2$. Where Δ is the difference between the targeted value and the OOC value of the parameters. Further, the CUSUM statistics for intercept are plotted against the $H_I^{(-,+)}$ and the CUSUM statistics for slope and error variance are plotted against the $H_S^{(-,+)}$ and $H_E^{(-,+)}$, respectively.

It is noted that the control charting constants and the limits of existing simple linear profile methods are reported in Table 1 to achieve an overall $ARL_0 = 200$ (by setting an individual $ARL_0 = 584.5$).

B. THE PROPOSED ASSORTED_3 CHART

The simple linear profile methods under Shewhart structure were used to detect a large shift in the linear profile parameters while for the detection of small to moderate shifts, EWMA and CUSUM charts were used. Beyond these charts, Abbas, *et al.* [36] proposed a mechanism, which is compatible with all type of shifts and referred to the assorted_3 chart. Similarly, the assorted_3 chart for the linear profile setup is discussed below:

The plotting statistic $\left(T_{(I)j}\right)$ of the assorted_3 chart for the intercept parameter is described as follows:

$$T_{(I)j} = max\left[T_{1(I)j}, T_{2(I)j}^+, T_{2(I)j}^-, T_{3(I)j}\right], \tag{3}$$

where the Shewhart statistic $\left(T_{1(I)j}\right)$, CUSUM statistics $\left(T_{2(I)j}^+, T_{2(I)j}^-\right)$, and EWMA statistic $\left(T_{3(I)j}\right)$ are defined as follows:

$$T_{1(I)j} = \frac{1}{c_s}\left|\frac{b_{0j} - B_0}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]}}\right|,$$

$$T_{2(I)j}^+ = \frac{1}{h_c}\left(\frac{C_{(I)j}^+}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]}}\right), \quad T_{2(I)j}^- = \frac{1}{h_c}\left(\frac{C_{(I)j}^-}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]}}\right),$$

$$T_{3(I)j} = \frac{1}{L_e}\left|\frac{EWMA_{(I)j} - B_0}{\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]}\left[\sqrt{\frac{\lambda}{2-\lambda}\left\{1 - (1-\lambda)^{2j}\right\}}\right]}\right|,$$

where $C_{(I)j}^+$ and $C_{(I)j}^-$ with reference value k are defined as follows:

$$C_{(I)j}^+ = max\left[0, b_{0j} - B_0 - k\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]} + C_{(I)j-1}^+\right],$$

$$C_{(I)j}^- = max\left[0, -(b_{0j} - B_0) - k\sigma\sqrt{\left[\frac{1}{n} + \frac{\bar{X}'^2}{S_{XX}}\right]} + C_{(I)j-1}^-\right],$$

The plotting statistic $\left(T_{(S)j}\right)$ of the Assorted_3 chart for the slope parameter is defined as follows:

$$T_{(S)j} = max\left[T_{1(S)j}, T_{2(S)j}^+, T_{2(S)j}^-, T_{3(S)j}\right], \tag{4}$$

where the Shewhart statistic $\left(T_{1(S)j}\right)$, CUSUM statistics $\left(T_{2(S)j}^+, T_{2(S)j}^-\right)$, and EWMA statistic $\left(T_{3(S)j}\right)$ are represented as follows:

$$T_{1(S)j} = \frac{1}{c_s}\left|\frac{b_{1j} - B_1}{\sigma\sqrt{\frac{1}{S_{XX}}}}\right|,$$

$$T_{2(S)j}^+ = \frac{1}{h_c}\left(\frac{C_{(S)j}^+}{\sigma\sqrt{\frac{1}{S_{XX}}}}\right), \quad T_{2(S)i}^- = \frac{1}{h_c}\left(\frac{C_{(S)j}^-}{\sigma\sqrt{\frac{1}{S_{XX}}}}\right)$$

$$T_{3(S)j} = \frac{1}{L_e}\left|\frac{EWMA_{(S)j} - B_1}{\sigma\sqrt{\frac{1}{S_{XX}}}\left[\sqrt{\frac{\lambda}{2-\lambda}\left\{1 - (1-\lambda)^{2j}\right\}}\right]}\right|,$$

TABLE 2. Choice of sensitivity parameters for different categories of shift.

Sensitivity Parameter	Shift Size		
	Small	Medium	Large
λ	0.03 *to* 0.20	0.21 *to* 0.50	0.51 *to* 1.00
k	0.1 *to* 0.75	0.76 *to* 1.50	*more than* 1.50

where $C^{+}_{(S)j}$ and $C^{-}_{(S)j}$ with reference value k are defined as follows:

$$C^{+}_{(S)j} = max\left[0, b_{1j} - B_1 - k\sigma\sqrt{\frac{1}{S_{XX}}} + C^{+}_{(S)j-1}\right],$$

$$C^{-}_{(S)j} = max\left[0, -(b_{1j} - B_1) - k\sigma\sqrt{\frac{1}{S_{XX}}} + C^{-}_{(S)j-1}\right],$$

The plotting statistic $\left(T_{(E)j}\right)$ of the Assorted_3 chart for the error variance is defined as follows:

$$T_{(E)j} = max\left[T_{1(E)j}, T^{+}_{2(E)j}, T^{-}_{2(E)j}, T_{3(E)j}\right], \quad (5)$$

where the Shewhart statistic $\left(T_{1(E)j}\right)$, CUSUM statistics $\left(T^{+}_{2(E)j}, T^{-}_{2(E)j}\right)$, and EWMA statistic $\left(T_{3(E)j}\right)$ are represented as follows:

$$T_{1(E)j} = \frac{1}{c_s}\left(\frac{T.MSE_j}{\sigma}\right),$$

$$T^{+}_{2(E)j} = \frac{1}{h_c}\left(\frac{C^{+}_{(E)j}}{\sigma}\right), \quad T^{-}_{2(E)j} = \frac{1}{h_c}\left(\frac{C^{-}_{(E)j}}{\sigma}\right),$$

$$T_{3(E)j} = \frac{1}{L_e}\left|\frac{EWMA_{(E)j} - B_1}{\sigma\sqrt{\frac{\lambda}{2-\lambda}\left\{1-(1-\lambda)^{2j}\right\}}}\right|,$$

where $T.MSE_j$ is the transformed mean square error, which equals to $-0.7882 + 2.1089 \times \log_e\left(\frac{mse_{ij}}{n-2} + 0.6261\right)$ and CUSUM statistics are given below:

$$C^{+}_{(E)j} = max\left[0, MSE - k\sigma + C^{+}_{(E)j-1}\right],$$

$$C^{-}_{(E)j} = max\left[0, -MSE - k\sigma + C^{-}_{(E)j-1}\right].$$

In the above-mentioned expressions, the c_s, h_c and L_e are the charting constants and k is the reference value, which are equal to $\Delta/2$. Where Δ is the difference between the targeted value and the OOC value of the simple linear profile parameters. Hence, the final plotting statistic for the assorted_3 control chart is given below:

$$T_j = max\left[T_{(I)j}, T_{(S)j}, T_{(E)j}\right]. \quad (6)$$

The plotting statistic T_j has only an upper control limit which is defined as follow:

$$UCL = 1. \quad (7)$$

When $T_j > 1$ then an OOC signal is observed in the process intercept and/or slope and/or error variance. The rationale for selecting the UCL equal to is outlined as follow:

The $T_j > 1$ implies the following:

(i) either $T_{1(I)j} > 1$, and/or $T_{1(S)j} > 1$ and/or $T_{1(E)j} > 1$ (cf. Eq. (3), (4) and (5)),
$\Rightarrow$ the Shewhart statistic exceeds its corresponding control limit c_s for linear profile parameters;

(ii) and/or $T^{+}_{2(I)j}$ *or* $T^{-}_{2(I)j} > 1$, and/or $T^{+}_{2(S)j}$ *or* $T^{-}_{2(S)j} > 1$ and/or $T^{+}_{2(E)j}$ *or* $T^{-}_{2(E)j} > 1$ (cf. (3), (4) and (5)),
$\Rightarrow$ the CUSUM statistic exceeds its corresponding control limit h_c for linear profile parameters;

(iii) and/or $T_{3(I)j} > 1$, and/or $T_{3(S)j} > 1$ and/or $T_{3(E)j} > 1$ (cf. (3), (4) and (5)),
$\Rightarrow$ the EWMA statistics exceeds its respective control limit L_e for linear profile parameters.

