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RESEARCH ARTICLE

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Structural analysis and design of irregular shaped footings subjected to eccentric loading

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Abstract

This article presents a simplified analytical model for designing irregular shaped reinforced concrete (RC) footings supporting square column and subjected to eccentric loading, that is, axial load **P** and biaxial moments, ***Mx*** in *(x-x)* axis and ***My*** in *(y-y)* axis, respectively. In this study, four design variations of footing sections are considered, namely, square, triangular, circular, and trapezoidal. Seven different footings (F-1 to F-7), each with a different loading condition, are used to analyze and design each of the selected irregular footings with the goal of getting the optimum footing section. The required reinforcing area of steel ***(As)*** is obtained using the SDM method in each selected footing which is then compared using finite element software (SAFE). The percentage difference of area of steel *(As)* for simplified method with the finite element software ranges within 1% to 13%. Moreover, the concrete volume results show that the circular and triangular footings prove to be the most economical footings followed by square and trapezoidal shaped footing sections. However, the results show that triangular shaped footings under heavy loads require a larger steel area *(As)* as in footing F-7, which is not economical for heavy loads.

KEYWORDS

eccentric loading, irregular footings, optimum footing section, steel area

1 | INTRODUCTION

Foundations are most important member of structure, which transmit the structural load to the soil. Foundations are classified as shallow or deep foundations, which defer in terms of geometry, soil behavior, and structure capability.[1-3] Different types of shallow foundations[4] are available according to their functionality; which are isolated, combined, strip, and mat footings.

Footing sizes[5-7] are mostly governed by their loading parameters which are; axial load P, Biaxial moments Mx and My, allowable soil pressure Qa, unit weight of concrete γ_c, soil unit weight γ_s and the depth of the footing base below the final grade D_f as shown in Figure 1. Similarly, soil pressure distribution under a footing is normally a function of type of soil, relative rigidity of soil and footing, and depth of footing. For structural design purpose, it is quite common to assume the linearly distributed soil pressure to the footing surface.[8]

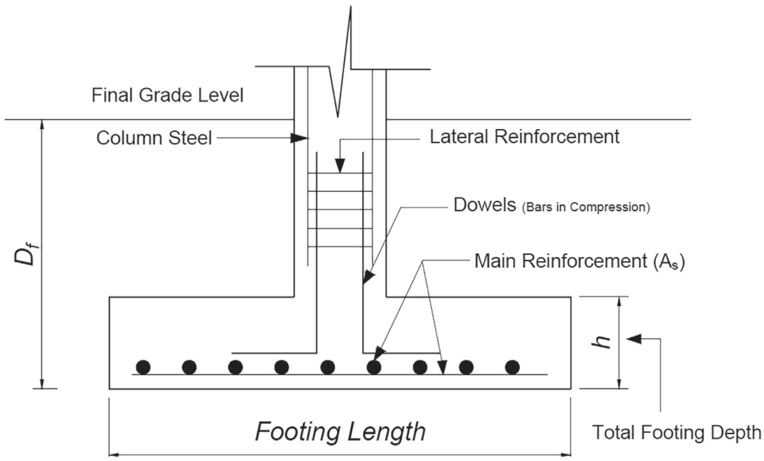


FIGURE 1 Isolated footing dimensions and reinforcement detailing[5]

Isolated footings are normally subjected to three different loading scenarios: (1) The footings subjected to axial load ***(P)*** only, (2) footing subjected to axial load ***(P)*** and unidirectional bending (***Mx***, moment in one direction only), and (3) footing subjected to axial load ***(P)*** and bi-directional bending ***(Mx*** and ***My***, moments in both directions).[9]

Square and rectangular footings are the most common shaped of the isolated RC footings in the construction industry, but other irregular shaped footings do exist such as circular, triangular and trapezoidal, depending on different scenarios in the construction field. Different mathematical models are presented in numerous studies[10-17] to structurally analyze and design the irregular shaped footing sections under the provisions of ACI building code of design (ACI 318-14).[18]

There are limited studies, exploring the detailed design for the irregular shaped footing sections. Stone et al[19] studied the response of triangular footings subjected to centric and eccentric loading. Their study analyzed model tests with the derivation of equivalent rectangular section using the conventional bearing capacity theory. Huat et al[20] studied the performance of triangular shell footings using finite element and field model test. The study concluded that the triangular shell is more efficient in carrying the load as compared to the traditional flat strip footing. Rojas[21] proposed mathematical model of circular footing subjected to axial load and biaxial bending. He concluded that this new circular footing model is more economical and adjustable to actual soil conditions.

The previous research studies consisted of complicated mathematical models for the analysis and design of irregular shaped footing section. In most cases, the proposed models of footing sections do not consider the effect of bi-axial moments. They are analyzed and designed based on the axial load values only.

This article, however, presents simplified analytical model for designing irregular shaped reinforced concrete footings, supporting square column, and subjected to eccentric loading, that is, axial load ***P*** and biaxial moments; ***Mx*** in *(X-X)* axis and ***My*** in *(Y-Y)* axis, respectively. In this study, four design variations of footing sections are considered, that is, square, triangular, circular, and trapezoidal to be analyzed using the simplified method approach. Seven different footings (*F-1* to *F-7*), each with a different loading condition, are used to analyze and design each of the selected irregular footings.

There are limited studies for the reinforced design of irregular shaped footings. This study will provide quick and easy approach to design such footings and will be useful for the students in their undergraduate and graduate courses as well as research related work.

Figure 2 includes the irregular shaped footing sections, which are studied in this research. Eccentric, shear and moment formulas are derived for each of these irregular shaped footings (square, circular, triangular, and trapezoidal). Mathcad software[22] is used for all the necessary calculations needed for the simplified design method (SDM). The design results of this method are also compared with the computer aided software (SAFE software). The comparison mainly includes the footing dimensions, total steel rebar areas and concrete volume from safety and economic perspectives.

2 | SHEAR AND FLEXURAL DESIGN FORMULAS

In general, the required footing area ($\boldsymbol{F_A}$) is computed based on the axial load ***P*** and the effective soil pressure ***Qe***. The equation is obtained from (ACI -318R-14).

