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A multi-objective planning and scheduling model for elective and emergency cases in the operating room under uncertainty

Yasaman Fallahpour [a], Majid Rafiee [b], Adel Elomri [c], Vahid Kayvanfar [c,*], Abdelfatteh El Omri [d,e,*]

[a] Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran
[b] Department of Industrial Engineering, Sharif University of Technology, Tehran, Iran
[c] Division of Engineering Management and Decision Sciences, College of Science and Engineering, Hamad Bin Khalifa University, Qatar Foundation, Doha 34110, Qatar
[d] Surgical Research Section, Department of Surgery, Hamad Medical Corporation, Doha 3050, Qatar
[e] Vice President for Medical and Health Sciences Office QU-Health, Qatar University, P.O. Box 2713 Doha, Qatar

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ABSTRACT

Hospitals are paramount hubs for delivering healthcare services, with their Operating Rooms (ORs) as a pivotal and financially substantial component. Efficient surgery ward planning is crucial in healthcare institutions, aiming to improve medical service quality while reducing costs. This research delves into the intricacies of integrated OR planning and scheduling, focusing on elective and emergency patients in an uncertain environment. To address these challenges, a mixed integer programming (MIP) framework is developed to minimize inactivity and patient wait times while optimizing high-priority resource allocation. Both upstream and downstream units of the ward, the Pre-operative Holding Unit (PHU), Post Anesthesia Care Unit (PACU), and Intensive Care Unit (ICU) are included. The inherently uncertain aspects of surgery, including surgical duration, Length of Stay (LOS), and the influx of emergency patients, demand an intelligent optimization approach. Consequently, a robust optimization strategy is harnessed to effectively grapple with this pervasive uncertainty. A deterministic model is introduced and improved using an enhanced epsilon constraint method. The culmination of this analytical journey yields a collection of Pareto-optimal solutions. Empirical results, supported by managerial insights, highlight the superiority of the proposed method over the traditional weighting approach.

1. Introduction

Contemporary healthcare facilities place paramount significance on adept management strategies to effectively deliver patient services within hospitals. With a dual objective of cost reduction and enhanced benefits, hospitals strive to streamline operational resources without compromising patient satisfaction. Central to this endeavor is the operating room (OR), acknowledged as a pivotal locus of both expenditure and revenue within hospital operations [1]. One way to increase OR efficiency is to effectively use OR time, which is related to surgical timing. A scheduled time is given to the patient for the surgery, and it is clear that it is preferred that the surgery starts at the scheduled time. But due to unpredictable factors, an OR may not be available at the scheduled start time, so the patient has to wait, which reduces patient and surgeon satisfaction [2]. Functioning as the cornerstone of hospital activity, the OR warrants meticulous attention due to its pivotal role in financial dynamics. Consequently, optimizing efficiency and productivity within the surgical domain yields multifaceted advantages, including prolonged patient longevity, heightened survival rates, and enhanced stakeholder satisfaction [3]. The increase in healthcare costs in recent decades has caused the importance of this issue to increase, and the attention of more researchers has been drawn to study and research in this field.

Enhancing the productivity and efficiency of hospitals, particularly in ORs, alongside ensuring timely treatment, necessitates proficient management strategies [4]. Medical services should be fairly provided to the vulnerable groups of the society. This is what health justice emphasizes [5]. Due to the aging of the population, the increase in chronic pathologies and the increase in life expectancy, hospitals are facing a growing demand for health care. This causes their capacity to be more saturated than before, and health care costs continue to rise [6].

The optimization of OR efficiencies can be facilitated through the application of optimization theories and information technologies, although manual planning persists in certain healthcare settings [7]. Typically, the demand for surgical interventions surpasses the available capacity, resulting in prolonged wait times, diminished patient satisfaction, and compromised service quality [8].

* Corresponding authors.
E-mail address: valikayvanfar@hbku.edu.qa (V. Kayvanfar).

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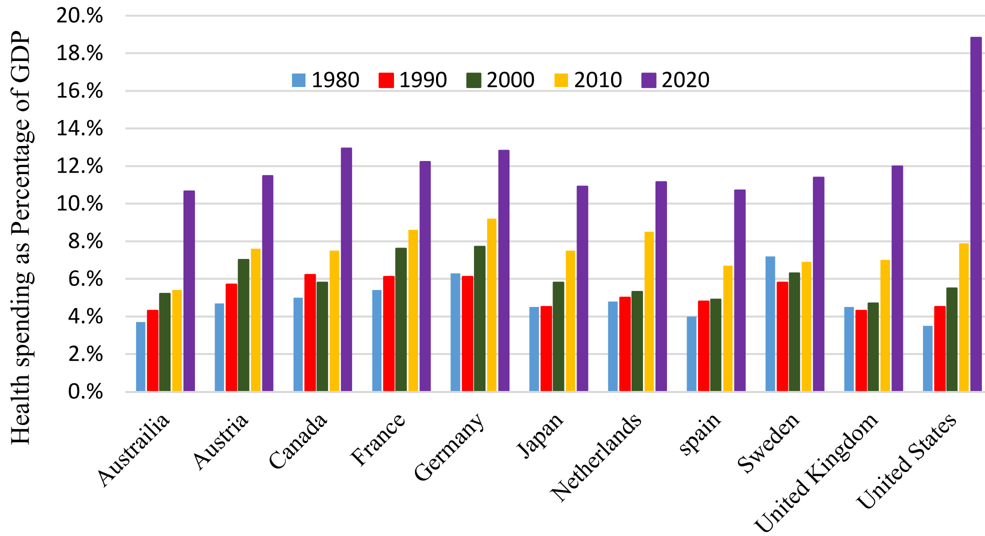

Fig. 1. Health spending as a percentage of GDP (from 1980 to 2020).

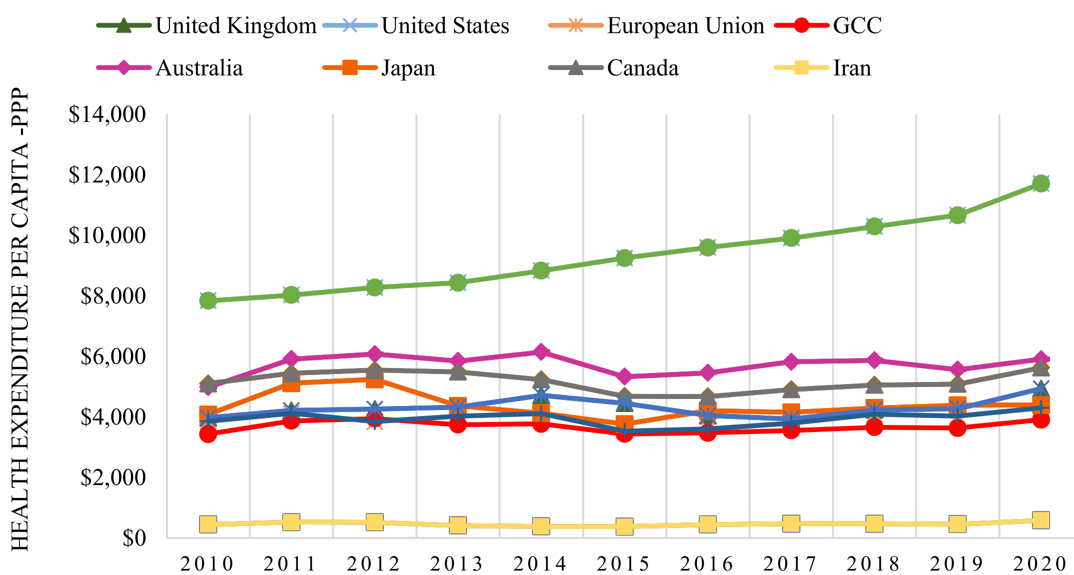

Fig. 2. Health expenditure per capita.

Given that approximately 60 to 70% of hospital admissions are associated with OR utilization, improvements in OR efficiency yield a positive impact on enhancing patient satisfaction levels, given the interconnected nature of ORs with other hospital functions [9]. According to statistics provided by the Organization for Economic Cooperation and Development (OECD), the costs related to hygiene and treatment have escalated in developed nations.

Fig. 1 illustrates a consistent upward trajectory in GDP-related costs, with Zhao and Li [10] forecasting a continuation of this trend in the coming years. Consequently, it becomes evident that the development and implementation of an effective program, particularly within the surgical ward, is imperative and justified. The proportion of healthcare spending relative to GDP has seen a rise in developed nations, reaching 12.4% in 2015, as indicated by the World Bank [11]. GCC countries also spend about 2 to 4% of their GDP on healthcare. Fig. 2, utilizing World Bank statistics, demonstrates a concurrent increase in health expenditure per capita, a metric encompassing the end-use of health goods and services for each individual. This encompasses spending from both public and private sources on medical services, goods, public health initiatives, prevention programs, and administration (OECD). Such statistics underscore the necessity for thorough investigation and analysis in this domain. Over the past few decades, Qatar's government has been investing heavily in developing an integrated healthcare system that offers high-quality services. As a consequence, the healthcare industry has witnessed unprecedented growth and is estimated to reach $12 billion in 2024, reflecting an incredible growth of 360% compared to 2010 as indicated by Mashreq [12]. According to the OECD, patients have historically experienced prolonged hospitalization durations on average. This is reflected in the extended Length of Stay (LOS) for patients in both upstream and downstream sectors. Fig. 3 visually depicts the average number of hospitalization days (from 2003 to 2021) for patients in both upstream and downstream units.