IV. PERFORMANCE EVALUATIONS

This section consists of the discussion on the performance evaluation of proposed Assorted_3 chart and comparative analysis with Shewhart_3, Hotelling's T^2, CUSUM_3, EWMA/R, and EWMA_3 charts.

A. IC SIMPLE LINEAR PROFILE MODEL

In the simulation study, we considered IC simple linear profile model with $\beta_0 = 3$ and $\beta_1 = 2$ (following Kang and Albin [6]) and the original model is given in Eq. (1) can be written as:

$$Y_{ij} = 3 + 2X_i + \epsilon_{ij}; \quad i = 1, 2, \ldots, 4$$

where X_i are chosen as 2, 4, 6 and 8 while ϵ_{ij} follows a standard normal distribution. Furthermore, the coded (transformed) model presented in Eq. (2) can be expressed as:

$$Y_{ij} = B_0 + B_1 X'_i + \epsilon_{ij},$$

where $B_0 = 13 + 5\,(\beta\sigma)$, $B_1 = (2 + \beta\sigma)X'_i$ and X'_i are equals to -3, -1, 1 and 3.

B. SHIFTS FOR SIMPLE LINEAR PROFILE MODEL

For evaluating the performance of simple linear profile methods, we have considered several amounts of shifts in simple linear profile parameters which are given as follows:

- Shifts in intercept parameter (B_0 *to* $B_0 + \varphi\left(\sigma/\sqrt{n}\right)$),
- Shifts in slope parameter (β_1 *to* $\beta_1 + \beta\left(\sigma/\sqrt{S_{XX}}\right)$),
- Shifts in slope parameter (B_1 *to* $B_1 + \delta\left(\sigma/\sqrt{S_{XX}}\right)$),
- Shifts in error variance (σ^2 *to* $\gamma\sigma$),

TABLE 3. Charting constants of the assorted_3 chart at fixed $ARL_0 = 200$.

Case	k	λ	h_c	L_e	c_s
1		0.25	11.57075	3.461273	3.518018
2	0.25	0.38	11.57075	3.495503	3.518018
3		0.55	11.57075	3.511677	3.518018
4		0.25	6.421674	3.414323	3.473969
5	0.50	0.38	6.431839	3.452829	3.476706
6		0.55	6.431839	3.469855	3.476706
7		0.05	4.566855	3.182446	3.523721
8		0.13	4.439271	3.321801	3.472589
9	0.75	0.25	4.370697	3.383178	3.444825
10		0.38	4.370697	3.419843	3.444825
11		0.55	4.380133	3.441441	3.448658
12		0.05	3.446544	3.189418	3.529296
13	1.00	0.13	3.367723	3.338557	3.487365
14		0.25	3.281446	3.379014	3.440934
15		**0.05**	**2.722548**	**3.188036**	**3.528191**
16	**1.25**	0.13	2.65931	3.336629	3.485664
17		0.25	2.594751	3.379852	3.441717

where the size of shifts are quantified as: for intercept parameter: $\lambda = 0.2 - 2.0$ with jump of 0.2; for slope parameter: $\beta = 0.025 - 0.25$ with jump of 0.025; for slope parameter: $\delta = 0.2 - 1.0$ with jump of 0.1, and for error variance: $\gamma = 1.2 - 3.0$ with jump of 0.2. It is to be noted that $\varphi = \beta = \delta = 0$ and $\gamma = 1$ corresponds to an in-control (IC) situation; whereas $\varphi = \beta = \delta \neq 0$ and $\gamma \neq 1$ refers to an OOC situation.

C. PERFORMANCE MEASURES

Control charts performance is provided by using some useful performance measures which are briefly outlined as follows:

Average Run Length (ARL): The number of points until an OOC signal appeared is called run length (RL) and the average number of points until an OOC signal indicated is known by average run length (ARL). Further, ARL is observed under two known states, namely IC state and OOC state. The ARL under IC state is represented by ARL_0 while under the OOC state, it is referred to ARL_1. The objective of the maximized ARL_0 is to delay the false alarms as far as feasible while ARL_1 is required to be minimized to detect the signal at the earliest for OOC process.

Extra Quadratic Loss (EQL): The EQL is the weighted average RL with respect to a range of shifts (δ_{min} to δ_{max}). In this measure, a square of shift (δ^2) is considered as a weight. Mathematically, EQL is described as:

$$EQL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \delta^2 ARL(\delta)\, d\delta,$$

Sequential extra quadratic loss ($SEQL$): The $SEQL$ is the extended form of the EQL up to a particular shift (δ_i) and defined as follow:

$$SEQL_i = \frac{1}{\delta_i - \delta_{min}} \int_{\delta_{min}}^{\delta_i} \delta^2 ARL(\delta)\, d\delta; \quad i = 2, 3, \ldots\ldots, \delta_{max}.$$

Relative Average Run Length $RARL()$: The $RARL$ measure is used to address the efficacy of the control chart comparative to a benchmark control chart (cf. Wu, *et al.* [39]). The mathematical expression of the $RARL$ is defined as follow:

$$RARL = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \frac{ARL(\delta)}{ARL_{benchmark}(\delta)} d\delta,$$

where $ARL(\delta)$ and $ARL_{benchmark}(\delta)$ denotes the ARL at shift δ for a specific chart and benchmark chart, respectively. A chart with a least EQL is generally regarded as a benchmark

TABLE 4. ARL_1 and EQL of the proposed assorted_3 chart.

Case	ARL_1										EQL
	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00	
1	51.842	18.303	9.394	5.560	3.702	2.733	2.147	1.765	1.501	1.310	3.861
2	52.326	19.120	10.323	6.182	4.038	2.889	2.222	1.811	1.525	1.328	4.060
3	53.236	19.829	11.172	7.053	4.624	3.203	2.373	1.875	1.554	1.341	4.350
4	68.363	17.603	8.535	5.237	3.600	2.663	2.102	1.732	1.475	1.289	3.770
5	69.446	18.053	8.872	5.512	3.801	2.797	2.175	1.776	1.498	1.308	3.900
6	70.259	18.382	9.165	5.824	4.055	2.984	2.291	1.831	1.532	1.320	4.050
7	49.246	15.633	7.533	4.517	3.161	2.385	1.891	1.581	1.362	1.212	3.360
8	68.679	17.428	8.062	4.867	3.369	2.533	2.011	1.669	1.431	1.258	3.610
9	87.020	21.154	8.774	5.143	3.525	2.623	2.075	1.709	1.459	1.280	3.840
10	91.783	22.193	9.076	5.304	3.658	2.717	2.143	1.746	1.481	1.297	3.960
11	94.632	22.807	9.323	5.467	3.789	2.817	2.209	1.796	1.510	1.309	4.070
12	49.701	15.701	7.532	4.523	3.157	2.384	1.905	1.590	1.367	1.213	3.370
13	70.171	17.706	8.140	4.918	3.401	2.550	2.023	1.676	1.434	1.263	3.640
14	93.492	23.822	9.353	5.241	3.517	2.625	2.071	1.705	1.454	1.279	3.930
15	**48.717**	**14.696**	**7.337**	**4.511**	**3.154**	**2.383**	**1.905**	**1.585**	**1.367**	**1.215**	**3.340**
16	69.939	17.721	8.147	4.904	3.394	2.545	2.022	1.677	1.434	1.264	3.640
17	95.000	24.040	9.447	5.287	3.547	2.626	2.072	1.710	1.458	1.280	3.950

chart and have $RARL = 1$. All other charts have $RARL > 1$ indicates lower performance than the benchmark chart.