$$F_A = \frac{P}{Q_e} = \frac{\mathrm{DL} + \mathrm{LL}}{Q_e}. \tag{1}$$

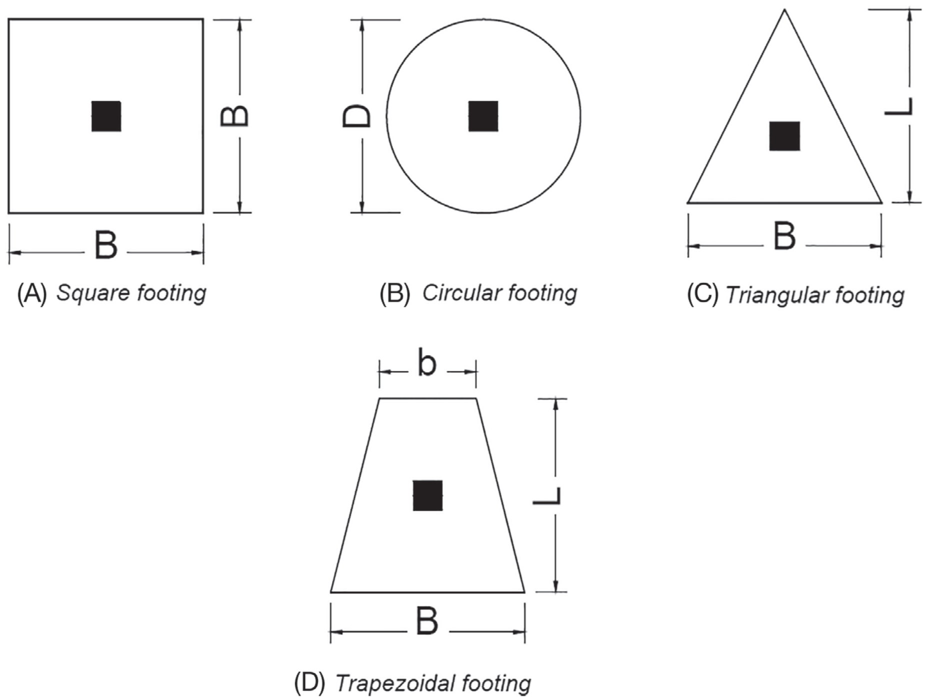


FIGURE 2 Cross section of different shaped footings

Also,

$$Qe = Qa - Wc - Ws \tag{1a}$$

$$Wc = \gamma_c \times h \tag{1b}$$

$$Ws = \gamma_c \times D_f \tag{1c}$$

$$h = d + d\prime \tag{1d}$$

Where
$\boldsymbol{F_A}$ = Footing area $\boldsymbol{Qa}$ = Allowable soil pressure
$\boldsymbol{Qe}$ = Effective soil pressure $\boldsymbol{Wc}$ = Concrete weight
$\boldsymbol{Ws}$ = Soil weight $\boldsymbol{h}$ = Total footing depth
$\boldsymbol{d}$ = Effective depth $\boldsymbol{d'}$ = Cover to steel centroid.

2.1 | Effective depth calculation

Both one-way and two-way shear are considered for estimating the footing's effective depth. The critical section for one way and two-way shear to estimate the effective depth for each shape is shown in Figure 3. The ACI building design code (ACI -318R-14) formula is used for calculating one-way shear depth:

$$\boldsymbol{d_{one\ way}} = \frac{\boldsymbol{Vu}}{\boldsymbol{\varphi_s \ \nu_c \ \ b_w}} \tag{2}$$

Where
$\boldsymbol{Vu}$ = Factored shear force,
$\boldsymbol{\varphi_s}$ = Shear reduction factor,
$\boldsymbol{\nu_c}$ = Shear stress carried by the concrete,
$\boldsymbol{b_w}$ = Footing width.

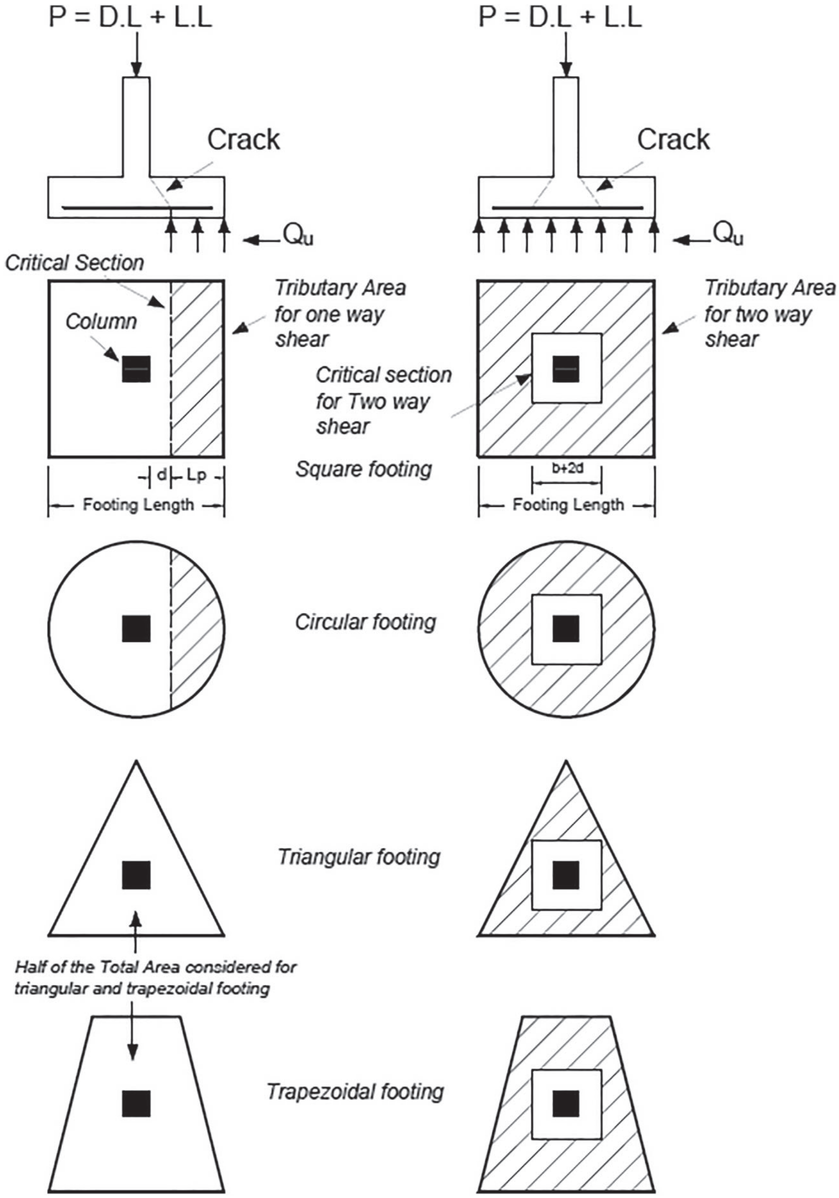


FIGURE 3 One- and two-way shear for different shaped footings

For two-way shear depth, the largest value is to be selected from the following ACI code (ACI-318-14) equations:

$$d_{\mathbf{2w(1)}} = \frac{6\ Vu}{\varphi_s \left(1 + \frac{8}{\beta_C}\right) \sqrt{f_c'}\ b_o} \quad (3)$$

$$d_{\mathbf{2w(2)}} = \frac{12\ Vu}{\varphi_s \left(2 + \frac{\propto_S}{b_o}\right) \sqrt{f_c'}\ b_o} \quad (4)$$

$$d_{\mathbf{2w(3)}} = \frac{3\ Vu}{\varphi_s \sqrt{f_c'}\ b_o} \quad (5)$$

Where
$\boldsymbol{\beta_C} =$ Ratio of long side of the column to the short of the column,
$\boldsymbol{f_c'}$ = Specified compression strength of concrete,
$\boldsymbol{b_o}$ = Perimeter around the punching area,
$\boldsymbol{\propto_S}$ = Ratio equals to 40, 30, and 20 for interior column, edge column, and corner column, respectively.