ORs represent significant financial investments, with surgical procedures accounting for over 40% of hospitals' expenditures [13]. The planning and scheduling of OR activities pose considerable challenges, primarily due to two factors: firstly, the complexity of resource allocation (including ORs, surgical staff, etc.) for surgeries and their sequencing, and secondly, the inherent uncertainty surrounding related activities [14]. Critical resources essential for OR planning and scheduling, such as personnel (surgeons, nurses, etc.), surgical equipment, and bed availability in units such as the Pre-Operative Holding Unit (PHU), Post Anesthesia Care Unit (PACU), Intensive Care Unit

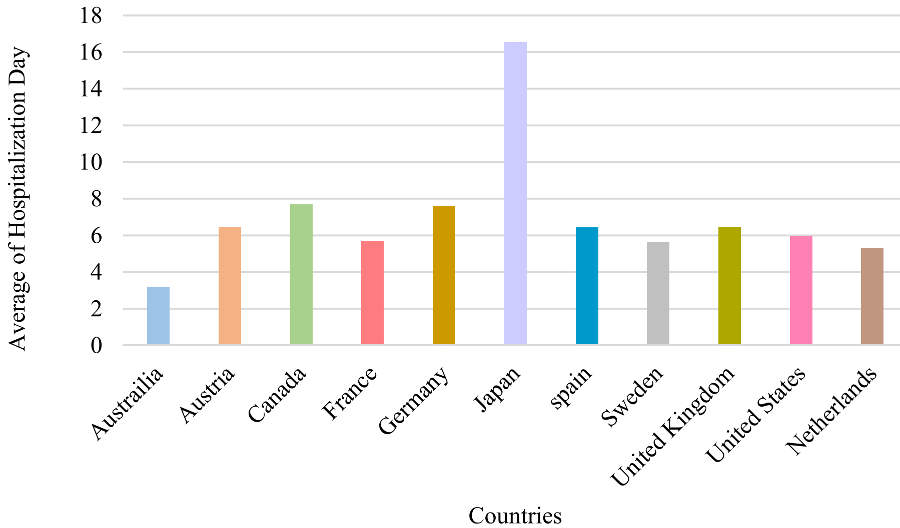

Fig. 3. The Average number of hospitalization days in upstream and downstream units for patients.

(ICU), and general wards, must be meticulously coordinated to ensure surgical readiness [15]. Recognizing capacity constraints and strategically identifying supplementary resources are vital for optimizing the utilization of available assets [16]. Furthermore, external factors such as high-pressure work environments (exemplified by pandemics like COVID-19 and mass casualty incidents) and resource scarcity persist as significant challenges [17]. Consequently, it becomes evident that the consideration of both upstream and downstream units holds paramount importance in comprehensive planning efforts.

The presence of stochastic elements is inherent to surgery scheduling dilemmas [18]. As underscored by Min and Yih [19], Shore [20], and Shehadeh and Padman [21], the variability in surgery duration and Length of Stay (LOS) within downstream units serves as a primary source of disruption to daily scheduling routines and patient flow downstream [22]. Fluctuations in LOS within units such as the PACU and downstream facilities engender unpredictable availability of recovery beds and contribute to congestion issues [23]. Fourati et al. [24] suggested an integrated dynamic satisfaction function and Lexicographic goal programming (GP) model so as to handle the scheduling of nurses, considering two related aspects of hospital regulation limitations. In practical scenarios, certain parameters remain subject to uncertainty, including surgery duration, LOS within upstream and downstream units, and emergency demand, all of which significantly impact operational dynamics. Effective and efficient management within ORs necessitates the explicit consideration of these uncertainties inherent to the surgical process. However, OR planning and scheduling have traditionally received limited attention due to the increased complexity introduced by such uncertainties [25]. Consequently, this paper adopts an integrated approach to address this multifaceted challenge. Moreover, the inclusion of emergency patients further compounds the complexity of the problem, given their potential arrival at any time of the day or night [26].

This study delves into the intricacies of integrated planning and scheduling within ORs, considering several constraints including OR availability, surgeon availability, and bed availability in both upstream and downstream units (PHU, PACU, ICU, and ward), alongside constraints on the maximum number of time slots allotted for each surgeon's surgeries within the planning horizon. The scheduling process encompasses both elective and emergency patients, employing an open strategy. In the open strategy, the available time blocks are not specialized to any surgeon or surgical group. Surgeons provide the list of their operations to the surgery department a few days in advance, and patients are scheduled according to the schedule and requests of all surgeons. Notably, the uncertainty surrounding surgery duration, Length of Stay (LOS) in upstream and downstream units, and emergency demands is duly acknowledged. A mixed-integer programming (MIP) model is formulated to address this complex scenario, with considerations given to idle time, waiting time, and allocation priorities for high-priority patients. Subsequently, a robust optimization approach is adopted to tackle the inherent uncertainty in the problem. To facilitate comparison of results, both weighting and epsilon-constraint methods are employed. The literature motivates us to explore why integrated OR review considering elective and emergency patients is important by addressing the following research questions.

(1) How does adding emergency patients affect the planning and scheduling of elective patients?
(2) How can the OR problem be formulated and solved by considering the upstream and downstream units and the inherent uncertainties in the surgical procedures?

The rest of the contributions of this paper are outlined as follows:

- Development of an innovative mixed-integer programming (MIP) model tailored for integrated OR planning and scheduling.
- Inclusion of considerations for both upstream and downstream units, encompassing the PHU, PACU, ICU, and ward.
- Accounting for uncertainty pertaining to surgery duration, LOS in upstream and downstream units, and emergency demand.
- Introduction of a robust optimization approach designed to effectively manage and mitigate uncertainty within the scheduling framework.

Each of the following sections in this paper investigates these contents: Section 2, the related literature review; Section 3, the problem description; Section 4, defining the proposed solution methods; Section 5, computational results and analysis; Section 6, achievements of managerial insights, and finally Section 7 conclusion.

2. Literature review

Addressing healthcare and treatment provision stands as a paramount concern, necessitating a scientific and pragmatic approach to mitigate rising costs and optimize resource utilization. This section aims to review pertinent literature focusing on models that incorporate uncertainty, particularly within the realm of OR scheduling, a prominent area in medical and operational research. Planning, defined as aligning process supply with demand, and scheduling, involving activity sequencing and timing, are fundamental aspects [27]. Erdogan and Denton [28], Cardoen et al. [1], and Guerriero and Guido

[29] have conducted extensive reviews on surgery scheduling issues. Molina-Pariente et al. [30] devised an integer programming model, incorporating surgeons' expertise's influence on surgery duration. Shylo et al. [31] tackled scheduling complexities through a block strategy, employing chance-constrained planning to manage OR overtime. Denton et al. [13] proposed a two-stage stochastic programming model accounting for surgery duration uncertainty. Moosavi and Ebrahimnejad [32] introduced a multi-objective mathematical model considering uncertain parameters such as surgery duration and Length of Stay (LOS) in upstream and downstream units, along with emergency demand. M'Hallah and Visintin [33] developed a stochastic model specifying surgery type and number, addressing stochastic elements like surgery and post-surgery LOS. Min and Yih [19] addressed scheduling incorporating patient preferences, framed as a stochastic dynamic programming model.

Kroer et al. [34] devised a stochastic model accommodating variable operation durations and unknown arrival times for emergency patients. Kamran et al. [35] addressed OR planning and scheduling, employing a stochastic MILP model capable of handling randomly distributed surgery durations alongside patient, staff, and surgeon preferences. Lee and Yih [36] incorporated constraints on recovery bed access and surgery duration uncertainty in their OR scheduling model. Vali-siar et al. [4]) investigated a multi-period, multi-resource OR integrated planning and scheduling problem, devising a MILP model. Kayvanfar et al. [37] proposed an optimization plan aimed at minimizing OR idle times and maximizing scheduled surgeries within optimal time windows. Mazloumian et al. [25] recently introduced an integrated OR planning and scheduling model, optimizing OR utilization rates and reducing patient waiting times while considering uncertain surgery durations and emergency arrivals.

The inclusion of constraints pertaining to human resources, equipment, and bed availability in units such as the PHU, PACU, ICU, and ward was paramount. Eun et al. [38] introduced the concept of maximizing the minimum patient health condition, considering uncertainty in operation durations. Jebali and Diabat [39] devised a two-stage chance-constrained stochastic programming model, accounting for random surgery durations, ICU LOS, and resources earmarked for emergencies.

Zhang et al. [40] investigated a stochastic elective surgery problem under downstream constraints, presenting a two-level optimization model integrating Markov decision processes (MDP) and stochastic programming. Khaniyev et al. [41] addressed the next-day scheduling dilemma in a single OR, incorporating uncertain durations for elective surgeries through a hybrid heuristic approach. Breuer et al. [42] recently proposed a robust optimization model to accommodate uncertainty and variations in procedure durations and surgeon availability. Rachuba et al. [43] integrated planning and scheduling challenges across ORs, clinical staff, and patient assignments. Their innovative chance-constraint optimization model addressed unscheduled arrivals, including emergencies, alongside uncertainties in surgery duration and LOS in ICU. They introduced a comprehensive planning framework incorporating both the optimization model and a simulation tool. Agrawal et al. [2] formulated a mathematical model considering stochastic surgery durations, proposing a suite of heuristics and employing Monte Carlo simulation techniques for problem resolution. Recently, Maleki et al. [44] studied a multi-stage linear mixed integer optimization model in order to optimize the operation time and resource allocation. They used the decision tree method to generate scenarios. They also applied Robust optimization and Upper partial moments methods to proposed efficient solutions. Bargetto et al. [45] addressed a new model of the sequence-dependent OR cleaning times. To solve the proposed planning and scheduling mathematical model, they devised a branch-and-price-and-cut algorithm. Yang et al. [46] focused on a resource-constrained OR scheduling problem for elective patients. They presented a mixed integer programming model to uncover the impact of no-wait and resources on the problem and used a slack speed-up based discrete artificial bee colony to solve the problem.

