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Measuring the fall of antihydrogen: the AEgIS experiment at CERN

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Abstract

Considerable efforts have been made and are still being made to verify the validity of the principle of the equivalence of gravitational and inertial mass, one of the cornerstones of the classical theory of general relativity. Specific attempts at quantum-mechanical formulations of gravity allow for non-Newtonian contributions, which might lead to a difference in the gravitational force on matter and antimatter. While it is widely expected that the gravitational interaction is independent of the composition of bodies, this has only been tested for matter systems, but never yet for antimatter systems. By combining techniques from different fields, and relying on recent developments in the production of Positronium and ongoing work to laser-excite Positronium to Rydberg states, such a test with neutral antimatter has become feasible. The primary goal of the AEGIS experiment being built at the Antiproton Decelerator at CERN is to carry out the first direct measurement of the Earth`s gravitational acceleration on antihydrogen by means of a classical Moiré deflectometer.

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Keywords: AEGIS; Antihydrogen; Gravity

1. Main text

A hypothetical quantum theory of gravitation necessarily constitutes a departure from the Einsteinian view of gravity as a geometric phenomenon and could potentially constitute a violation of the weak equivalence principle. Attempts at formulating a quantum theory of gravity have mainly been hampered by the fact that such a theory is non-renormalizable, though a renormalization within the framework of an effective field theory may turn out to be feasible [1]. As in other quantum field theories, the interaction is mediated by exchange particles. The spins of these exchange bosons as well as the signs of the charges to which they couple determine whether a force is repulsive or attractive. Modern theories of gravity that attempt to unify gravity with the other forces of nature allow that, at least in principle, antimatter may fall differently from normal matter in the Earth`s gravitational field. Specifically, as pointed out by Sherk [2], N=2,..,8 theories of supergravity (where N is the number of supersymmetries) lead to the possibility of (massive, and thus finite range) Kaluza-Klein graviscalar and gravivector components to the gravitational interaction which may lead to different couplings for matter and antimatter.

While it is possible to constrain deviations from standard gravity through e.g. effects of virtual antiparticles in ordinary matter, most of the quantitative limits [3-5] do not apply to more elaborate models involving vector and scalar gravitons [6]. If the hypothetical vector and scalar charges, as well as their masses (and thus the ranges of the interactions) are carefully chosen, such contributions can be strongly suppressed in ordinary matter. The most stringent limits on such additional components come from an analysis of the Eöt-Wash experiment [7], an Eötvös-type torsion-balance experiment using test bodies of different compositions. Unlike ordinary matter [8], the behavior of antimatter particles in a gravitational field has never been tested experimentally. Two attempts, at Stanford [9] and CERN`s Low-Energy Antiproton Ring [10] were thwarted by the overwhelming effect of stray electric and magnetic fields upon the electrically charged test particles. The recent production of copious amounts of cold antihydrogen at CERN`s Antiproton Decelerator (AD) [11,12] has paved the way for high-precision gravity experiments with neutral antimatter. We have proposed the AEGIS experiment (Antimatter Experiment: Gravity, Interferometry, Spectroscopy), to be performed at CERN/AD, in order to address this important question [13].

The primary scientific goal of AEGIS is the direct measurement of the Earth`s local gravitational acceleration $\boldsymbol{g}$ on antihydrogen. In a first phase of the experiment, a gravity measurement with 1% relative precision will be carried out by observing the vertical displacement (using a high-resolution position sensitive detector) of the shadow image produced by an antihydrogen beam, formed by its passage though a Moiré deflectometer, the classical counterpart of a matter wave interferometer. This measurement will represent the first direct determination of the effect of gravity on antimatter.

The essential steps leading to the production of a pulsed cold beam of antihydrogen and the measurement of ***g*** with AEGIS are the following (Fig. 1) production of positrons (e$^+$) from a Surko-type source and accumulator; ii) capture and accumulation of antiprotons from the AD in a cylindrical Penning trap; iii) cooling of the antiprotons to sub-K temperatures; iv) production of positronium (Ps) by bombardment of a cryogenic nanoporous material with an intense e$^+$ pulse; v) excitation of the Ps to a Rydberg state with principal quantum number $n = 30 \sim 40$; vi) pulsed formation of antihydrogen by resonant charge exchange between Rydberg Ps and cold antiprotons; vii) pulsed formation of an antihydrogen beam by Stark acceleration with inhomogeneous electric fields; viii) determination of ***g*** in a two-grating Moiré deflectometer coupled with a position-sensitive detector

The feasibility of the first two points has been conclusively demonstrated by the ATHENA and ATRAP collaborations (see, in particular [14,15]). In the following, we will discuss the remaining aspects of the proposed technique in more detail.