D. SENSITIVITY ANALYSIS OF ASSORTED_3 CHART

The choice of design parameters k and λ plays a vital role in the sensitivity of the proposed chart. To examine the sensitivity of the proposed method, we have considered 17 different cases of parameters against all type of shifts. The setting of parameters with respect to different type of shifts is reported in Table 2.

After the selection of different choices of sensitivity parameters, the next step is to find an optimal combination of the control limit coefficients (h_c, L_e, c_s). The adopted optimality criteria is discussed below:

Objective function: $\min(EQL)$

Subject to: $ARL_0 = \tau$, where τ is the pre-specified ARL_0.

such that $ARL_s = ARL_e = ARL_c$

where ARL_s, ARL_e and ARL_c refers to the ARL of the Shewhart, EWMA and CUSUM charts, respectively.

On the fixed overall $ARL_0 = 200$, control limit coefficients of the twelve individual charts are selected in such a way that all posses same individual ARL. The assumption of similar individual ARL is considered to avoid the redundancy of any single chart. Further, the resulting control charting constants (h_c, L_e, c_s) are provided in Table 3.

E. PERFORMANCE ANALYSIS OF ASSORTED_3 CONTROL CHART

The efficiency of the assorted_3 chart is assessed by using the ARL and EQL for different combinations of k, λ and φ. The outcomes are provided in Table 4 at fixed $ARL_0 = 200$. The result reveals the following findings:

- The assorted_3 chart is sensitive to the small, moderate and large shifts.
- Case 15 with sensitivity parameters such as $k = 1.25$ and $\lambda = 0.05$ and the charting constants ($h_c = 2.722548$, $L_e = 3.188036$, $c_s = 3.528191$) is an optimal choice, because it has minimum EQL equals to 3.340.

TABLE 5. Performance comparison under the shifts in intercept (B_0 to $B_0 + \varphi\sigma$).

Chart	Measure	φ									
		0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
assorted_3	ARL	48.70	14.68	**7.31**	**4.52**	3.16	**2.39**	**1.90**	**1.58**	**1.37**	**1.21**
	SEQL	0.97	1.56	1.87	2.09	2.28	2.45	2.61	2.77	2.93	**3.11**
	RARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	**1.00**
Shewhart_3	ARL	151.40	77.90	33.80	15.50	7.70	4.30	2.70	1.90	1.50	1.20
	SEQL	3.03	6.14	8.20	8.91	8.89	8.57	8.16	7.78	7.45	7.19
	RARL	2.05	3.13	3.74	3.81	3.64	3.39	3.13	2.90	2.71	2.54
Hotelling's T^2	ARL	137.70	63.50	28.00	13.20	6.90	4.00	2.60	1.80	1.50	1.20
	SEQL	2.75	5.29	6.90	7.49	7.53	7.33	7.06	6.78	6.55	6.38
	RARL	1.91	2.75	3.19	3.24	3.10	2.90	2.71	2.53	2.37	2.24
EWMA/R	ARL	66.50	17.70	8.40	5.40	3.90	3.20	2.70	2.30	2.10	1.90
	SEQL	1.33	2.04	2.33	2.56	2.78	3.03	3.30	3.59	3.90	4.23
	RARL	1.18	1.23	1.22	1.20	1.21	1.22	1.24	1.27	1.29	1.32
EWMA_3	ARL	59.10	16.20	7.90	5.10	3.80	3.10	2.60	2.30	2.10	1.90
	SEQL	1.18	1.83	2.13	2.36	2.59	2.85	3.13	3.42	3.75	4.09
	RARL	1.11	1.13	1.12	1.12	1.13	1.15	1.17	1.20	1.24	1.27
CUSUM_3	ARL	72.10	20.30	8.20	4.60	**3.10**	2.40	1.91	1.60	1.40	1.30
	SEQL	1.44	2.25	2.54	2.64	2.72	2.81	2.92	3.04	3.19	3.35
	RARL	1.24	1.34	1.31	1.25	1.20	1.16	1.14	1.12	1.11	1.11

- The sensitivity of assorted_3 chart rises with a reduction in λ at a specific choice of k, and it is valid for all k values.
- The sensitivity of assorted_3 chart upturns with a decline in k at a particular selection of λ and it is correct for all λ values.

V. COMPARATIVE ANALYSIS

In addition to *ARL* (used as performance measurement at a specific shift), other significant measures such as *EQL*, *SEQL* and *RARL* are also used to assess the general performance of the chart. The performance of all charts under consideration are discussed in the following subsections.

A. A SHIFT IN INTERCEPT OF TRANSFORMED MODEL

When the shift occurs in the intercept of the transformed model, the outcomes of distinct efficiency assessments are shown in Table 5. The findings show that:

- The assorted_3 chart is chosen as a benchmark chart due to minimum *EQL* (i.e., 3.11). The *EQL′s* for Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts are 7.19, 6.38, 4.23, 4.09 and 3.35, respectively.
- The *RARL* of the assorted_3 chart is equaled to 1 while the *RARL* for Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts are 2.54, 2.24, 1.32, 1.27 and 1.11, respectively. These results indicate that the assorted_3 chart's detection ability is greater than all other charts listed in this research.
- *SEQLs* behavior also demonstrate that the performance of assorted_3 chart is greater than others competing charts, over different amounts of shifts (cf. Table 5 and Figure 1(a)). For example, at $\varphi = 1.20$, the *SEQL* values for the assorted_3, Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts are 2.45, 8.57, 7.33, 3.03, 2.85 and 2.81 respectively.

B. A SHIFT IN SLOPE OF ORIGINAL MODEL

When shifts are introduced in the slope of the model given in Eq. (1). The findings of the assorted_3 chart and competing are provided in Table 6. The main results are as follows:

- The assorted_3 chart is selected as a benchmark chart due to minimum *EQL* (i.e., 0.096) while *EQL′s* are

TABLE 6. Performance comparison under the shifts in the slope of the original model (β_1 to $\beta_1 + \beta\sigma$).

Chart	Measure	β									
		0.025	**0.050**	**0.075**	**0.100**	**0.125**	**0.150**	**0.175**	**0.200**	**0.225**	**0.250**
assorted_3	ARL	90.84	31.11	15.53	9.509	**6.552**	**4.834**	**3.771**	**3.05**	**2.55**	**2.186**
	SEQL	0.028	0.048	0.059	0.067	0.074	0.079	0.084	0.088	0.092	**0.096**
	RARL	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	**1.000**
Shewhart_3	ARL	178.3	125.0	79.20	46.70	27.90	17.10	10.90	7.100	5.000	3.600
	SEQL	0.056	0.134	0.216	0.276	0.311	0.327	0.332	0.329	0.322	0.314
	RARL	1.481	2.236	3.010	3.509	3.724	3.753	3.676	3.542	3.387	3.228
Hotelling's T^2	ARL	166.0	105.6	60.70	34.50	20.10	12.20	7.800	5.200	3.700	2.700
	SEQL	0.052	0.118	0.179	0.220	0.242	0.251	0.252	0.248	0.243	0.236
	RARL	1.414	2.012	2.558	2.861	2.958	2.931	2.840	2.721	2.594	2.469
EWMA/R	ARL	119.0	43.90	19.80	11.30	7.700	5.800	4.700	3.900	3.400	3.000
	SEQL	0.037	0.065	0.080	0.088	0.094	0.099	0.104	0.110	0.116	0.123
	RARL	1.414	1.258	1.286	1.272	1.254	1.243	1.240	1.243	1.250	1.260
EWMA_3	ARL	101.6	36.5	17.00	10.30	7.200	5.500	4.500	3.800	3.300	2.900
	SEQL	0.032	0.055	0.068	0.075	0.082	0.088	0.094	0.100	0.107	0.114
	RARL	1.059	1.103	1.113	1.107	1.104	1.106	1.115	1.128	1.143	1.160
CUSUM_3	ARL	85.700	37.800	19.000	11.100	7.200	5.000	3.900	3.100	2.600	2.300
	SEQL	0.027	0.050	0.067	0.078	0.084	0.089	0.093	0.097	0.100	0.104
	RARL	0.972	1.025	1.090	1.116	1.120	1.111	1.100	1.090	1.082	1.078

TABLE 7. Performance comparison under the shifts in the slope of the transformed model (B_1 to $B_1 + \delta\sigma$).