2.2 | Footing moments and reinforcement calculation

Footing bending moments (***Mu)*** in both axes are considered at the face of the column (Figure 4).

$$\boldsymbol{Mu} = \frac{\boldsymbol{L_p^2}}{2}\boldsymbol{q_u b_w} \tag{6}$$

and

$$\boldsymbol{qu} = \frac{\boldsymbol{Pu}}{\boldsymbol{F_A}} = \frac{\boldsymbol{DL \times DLF + LL \times LLF}}{\boldsymbol{F_A}} \tag{7}$$

Where
Mu = Fully factored bending moment,
$\boldsymbol{L_p}$ = Maximum projected length,
$\boldsymbol{q_u}$ = Bearing pressure for strength design,
DLF = Dead load factor equal 1.2,
LLF = Live load factor equal 1.6.
The reinforcement area ***As*** of the footing can be computed as:

$$\boldsymbol{As} = \frac{\boldsymbol{Mu}}{\boldsymbol{\varphi_b\ fy}\left(\boldsymbol{d} - \frac{\boldsymbol{a}}{2}\right)} \tag{8}$$

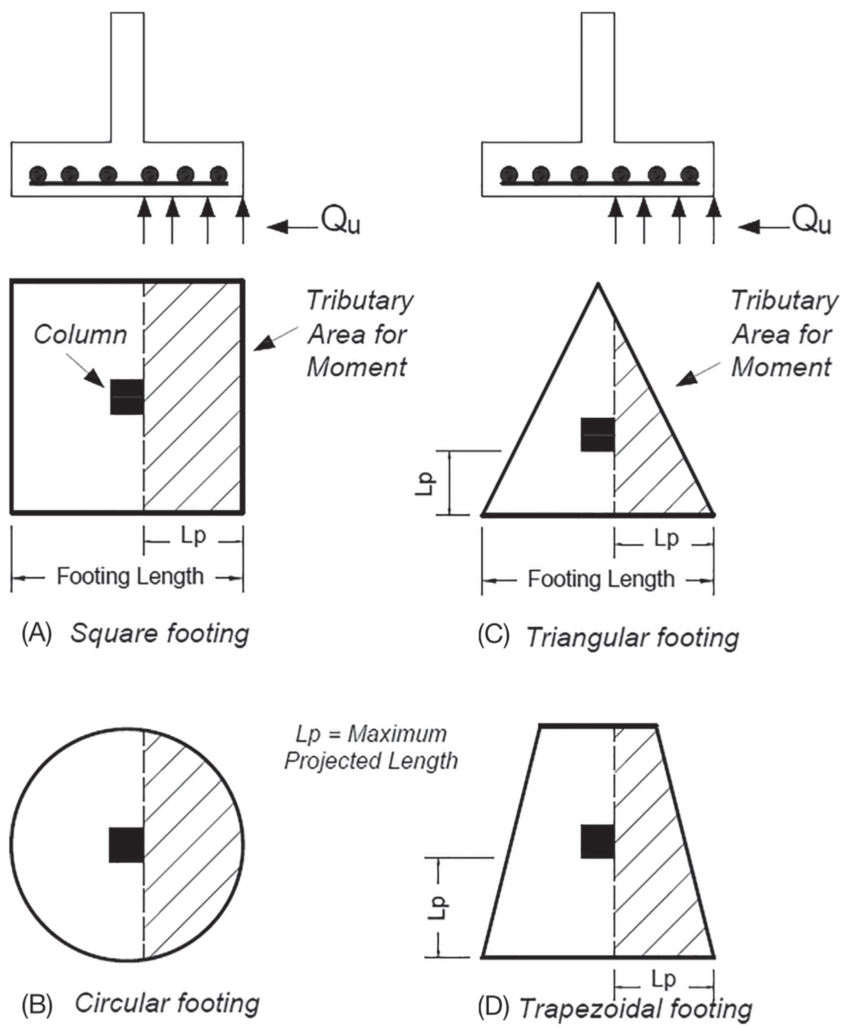


FIGURE 4 Tributary area for moments in different footings

Where
φ_b = Bending reduction factor,
fy = Specified yield strength of non-prestressed reinforcing,
As = Area of tension steel,
d = Effective depth,
a = Depth of the compression block.
Also,

$$d_B^L \le d \le d_B^U \tag{9}$$

$$As_S^{Mini} \le As \le As_S^{Max} \tag{10}$$

$$As^{Max} = 0.75 \times \beta 1 \times \frac{f_c'}{fy}\left(\frac{600}{600 + fy}\right) bd \tag{11}$$

$$As^{Mini} = \left(\frac{1.4}{fy}\right) bd \tag{12}$$

$$\beta 1 = 0.85 \ \ for\ \boldsymbol{f_c'} \le \mathbf{30}\ \boldsymbol{MPa}$$

$$\beta 1 = 0.85 - 0.008(\boldsymbol{f_c'} - 30) \ge 0.65 \ \ for\ \boldsymbol{f_c'} > \mathbf{30}\ MPa$$

Where $\boldsymbol{d_B^L}$ and $\boldsymbol{d_B^U}$ are footing depth lower and upper bounds, and $\boldsymbol{As_B^{Mini}}$ and $\boldsymbol{As_B^{Max}}$ are footing steel reinforcement area lower and upper bounds, respectively.

The reinforcing bars must have the required length to provide enough strength. In other words, the bars must extend developmental length $\boldsymbol{L_d}$ from the face of the column (ACI 318-14).

$$L_d < L_d T_{Available} \tag{13}$$

Where
L_d = Required bar developmental length,
$L_d T_{Available}$ = Available length in tension.
For the dowel bars under compression:

$$\mathrm{As}_{\boldsymbol{dowels}} \ge \mathbf{0.005}\ A_{\boldsymbol{Column}} \tag{14}$$

$$L_{d\boldsymbol{Comp}} < L_d C_{\boldsymbol{Available}} \tag{15}$$

$$h > L_{d\boldsymbol{Comp}} + \boldsymbol{Cover} + \mathbf{2}d_b \tag{16}$$

Where
$\boldsymbol{As_{dowels}}$ = Steel area of the dowels,
$\boldsymbol{A_{Column}}$ = Column area,
$\boldsymbol{L_{dComp}}$ = Required bar developmental length in compression,
$\boldsymbol{L_d\ C_{Available}}$ = Available length in compression,
$\boldsymbol{h}$ = Total footing depth,
$\boldsymbol{Cover}$ = Concrete cover thickness,
$\boldsymbol{d_b}$ = Bar diameter.

2.3 | Eccentric formulation

Eccentric footing is the footing that is subjected to axial load $\boldsymbol{P}$ and biaxial moments $\boldsymbol{Mx}$ and $\boldsymbol{My}$ about x and y axes as shown in Figure 5A.