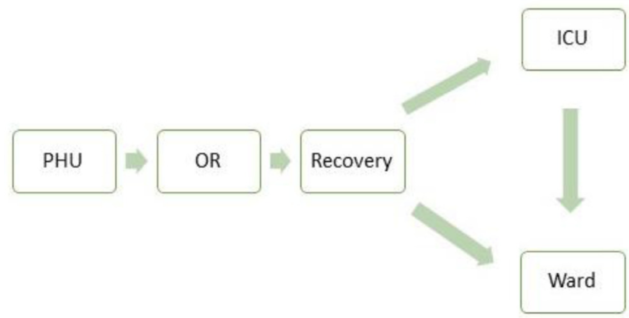

Fig. 4. The surgery process.

To facilitate comparison with recently published studies, their key characteristics have been synthesized and juxtaposed in Table 1. The literature review underscores a relative paucity of attention directed towards emergency patient management, constraints within upstream and downstream units, and uncertainty across all stages simultaneously, with only a few studies addressing these aspects collectively. Notably, meta-heuristic approaches have been predominantly employed for model resolution, with fewer instances of definitive solutions. To our knowledge, none of the existing literature has utilized constraint-based methods. In addressing these gaps, our objective is to augment the literature by providing a more precise and effective solution through rigorous examination.

The current investigation endeavors to comprehensively address significant uncertainties inherent in surgical processes, encompassing variables such as surgery duration, LOS in both upstream and downstream units, and the unpredictable arrival of emergency patients. Within this framework, a multi-objective model is proposed for orchestrating the planning and scheduling of ORs, with the primary aims of minimizing idle periods and patient waiting times, while maximizing the allocation priority for high-priority patients.

3. Problem description

The surgery process for each patient consists of three stages. In the first step, a bed is allocated to a patient, where a nurse checks the patient's condition. In the second phase, the patients are transported to the OR. In most hospitals, there are different ORs with different features. No difference is considered between surgeons and only their availability is important.

A suitable OR is assigned to a surgery by its type. Finally, after the surgery, the patient is transferred back to recovery. Depending on the patient's health condition, he/she is transported to ICU or ward. It should be mentioned that some patients do not need to be hospitalized and, shortly after surgery, can be discharged. Fig. 4 shows the different stages of the surgery process. In this study, emergency surgeries are included. They can affect the schedule and make changes to scheduled elective patients. In this regard, a MIP model is designed and proposed in this research.

The following assumptions are taken into account in this research:

1. Upstream and downstream units, including PHU, PACU, ICU and ward, are all considered.
2. The elective patients have priority. Considering that the list of elective surgeries and the status of people are known at the beginning of the planning, so the patients who have priority are known.
3. Emergency patients are taken into account. The condition of admitting an emergency patient is that if it takes less than one hour from the time of the patient's arrival to the emptying of the appropriate OR, the patient will be admitted, otherwise he/she will not be admitted.

Table 1
A summary of features of the selected studies after 2010.

Authors and years	Problem		Resources				Uncertainty				Patient	
	Planning	Scheduling	PHU	PACU	ICU	Ward	Surgery	Recovery	LOS	Emergency	Elective	Emergency
Min and Yih [19]	✔	✔			✔		✔					
Shylo et al. [31]		✔					✔				✔	
Lee and Yih [36]	✔	✔		✔			✔					
Molina-Pariente et al. [30]	✔	✔					✔				✔	
Jebali and Diabat [39]	✔				✔		✔		✔	✔	✔	✔
Kroer et al. [34]	✔	✔					✔			✔	✔	✔
Eun et al. [38]		✔					✔				✔	
Moosavi and Ebrahimnejad [32]		✔			✔	✔	✔		✔	✔	✔	✔
Vali-siar et al. [4]	✔	✔	✔	✔	✔	✔	✔	✔	✔		✔	
M'Hallah and Visintin [33]		✔			✔	✔	✔		✔		✔	
Zhang et al. [40]		✔			✔		✔		✔		✔	
Kamran et al. [35]	✔	✔					✔			✔	✔	
Breuer et al. [42]	✔	✔					✔			✔		
Khaniyev et al. [41]		✔					✔				✔	
Rachuba et al. [43]	✔				✔			✔	✔	✔	✔	✔
Mazlooumian et al. [25]	✔				✔		✔			✔	✔	✔
Agrawal et al. [2]		✔					✔				✔	
Maleki et al. [44]	✔	✔	✔	✔			✔				✔	✔
Bargetto et al. [45]	✔	✔									✔	
Yang et al. [46]		✔	✔	✔							✔	
Current investigation	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔

4. The same resources are considered for elective and emergency patients.
5. The ORs are different.
6. The duration of surgery (without considering cleaning and set up time), LOS in upstream and downstream units, as well as emergency demand are all considered uncertain.
7. The roster of elective patients is determined.
8. The time is divided into 20-min intervals.
9. Based on the condition of the patient after surgery, he/she is transferred to the intensive care unit or ward.

In order to introduce the model, notations, including indices, should be first defined.

3.1. Sets

p, b	Index for patients p=1,2,3,…,P b=1,2,3,…,B (b is just for showing the sequence)
$i(1,2,3)$	Index for PHU (i=1), operation (i=2), recovery (i=3)
$j(1,2)$	Index for ICU (j=1), ward (j=2)
o	Index for ORs o=1,2,3,…,O
s	Index for surgeons s=1,2,3,…,S
t	Index for time slots t=1,2,3,…,T
d, d'	Index for days d=1,2,3,…,D
e	Index for emergency patients e=1,2,3,…,E
o'	The rooms that are free to schedule the emergency patient.
s'	The surgeons that are free to do the operation on the emergency patient.

3.2. Parameters

lp	A binary parameter; if patient p needs ICU, =1, otherwise =0.
lw_p	A binary parameter; if patient p needs ward, =1, otherwise =0.
Bed_i	Total number of beds in stage i
be_p	The beginning of the best period of scheduling surgery for patient p
fi_p	The finishing of the best period of scheduling surgery of patient p
C_p	The point of priority of patient p(based on the urgency of the patient's condition)
M	A very large number
$Hmax$	The maximum time slots that a surgeon can do surgeries in the planning horizon
Nt	The normal time (last slot) for scheduling surgeries (OR opening hours)
Ot	The last time slot in opening hours (when no operation is planned for selected patients after that)
α	The occupancy level coefficient
du^1_{pi}	The duration of stage i for patient p
du^2_{pj}	The duration of stage j for patient p
Ca_{dj}	The capacity of stage j in day d
av_{sdt}	A binary parameter; =1 if surgeon s is not available at time t on day d; otherwise =0
n_{pos}	A binary parameter; =1 if the surgery of patient p can be scheduled in room o and surgeon s; otherwise = 0
ar_e	The arrival time of emergency patient e
da_e	The day of arrival emergency patient e
num_d	The number of emergency patients on day d
a_e	=1 if patient e needs to be in ICU; otherwise, = 0.
aw_e	=1 if patient e needs to be in the ward; otherwise, = 0.
len^1_{ei}	The duration of stage i for patient e
len^2_{ej}	The duration of stage j for patient e
lo	The earliest time that an OR is free.

3.3. Decision variables

x_{postd}	A binary variable; equals 1 if the surgery of patient p is scheduled in o$^{\text{th}}$ OR with surgeon s at time t on day d; otherwise =0
t^1_{pi}	The start time of stage i for patient p
t^2_{pj}	The finishing time of stage i for patient p
λ_{ptdi}	A binary variable; =1 if stage i for patient p in time t and day d is scheduled; otherwise =0.
xi_{pd}	A binary variable; =1 if patient p needs to be in ICU on day d; otherwise = 0.
xw_{pd}	A binary variable; =1 if patient p needs to be in the ward on day d; otherwise = 0.

idt_{od}	The idle time of the OR o in day d
R_p	A binary variable; =1 if patient p is scheduled in the best period; otherwise = 0.
f_{pbd}	A binary variable; =1 if patient b is operated after p on day d; otherwise = 0.
w_{pbd}	A binary variable; =1 if patient b goes to recovery after p on day d; otherwise = 0.
y_{eostd}	A binary variable; =1 if patient e is scheduled in oth OR with surgeon s on day d; otherwise = 0.
tb_{ei}	The start time of stage i for patient e
tf_{ei}	The finishing time of stage j for patient e
yi_{ed}	A binary variable; =1 if patient e needs to be in ICU; otherwise = 0.
yw_{ed}	A binary variable; =1 if patient e needs to be in the ward; otherwise = 0.

