

NONLINEAR ADAPTIVE CONTROL OF A PERMANENT MAGNET STEPPER MOTOR

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ABSTRACT

This paper considers adaptive position control of a permanent magnet (PM) stepper motor using exact linearization via static state-feedback. Physical outputs are chosen in such a way that the resulting closed-loop system is linearized and decoupled. The objective of the adaptive controller for a PM stepper motor is to achieve output tracking between the motor outputs and prescribed reference signals in presence of parameter uncertainties. This will be achieved by incorporating a parameter identification scheme in the nonlinear state-feedback control law. Simulation results are presented and discussed.

INTRODUCTION

Stepper motors are electromechanical devices that convert input digital pulses into an output analog motion. Permanent magnet stepper motors are simple, low cost, and reliable motors compared with the widely used dc motors. Moreover, compared with other control systems, a control system using a stepper motor has the advantages that no feedback is normally required for either position or speed control and the position error is non-cumulative. However, the performance of a PM stepper motor is limited using open-loop drive. For instance a stepper motor driven in the open-loop mode may fail to follow a pulse command when the pulse train frequency is too high or the inertial load is too heavy. Also, the motor performance tends to be oscillatory in the open-loop mode. The performance of a stepper motor can be improved to a great extent by employing either position or velocity feedback. Closed-loop control of a PM stepper motor is advantageous over open-loop control not only in that the step failure never occurs but also the dynamic behavior is much quicker and smoother [1].

Recently, there has been a great deal of interest in the use of state feedback to linearize different electromechanical systems. A feedback linearizing control of switched reluctance motor is investigated in [2]. De luca and Ulivi [3] have used a state feedback linearizing control technique to design a controller for induction motor torque and flux. Chiasson [4] has developed a feedback linearization of a sixth -order nonlinear dynamic model of induction motor. A two-time scale nonlinear control design technique along with a linearizing controller [5] are used to control a switched reluctance motor where the linearizing controller reduces the motor torque ripple. Position control of a PM stepper motor by exact linearization is presented in [6]. Nonlinear full and reduced -order speed observers for a PM stepper motors are developed in [7].

In this paper, adaptive position control of a PM stepper motor using an exact linearizing technique is considered. The paper is organized as follows: in section II, mathematical modeling of the PM stepper motor is presented. The nonlinear control design technique is presented in section III. The adaptive control design of the motor using the linearizable static state-feed back is outlined in section IV. The main result is given in section V. Simulation results and conclusions are given in sections VI and VII respectively.

MATHEMATICAL MODEL OF THE PM STEPPER MOTOR

A dynamic model of a two-phase PM stepper motor can be described by the following equations [1]:

$$\left. \begin{aligned}
 \frac{d i_a}{dt} &= \frac{1}{L} [V_a - R i_a + K_m \omega_m \sin(N_r \theta_m)] \\
 \frac{d i_b}{dt} &= \frac{1}{L} [V_b - R i_b - K_m \omega_m \cos(N_r \theta_m)] \\
 \frac{d \omega_m}{dt} &= \frac{1}{J} [K_m i_b \cos(N_r \theta_m) - K_m i_a \sin(N_r \theta_m) - D \omega_m] \\
 \frac{d \theta_m}{dt} &= \omega_m
 \end{aligned} \right\} \quad (1)$$

where

- V_a : voltage applied to phase a
- V_b : voltage applied to phase b
- i_a : current in phase a
- i_b : current in phase b
- ω_m : motor angular speed
- θ_m : motor angular position
- R : stator winding resistance per phase
- L : stator winding inductance per phase
- D : Viscous friction coefficient
- J : motor and load inertia
- K_m : motor torque constant
- N_r : number of rotor teeth

For convenience, let $K_1 = -R/L$, $K_2 = K_m/L$, $K_3 = K_m/J$ and $K_4 = -D/J$.

If the state vector x and the control input u are defined as $x^T = [i_a \cdot i_b \cdot \omega_m \cdot \theta_m]$

and $u^T = (1/L) [V_a \cdot V_b]$ then equations (1) can be put in the form :

$$\dot{x} = f(x) + g_1 u_1 + g_2 u_2 \quad (2)$$

where

$$f(x) = \begin{bmatrix} K_1 x_1 + K_2 x_3 \sin(N_r x_4) \\ K_1 x_2 - K_2 x_3 \cos(N_r x_4) \\ -K_3 x_1 \sin(N_r x_4) + K_3 x_2 \cos(N_r x_4) + K_4 x_3 \\ x_3 \end{bmatrix},$$

$$g_1^T = [1 \ 0 \ 0 \ 0] \text{ and } g_2^T = [0 \ 1 \ 0 \ 0]$$

Equations (2) are nonlinear coupled differential equations representing the dynamics of a PM stepper motor.