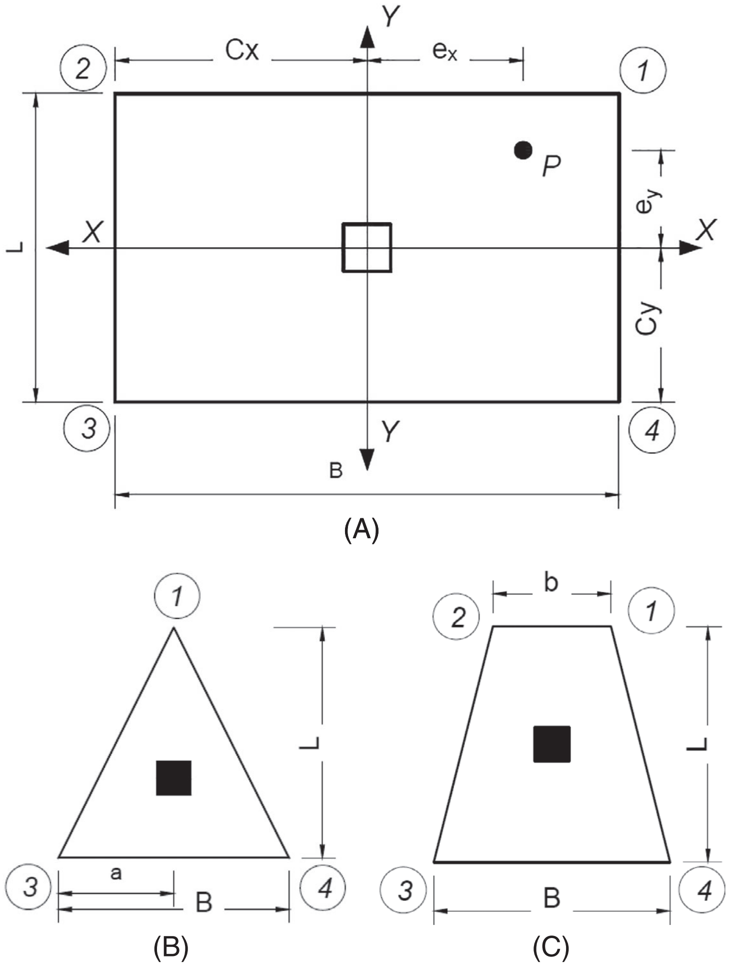


FIGURE 5 A, Footing subject to axial load P and biaxial moments ***Mx*** and ***My***. B, Triangular section. C, Trapezoidal section

The soil pressure at the footing corners 1, 2, 3, and 4 is given by the following formula.[8]

$$Q_{CORNERS} = \frac{-P}{A} \mp \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y} \quad (17)$$

Where
P = Axial Load,
Mx = Moment about the x axis ($P \times e_y$),
My = Moment about the y axis ($P \times e_x$),
A = Footing area,
Ix = Moment of inertia about the x axis,
Iy = Moment of inertia about the y axis,
Cx = Centroid coordinates on the x axis,
Cy = Centroid coordinates on the y axis.
Load ***P*** acts at distance e_x from the y axis and a distance e_y from the x axis, therefore,

$$Q_{CORNERS} = \frac{-P}{A} \mp \frac{P\, e_y\, C_y}{I_x} \pm \frac{P\, e_x C_x}{I_y}. \quad (18)$$

Substituting for ***Cy***, ***Cx***, ***Iy***, and ***Ix*** for rectangular footing in equation −18, the soil pressure equation becomes

$$Q_{CORNERS} = \frac{-P}{A}\left[1 \mp \frac{6\, e_y}{L} \mp \frac{6\, e_x}{B}\right]. \quad (19)$$

Similarly, for square footing

$$Q_{CORNERS} = \frac{-P}{A}\left[1 \mp \frac{6\,e_y}{B} \mp \frac{6\,e_x}{B}\right]. \tag{20}$$

For circular footing

$$Q_{CORNERS} = \frac{-P}{A}\left[1 \mp \frac{8\,e_y}{D} \mp \frac{8\,e_x}{D}\right]. \tag{21}$$

Also, the following two equations can also be used for circular footings

$$Q_{max} = \frac{-P}{A}\left[1 - \frac{8 \times e}{D}\right] \tag{21a}$$

$$Q_{min} = \frac{-P}{A}\left[1 + \frac{8 \times e}{D}\right] \tag{21b}$$

Where

$$M = \sqrt{Mx^2 + My^2} \quad \text{and} \quad e = {}^{M}/_{P}.$$

For triangular footing with equal lengths (Figure 5B), the soil pressure equation at each corner is

$$Q_{CORNER\,(1)} = \frac{-P}{A}\left[1 - \frac{12 \times e_y}{L} - 0\right] \tag{22}$$

$$Q_{CORNER\,(3)} = \frac{-P}{A}\left[1 + \frac{18 \times e_x}{a^2 - aB + B^2} + \frac{12 \times e_y}{L}\right] \tag{23}$$

$$Q_{CORNER\,(4)} = \frac{-P}{A}\left[1 - \frac{18 \times e_x}{a^2 - aB + B^2} + \frac{12 \times e_y}{L}\right] \tag{24}$$

For trapezoidal footing (Figure 5C), the soil pressure equation at each corner is

$$Q_{CORNER\,(1)} = \frac{-P}{A}\left[1 - \frac{e_x(12B)}{(b^2 + B^2)} - \frac{e_y(24b + 12B\,)\,(b + B)}{2L(b^2 + 4bB + B^2)}\right] \tag{25}$$

$$Q_{CORNER\,(2)} = \frac{-P}{A}\left[1 - \frac{e_x(12B)}{(b^2 + B^2)} + \frac{e_y(24b + 12B\,)\,(b + B)}{2L(b^2 + 4bB + B^2)}\right] \tag{26}$$

$$Q_{CORNER\,(3)} = \frac{-P}{A}\left[1 + \frac{e_x(12b)}{(b^2 + B^2)} + \frac{e_y(24B + 12b\,)\,(b + B)}{2L(b^2 + 4bB + B^2)}\right] \tag{27}$$

$$Q_{CORNER\,(4)} = \frac{-P}{A}\left[1 + \frac{e_x(12b)}{(b^2 + B^2)} - \frac{e_y(24B + 12b\,)\,(b + B)}{2L(b^2 + 4bB + B^2)}\right]. \tag{28}$$

Moreover, the soil pressure at the corners must be in compression state and less than the effective soil pressure to determine the required footing size.

3 | FOOTING DESIGN PROCEDURE

The following steps need to be followed in the SDM for an economical design of the eccentric footings.

Step 1: Determine the Effective Soil Pressure ***Qe*** *(Equation (1a))*.
Step 2: Determine the initial footing dimensions based on the area of the footing $\boldsymbol{F_A}$ *(Equation (1))*.

TABLE 1 Footing design loads

Footing	Axial load P_{DL} (kN)	Axial load P_{LL} (kN)	M_{ux} (kN-m)	M_{uy} (kN-m)	Column size (mm × mm)
F1	200	100	60	40	300 × 300
F2	120	70	30	50	270 × 270
F3	1000	800	200	150	500 × 500
F4	500	200	90	50	320 × 320
F5	2000	1300	150	250	550 × 550
F6	800	600	100	30	350 × 350
F7	3000	2000	350	280	700 × 700

Step 3: Determine the final footing dimensions based on the appropriate eccentric formula *(Equations (18) to (28)), based on the footing shape).*
Step 4: Determine the required depth for one-way shear $\boldsymbol{d_{one\ way}}$ *(Equation (2)).*
Step 5: Determine the required depth for two-way shear $\boldsymbol{d_{2-way}}$.
Step 6: Determine footing reinforcement ***As*** *(Equation (8)).*
Step 7: Determine the required bar developmental length ***Ld*** *(Equation (13)).*
Step 8: Determine the required bar developmental length for compression $\boldsymbol{Ld_{Comp}}$ *(Equations (14) to (16)).*
Step 9: Check if the total thickness ***h*** satisfies Equation (16).
Step 10: Detailing of the footing.

4 | NUMERICAL ANALYSIS

The input values for all the selected seven footings (F-1 to F-7) are described in Table 1. The analysis and design of the seven footings for each shape are presented and the results obtained are compared with the finite element software (SAFE).

The common design parameter used for all footings are as follows;

$fy = 400$ MPa $\quad fc' = 30$ MPa
$Qa = 200$ kPa $\quad \gamma_c = 25$ kN/m^3
$\gamma_s = 15$ kN/m^3 $\quad d' = 75$ mm
$Df = 1$ m.

5 | RESULTS AND DISCUSSION

The selected seven footings (F-1 to F-7) are designed individually as four different shaped footings (square, circular, triangular, and trapezoidal) using the simplified irregular design method approach. For each footing shape, they are further analyzed and designed by SAFE software. The results obtained are illustrated in Tables 2 to 5, respectively.