3.4. Mathematical model

$$Min \sum_o \sum_d idt_{od}$$
$$Min \sum_p \sum_o \sum_s \sum_t \sum_d x_{postd} * d \tag{1}$$
$$Max \sum_p C_p R_p$$

Subject to:

$$\sum_o \sum_s \sum_d x_{postd} \le 1 \qquad \forall p.t \tag{2}$$
$$\sum_p \sum_o x_{postd} \le 1 \qquad \forall s.t.d \tag{3}$$
$$\sum_p \sum_o \sum_s \sum_t \sum_d x_{postd} = 0 \qquad \forall t > Ot \tag{4}$$
$$\sum_o x_{postd} \le \left(1 - av_{std}\right) \qquad \forall p.s.t.d \tag{5}$$
$$\sum_t \sum_d x_{postd} \le n_{pos} \qquad \forall p.o.s \tag{6}$$
$$t^1_{pi} = t^2_{p.i-1} \qquad \forall p.i \ne 1 \tag{7}$$
$$t^2_{pi} \ge t^1_{pi} + du^1_{pi} \sum_o \sum_s \sum_t \sum_d x_{postd} \qquad \forall p.i \tag{8}$$
$$\sum_p \delta_{ptdi} \le Bed_i \qquad \forall t.d.i \ne 2 \tag{9}$$
$$idt_{od} = NT - \sum_p \sum_s \sum_t x_{postd} \qquad \forall o.d.t \le NT \tag{10}$$
$$\sum_p l_p \sum_p \sum_o \sum_s \sum_t x_{postd} \le \alpha Ca_{dj} - \sum_p xi_{pd} \qquad \forall d.j = 1 \tag{11}$$
$$xi_{pd} \ge \sum_o \sum_s \sum_t x_{postd'} l_p \qquad \forall p.d.d'.j.d' \le d < d' + du^2_{pj} \tag{12}$$
$$t^1_{pi} \le M\left(\sum_o \sum_s \sum_t \sum_d x_{postd}\right) \qquad \forall p.i \tag{13}$$
$$\sum_t \sum_d \delta_{ptdi} = t^2_{pi} - t^1_{pi} \qquad \forall p.i \tag{14}$$
$$\sum_p \sum_o \sum_t \sum_d x_{postd} \le Hmax \qquad \forall s \tag{15}$$
$$xw_{pd} \ge \sum_o \sum_s \sum_t x_{postd'} lw_p \qquad \forall p.d.d'.j = 2.d' \le d < d' + du^2_{pj} \tag{16}$$
$$\sum_p \sum_o \sum_s \sum_t x_{postd} \le Ca_{dj} - \sum_p xw_{pd} \qquad \forall d.j = 2 \tag{17}$$
$$\sum_o \sum_s \sum_t \sum_d x_{postd} = R_p \qquad \forall p.be_p < d < fi_p \tag{18}$$
$$f_{pbd} + f_{bpd} \le 1 + M \times \left(2 - \sum_o \sum_s \sum_t x_{postd} - \sum_o \sum_s \sum_t x_{bostd}\right) \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{19}$$
$$f_{pbd} + f_{bpd} \ge 1 - M \times \left(2 - \sum_o \sum_s \sum_t x_{postd} - \sum_o \sum_s \sum_t x_{bostd}\right) \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{20}$$
$$w_{pbd} + w_{bpd} \le 1 + M \times \left(2 - \sum_o \sum_s \sum_t x_{postd} - \sum_o \sum_s \sum_t x_{bostd}\right) \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{21}$$
$$w_{pbd} + w_{bpd} \ge 1 - M \times \left(2 - \sum_o \sum_s \sum_t x_{postd} - \sum_o \sum_s \sum_t x_{bostd}\right) \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{22}$$
$$f_{pbd} + w_{pbd} \le 2 \sum_o \sum_s \sum_t x_{postd} \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{23}$$
$$f_{pbd} + w_{pbd} \le 2 \sum_o \sum_s \sum_t x_{bostd} \qquad \forall p.b.p \ne b.p < b.\ \forall d \tag{24}$$
$$\sum_o \sum_s y_{eostd} = 1 \qquad \forall e.t.\,if\ o \in o'.\,if\ s \in s'.d = da_e \tag{25}$$
$$lo - ar_e \le 1 + M\left(1 - \sum_o \sum_s \sum_t y_{eostd}\right) \qquad \forall e.d.\,if\ o' = 0 \tag{26}$$
$$lo - ar_e \ge 1 - M\left(\sum_o \sum_s \sum_t y_{eostd}\right) \qquad \forall e.d.\,if\ o' = 0 \tag{27}$$
$$tf_{ei} \ge tb_{ei} + len^1_{ei} \sum_o \sum_s \sum_t \sum_d y_{eostd} \qquad \forall e.i \tag{28}$$
$$y_{eostd} = 0 \qquad \forall d \ne da_e.e.o.s.t.d \tag{29}$$
$$tb_{ei} = tf_{ei-1} \qquad \forall e.i \ne 1 \tag{30}$$
$$yi_{ed} \ge \sum_o \sum_s \sum_t y_{eostd'} * a_e \qquad \forall e.d.d'.j = 1.d' \le d < d' + len^2_{ej} \tag{31}$$
$$yw_{ed} \ge \sum_o \sum_s \sum_t y_{eostd'} * aw_e \qquad \forall e.d.d'.j = 2.d' \le d < d' + len^2_{ej} \tag{32}$$
$$\sum_e \sum_o \sum_s \sum_t y_{eostd} \le num_d \qquad \forall d \tag{33}$$
$$\sum_p \sum_s x_{postd} + \sum_e \sum_s y_{eostd} \le 1 \qquad \forall o.t.d \tag{34}$$
$$tb_{ei} \le M\left(\sum_o \sum_s \sum_t \sum_d y_{eostd}\right) \qquad \forall e.i \tag{35}$$
$$x_{postd}, \sigma_{ptdi}, xi_{pd}, R_p, f_{pbd}, w_{pbd}, y_{eostd}, yi_{ed}, yi_{ed}, \in \{0.1\}$$
$$t^1_{pi}, t^2_{pi}, idt_{od}, tb_{ei}, tf_{ei} \ge 0 \tag{36}$$

The objective function presented in Eq. set (1) consists of minimizing idle and waiting times as well as maximizing the allocation points of high-priority patients. Constraint (2) ensures that only one OR and one surgeon can be allocated to each patient at each time slot. Constraint (3) states that each surgeon can do one surgery at a time slot per day. Constraint (4) ensures that after the last time slot, no surgery can be performed. Constraint (5) states that the surgeon exists at the scheduled time. Constraint (6) ensures that the assigned OR for each surgery is suitable. Constraint (7) is related to the sequence of each stage. Constraint (8) determines the finish times for each stage. Constraint (9) ensures that in each slot, the allocated patients to PACU or PHU are less than the maximum number of beds per day. Constraint (10) computes the idle time for each OR on each day. Constraint (11) ensures that the number of patients in the ICU should not exceed the number of available beds. Constraint (12) states that the patient will stay in ICU if the healing period is not over. Constraint (13) ensures that if the patient's surgery is not scheduled, the start time will be zero. Constraint (14) guarantees that the difference between the beginning and finishing times equals the assigned time slots. Constraint (15) determines the maximum time slots that each surgeon can do surgeries during the planning horizon. Constraint (16) confirms that the patient will stay in the ward if the healing period is not over. Constraint (17) limits the number of patients in the ward by considering the available capacity each day. Constraint (18) computes the patients who are scheduled in their allowed period. Constraints (19) and (20) show the condition of patients relative to each other in the OR. Constraints (21) and (22) show the condition of patients relative to each other in recovery. Constraints (23) and (24) ensure that when a patient is scheduled, its corresponding sequence variable will take the value of one. Constraint (25) ensures that if, at the time of the emergency patient's arrival, at least one OR is empty, the patient will be accepted. Constraint (26) and (27) state that when all ORs are full at the arrival time, the patient will be accepted if at least one OR be free for less than one hour. Constraint (28) computes the finish time of each stage. Constraint (29) ensures that in case of accepting an emergency patient, he/she will be operated in the arrival day. Constraint (30) is related to the sequence of stages. Constraint (31) ensures that the emergency patient will stay in ICU if the healing period is not over. Constraint (32) guarantees that the patient will stay in the ward if the healing period is not over. Constraint (33) confirms that the number of operated emergency demands will be less than or equal to the whole demand. Constraint (34) ensures that just one patient of elective or emergency will be scheduled in a one-time slot in a day in one OR. Constraint (35) ensures that if the patient is not operated on, the start time will be zero. Constraint (36) delineates the type of decision variables.

Constraints (25) to (35) are related to emergency patients. After adding emergency patients, Eq. (10), which measures the idle time, should consider both elective and emergency patients. So, it is changed to constraint (37) as follows:

$$idt_{od} = NT - \sum_{p}\sum_{s}\sum_{t} x_{postd} - \sum_{e}\sum_{s}\sum_{t} y_{eostd} \qquad \forall o.d.t \le NT \tag{37}$$

To solve the model considering both kinds of elective and emergency patients, all of the above constraints ((2) to (35)) must be considered.