Chart	Measure	δ								
		0.20	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**
assorted_3	ARL	12.1	**6.05**	**3.76**	**2.66**	**2.02**	**1.63**	**1.38**	**1.21**	**1.1**
	SEQL	0.24	0.33	0.39	0.44	0.48	0.52	0.56	0.6	**0.65**
	RARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Shewhart_3	ARL	64.29	25.29	11.08	5.42	3.06	2.03	1.49	1.24	1.10
	SEQL	1.28	1.66	1.75	1.72	1.63	1.55	1.47	1.42	1.38
	RARL	3.15	3.68	3.65	3.42	3.15	2.89	2.67	2.50	2.34
Hotelling's T^2	ARL	52.2	21.2	9.6	4.9	2.9	1.9	1.5	1.2	1.1
	SEQL	1.04	1.36	1.45	1.44	1.39	1.33	1.28	1.25	1.23
	RARL	2.66	3.07	3.06	2.89	2.68	2.48	2.31	2.17	2.05
EWMA/R	ARL	76.7	33.7	15.3	7.5	4.2	2.6	1.8	1.4	1.2
	SEQL	1.53	2.04	2.21	2.2	2.12	2.02	1.92	1.83	1.76
	RARL	3.67	4.43	4.53	4.31	4	3.69	3.41	3.17	2.96
EWMA_3	ARL	13.1	6.6	4.4	3.3	2.7	2.3	2.1	1.9	1.7
	SEQL	0.26	0.36	0.43	0.5	0.57	0.63	0.71	0.79	0.87
	RARL	1.04	1.06	1.07	1.1	1.13	1.17	1.2	1.24	1.27
CUSUM_3	ARL	12.4	7.9	5.8	4.6	3.8	3.3	2.9	2.6	2.4
	SEQL	0.26	0.37	0.48	0.59	0.7	0.82	0.93	1.05	1.17
	RARL	1.01	1.06	1.15	1.25	1.34	1.43	1.51	1.58	1.64

reported as 0.314, 0.236, 0.123, 0.114 and 0.104 for the Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts, respectively.

- All competing charts (i.e., Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts) have *RARL* greater than 1 (i.e., 3.228, 2.469, 1.26,

TABLE 8. Performance comparison under the shifts in error variance (σ to $\gamma\sigma$).

Chart	Measure	γ									
		1.20	**1.40**	**1.60**	**1.80**	**2.00**	**2.20**	**2.40**	**2.60**	**2.80**	**3.00**
assorted_3	ARL	26.90	**8.84**	**4.70**	**3.11**	**2.37**	**1.95**	**1.69**	1.52	**1.39**	1.31
	SEQL	119.37	73.69	54.02	43.28	36.58	32.06	28.85	26.49	24.72	**23.38**
	RARL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	**1.00**
Shewhart_3	ARL	40.10	13.50	6.50	4.00	2.80	2.20	1.80	1.60	1.50	1.40
	SEQL	128.87	85.49	64.17	51.83	43.88	38.39	34.41	31.43	29.19	27.49
	RARL	1.25	1.38	1.40	1.39	1.36	1.32	1.29	1.26	1.24	1.22
Hotelling's T^2	ARL	39.60	14.90	7.90	5.10	3.80	3.00	2.50	2.20	2.00	1.80
	SEQL	128.51	85.81	65.45	53.68	46.12	40.91	37.13	34.32	32.20	30.57
	RARL	1.24	1.41	1.50	1.54	1.56	1.56	1.55	1.54	1.53	1.52
EWMA/R	ARL	34.30	12.00	6.10	3.90	2.90	2.30	1.90	1.70	1.50	1.40
	SEQL	124.70	80.58	60.24	48.71	41.39	36.39	32.77	30.07	28.02	26.44
	RARL	1.14	1.23	1.26	1.26	1.25	1.25	1.24	1.22	1.21	1.19
EWMA_3	ARL	33.50	12.70	7.20	5.10	3.90	3.20	2.80	2.50	2.30	2.10
	SEQL	124.12	80.34	60.78	49.96	43.18	38.57	35.32	32.97	31.25	29.97
	RARL	1.12	1.23	1.32	1.38	1.44	1.47	1.50	1.52	1.53	1.54
CUSUM_3	ARL	31.20	9.40	4.80	3.20	2.40	2.00	1.70	**1.50**	1.40	**1.30**
	SEQL	122.46	77.07	56.49	45.20	38.16	33.40	30.02	27.51	25.63	24.20
	RARL	1.08	1.10	1.08	1.07	1.06	1.05	1.05	1.04	1.03	1.03

1.16 and 1.078), which shows the superiority of assorted_3 chart.

- The assorted_3 chart performed well for the detection of moderate to large shifts. For instance, at $\beta = 0.125$, the ARL_1 of assorted_3 and competing charts namely Shewhart_3, Hotelling's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts are 6.552, 27.90, 20.1, 7.7, 7.2 and 7.2, respectively. Moreover, *SEQL* are presented in Figure 1(b), which reveals that the assorted_3 chart has superior performance as compared to all other charts.

C. A SHIFT IN SLOPE OF TRANSFORMED MODEL

For the evaluation of simple linear profile methods, shifts are introduced in the slope of the transformed model given in Eq. (2). The findings are reported in Table 7, and the main results reveal:

- The detection ability of the assorted_3 chart at small, moderate and large shifts is better than Shewhart_3, Hotellin's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts. The assorted_3 chart has the lowest EQL, which equals to 0.65. The *EQLs* of the Shewhart_3, Hotellin's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts are reported as 1.38, 1.23, 1.76, 0.87 and 1.17, respectively.
- The assorted_3 chart is considered a benchmark chart, so it's RARL is equals to 1. All other charts (i.e., Shewhart_3, Hotellin's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts) have RARL greater than 1 (i.e., 2.34, 2.05, 2.96, 1.27, 1.64 and 1.21), which shows their inferiority to detect shifts in the slope of the transformed model.
- The graphical representation of *SEQL* measure is plotted in Figure 1(c), and results reveal that the assorted_3 chart has superior performance as compared to all other charts.