NONLINEAR CONTROL DESIGN TECHNIQUE

Consider a square multi-input multi-output nonlinear plant described by

$$\left. \begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^m g_i(x) u_i \\ y &= h(x) = [h_1(x), \dots, h_m(x)]^T \end{aligned} \right\} \quad (3)$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$, $y \in \mathcal{R}^m$ and f , g_i and h_i , $i = 1, 2, \dots, m$ are smooth functions, i.e. continuously differentiable. A common approach to control such a system, based on the differential geometric control theory [8], is

to look for a nonlinear state transformation $Z = T(x) \in \mathcal{R}^n$ which maps the nonlinear control problem into a linearized one via state-feedback. In the transformed new coordinates, one may try to find a state-feedback law of the form

$$u = A(x) + B(x)v \quad (4)$$

with $B(x)$ nonsingular such that the nonlinearities are canceled and the resulting closed-loop system is linear and decoupled. Now define $y_j^{\gamma_j}$ to be the γ_j th derivative of y_j with respect to time and γ_j , $j = 1, \dots, m$ to be the smallest integer such that at least one of the inputs appears in $y_j^{\gamma_j}$, i.e.

$$y_j^{\gamma_j} = L_f^{\gamma_j} h_j + \sum_{i=1}^m L_{g_i} (L_f^{\gamma_j-1} h_j) u_i \quad (5)$$

with at least one of the $L_{g_i} (L_f^{\gamma_j-1} h_j) u_i \neq 0$ for some x .

Equations (5) may be written for $j = 1, \dots, m$ in the following form

$$\begin{bmatrix} y_1^{\gamma_1} \\ \vdots \\ y_m^{\gamma_m} \end{bmatrix} = \begin{bmatrix} L_f^{\gamma_1} h_1(x) \\ \vdots \\ L_f^{\gamma_m} h_m(x) \end{bmatrix} + \Lambda(x) \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} \quad (6)$$

where

$$\Lambda(x) = \begin{bmatrix} L_{g_1} L_f^{\gamma_1-1} h_1(x) & \dots & L_{g_m} L_f^{\gamma_1-1} h_1(x) \\ \vdots & & \vdots \\ L_{g_1} L_f^{\gamma_m-1} h_m(x) & \dots & L_{g_m} L_f^{\gamma_m-1} h_m(x) \end{bmatrix}$$

1. In (5) and (6), $L_f h(x) = \frac{\partial h}{\partial x} f(x) : \mathfrak{R}^n \rightarrow \mathfrak{R}$ stands for the Lie derivative of $h(x)$ with respect to f [8] and is given by

$$L_f h(x) = \sum_{i=1}^m \frac{\partial h(x)}{\partial x} f(x). \quad \text{The Lie derivative of } h(x) \text{ repeated } \kappa$$

times is denoted by $L_f^\kappa h(x)$. If the matrix $\Lambda(x) \in \mathfrak{R}^{m \times m}$ is non-singular for all x , then the state-feedback law (4) can be chosen from (6) as

$$u = -\Lambda^{-1}(x) \left\{ \begin{bmatrix} L_f^{\gamma_1} h_1 \\ \vdots \\ L_f^{\gamma_m} h_m \end{bmatrix} - \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \right\} \quad (7)$$

The control law (7) yields the following linear and decoupled closed-loop system

$$\begin{bmatrix} y_1^{\gamma_1} \\ \vdots \\ y_m^{\gamma_m} \end{bmatrix} = \begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix} \tag{8}$$

where $v \in \mathcal{R}^m$ is a reference input vector. If the system (3) has strong relative degrees $\gamma_1, \gamma_2, \dots, \gamma_m$ where

$$\sum_{i=1}^m \gamma_i = n \tag{9}$$

then the transformations

$$\left. \begin{aligned} T_1(x) &= h_1(x) \\ &\vdots \\ T_{\gamma_1}(x) &= L_f^{\gamma_1-1} h_1(x) \\ T_{\gamma_1+1}(x) &= h_2(x) \\ &\vdots \\ T_{\gamma_1+\gamma_2}(x) &= L_f^{\gamma_2-1} h_2(x) \\ &\vdots \\ T_{\gamma_1+\gamma_2+\dots+\gamma_{m-1}+1}(x) &= h_m(x) \\ &\vdots \\ T_{\gamma_1+\gamma_2+\dots+\gamma_{m-1}+\gamma_m}(x) &= T_n(x) = L_f^{\gamma_m-1} h_m(x) \end{aligned} \right\} \tag{10}$$

represent a diffeomorphism of the state variables x [9], [10]. Moreover, with the condition (9) satisfied, system (3) has no zero dynamics and therefore is a minimum phase. In fact the outputs $h_1(x), \dots, h_m(x)$ have a set of relative degrees equal to Kronecker indices [11].