3.5. The model under uncertainty

In this study, the duration of surgery, LOS in upstream and downstream units (PHU, PACU, ICU, and ward) and the emergency demand are considered uncertain. To handle such uncertainty, a robust optimization approach is employed, proposed first by Soyster [47]. Among those who used this approach, one can mention Wang et al. [48], which applied a robust linear programming approach to deal with the surgery scheduling problem with uncertainty. Also, Liu et al. [49] proposed a scenario-based robust optimization approach to handle the uncertainty in surgery duration. We used the Soyster method because, to the best of our knowledge, this method has not been used in any research in this field, and this method helps the hospital management to prepare itself in the best way by considering the worst conditions. Fig. 5 illustrates a method with high conservatism, so-called the worst-case scenario. Considering that unpredictable occurrences could have happened in OR against the already planned schedule, one can handle it through choosing this method. In this regard, we try to examine the result by considering the worst-case scenario and providing a solution for it. Based on Soyster's method, one can have:

$$U_{\infty} = \{\xi \| \|\xi\| \le \varphi\} = \{\xi \| \|\xi_j\| \le \varphi \forall j \in J_i\}$$
$$\varphi = 1, \xi[-1.1]$$

In the proposed model in Section 3, constraints 8, 12, 16, 26, 27, 28, 31, 32, and 33 include uncertain parameters, comprising uncertain parameters du^1_{pi}, du^2_{pj}, ar_e, len^1_{ei}, len^2_{ej}, num_d. For example, in constraint (38), the du^1_{pi} is assumed uncertain. So, one can rewrite the constraints (38) to (44), including uncertain parameters as follows.

$$\widetilde{du}^1_{pi} = du^1_{pi} + \xi_{pi} * \widehat{du}^1_{pi} \tag{38}$$

Where du^1_{pi} is called the nominal value, and du_{pi} is the deviation from the nominal value. $\xi \in [-1,1]$, and the value of Ψ is considered 1.

$$\widetilde{num}_d = num_d + \xi_d * \widehat{num}_d \tag{39}$$

$$\widetilde{du}^2_{pj} = du^2_{pj} + \xi_{pj} * \widehat{du}^2_{pj} \qquad \forall p.j = 1 \tag{40}$$

$$lo - ar_e \le 1 + M\left(1 - \sum_{o}\sum_{s}\sum_{t} y_{eostd}\right) \qquad \forall e.d.o' = 0.\ \widetilde{ar}_e = ar_e + \xi_e * \widehat{ar}_e \tag{41}$$

$$-lo + ar_e + 1 \le M\left(\sum_{o}\sum_{s}\sum_{t} y_{eostd}\right) \qquad \forall e.d.o' = 0.\ \widetilde{ar}_e = ar_e + \xi_e * \widehat{ar}_e \tag{42}$$

$$-tf_{ei} \le -tb_{ei} - len^1_{ei}\sum_{o}\sum_{s}\sum_{t}\sum_{d} y_{eostd} \qquad \forall e.i.\ \widetilde{len}^1_{ei} = len^1_{ei} + \xi_{ei} * \widehat{len}^1_{ei} \tag{43}$$

$$d < d' + len^2_{ej} \qquad \forall e.j = 1.\ \widetilde{len}^2_{ej} = len^2_{ej} + \xi_{ei} * \widehat{len}^2_{ej} \tag{44}$$

4. Solution method

The presented MIP model studied in Section 3 is based on the approach proposed by Vali-siar et al. [4]. They studied a multi-period multi-resource OR integrated planning and scheduling model and considered the constraints related to human resources, downstream and upstream units' beds, as well as equipment. They also assumed that PHU, surgery and recovery duration is uncertain and used a robust optimization approach to handle the uncertainty. Then, they used a metaheuristic method based on the genetic algorithm (GA) to solve their on-hand problem on a large scale.

The surgery planning and scheduling problem addressed in this paper is related to the study conducted by Vali-siar et al. [4]. The main difference between the two papers is that we attempted not only to assign elective patients, but also to consider emergency patients. Also, we considered priority for elective patients as well. Another difference between the two papers is the objective function. We considered minimizing the idle and waiting time and maximizing the allocation points of high-priority patients, while [4] aimed to minimize the tardiness, idle and over time. The last difference is that we proposed the occupancy level coefficient for ICU which has not been studied in literature so far. Approaching the Pareto-optimal solutions is the main goal of the multi-objective procedure [50]. For solving such a complex problem, an improved version of the eps-constraint method (Augmented eps-constraints) is used. Since there are three objectives in the proposed model, the improved version of the eps-constraint could be an appropriate method. First, the suggested model considering certainty is presented and solved. Then, a robust optimization method, as a decision-making approach under uncertainty, is used to manage the existing uncertainty. Since, in stochastic programming, distributional information about the uncertain parameters is needed, this approach is appropriate when the probability distribution of uncertain parameters is difficult to estimate.

$$\sum_{j} a_{ij}x_j + \left[\max_{\xi \in U}\left[\sum_{j \in J_i} \xi_{ij}\hat{a}_{ij}x_j\right]\right] \le b_i \tag{45}$$

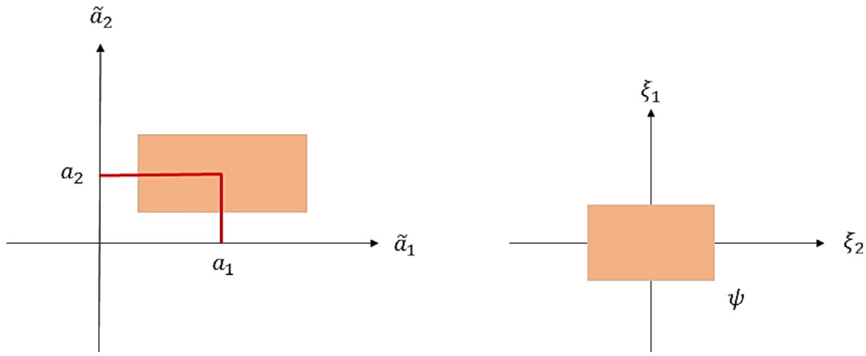

Fig. 5. U set-in box uncertainty set.

$$\sum_j a_{ij}x_j + \left[\Psi \sum_{j\in j_i} \hat{a}_{ij}\left|x_j\right|\right] \le b_i \tag{46}$$

Constraints (45) and (46) show the generality of the robust optimization method. The value of Ψ is considered 1 in Soyster's method. We could successfully handle the problem of up to 40 patients, and one can say that solving such a OR planning and scheduling problem with a high number of constraints is prominent. The computations for up to 27 patients were performed on a computer with 8 GB RAM and 2.20 GHz, running on Windows 7 (64-bit), in an acceptable computational time, while for 30–40 patients, the NEOS Server was used. To compare the results, the models were solved by the eps-constraint method and weighting approach. The capacity of each part increased by growing the number of patients, which means that capacities are assumed to be proportional to the demands. This is more interesting when one knows that in real-world conditions, finding acceptable and suitable answers is necessary.

4.1. Eps-constraint method

Multi-objective programming has more than one objective function, and there is no optimal solution to optimize all the objective functions simultaneously. A practical technique to solve these problems is the ε-constraint method. To search for the effects of the desired functions on each other, as well as to make a fair compromise between them, an augmented version of the epsilon-constraint technique [51] is employed to solve the proposed multi-objective surgical ward planning and scheduling problem. The eps-constraint method is for producing efficient Pareto-optimal non-dominated solutions in multi-objective problems. It has been proved that the generated optimal solutions are efficient solutions to the multi-objective problem. The eps-constraint method consists of two phases: (1) creation of the pay-off table (Table 2), (2) use of the ranges from the pay-off table in order to apply this method. The algorithm can also work with MIP models [51] since the eps-constraint method transfers a multi-objective model to a single objective model.

The ε-constraint is an optimization algorithm working with the predefined virtual grid in the objective space and solving different single objective functions [52]. The readers can refer to Mavrotas [51] for more explanations about this method.

5. Computational experiments

In this section, the proposed method and mathematical model have been applied to 12 sample problems on different scales. The number of patients, ORs, surgeons, days, beds of upstream and downstream units and ORs' active hours are changed proportionately with the model's size. The duration of surgery varies between 20 to 240 min, and LOS in ICU and ward ranges between 0 to 4 days. Vali-siar et al. [4] estimated the durations of surgeries via log-normal distribution. They also estimated the duration of the patient's recovery by log-normal similar to Jebali et al. [53]. In this study, all durations are generated to the minute and then rounded since 20-min time slots are taken into account.

Table 2
Datasets used for computational analysis.

Total number of allowed time slots	12–24
The duration of PHU, surgery and PACU	20–240 (min)
The occupancy level coefficient	1
LOS in ICU	0–4 days
LOS in ward	0–4 days
Priority of patients	1–10
Number of emergency patients in each day	0–2
Hmax	60–80

In each surgical specialty, the percentage of patients is selected randomly. Also, the number of ORs, beds in upstream and downstream units, the duration of surgeries and LOS are generated randomly to find the best answer. The numerical examples are used to compare the results of certain and uncertain models, where the results are compared in three different parts. First, a comparison is conducted between the results of the certain model with elective patients. In the second part, the outcomes of the model for elective patients under uncertainty are reported at first, and then a robust optimization technique is used to manage uncertainty. Next, the proposed model for elective and emergency patients under uncertainty is presented. The MILP model is coded in GAMS 24.1.2, and the improved version of the eps-constraint is used to solve the problem. Tables 2 and 3 show the datasets used in this problem.

In Table 4, the solution results of certain models are presented in terms of the number of scheduled patients, the value of the objective functions, and the CPU time. As long as we do not know the priorities of the decision-maker, all Pareto-optimal solutions would be the same. Since in a multi-objective optimization problem, the probability of optimizing all objective functions simultaneously is very low, when there is a conflict between the objectives, improving one of them worsens the other ones.

In Table 5, the results of elective patients under uncertainty are provided. Given that Soyster's method is the most pessimistic and conservative, the number of scheduled patients has decreased and, in some cases, has not changed.

Finally, in Table 6, the results of the elective and emergency patients' model under uncertainty are reported. As can be seen, and according to the obtained results, one can say that the number of scheduled patients in each model is acceptable.

As can be seen in Figs. 6 and 7, and as expected, by increasing the number of patients and resources, the execution time is increased. By adding emergency patients, the number of scheduled elective patients is decreased. Fig. 8 shows this downward trend.

Table 3
The datasets used in the problems.