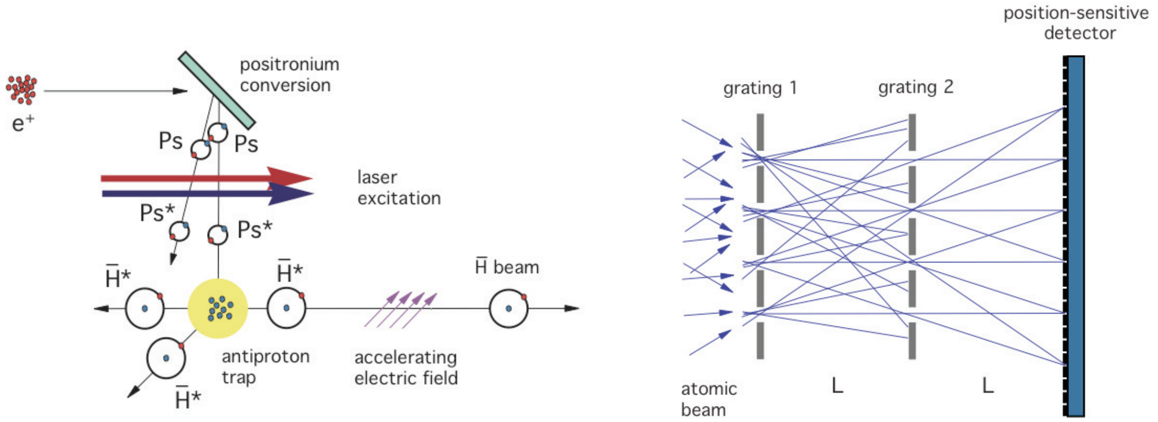

Figure 1. Proposed method for the production of a pulsed beam of cold antihydrogen atoms. Left side: formation and acceleration of antihydrogen atoms. Right side: sketch of the functionality of the Moiré deflectometer which produces a periodic pattern of impact points of the atoms in the beam of antihydrogen at the detector plane.

2. Method

Positronium production and excitation

In recent years, the potential for nanoporous insulator materials to be used as highly efficient Ps converters has been recognized, and the relevant formation mechanisms have been studied extensively [16]. A strategy to obtain Ps in vacuum is using porous materials with pores open to the surface. Porous materials are necessary not only to form a high yield of Ps atoms but also to cool Ps through collisions with the inner walls of the pores. When e$^+$ are implanted into such a material at kinetic energies ranging from several 100 eV to a few keV, they scatter off atoms and electrons in the bulk and are slowed to eV energies within a few ps. With efficiencies ranging from 10% to 50%, the slow e$^+$ capture either bound e$^-$ or those liberated in prior collisions and form Ps. In the pores, Ps repeatedly bounces off the cavity walls and eventually approaches complete thermalization with the target material. The formation and cooling of Ps has been extensively discussed in [17].

While some ortho-Ps are lost due to so-called pick-off annihilations of e$^+$ with the molecular e$^-$ of the cavity walls, a sizable fraction diffuses out of the film at thermal energies. The overall ortho-Ps yield, as well as the final velocity distribution, depends upon the characteristics of the target material (in particular, its pore structure), the implantation depth, and the target temperature. For the AEGIS experiment, the precise degree of thermalization is critical, since at too low a Ps temperature, annihilation in the target material will dominate, while at too high a Ps temperature, the subsequent charge exchange cross section (to form antihydrogen) drops rapidly. The velocity distribution of the Ps atoms coming out of the target should be the order of 10^4 m·s^{-1} to allow Ps laser excitation to a Rydberg state (Ps*) and for efficient antihydrogen formation, which requires that the relative velocity of antiprotons and Ps* be comparable to the classical orbital velocity of the positron in the Rydberg Ps atom. By carefully tailoring the topology of the target material's pores, a degree of control of the temperature of the Ps emitted into vacuum appears possible [18].