5.1 | Square footing

In this section, the selected seven footings are designed and analyzed as eccentric square footing. This method gives safe and optimum dimensions for the assigned load. These footings are also analyzed using the computer software (SAFE) and the results obtained are presented in Table 2.

The reinforcement results obtained from the safe software show similarity with the SDM results regarding the area of steel required with a percentage difference of 2% to 13%. This shows the accuracy of proposed design formulas of square footing using SDM method. The required steel area results obtained are also displayed in bar chart (Figure 6). Moreover, the settlement contours obtained from the SAFE software are shown in Figure 7.

TABLE 2 Square footing results

Footing	SDM method				SAFE software	
	Width (B) (m)	Thickness (h) (mm)	As (mm^2)	Concrete volume (m^3)	As (mm^2)	Settlement (mm)
F1	2.2	350	2118	1.69	2121	5.32
F2	2.8	350	2695	2.74	2670	2.98
F3	3.7	550	6406	7.53	6390	12.98
F4	2.5	360	2569	2.25	2547	10.24
F5	4.7	730	11 560	16.13	9854	12.04
F6	3.1	500	4611	4.81	4260	11.22
F7	5.7	870	15 860	28.27	15 701	11.57

TABLE 3 Circular footing results

Footing	SDM method				SAFE software	
	Diameter (D) (m)	Thickness (h) (mm)	As (mm^2)	Concrete volume (m^3)	As (mm^2)	Settlement (mm)
F1	2.1	350	2021	1.2	2042	6.20
F2	2.6	350	2502	1.86	2513	2.91
F3	4.1	530	6529	6.99	6521	9.88
F4	2.7	350	2600	2.0	2650	8.71
F5	5.2	730	11 920	15.5	11 938	11.47
F6	3.5	510	5329	4.91	5654	10.40
F7	6.4	870	17 810	27.9	17 907	11.83

TABLE 4 Triangular footing results

Footing	SDM method					SAFE software	
	Width (B) (m)	Length (L) (m)	Thickness (h) (mm)	As (mm^2)	Concrete volume (m^3)	As (mm^2)	Settlement (mm)
F1	2.8	2.8	350	2695	1.372	2748	4.92
F2	2.5	2.5	350	2406	1.094	2434	3.87
F3	5.2	5.2	530	11 290	7.166	10 681	9.77
F4	3.6	3.6	350	4871	2.268	4398	7.67
F5	6.5	6.5	720	18 330	15.21	16 366	11.62
F6	4.4	4.4	500	8145	4.84	8042	10.37
F7	8.1	8.1	870	28 020	28.54	24 504	11.97

TABLE 5 Trapezoidal footing results

Footing	SDM method						SAFE software	
	Width (B) (m)	Length (L) (m)	Least width (b) (mm)	Thickness (h) (mm)	As (mm^2)	Concrete volume (m^3)	As (mm^2)	Settlement (mm)
F1	2.5	2.5	1.25	350	2406	1.64	2434	5.74
F2	3.5	3.5	1.75	350	3369	3.22	3377	2.15
F3	4.3	4.3	2.15	750	10 160	10.4	10 053	12.08
F4	2.9	2.9	1.45	500	4314	3.15	4398	10.13
F5	5.5	5.5	2.75	1000	17 810	22.7	17 907	13.74
F6	3.6	3.6	1.8	700	7245	6.8	7854	12.98
F7	6.6	6.6	3.3	800	16 750	26.14	18 477	13.30

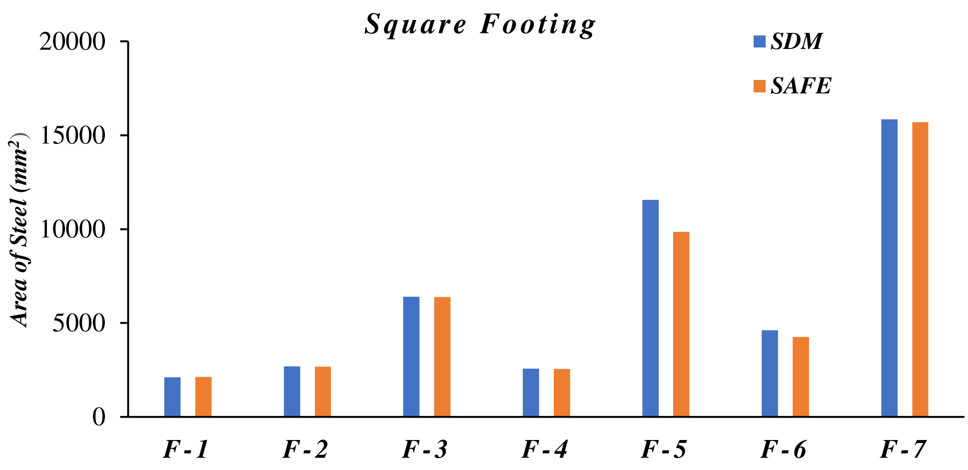


FIGURE 6 Area of Steel (*As*) comparison for square footing

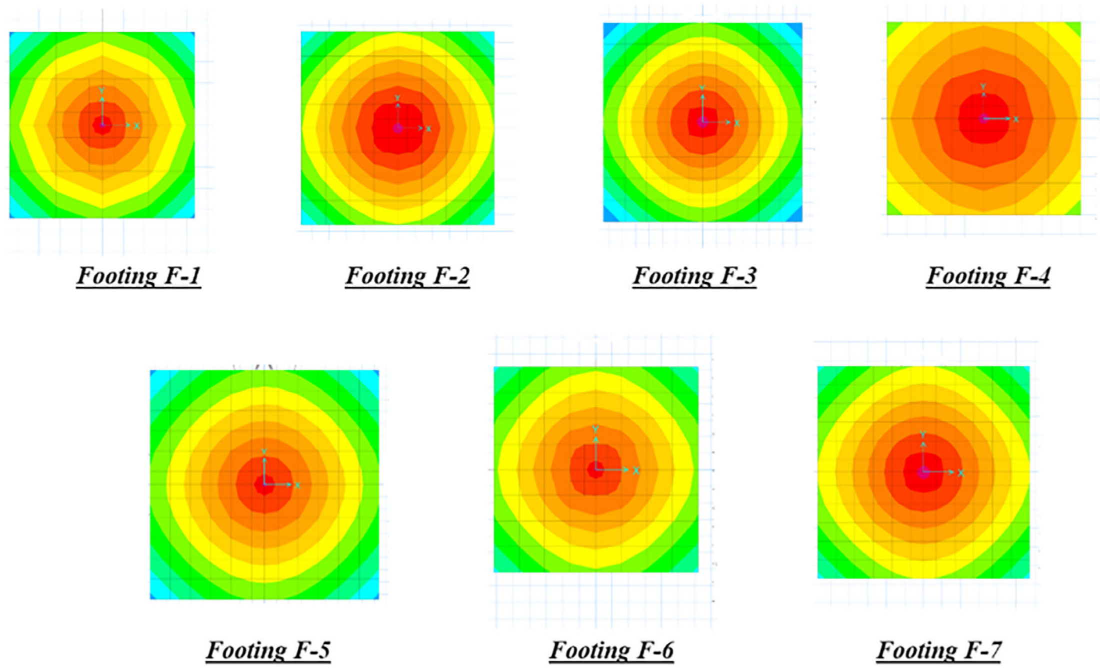


FIGURE 7 Settlement contours of square footings

5.2 | Circular footing

In this section, the selected seven footings are designed and analyzed as eccentric circular footing. The diameter obtained for each circular footing gives safe and economical design against the applied load. These circular footings are analyzed using the computer software (SAFE) as well and results obtained are displayed in Table 3.