D. A SHIFT IN ERROR VARIANCE OF ORIGINAL MODEL

The findings of the shifts in the error variance parameter are reported in Table 8. The notable outcomes are:

- The detection ability of assorted_3 chart at small and moderate shifts is significantly better among all other charts. For instance, at $\gamma = 1.40$, the *ARL* values of assorted_3, Shewhart_3, Hotellin's T^2, EWMA/R, EWMA_3 and CUSUM_3 charts were 8.84, 13.50, 14.90, 12.00, 12.70 and 9.40, respectively.
- Since assorted_3 chart has lowest *EQL*, which equals to 23.38. Therefore, it is considered as a benchmark chart. The $EQL's$ of the Shewhart_3, Hotellin's T^2, EWMA/R, EWMA_3 and CUSUM_3 chart are reported as 27.49, 30.57, 26.44, 29.97 and 24.94.
- The *RARL* of the assorted_3 chart is equaled to 1 while other competing charts have RARL greater than 1. The second-best chart is the CUSUM_3 with $RARL = 1.03$.
- The *SEQL* values of all charts against shifts in error variance are drawn in Figure 1(d). The assorted_3 chart

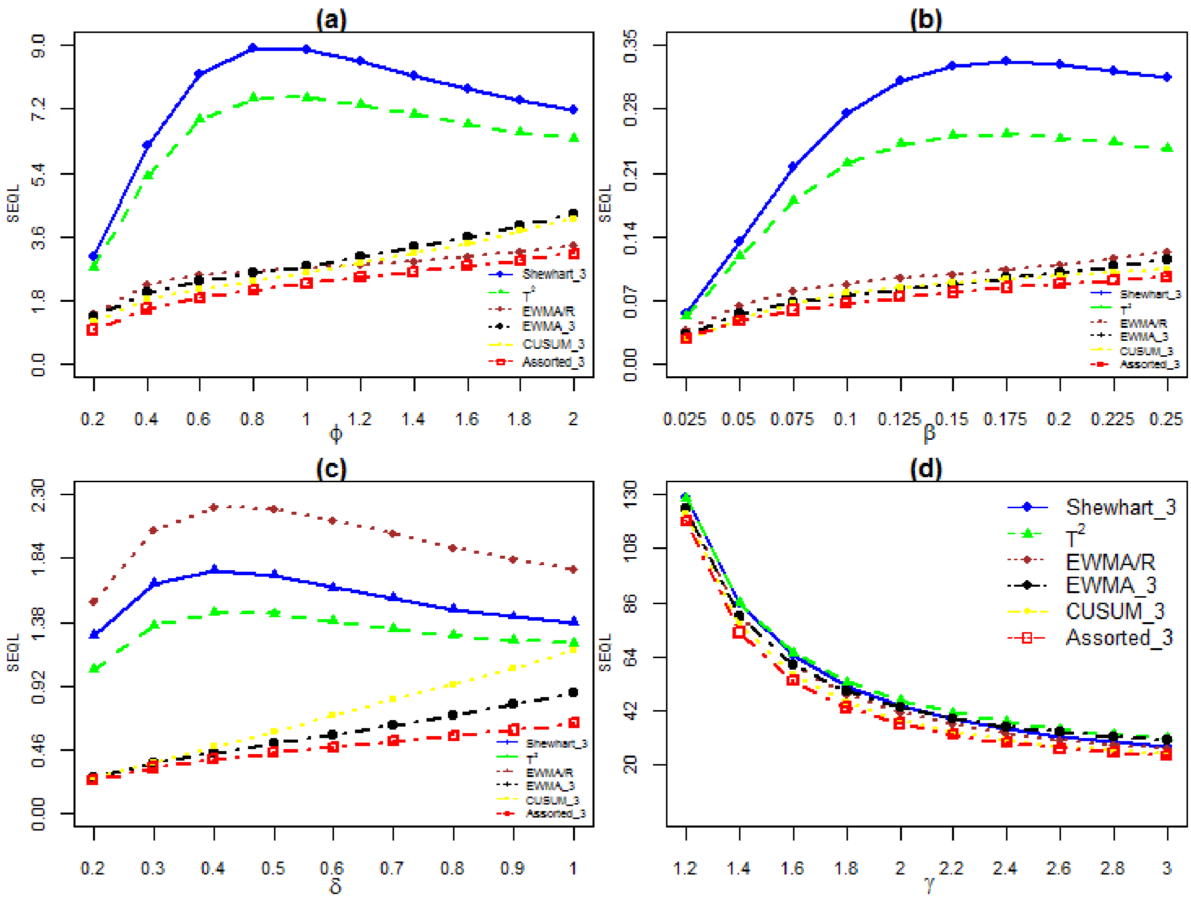

FIGURE 1. SEQL comparison: (a) shift in the intercept of the transformed model; (b) shift in the slope of original model (c) shift in the slope of the transformed model; (d) shift in error variance of original model.

showed superior performance as compared to all other charts at small, moderate and large shifts in the error variance of the original model.

VI. MONITORING THERMAL MANAGEMENT OF DIAMOND-COPPER COMPOSITES

Thermal management of high-performance electronic devices is the key to their efficient and continued working. The average size of electronic devices is decreasing day by day with the decreasing size of the transistor. Each electronic process produces waste heat in the component.

High thermal conductivity metals like copper, silver or aluminum (for copper ∼ 400 W/mK) seems a good solution for the substrate material. The thermal expansion coefficient of electronic devices has low value (for silicon ∼ 5 m/mK) while that of the mentioned metals is very high in comparison (for copper ∼ 16 m/mK). Ceramic materials are generally very low in their thermal expansion coefficient; for example, diamond has a very high thermal conductivity of 2000 W/mK with a thermal expansion coefficient of only ∼ 2 m/mK. A composite of diamond particles and copper metal may produce a combination of high thermal conductivity and a thermal expansion coefficient comparable to that of electronic devices. The effective thermal conductivity and thermal expansion coefficients are mainly affected by the volume fraction of diamond and the densification of the composite. Densification is the ratio of actual density to the theoretical density of the composite sample.

A. DATA DESCRIPTION

In this study, diamond-copper composites were produced by conventional sintering route. The pressure of cold compaction (PCC) is an important parameter which affects the final properties of the composite. The composite samples were sintered following the same sintering cycle. The volume fraction of diamond particles was 10%, and the sintering was carried out at 900 °C for 2 hours in a vacuum environment. The only independent variable was PCC. The composite samples were cold compacted at five different levels of pressure, i.e. 425, 450, 475, 500 and 525 MPa. The dependent variable was the densification of the diamond-copper composite. The densification was measured 24 times by an apparatus based on Archimedes' principle.

B. IMPLEMENTATION OF ASSORTED_3 CHART

In this study, we have considered the explanatory variable (PCC) and its values are fixed as (X = 425, 450, 475, 500 and 525) while the densification (Y) is