No. of patients	No. of rooms	ICU beds	Ward beds	Surgeons	days	Time slots
10	4	4	7	4	3	12
13	4	4	7	4	3	12
15	4	4	7	4	3	12
17	4	5	7	4	4	14
	4	5	7	4	4	14
23	5	8	10	5	4	20
25	5	9	12	5	4	20
27	6	10	11	5	4	20
30	6	11	12	5	4	20
33	6	10	11	5	5	24
35	6	12	14	5	5	24
40	6	12	14	6	5	24

Table 4
The outputs of the certain model.

No. of patients	Scheduled patients	Objective function #1	Objective function #2	Objective function #3	Execution time
10	8	135	18	59	477.566
		136	15	59	
		137	12	52	
		138	9	49	
		140	5	38	
		141	3	28	
13	9	135	18	62	726.746
		136	15	71	
		137	12	51	
		138	9	47	
		140	5	40	
		141	3	30	
15	11	132	24	98	860
		134	18	92	
		135	15	62	
		136	12	76	
		138	8	36	
17	14	204	50	99	1026.829
		208	34	63	
		211	24	97	
		216	11	58	
20	16	172	50	87	1092.69
		176	34	113	
		179	24	77	
		184	11	36	
23	19	348	80	83	1408.75
		353	60	120	
		359	39	116	
		366	20	88	
25	21	372	70	88	4609.62
		377	50	105	
		382	33	137	
		388	17	61	
27	24	440	100	132	4885.826
		447	72	139	
		454	48	136	
30	27	440	83	167	5553.541
		447	75	199	
33	28	425	80	217	4069.956
		452	67	193	
35	28	555	135	177	12155.514
		600	107	217	
40	29	436	110	204	12015.821
		480	77	141	

Figs. 9, 10, 11, and 12 show the Pareto-optimal front and Pareto surface of two instances. Overall, it can be concluded that coding a model with three functions and solving it via the improved eps-constraint method can provide a suitable outcome. By increasing the size of the examples, the number of cuts reduces, so the Pareto-optimal solutions will be reduced. Vali-siar et al. [4] assumed deterministic LOS in the ICU and ward, while the scheduling can be affected by uncertainty in this stage. Since the capacity of the ICU and ward are limited, and the LOS in these stages are uncertain in this study, these parameters are considered uncertain in order to make the conditions as realistic as possible.

Table 5
The output of the robust model for elective patients.

No. of patients	Scheduled patients	Objective function #1	Objective function #2	Objective function #3	Execution time
10	8	135 136 137 138 140 141	18 15 12 9 5 3	52 43 51 45 36 28	365.619
13	9	133 135 136 137 139 141	21 15 12 10 6 3	70 60 53 53 46 31	947.550
15	10	132 134 136 138 140	24 18 12 8 4	58 85 58 53 29	899.392
17	14	206 209 212 217	43 31 21 9	54 73 74 66	2320.749
20	13	368 372 376 382 388	80 64 48 30 16	65 95 72 93 52	6397.92
23	14	348 352 356 362 368	80 64 48 30 16	81 73 57 84 56	5195.816
25	17	364 370 376 385	90 66 45 21	72 111 68 77	5006.586
27	17	447 452 458 465	80 59 39 20	110 123 118 37	7689.804
30	18	446 448	82 73	144 139	8561.82

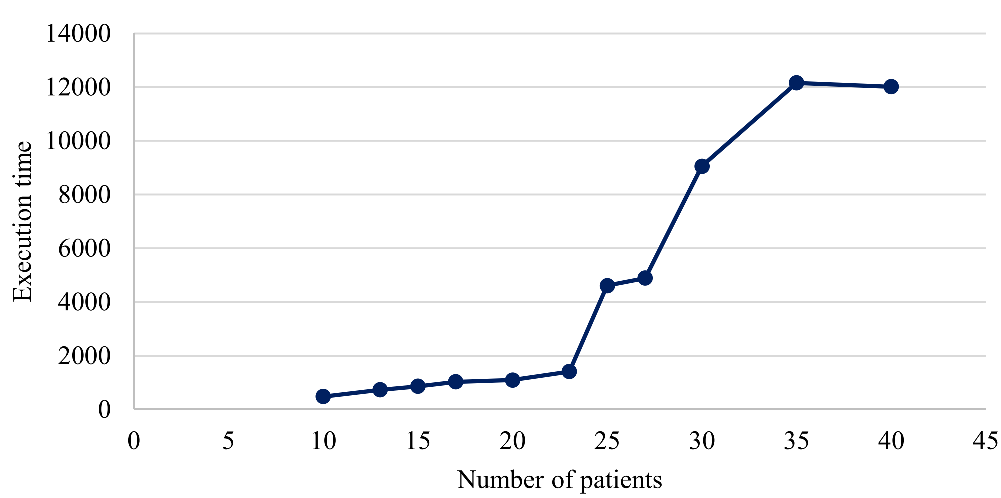

Fig. 6. The execution time of the certain model.

According to the capacity of the ICU and its effect on the scheduled patients and performed sensitivity analysis, it can be concluded that there is a bottleneck in the system/model which can affect the number of scheduled patients. The conducted sensitivity analysis is carried out to yield an appropriate vision for the managers and practitioners in the field, including the existing bottlenecks in the system (see Table 7).

Figs. 14 and 15 show the contradiction between the objective functions. As it is obvious, improving one objective function makes

Table 6
The output of the robust model for elective and emergency patients.

No. of elective patients	No. of scheduled elective and emergency patients	Objective function #1	Objective function #2	Objective function #3	Execution time
10	8+4	128	19	64	1423.369
		130	13	65	
		131	10	55	
		132	8	54	
		133	6	41	
		135	3	30	
13	9+4	127	21	71	1013.196
		129	15	63	
		130	12	53	
		131	10	53	
		133	6	49	
		135	3	29	
15	10+3	88	26	82	1136.857
		90	20	91	
		92	14	86	
		94	10	68	
		97	5	51	
17	10+6	197	44	56	1280.878
		200	32	73	
		203	22	92	
		208	10	68	
20	11+6	226	53	72	5852.643
		230	37	98	
		234	24	75	
		239	12	77	
23	14+6	340	80	56	6464.488
		344	64	89	
		348	48	77	
		354	31	103	
25	16+6	204	70	80	6513.962
		207	58	78	
		214	33	107	
		223	11	56	
27	18+5	437	83	113	10121.127
		443	59	138	
		449	40	135	
		457	19	53	
30	17+5	440	82	139	13741.123
		472	73	144	
33	19+10	665	84	154	13863.854
		675	80	133	
35	25+5	535	91	193	20448.31
		653	100	168	

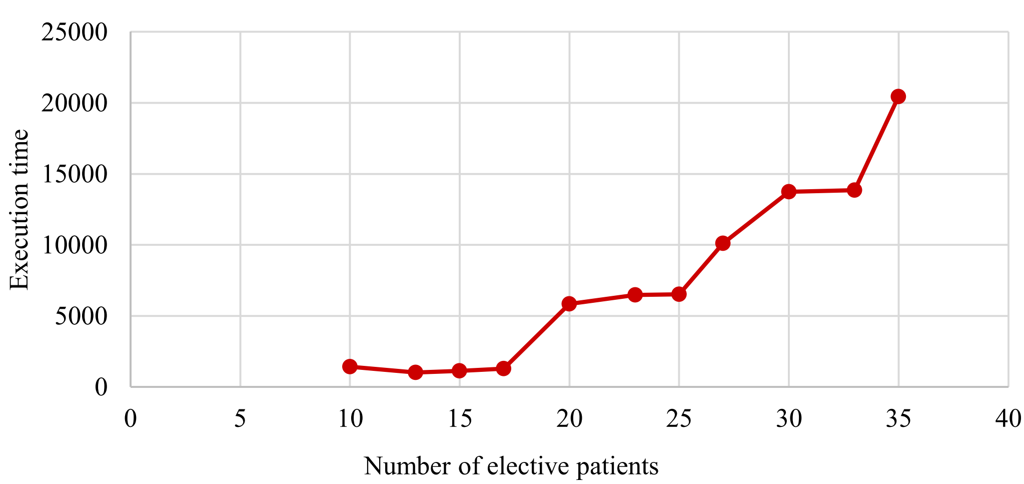

Fig. 7. The execution time of the robust model for elective and emergency patients.

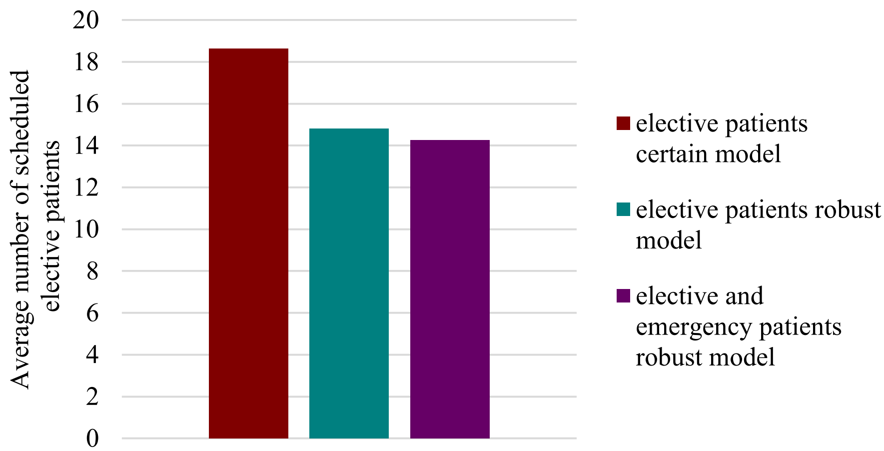

Fig. 8. Comparison of the mean number of scheduled patients.