The reinforcement results obtained from the safe software are quite close to the ones obtained from the SDM approach with a percentage difference of 1% to 5%. This indicates the accuracy of the design formulas of circular footing using the SDM method. The required steel area results obtained are also displayed in bar chart (Figure 8). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 9.

5.3 | Triangular footing

This section includes the analysis and design of the selected seven footings as eccentric triangular footings. In this study, both sides of the triangle are kept equal for the design and analysis purposes. Also, the proposed formulas using the SDM

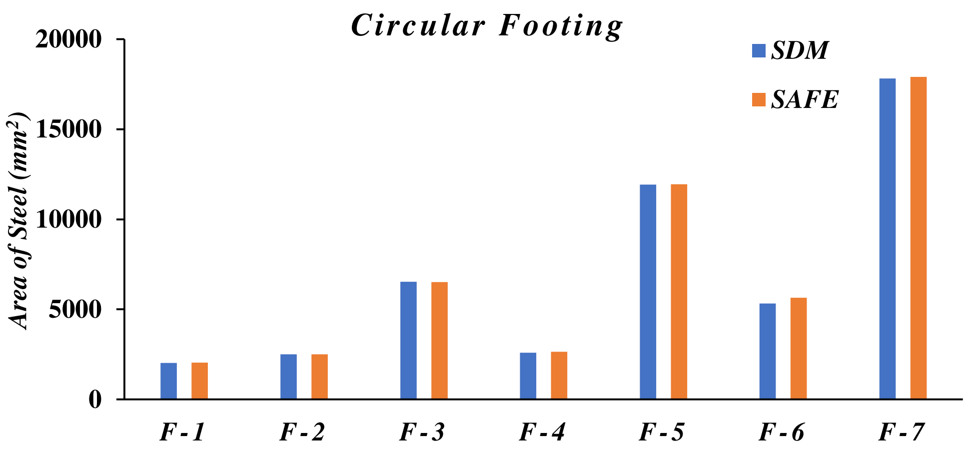


FIGURE 8 Area of steel *(As)* comparison for circular footing

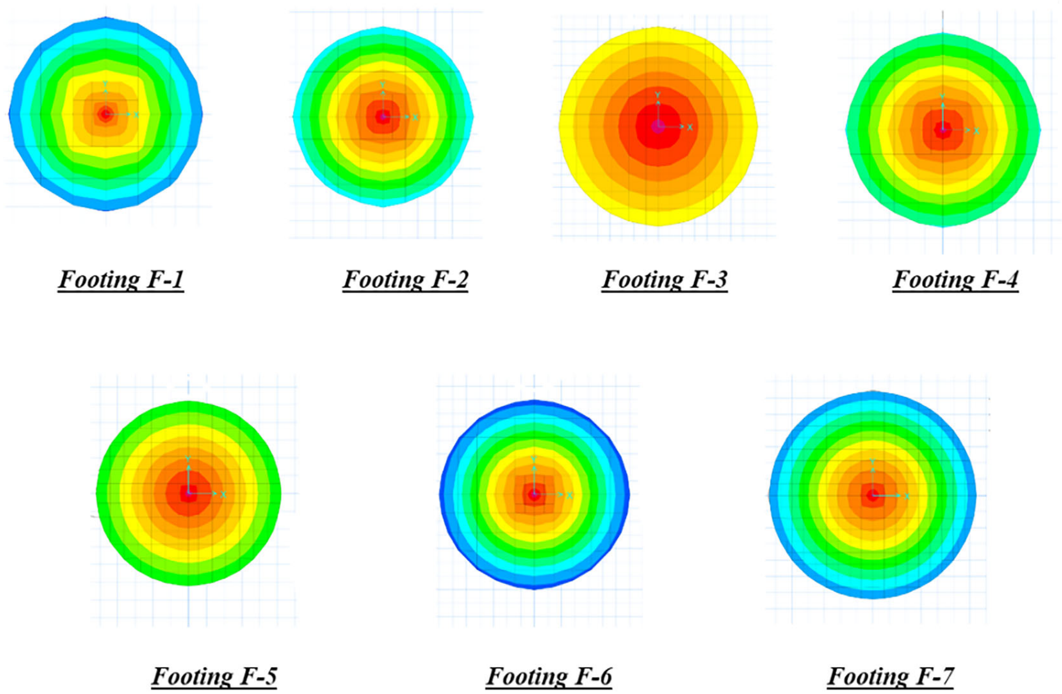


FIGURE 9 Settlement contours of circular footings

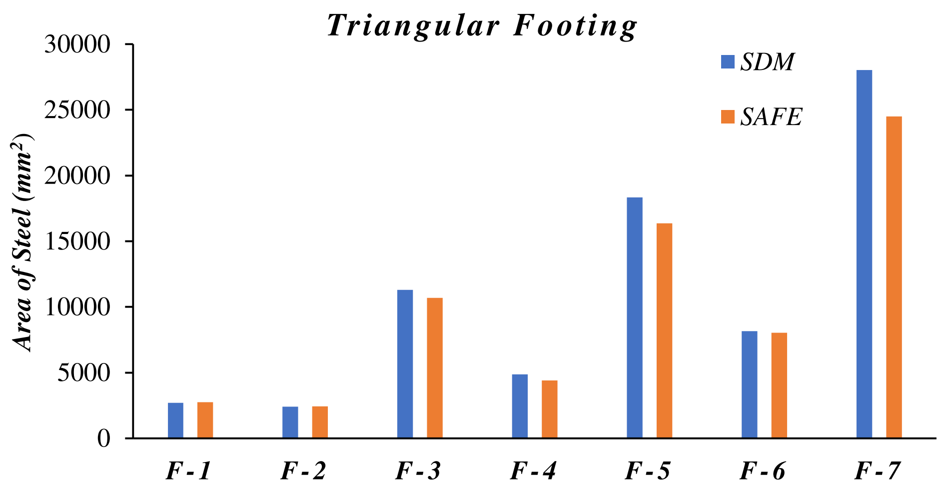
method work only for triangles with equal sides as this is the most common shaped triangular footing used in construction industry. For unequal legs, the formulas are needed to be changed accordingly. These triangular footings are also analyzed using the computer software (SAFE) and the results obtained are illustrated in Table 4.

The reinforcement results obtained from the safe software are corresponding to the ones obtained from the SDM approach with a percentage difference of 1% to 12%. This indicates the precision of the design formulas for equal leg triangular footing sections using the SDM method. The required steel area results obtained are also displayed in bar chart (Figure 10). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 11.

5.4 | Trapezoidal footing

This section includes the analysis and design of the selected seven footings as eccentric trapezoidal footings using SDM. The width (B) and length (L) of the trapezoidal section as shown in Figure 2 are kept similar in this study. The triangular section is a variant of the trapezoidal section in which the least width dimension (b) of trapezoidal section is set as zero.