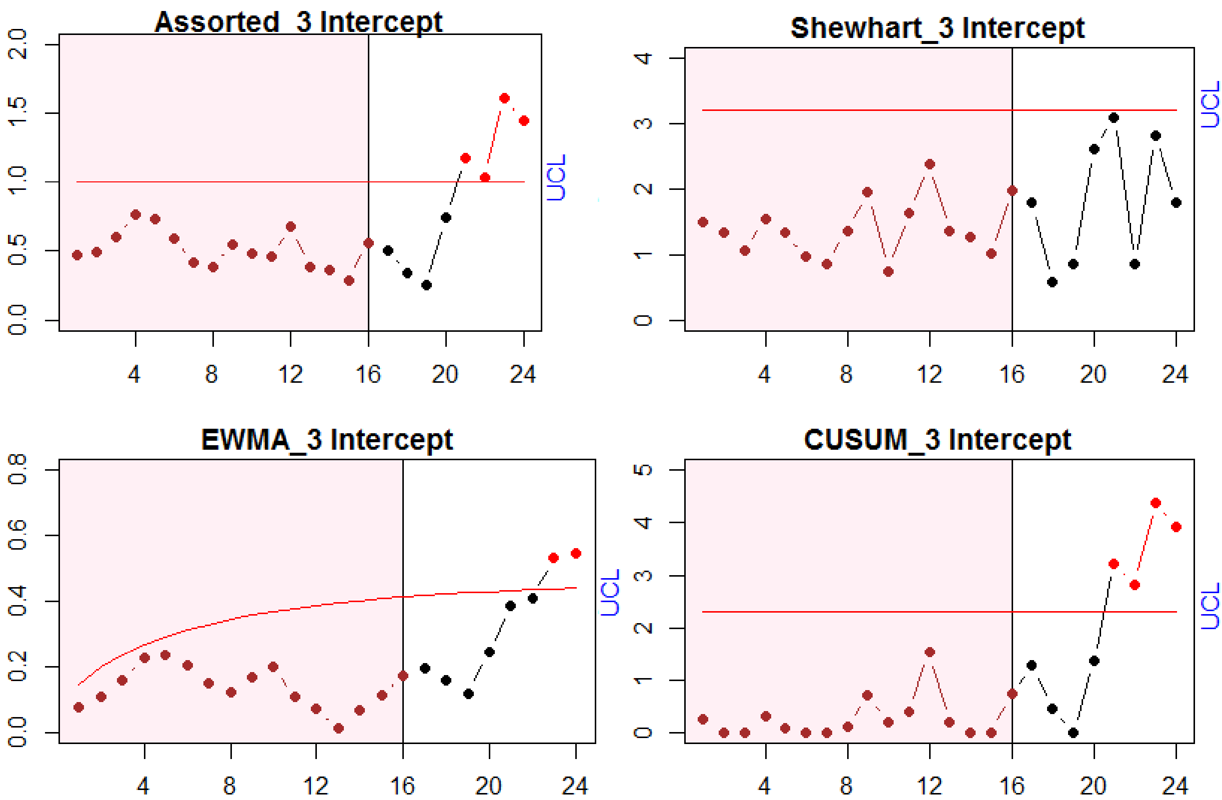

FIGURE 2. (a) shifted intercept of Assorted_3; (b) shifted intercept of Shewhat_3; (c) shifted intercept of EWMA_3; (d) shifted intercept of CUSUM_3.

considered as a predictor variable. The implementation of the assorted _3 chart needs the following steps:

Step 1: We have a complete set of 120 observations (i.e., 24 profiles). The IC regression model centered on 24 profiles is expressed as:

$$Y = 77.999 + 0.0297X + \varepsilon. \quad \text{(original model)}$$

Step 2: In addition, to obtain the coded model, we converted X into X' by using $X' = X - \bar{X}$,

$$X' = -50, -25, 0, 25, 50$$

and the transformed model is represented as,

$$Y = 91.518 + 0.0297X' + \varepsilon. \quad \text{(transformed model)}$$

Step 3: The charting constants are chosen for the Assorted_3, Shewhart_3, EWMA_3 and CUSUM_3 charts were provided below:

$$\text{For Assorted\_3:} \quad \begin{cases} k = 1.25, \lambda = 0.05 \\ h_c = 2.7225 \\ L_e = 3.1880 \\ c_s = 3.5281 \\ UCL = 1, \end{cases}$$

$$\text{For Shewhart\_3:} \quad \begin{cases} c_s = 3.215 \\ UCL = 3.215 \end{cases}$$

$$\text{For EWMA\_3:} \quad \begin{cases} k = 1.25, \lambda = 0.05 \\ L_e = 2.873 \\ UCL = 0.44 \end{cases}$$

$$\text{For CUSUM\_3:} \quad \begin{cases} k = 1.25, \lambda = 0.05 \\ h_c = 2.302 \\ UCL = 2.302 \end{cases}$$

Step 4: The proposed statistics for intercept, slope and error variance are plotted against their upper control limit.

Step 5: A shift is noted in the densification of the diamond-copper composite due to raising in the PCC after 16$^{\text{th}}$ sample profile. We will evaluate the performance of our proposed assorted_3 chart versus CUSUM_3, EWMA_3 and Shewhart_3 charts in Figures 2-4, respectively. The summary of the detection ability of these charts is presented in Table 9. The results reveal that the assorted_3 chart has superior detection ability to monitor simple linear profile parameters.

It is obvious from the detection ability that the Shewhat_3 and EWMA_3 charts are the lowest effective charts.

The detection ability of Assorted_3 chart is best among the other counterpart charts. This order of superiority refers in terms of shift amount in the process. Since the purpose of the assorted_3 chart is to detect any amount of shift (small, moderate and large) in the process, the detection of OOC situations requires precedence over other charts.

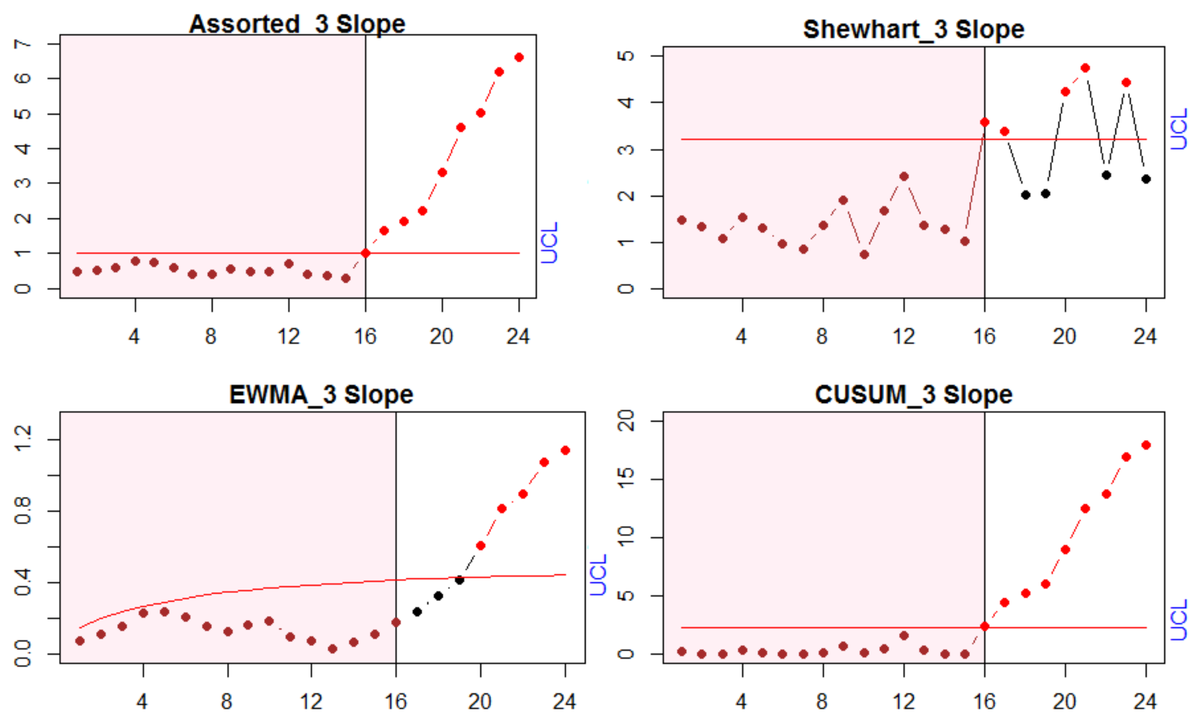

FIGURE 3. (a) shifted slope of Assorted_3; (b) shifted slope of Shewhat_3; (c) shifted slope of EWMA_3; (d) shifted slope of CUSUM_3.