Table 7
The scheduled patients with $\alpha = 0.8$.

No. of patients	No. of scheduled patients	Objective function #1	Objective function #2	Objective function #3	Execution time
15	9	136 137 138 140 141	15 12 9 5 3	60 31 40 38 26	735.186
17	12	208 210 212 214 216	40 32 24 18 12	85 78 90 88 41	1625.983
27	23	448 453 459 466	80 60 39 20	111 77 130 84	6608.920
30	26	452 454	80 59	199 175	7727.782

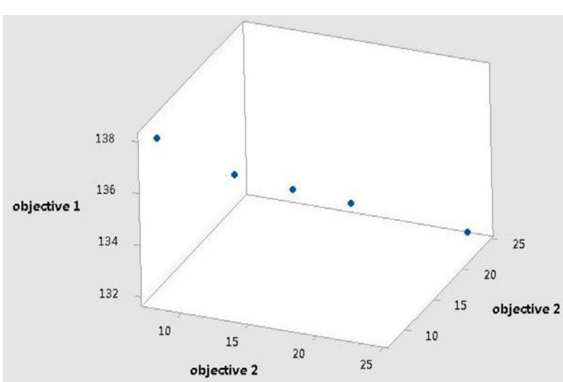

Fig. 9. Pareto front for 15 patients (certain).

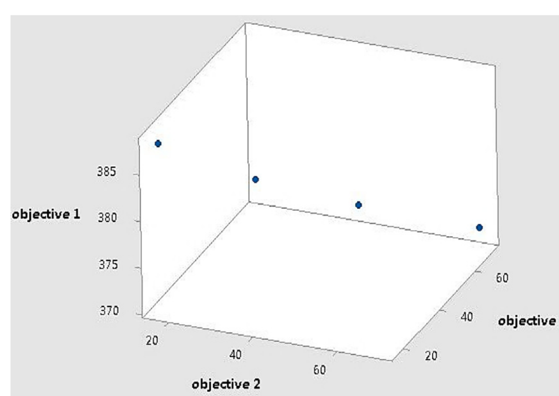

Fig. 10. Pareto front for 25 patients (certain).

another worse. According to the authors' exploration of the literature, it sounds that the majority of researchers have used a weighting approach in these conditions. To conduct an analysis, the obtained results of the improved version of the eps-constraint and weighting approach are compared. By doing so, the decision maker's priority is taken into account from the beginning.

In order to more closely examine the impact of ICU capacity on the number of patients scheduled for surgery, in Fig. 16, we considered issues with the number of different capacities. First, we solved the problem considering the capacity of 2 and then increased the capacity to 5. As it is clear from the results and we expected the number of planned patients would increase. In fact, based on Figs. 13, 14, 15, and

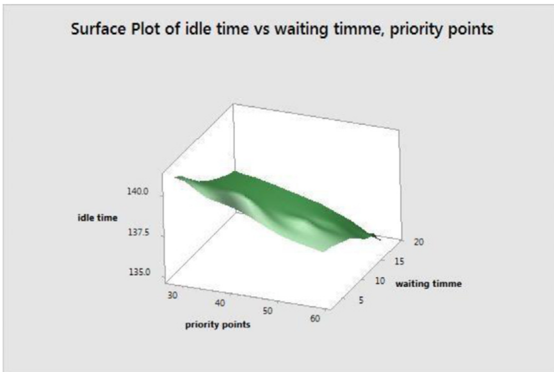

Fig. 11. Pareto front surface for 15 patients.

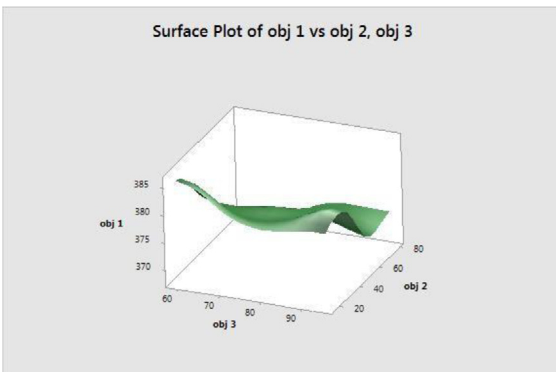

Fig. 12. Pareto front surface for 25 patients.

16, it can conclude that it acts like a bottleneck in the model, and by allocating a suitable and reasonable capacity based on the opinion of the hospital management and considering economic issues, the impact of this bottleneck on the number of planned patients can be greatly reduced.

In this regard, different weights are considered to achieve more results, as reported in Tables 8 and 9. As one can imply, when the assigned weight to the second objective increases, the number of selected patients for surgery reduces, and when the weights of objective functions #1 and #3 increase, the number of patients increases, too. However, totally the obtained results of the weighting approach demonstrate that the average number of scheduled patients in comparison to the eps-constraint is lower. By using the eps-constraint method, Pareto-optimal fronts are attained for each sample, and accordingly, this method is proposed.

5.1. Managerial insights

To have a better sense of the impact of this research, some important managerial insights could be presented according to the obtained results. The conducted analyses could be useful for either helping managers to make a specific decision or avoiding a wrong option, etc. The findings of the current study have fundamental managerial implications. Since the ORs review is considered in relation to the upstream and downstream units, the study of ORs should not be carried out as an isolated unit. By doing so, hospitals can benefit from the findings of such analyses.

As mentioned, ICU has been detected as a bottleneck in such a model, and accordingly, as one of the most important implications, it is strongly recommended to start the planning from the ICU since it would be more efficient altogether. Second, it would be effective to take priority for the objective functions into account since it can reduce the number of patients and also can overshadow other features, such as increasing the idle time of ORs and the number of surgeries carried out by each surgeon.

Table 8
The used weights in the weighting method (elective patients' certain model).

W1	W2	W3	No. of scheduled patients
		10 patients	
0.8	0.1	0.1	7
0.7	0.1	0.2	7
0.6	0.1	0.3	7
0.6	0.3	0.1	5
0.3	0.5	0.2	5
0.2	0.6	0.2	5
0.1	0.7	0.2	4
0.1	0.1	0.8	7
		15 patients	
0.8	0.1	0.1	11
0.7	0.1	0.2	11
0.6	0.1	0.3	11
0.6	0.3	0.1	11
0.3	0.5	0.2	6
0.2	0.6	0.2	5
0.1	0.7	0.2	5
0.1	0.1	0.8	11
		20 patients	
0.8	0.1	0.1	19
0.7	0.1	0.2	19
0.6	0.1	0.3	19
0.6	0.3	0.1	17
0.3	0.5	0.2	14
0.2	0.6	0.2	9
0.1	0.7	0.2	8
0.1	0.1	0.8	19
		25 patients	
0.8	0.1	0.1	22
0.7	0.1	0.2	22
0.6	0.1	0.3	21
0.6	0.3	0.1	21
0.3	0.5	0.2	16
0.2	0.6	0.2	12
0.1	0.7	0.2	9
0.1	0.1	0.8	21
		30 patients	
0.8	0.1	0.1	28
0.7	0.1	0.2	26
0.6	0.1	0.3	26
0.6	0.3	0.1	20
0.3	0.5	0.2	21
0.2	0.6	0.2	14
0.1	0.7	0.2	26
0.1	0.1	0.8	26

6. Conclusion and future streams

Surgical ward planning and scheduling are one of the most important issues among healthcare problems. Many researchers have proposed several different methods to improve the quality of planning and optimal use of resources. In this paper, a multi-objective mathematical model was presented for planning and scheduling elective and emergency patients by considering integrated ORs under uncertainty. Beds of the Pre-Operative Holding Unit (PHU), recovery, Intensive Care Unit (ICU), and ward were taken into account in this study. To handle the on-hand problem, a mixed integer programming (MIP) model was also formulated. Constraints related to the patients' priority, downstream and upstream limitations, different ORs and surgeons' availability were taken into consideration, as well. The model consists of three objectives, where the first one minimizes idle time, the second one minimizes waiting time, and the last one maximizes the allocation

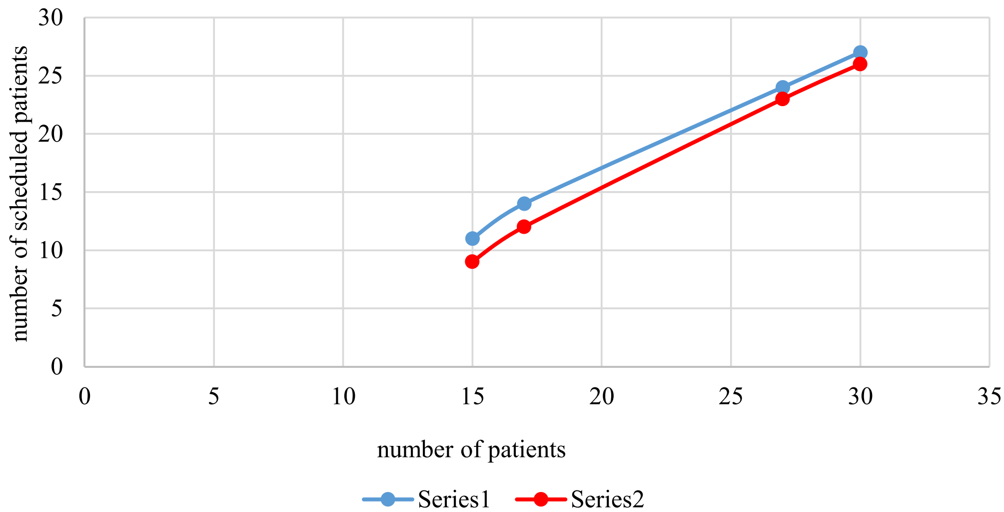

Fig. 13. The scheduled patients with $\alpha = 1$ and $\alpha = 0.8$.