FIGURE 10 Area of steel *(As)* comparison for triangular footing



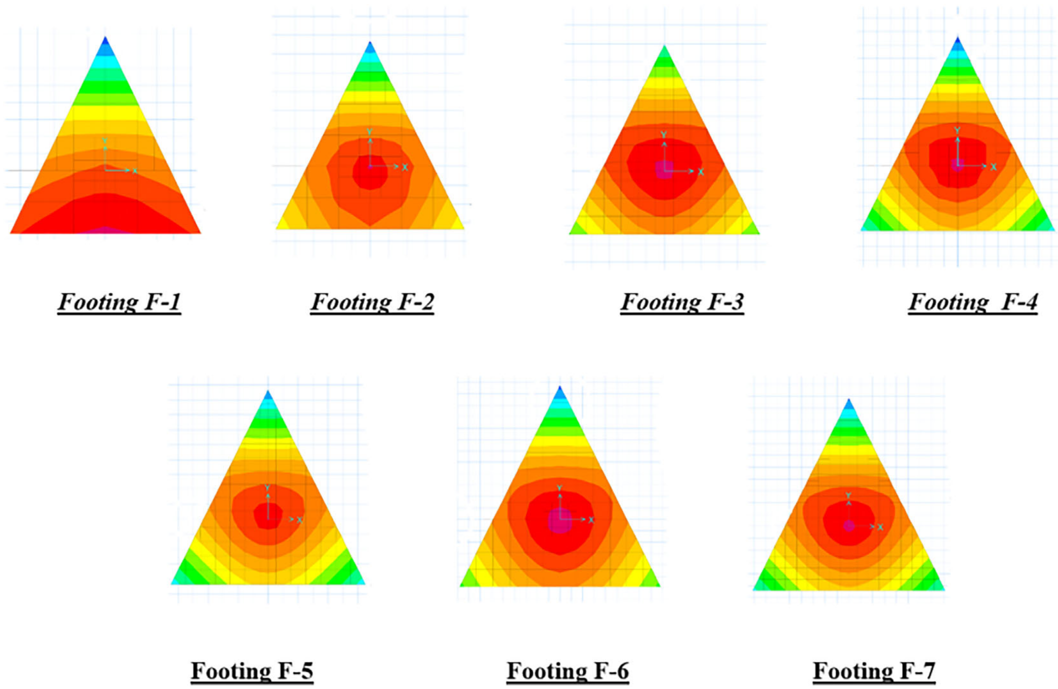


FIGURE 11 Settlement contours of triangular footings

These selected footings are analyzed using the computer software (SAFE) as well and the results obtained are illustrated in Table 5.

The results obtained from the safe software and SDM are quite alike, with a percentage difference of 2% to 9%. The efficiency of design formulas of trapezoidal sections using the SDM method is thus further confirmed. The area of required steel results obtained is also displayed in bar chart (Figure 12). Moreover, the deflection contours obtained from the SAFE software are shown in Figure 13.

The results obtained from all of the selected shaped footing indicate that the SDM is an easy approach to design and analyze the irregular shaped footings (square, circular, triangular, and trapezoidal). Moreover, a cost comparison is also made to select the best optimum footing shape design against the applied loading.

The concrete cost is to be calculated against the volume of concrete required in each footing in addition to the required area of steel. The required concrete volume in each footing is calculated and mentioned in Tables 2 to 5 for square, circular, triangular, and trapezoidal shaped footing, respectively. The footing shape with higher concrete volume will have higher construction cost in comparison with the lower concrete volume footing. The concrete volume results obtained from each footing sections (Figure 14) reveal that the circular and triangular footings prove to be the most economical footings