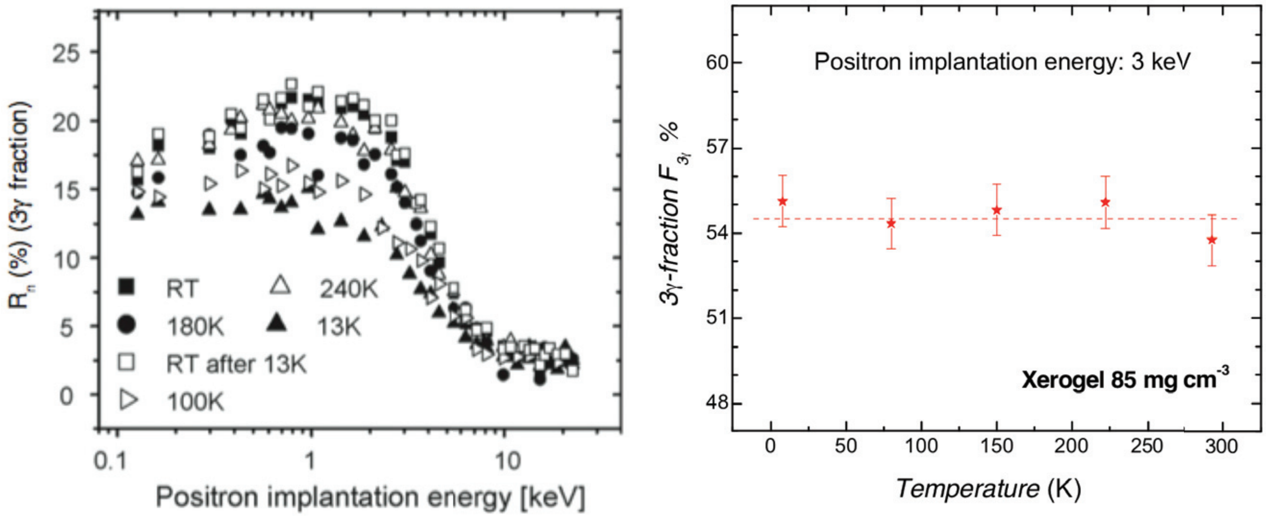

Figure 2. Left: ortho-Ps fraction as a function of implantation energy for different target materials and temperatures. Right: Ps yield as a function of the temperature in a Xerogel sample.

Measurements using positron annihilation lifetime spectroscopy have shown that the ortho-Ps fraction (outside the sample) can exceed 20% in silicon-based polymer materials cooled to 50 K [19]. This is shown in Fig. 2, where the ortho-Ps fraction is displayed as a function of implantation energy for different target materials and temperatures. In other experimental work, it was shown that the energy profile of Ps emitted from the surface of a silica film at room temperature followed a Maxwell-Boltzmann distribution and was compatible with the Ps being fully thermalized [20]. We are currently conducting experiments in order to determine the optimal converter material and e^+ energy in terms of ortho-Ps yield. Furthermore, we are investigating how well the emitted Ps is thermalized at very low target temperatures, as the AEgGIS experiment requires a cryogenic temperature (~100 mK) target. The low temperature of the sample contributes to Ps thermalization [21, 22]. Figure 2 shows the Ps yield as a function of the temperature in a Xerogel sample (a variant of aerogel) of 85 mg cm^{-3} for positrons implanted in the sample at 3 keV [23]. These results indicate that in silica-based materials, the high Ps production rate does not depend on temperature. A similar result was found in silica films [24].

The photo-excitation of Ps to Rydberg states requires photon energies close to the binding energy of 6.8 eV. Laser systems at the corresponding wavelengths (180 nm) are not commercially available. We are therefore planning to perform a two-step excitation, from the ground state to the n = 3 state (λ= 205 nm), and then to the n = 35 Rydberg band (λ ~ 1670 nm) [24]. Two pulsed-laser systems are currently under development. Both systems must provide sufficient power to excite the emitted Ps within a few ns and must be geometrically matched to the expanding cloud. Furthermore, the bandwidths of the lasers must be tailored to the transition line width broadened due to the Doppler effect as well as level splitting due to the motional Stark effect and the linear and quadratic Zeeman effect. The pump laser of the whole system will be a 650 mJ Q-switched Nd:YAG laser delivering a 4 ns pulse. The radiations are produced through second-order polarization in optical crystals (Fig. 3). The 205 nm radiation for the first transition is obtained by summing in a non- linear BBO crystal the 266 nm fourth-harmonic of the 1064 nm Nd:YAG pulse and the 894 nm radiation generated in an optical parametric generator (OPG) by down-conversion of the second- harmonic of the same laser. The other wavelength (around 1670 nm) is generated in a single step by an OPG starting from the same pump laser and then amplified by an optical parametric amplifier (OPA) system. The system has been completely designed and is partly completed. The energy required to saturate this second transition of 174 μJ [25] has been exceeded, 3 mJ having been reached after an OPA using a temporary Nd:YAG laser of only 80 mJ energy per pulse. Also for the first transition, we have reached the required 200 μJ at 205 nm.