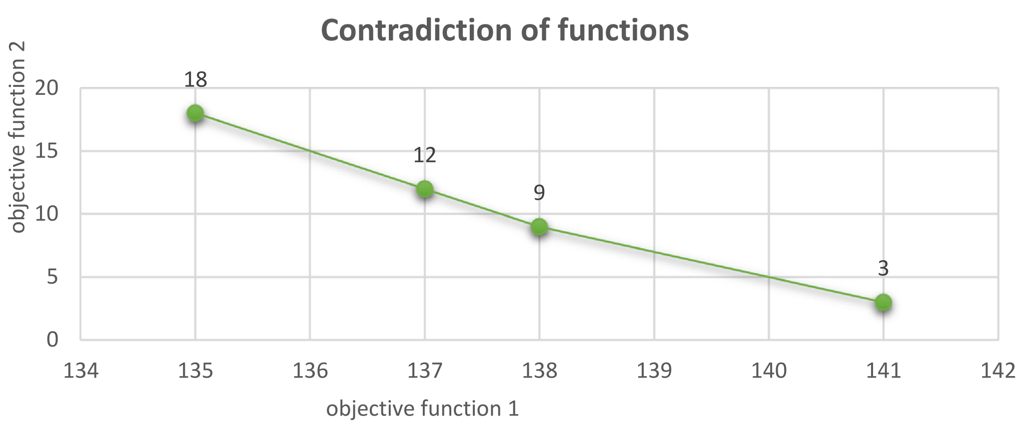

Fig. 14. Contradiction between the objective functions #1 and #2.

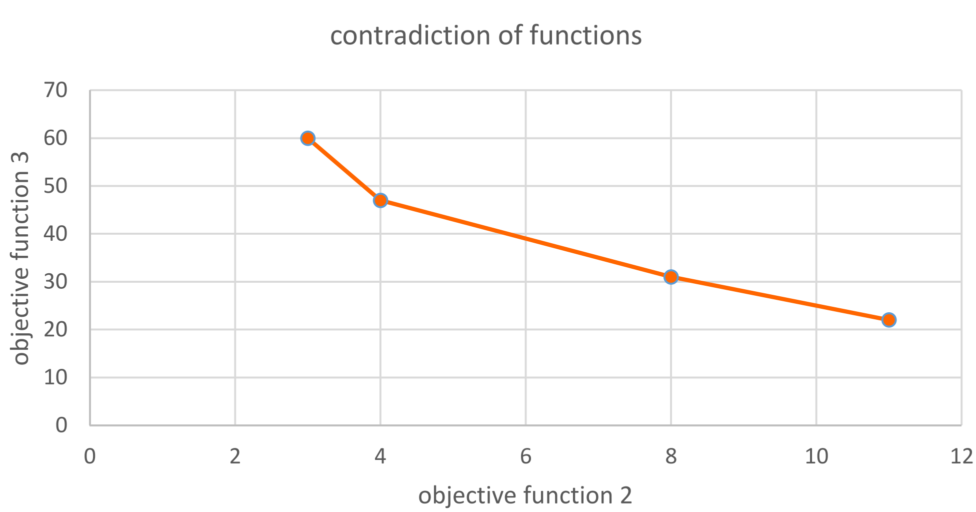

Fig. 15. Contradiction between the objective functions #2 and #3.

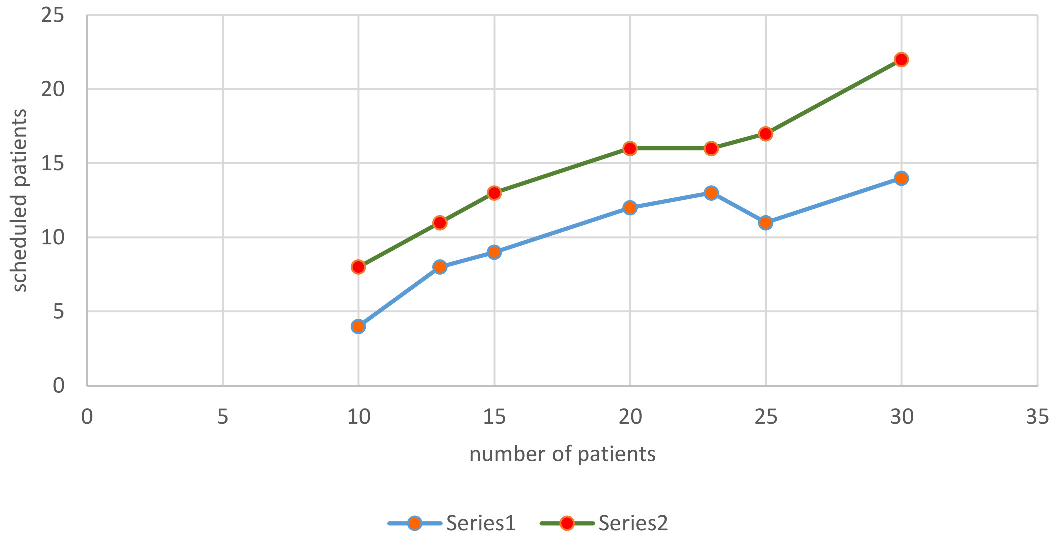

Fig. 16. Comparing the number of planned patients with changes in ICU capacity.

Table 9
The used weights in the weighting method (elective and emergency robust models).

W1	W2	W3	No. of scheduled patients (Elective, Emergency)
	10 patients		
0.8	0.1	0.1	8,4
0.7	0.1	0.2	9,5
0.6	0.1	0.3	9,5
0.6	0.3	0.1	5,5
0.3	0.5	0.2	5,5
0.2	0.6	0.2	4,5
0.1	0.7	0.2	4,4
0.1	0.1	0.8	9,4
	15 patients		
0.8	0.1	0.1	8,4
0.7	0.1	0.2	8,5
0.6	0.1	0.3	8,4
0.6	0.3	0.1	5,4
0.3	0.5	0.2	4,4
0.2	0.6	0.2	2,5
0.1	0.7	0.2	2,4
0.1	0.1	0.8	8,4
	20 patients		
0.8	0.1	0.1	10,4
0.7	0.1	0.2	10,4
0.6	0.1	0.3	11,4
0.6	0.3	0.1	8,6
0.3	0.5	0.2	8,6
0.2	0.6	0.2	8,5
0.1	0.7	0.2	8,5
0.1	0.1	0.8	11,4
	25 patients		
0.8	0.1	0.1	14,4
0.7	0.1	0.2	16,5
0.6	0.1	0.3	15,5
0.6	0.3	0.1	11,5
0.3	0.5	0.2	10,5
0.2	0.6	0.2	9,5
0.1	0.7	0.2	9,5
0.1	0.1	0.8	15,5
	30 patients		
0.8	0.1	0.1	18,4
0.7	0.1	0.2	16,4
0.6	0.1	0.3	15,4
0.6	0.3	0.1	11,6
0.3	0.5	0.2	15,5
0.2	0.6	0.2	11,6
0.1	0.7	0.2	6,6
0.1	0.1	0.8	18,5

points of high-priority patients. The surgery duration and length of stay (LOS) in PHU, PACU, ICU and ward were considered uncertain, and a robust optimization approach was then used to manage the uncertainty. Three models were presented in this study: elective patients' certain model, elective patients robust, as well as elective and emergency patients' robust model.

To evaluate the proposed MIP model, 12 instances for the elective patient's certain model, 9 samples for the elective patient's robust model, as well as 11 instances for elective and emergency patient's robust model were generated. Almost all instances were efficiently solved in terms of CPU time. Moreover, through the proposed arrangement, at least 60% of patients could successfully be planned, while in some examples, this rate touched 80 to 90%. It should be emphasized that in this study, we tried to find the exact answer to this problem. If the conditions for the admission of an emergency patient are not provided, he/she will be transferred to another hospital and will be out of schedule. As predicted, by taking the uncertainties into account, the number of scheduled patients was reduced. Having analyzed the model, the ICU was detected as the bottleneck, and based on this achievement, an efficient approach was provided, where the elective patients should be first planned which are in serious need of ICU according to its capacity, and then the left free capacity of ORs could be assigned for other operations. For solving the proposed models, an improved eps-constraint method and weighting approach were employed and conducted extensive analyses, including the objective function value, computational time, and the number of scheduled patients, were reported.

For the weighting method, different weights were considered to measure their effects on scheduled patients. By comparing the results, it could be concluded that the eps-constraint technique outperforms the weighting approach. The results indicated that the values of objective functions are satisfactory, and the number of scheduled patients and the execution time are acceptable. Taking the ICU into account as a bottleneck in this model, Occupancy Level Coefficient (OLC) was suggested to increase the readiness of hospitals when the system is faced with an emergency demand. By examining the results, $\alpha = 0.8$ was identified as a practical value for OLC.

This article acknowledges the limitations of the proposed model. While the model was designed as an integrated one, it was not feasible to solve it on a large scale with the proposed approach. Additionally, we only accounted for one surgeon per surgery, thereby excluding surgeries that require multiple surgeons. As a future direction, it is imperative to develop efficient methods for large-scale problem-solving, taking into account more resource constraints (such as nurses and equipment) and financial limitations. These areas present promising avenues for future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors confirm that the acquiring scheme of data supporting the findings of this study is available within the article.

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