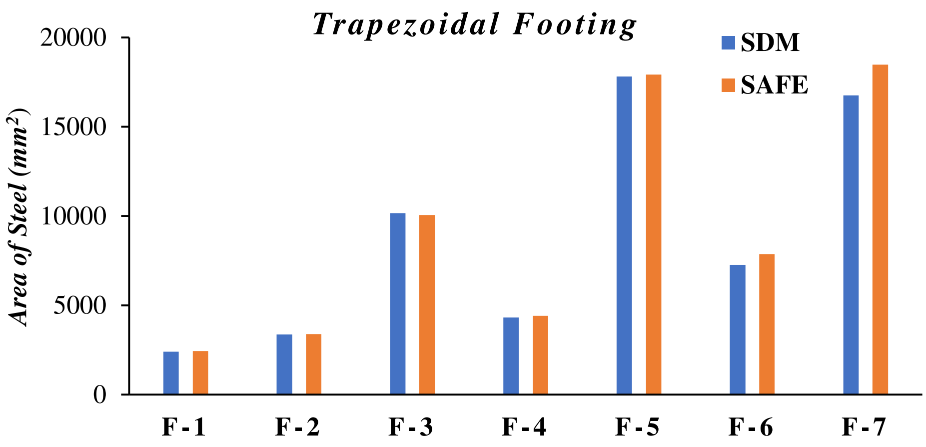


FIGURE 12 Area of steel **(As)** comparison for trapezoidal footings

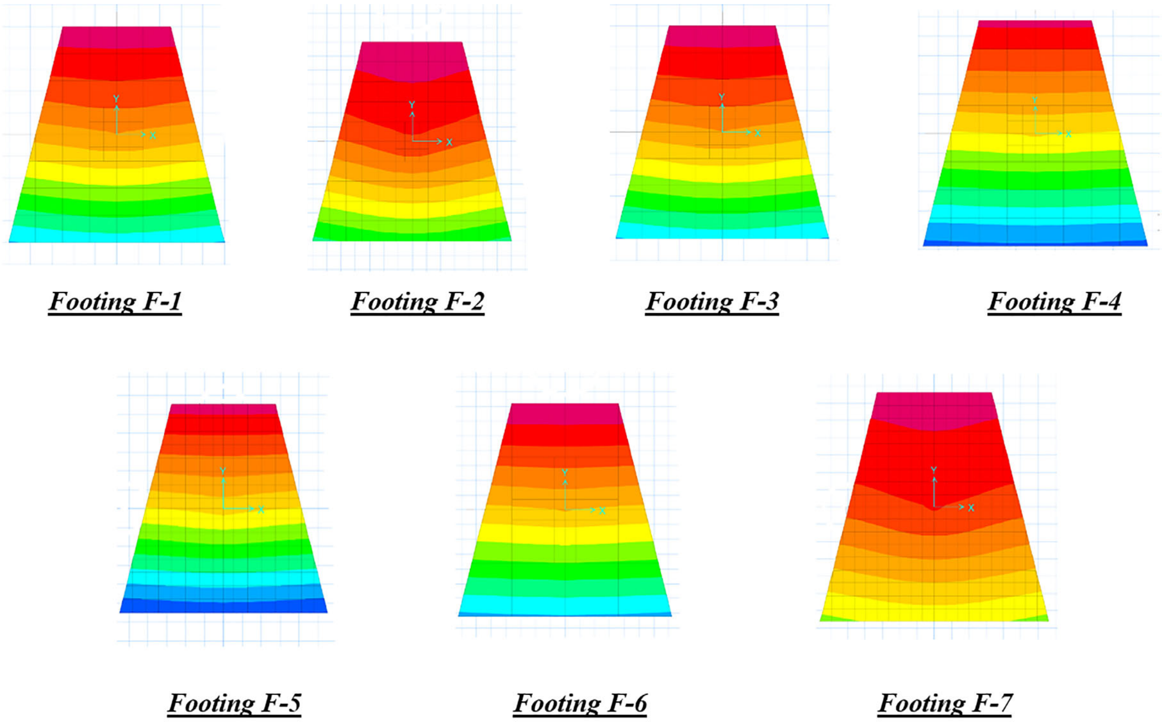


FIGURE 13 Settlement contours of trapezoidal footings

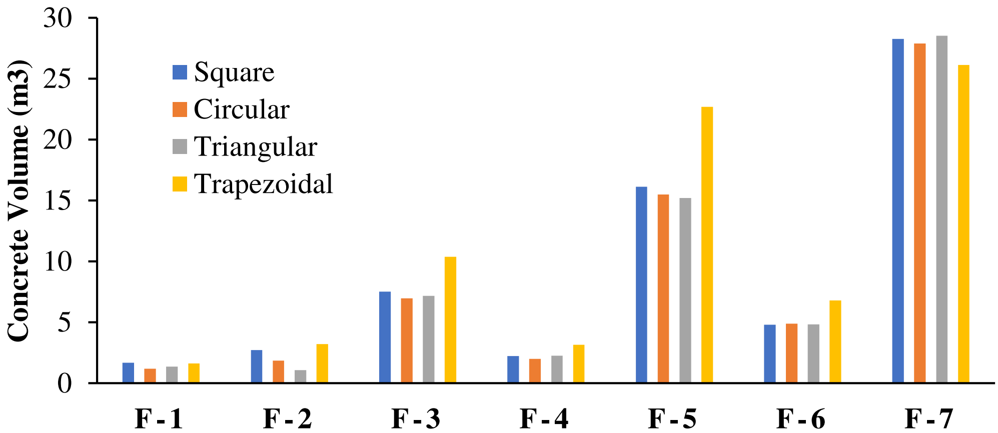


FIGURE 14 Concrete volume comparison for all footings

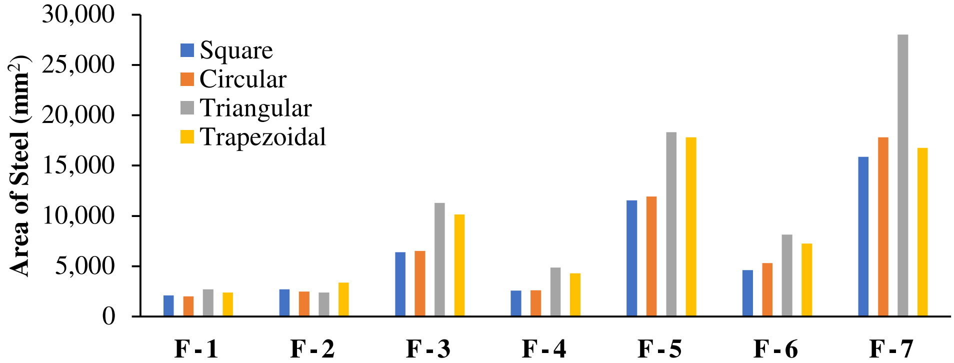


FIGURE 15 Steel area ***(As)*** comparison for all footings

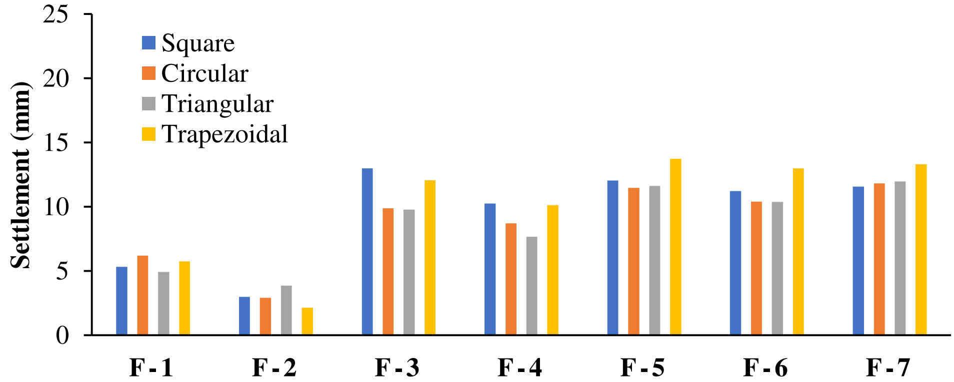


FIGURE 16 Settlement comparison for all footings

followed by square and trapezoidal shaped footing sections. Also, triangular footings under heavy load tend to have a larger steel area *(As)* as in footing F-7 so it is not economical for such load.

Moreover, the area of steel ***(As)*** comparison for each footing shape section is displayed in Figure 15.

The bar chart in Figure 16 shows that all footings displayed an acceptable settlement value which is less than the value allowable according to the ACI code of design (ACI 318-14) indicating safe design.

6 | CONCLUSION

This article presents the irregular shaped reinforced concrete footings supporting square column subjected to eccentric loading, that is, axial load P and biaxial moments; ***Mx*** in *(x-x)* axis and ***My*** in *(y-y)* axis, respectively, by a simplified analytical model. Footings with four different shapes (square, circular, triangular, and trapezoidal) are studied in this research. The SDM is used to derive the formulas needed to analyze and design these footings.

Seven footings *(F-1 to F-7)* with different loading conditions are analyzed and designed individually as square, circular, triangular, and trapezoidal shaped footings, respectively, to get the optimum shaped design footing based on concrete volume and steel weight represented by steel area *(As)*. The footings are analyzed and designed according to the ACI code of design (ACI 318R-14).

The area of steel required in each footing obtained from the SDM method showed promising results when compared with the finite element software (SAFE) with a percentage difference of 1% to 13%, respectively. Also, the concrete volume results obtained from each footing sections revealed that the circular and triangular footings prove to be the most economical footings followed by square and trapezoidal shaped footing sections.

However, triangular footings are not economical for heavy loads, as these require larger steel area as in footing F-7 under such loads. Even though square footing is not the most economical choice as far as concrete volume and steel weight ***(As)*** are concerned, yet it is used most commonly because it is easier to construct, saves time and labor work. All footings show a settlement value that is acceptable and less than the allowable limit indicating a secure and efficient design.

PEER REVIEW INFORMATION

Engineering Reports thanks the anonymous reviewers for their contribution to the peer review of this work.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this article.

AUTHOR CONTRIBUTIONS

Mohammed S. Al-Ansari: Data curation; formal analysis; investigation; methodology; supervision; validation; writing-original draft; writing-review and editing. **Muhammad S. Afzal:** Data curation; formal analysis; investigation; methodology; software; supervision; validation; writing-original draft; writing-review and editing.

NOTATIONS

F_A	footing area
Qa	allowable soil pressure
Qe	effective soil pressure
Wc	concrete weight
Ws	soil weight
h	total footing depth
d	effective depth
d'	cover to steel centroid
Vu	factored shear force
φ_s	shear reduction factor
v_c	shear stress carried by the concrete
b_w	footing width
β_C	ratio of long side of the column to the short of the column
f_c'	specified compression strength of concrete
b_o	perimeter around the punching area
$\propto_S$	ratio equals 40, 30, and 20 for interior column, edge column, and corner column respectively
Mu	fully factored bending moment
L_p	maximum projected length
q_u	bearing pressure for strength design
DLF	dead load factor equal 1.2
LLF	live load factor equal 1.6
φ_b	bending reduction factor
fy	specified yield strength of non-prestressed reinforcing
As	area of tension steel
d_b	bar diameter
a	depth of the compression block
L_d	Required bar developmental length
$L_dT_{Available}$	available length in tension
As_{dowels}	steel area of the dowels
A_{Column}	column area
L_{dComp}	required bar developmental length in compression
$L_dC_{Available}$	available length in compression

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