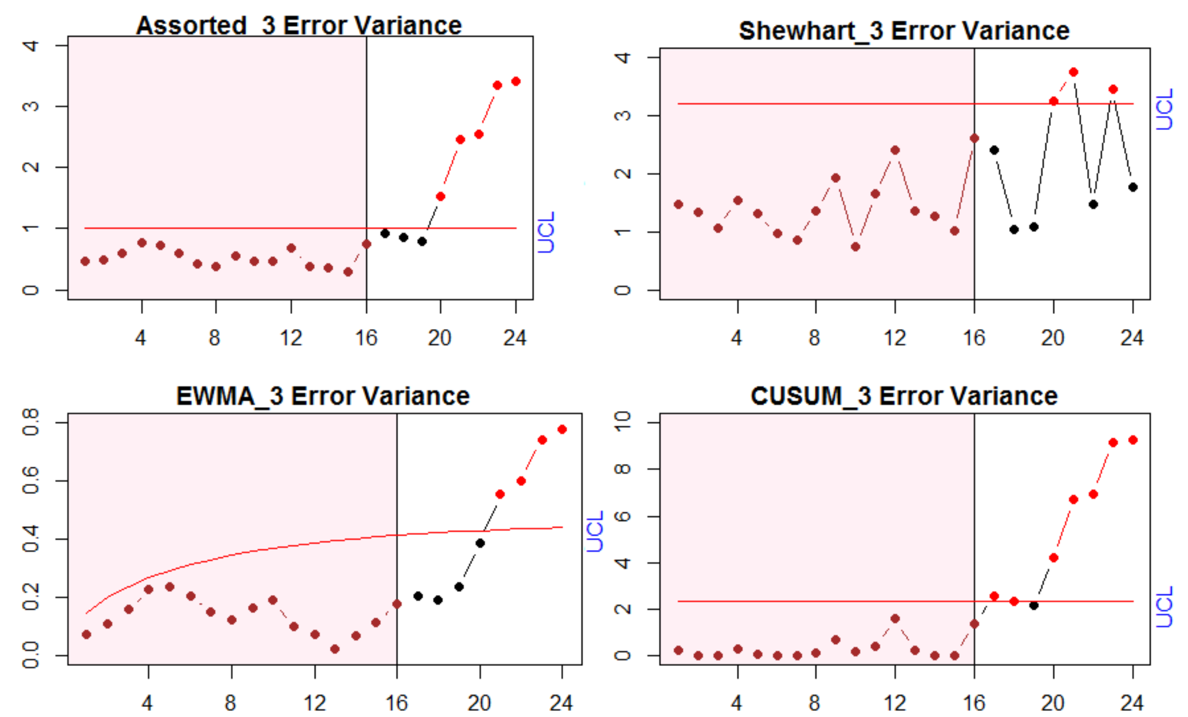

FIGURE 4. (a) shifted error variance of Assorted_3; (b) shifted error variance of Shewhart_3 (c) shifted error variance EWMA_3; (d) shifted error variance of CUSUM_3.

The diamond-copper composite is portrayed in Figure 5; (a) at 500 PCC while when PCC is increased, we can observe a blister on the diamond-copper composite in Figure 5 (b). Further, this blister is investigated by scanning electron microscopy (SEM) (cf. Figure 5 (c and d)) (cf. [38]).

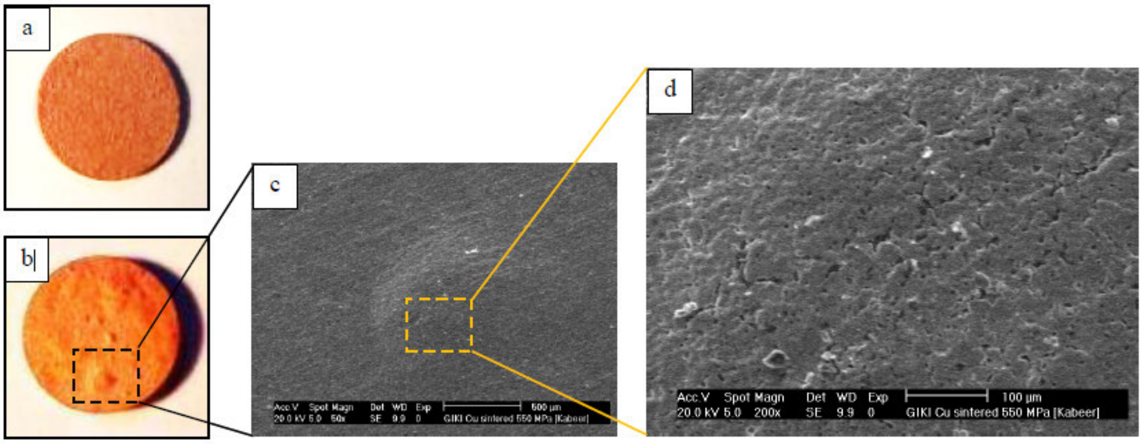

FIGURE 5. Blister investigation by SEM.

TABLE 9. Detection summary.

Control Charts	Intercept		Slope		Error Variance	
	False Alarms	OOC	False Alarms	OOC	False Alarms	OOC
Assorted_3	0	4	0	9	0	5
Shewhart_3	0	0	4	5	0	0
EWMA_3	0	2	0	5	0	0
CUSUM_3	0	4	0	9	1	7

VII. SUMMARY AND CONCLUSIONS

Monitoring methods based on simple linear profiles is an emerging area within SPC. Many control charting structures are available in the literature to monitor slope, intercept and error variance such as the Shewhart_3, Hotelling's T^2, EWMA_3, EWMA/R and CUSUM_3 charts. We have proposed a new assorted_3 approach for the monitoring of simple linear profile parameters in a single control charting setup. Using the performance measures such as *ARL*, *EQL*, *SEQL* and *RARL*, we have assessed and compared the efficiency of the proposed assorted_3 chart with some current equivalent existing counterpart charts.

Thorough performance analysis showed that the proposed assorted_3 chart is sensitive to monitor simple linear profile parameters at varying shift amounts. The performance of assorted_3 chart at $k = 1.25$ and $\lambda = 0.05$ is ideal in aspects of distinct run length properties. The *RARL* of the competing charts are greater than 1 which shows that the assorted_3 chart has better detection ability as compared to the Shewhart_3, Hotelling's T^2, EWMA_3, EWMA/R and CUSUM_3 charts. Furthermore, the *SEQL* is calculated to explore the efficiency of said charts at different amounts of shifts, and it also supports that the assorted_3 chart has outperformed all other charts. For thermal conductivity process, real implementation of the proposed and other competing charts is also provided. The real-life application supports the results in favor of our proposed assorted_3 technique to monitor simple linear profile parameters. It is noted that the scope of this research may be extended, using assorted approach, to monitor all kinds of shifts in non-linear and multivariate profiles.

REFERENCES

[1] B. J. Mandel, "The regression control chart," *J. Qual. Technol.*, vol. 1, no. 1, pp. 1–9, Jan. 1969.
[2] D. M. Hawkins, "Multivariate quality control based on regression-adiusted variables," *Technometrics*, vol. 33, no. 1, pp. 61–75, Feb. 1991.
[3] D. M. Hawkins, "Regression adjustment for variables in multivariate quality control," *J. Qual. Technol.*, vol. 25, no. 3, pp. 170–182, Jul. 1993.
[4] M. R. Wade and W. H. Woodall, "A review and analysis of cause-selecting control charts," *J. Qual. Technol.*, vol. 25, no. 3, pp. 161–169, Jul. 1993.
[5] D. J. Hauck, G. C. Runger, and D. C. Montgomery, "Multivariate statistical process monitoring and diagnosis with grouped regression-adjusted variables," *Commun. Statist. Simul. Comput.*, vol. 28, no. 2, pp. 309–328, Jan. 1999.
[6] L. Kang and S. L. Albin, "On-line monitoring when the process yields a linear profile," *J. Qual. Technol.*, vol. 32, no. 4, pp. 418–426, Oct. 2000.
[7] S. Gupta, D. C. Montgomery, and W. H. Woodall, "Performance evaluation of two methods for online monitoring of linear calibration profiles," *Int. J. Prod. Res.*, vol. 44, no. 10, pp. 1927–1942, May 2006.
[8] M. A. Mahmoud and W. H. Woodall, "Phase I analysis of linear profiles with calibration applications," *Technometrics*, vol. 46, no. 4, pp. 380–391, Nov. 2004.
[9] A. Yeh and Y. Zerehsaz, "Phase I control of simple linear profiles with individual observations," *Qual. Rel. Eng. Int.*, vol. 29, no. 6, pp. 829–840, Oct. 2013.
[10] R. Noorossana, A. Amiri, A. Vaghefi, and E. Roghanian, "Monitoring quality characteristics using linear profile," in *Proc. 3rd Int. Ind. Eng. Conf.*, 2004, pp. 246–255.
[11] R. Noorossana, S. Vaghefi, and A. Amiri, "The effect of non-normality on monitoring linear profiles," in *Proc. 2nd Int. Ind. Eng. Conf.*, Riyadh, Saudi Arabia, 2004, doi: 10.1109/IEEM.2008.4737871.