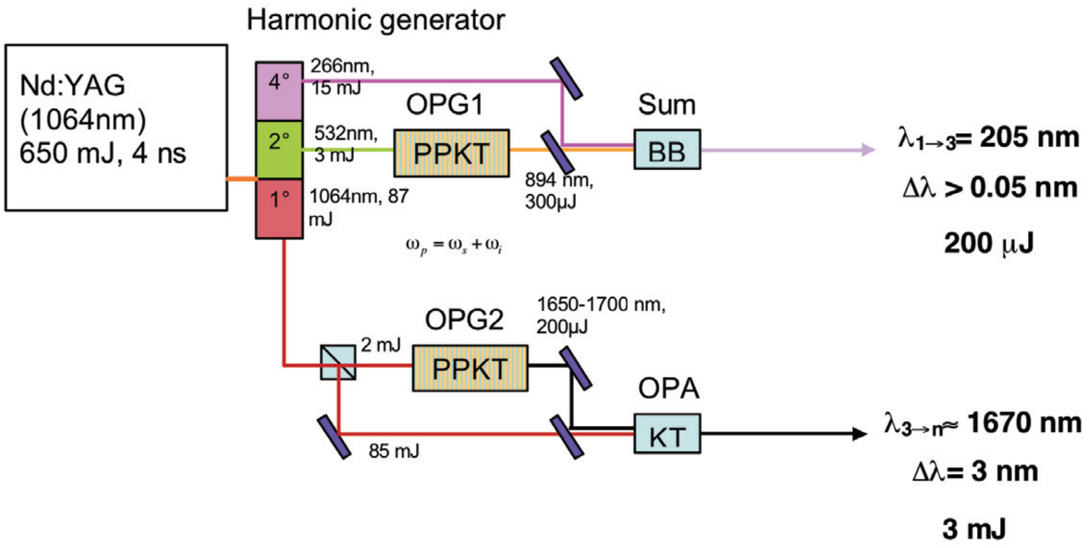

Figure 3. Sketch of the set up for the laser excitation of Rydberg Ps. The wavelength of the n=1→3 excitation is at 205 nm; the wavelength of the n=3→20..30 excitation is at about 1670 nm.

Antihydrogen recombination and beam formation

An antihydrogen recombination scheme based on resonant charge exchange with Ps was first proposed almost twenty years ago [26]. The reaction proceeds according to the equation $\mathrm{Ps}^* + \bar{p} \rightarrow \bar{\mathrm{H}}^* + e^-$, where the star denotes a highly excited Rydberg state. This reaction owes its appeal to the fact that the cross-section scales approximately with the fourth power of the principal quantum number. In addition, it creates antihydrogen in a narrow and well-defined band of final states. Most importantly, antihydrogen formed with antiprotons at rest is created with a velocity distribution dominated by the antiproton temperature, hence the surrounding (cryogenic) environment [27]. This is in stark contrast to the rather high antihydrogen temperature observed when using the nested-well technique pioneered by ATRAP and ATHENA [28,29] (although very recently, significantly colder antihydrogen atoms have been produced via this technique and trapped [30]). Our proposed technique is conceptually similar to a charge exchange technique based on Rydberg cesium [31] which has been successfully demonstrated by ATRAP [32], but offers greater control of the final state distribution of antihydrogen and, more importantly, allows pulsed production of antihydrogen.

The principle is illustrated in Fig. 1. The Ps emitted from the porous insulator material are excited to Rydberg states. They then traverse a Penning trap region in which ~ 10^5 antiprotons have been accumulated, stored and cooled to O(100 mK). The low temperature requirement on the antiprotons comes from the requirement that the antihydrogen atoms that will be formed will have a velocity that is low compared to the velocity of several 100 m·s^{-1} that they will achieve after acceleration. To reach such a low temperature, the Penning trap is coupled to a 50 mK dilution refrigerator, and the antiprotons are coupled to the low temperature environment by embedding them in an electron cloud. The later will cool through synchrotron radiation, as well as through a tuned circuit; furthermore, evaporative cooling of the pre-cooled antiprotons is being envisaged. The charge exchange cross-section is very large (~ 10^7 Å^2 for n = 35) and reaches a maximum when the e^+ and antiproton relative velocities are matched. Taking into account the corresponding kinetic energy, as well as a smaller contribution due to converted internal energy, antihydrogen is formed with velocities of 25 ~ 80 $m \cdot s^{-1}$.