[12] C. Zou, Y. Zhang, and Z. Wang, "A control chart based on a change-point model for monitoring linear profiles," *IIE Trans.*, vol. 38, no. 12, pp. 1093–1103, Dec. 2006.

[13] M. A. Mahmoud, P. A. Parker, W. H. Woodall, and D. M. Hawkins, "A change point method for linear profile data," *Qual. Rel. Eng. Int.*, vol. 23, no. 2, pp. 247–268, 2007.

[14] K. Kim, M. A. Mahmoud, and W. H. Woodall, "On the monitoring of linear profiles," *J. Qual. Technol.*, vol. 35, no. 3, pp. 317–328, 2003.

[15] A. Saghaei, M. Mehrjoo, and A. Amiri, "A CUSUM-based method for monitoring simple linear profiles," *Int. J. Adv. Manuf. Technol.*, vol. 45, nos. 11–12, pp. 1252–1260, Dec. 2009.

[16] R. Noorossana and A. Amiri, "Enhancement of linear profiles monitoring in phase II," *Amirkabir J. Sci. Technol.*, vol. 48, no. 66-B, pp. 19–27, 2007.

[17] W. H. Woodall, "Current research on profile monitoring," *Production*, vol. 17, no. 3, pp. 420–425, Dec. 2007.

[18] C. Zou, C. Zhou, Z. Wang, and F. Tsung, "A self-starting control chart for linear profiles," *J. Qual. Technol.*, vol. 39, no. 4, pp. 364–375, Oct. 2007.

[19] W. A. Jensen, J. B. Birch, and W. H. Woodall, "Monitoring correlation within linear profiles using mixed models," *J. Qual. Technol.*, vol. 40, no. 2, pp. 167–183, Apr. 2008.

[20] P. Soleimani, R. Noorossana, and A. Amiri, "Simple linear profiles monitoring in the presence of within profile autocorrelation," *Comput. Ind. Eng.*, vol. 57, no. 3, pp. 1015–1021, Oct. 2009.

[21] J. Zhang, Z. Li, and Z. Wang, "Control chart based on likelihood ratio for monitoring linear profiles," *Comput. Statist. Data Anal.*, vol. 53, no. 4, pp. 1440–1448, Feb. 2009.

[22] R. Noorossana, S. A. Fatemi, and Y. Zerehsaz, "Phase II monitoring of simple linear profiles with random explanatory variables," *Int. J. Adv. Manuf. Technol.*, vol. 76, nos. 5–8, pp. 779–787, Feb. 2015.

[23] T. Mahmood, M. Riaz, M. H. Omar, and M. Xie, "Alternative methods for the simultaneous monitoring of simple linear profile parameters," *Int. J. Adv. Manuf. Technol.*, vol. 97, nos. 5–8, pp. 2851–2871, Jul. 2018, doi: 10.1007/s00170-018-2149-9.

[24] U. Saeed, T. Mahmood, M. Riaz, and N. Abbas, "Simultaneous monitoring of linear profile parameters under progressive setup," *Comput. Ind. Eng.*, vol. 125, pp. 434–450, Nov. 2018, doi: 10.1016/j.cie.2018.09.013.

[25] M. Riaz and F. Touqeer, "On the performance of linear profile methodologies under runs rules schemes," *Qual. Rel. Eng. Int.*, vol. 31, no. 8, pp. 1473–1482, Dec. 2015.

[26] T. Abbas, T. Mahmood, M. Riaz, and M. Abid, "Improved linear profiling methods under classical and Bayesian setups: An application to chemical gas sensors," *Chemometric Intell. Lab. Syst.*, vol. 196, Jan. 2020, Art. no. 103908.

[27] M. Riaz, T. Mahmood, S. A. Abbasi, N. Abbas, and S. Ahmad, "Linear profile monitoring using EWMA structure under ranked set schemes," *Int. J. Adv. Manuf. Technol.*, vol. 91, nos. 5–8, pp. 2751–2775, Jul. 2017.

[28] M. Riaz, T. Mahmood, N. Abbas, and S. A. Abbasi, "On improved monitoring of linear profiles under modified successive sampling," *Qual. Rel. Eng. Int.*, vol. 35, pp. 2202–2227, May 2019, doi: 10.1002/qre.2498.

[29] T. Abbas, F. Rafique, T. Mahmood, and M. Riaz, "Efficient phase II monitoring methods for linear profiles under the random effect model," *IEEE Access*, vol. 7, pp. 148278–148296, 2019.

[30] T. Mahmood, S. A. Abbasi, M. Riaz, and N. Abbas, "An efficient phase I analysis of linear profiles with application in photo-voltaic system," *Arabian J. Sci. Eng.*, vol. 44, no. 3, pp. 2699–2716, Mar. 2019.

[31] S. A. Abbasi, T. Abbas, M. Riaz, and A.-S. Gomaa, "Bayesian monitoring of linear profiles using DEWMA control structures with random X," *IEEE Access*, vol. 6, pp. 78370–78385, 2018.

[32] T. Abbas, Z. Qian, S. Ahmad, and M. Riaz, "Bayesian monitoring of linear profile monitoring using DEWMA charts," *Qual. Rel. Eng. Int.*, vol. 33, no. 8, pp. 1783–1812, Dec. 2017.

[33] T. Abbas, S. Ahmad, M. Riaz, and Z. Qian, "A Bayesian way of monitoring the linear profiles using CUSUM control charts," *Commun. Statist. Simul. Comput.*, vol. 48, no. 1, pp. 126–149, Jan. 2019.

[34] T. Abbas, S. A. Abbasi, M. Riaz, and Z. Qian, "Phase II monitoring of linear profiles with random explanatory variable under Bayesian framework," *Comput. Ind. Eng.*, vol. 127, pp. 1115–1129, Jan. 2019.

[35] M. Faisal, R. F. Zafar, N. Abbas, M. Riaz, and T. Mahmood, "A modified CUSUM control chart for monitoring industrial processes," *Qual. Rel. Eng. Int.*, vol. 34, no. 6, pp. 1045–1058, Oct. 2018.

[36] N. Abbas, U. Saeed, and M. Riaz, "Assorted control charts: An efficient statistical approach to monitor pH values in ecotoxicology lab," *J. Chemometrics*, vol. 33, no. 6, p. e3129, Jun. 2019.

[37] D. C. Montgomery, *Introduction to Statistical Quality Control*, 6th ed. New York, NY, USA: Wiley, 2009.

[38] S. V. Crowder and M. D. Hamilton, "An EWMA for monitoring a process standard deviation," *J. Qual. Technol.*, vol. 24, no. 1, pp. 12–21, Jan. 1992.

[39] Z. Wu, S. H. Yeo, and H. Fan, "A comparative study of the CRL-type control charts," *Qual. Rel. Eng. Int.*, vol. 16, no. 4, pp. 269–279, 2000.

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