While neutral atoms are not sensitive (to first order) to constant electric fields, they do experience a force when their electric dipole moment is exposed to an electric-field gradient. Since the dipole moment scales approximately with the square of the principal quantum number, Rydberg atoms are especially amenable to being manipulated in this way. Such so-called Stark acceleration (and deceleration) has been successfully demonstrated, among others, by one of the AEGIS groups with (ordinary) hydrogen after excitation to the n = 22, 23, 24 states [33,34]. In these experiments, accelerations of $2 \cdot 10^8$ $m \cdot s^{-2}$ were achieved using two pairs of electrodes at right angles to each other. A hydrogen beam traveling at 700 $m \cdot s^{-1}$ was stopped within 5 μs over a distance of only 1.8 mm. We intend to use a similar field configuration, generated by axially split electrodes within the cylindrical geometry of a Penning trap, to accelerate the formed antihydrogen atoms to about 400 $m \cdot s^{-1}$ in the direction of the deflectometer apparatus.

Gravity measurement

In matter wave interferometers of the Mach-Zehnder type [35,36], three identical gratings are placed at equal distances L from each other. The first two gratings produce an interference pattern at the location of the third. That pattern has the same period d as the gratings, and its position perpendicular to the diffracted particle beam can be determined precisely by displacing the third grating and recording the overall transmission with a particle detector. Under the influence of gravity, the interference pattern is vertically displaced (it ''falls'') by a distance $\delta x = gt^2$ where g is the local gravitational acceleration and t is the time of flight L/v between each pair of gratings of a particle beam traveling at velocity v.

Contrary to such true interferometers, which place a very stringent limit on the acceptable beam divergence (and thus antihydrogen temperature at production), the so-called Moiré deflectometer, in which diffraction on the gratings is replaced by a (classical) shadow pattern of those particles that converge onto the third grating, works in the classical regime. Interestingly, the gross characteristics of the interferometer are retained [37], in particular, the vertical displacement of the interference pattern. A three-grating Moiré deflectometer has been used to measure the local gravitational acceleration to a relative precision of $2 \cdot 10^{-4}$ with a beam of argon atoms traveling at an average velocity of 750 $m \cdot s^{-1}$ [37]. In departing from the three-grating deflectometer, we intend to replace the third grating by a position-sensitive silicon strip detector (Fig. 1). Antihydrogen atoms impacting on the detector plane annihilate, and the impact point can be reconstructed by means of the energy deposited locally by the annihilation products.

The value of g is extracted from two primary observables (time of flight T and vertical displacement of the fringe pattern δx). The periodic nature of the arrangement means that for a given value of T, the impact point will have dropped by a well-defined amount $\delta x(T)$, modulo the grating period. By varying the accelerating field gradient (and thus varying the velocity of the antihydrogen atoms), or simply through the spread in velocity of the ensemble of antihydrogen atoms, the quadratic dependence of δx on T is probed.

Our simulations have shown that in order to perform a measurement of g to 1% relative precision, about 10^5 antihydrogen atoms at a temperature of 100 mK will be required, equivalent to several months of data taking at the AD. In these simulations, a grating period of 80 μm was used, and a finite detector resolution of 10 μm was taken into account. The experiment is charge symmetric, in the sense that if antiprotons are replaced with protons, it should be possible to produce an identical beam of hydrogen atoms, which can be used to evaluate systematic effects of the measurement. While their behavior is expected to be identical in almost all respects to that of antihydrogen, measuring the impact point of (ground state) hydrogen atoms with a precision of the order of a few micrometers represents a challenge that has not been solved to date; excited states however might be detectable to this accuracy. Which fraction of the produced Rydberg (anti)hydrogen atoms remain excited during their trajectory through the Moiré deflectometer is equally an open question.

Summary and outlook

Construction has started on the AEGIS experiment, whose design is based upon the broad experience gained with the ATHENA and ATRAP experiments at the AD, a series of ongoing related experiments, tests and developments,as well as extensive simulations of critical processes (charge exchange production of antihydrogen, Stark acceleration and propagation through the Moiré deflectometer, resolution of the position-sensitive detector located at the end of the deflectometer). The proposed gravity measurement merges in a single experimental apparatus technologies already demonstrated or based on reasonable additional development.

For the initial phase of the experiment, obtaining samples of anti-atoms at 100 mK is an essential requirement. Gravity measurements with even higher precision, as well as competitive CPT tests through spectroscopy, are desirable, but will necessitate the development of novel techniques to attain even colder antihydrogen ensembles. The experiment has been designed with flexibility of the apparatus in mind, in order to allow a number of techniques, which may lead to such physics topics, to be implemented. One natural extension of the modular design is to incorporate, in a future stage, a magnetic decelerator and trap for antihydrogen, which will be spatially separated from the region where the anti-atoms are produced, similar to the devices currently being used to trap and study hydrogen atoms [38,39]. The experience gained in the first phase of AEGIS with the formation of an antihydrogen beam will be used to optimize the design of such a trapping system.

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