ADAPTIVE CONTROL DESIGN OF A PM STEPPER MOTOR

Linearizable Feedback Control Law

The nonlinear transformation which converts the nonlinear equations of the motors (2) into a nonlinear control canonical form is given by [6]:

$$\left. \begin{aligned} Z_1 = T_1(x) &= x_4 / K_3 \\ Z_2 = T_2(x) &= L_f T_1 = x_3 / K_3 \\ Z_3 = T_3(x) &= L_f^2 T_1 = -x_1 \sin(N_r x_4) + x_2 \cos(N_r x_4) + (K_4 / K_3) x_3 \\ Z_4 = T_4(x) &= x_2 \sin(N_r x_4) + x_1 \cos(N_r x_4) \end{aligned} \right\} \quad (11)$$

Since the Kronecker indices of the PM stepper motor are $\gamma_1 = 3$ and $\gamma_2 = 1$, the motor equations (2) can be transformed to the following nonlinear control canonical form

$$\dot{x} = f^*(x) + B^*(x)u \quad (12)$$

where $f^*(x) = [Z_2, Z_3, L_f T_3(x), L_f T_4(x)]^T$ and

$$B^*(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ L_{g_1} T_3(x) & L_{g_2} T_3(x) \\ L_{g_1} T_4(x) & L_{g_2} T_4(x) \end{bmatrix}$$

Using (10), the motor outputs can be chosen as

$$\left. \begin{aligned} y_1 &= h_1(x) = T_1(x) = \frac{x_4}{K_3} \\ y_2 &= h_2(x) = T_4(x) = x_1 \cos(N_r x_4) + x_2 \sin(N_r x_4) \end{aligned} \right\} \quad (13)$$

where the output y_1 represents the motor angular position and the output y_2 represents the motor direct-axis current i_d . Now, equations (6) for $\gamma_1 = 3$ and $\gamma_2 = 1$ can be rewritten as

$$\begin{bmatrix} y_1^3 \\ y_2^1 \end{bmatrix} = \begin{bmatrix} L_f^3 h_1(x) \\ L_f h_2(x) \end{bmatrix} + \Lambda(x) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (14)$$

where

$$\Lambda(x) = \begin{bmatrix} L_{g_1} L_f^2 h_1(x) & L_{g_2} L_f^2 h_1(x) \\ L_{g_1} h_2(x) & L_{g_2} h_2(x) \end{bmatrix}$$

Performing the mathematical manipulation in (14) yields

$$\begin{bmatrix} y_1^3 \\ y_2^1 \end{bmatrix} = \begin{bmatrix} \alpha(x) \\ \beta(x) \end{bmatrix} + \Lambda(x) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (15)$$

where

$$\alpha(x) = (K_1 + K_4) i_q + \left(\frac{K_4^2}{K_3} - K_2\right) x_3 - N_r i_d x_3$$

$$\beta(x) = K_1 i_d + N_r i_q x_3 \quad ,$$

$$\Lambda(x) = \begin{bmatrix} -\sin(N_r x_4) & \cos(N_r x_4) \\ \cos(N_r x_4) & \sin(N_r x_4) \end{bmatrix},$$

i_d and i_q are the currents in the direct and quadrature axis of the motor and are given by

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(N_r x_4) & \sin(N_r x_4) \\ -\sin(N_r x_4) & \cos(N_r x_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (16)$$

Note that the matrix $\Lambda(x)$ represents the last two rows of the matrix $B^*(x)$ given in (12). The state-feedback linearizing control is determined from (15) as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \Lambda^{-1}(x) \left\{ \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \alpha(x) \\ \beta(x) \end{bmatrix} \right\} \quad (17)$$

or in a compact form

$$u = \Lambda^{-1}(x) \{v - Q(x)\} \quad (18)$$

where $v^T = [v_1 \quad v_2]$ and $Q^T(x) = [\alpha(x) \quad \beta(x)]$.

Adaptive Position Tracking

The objective of the adaptive control of the PM stepper motor considered in this paper is to make the motor outputs y_1 and y_2 track prescribed reference trajectories y_{m1} and y_{m2} in presence of parameter uncertainties. In this case, the control inputs v_1 and v_2 can be chosen to be

$$\left. \begin{aligned} v_1 &= y_{m1}^3 + a_2(y_{m1}^2 - y_1^2) + a_1(y_{m1}^1 - y_1^1) + a_0(y_{m1} - y_1) \\ v_2 &= y_{m2}^1 + b_0(y_{m2} - y_2) \end{aligned} \right\} \quad (19)$$

where

a_i for $i=0,1,2$ and b_0 are selected such that $(s^3 + a_2s^2 + a_1s + a_0)$ and $(s + b_0)$ are Hurwitz polynomials. In order to achieve the desired output tracking, the nonlinear system has to be minimum phase [9]. Since $\gamma_1 + \gamma_2 = n = 4$, the stepper motor model has no zero-dynamics and therefore is minimum phase. In presence of parameter uncertainties due to unknown friction and/or loads, the adaptive version of the control law (18) can be obtained by incorporating a parameter estimation scheme. This can be done by replacing (18) by

$$u = \Lambda^{-1}(x)\{v - Q_e(x)\} \quad (20)$$

where $Q_e(x)$ is the estimated value of the vector $Q(x)$. Substituting (20) into (15) gives

$$\begin{bmatrix} y_1^3 \\ y_2^1 \end{bmatrix} = v + \{Q(x) - Q_e(x)\} \quad (21)$$

Now, equations (19) are used in (21) to get

$$\begin{bmatrix} e_{0_1}^3 + a_2 e_{0_1}^2 + a_1 e_{0_1}^1 + a_0 e_{0_1} \\ e_{0_2}^1 + b_0 e_{0_2} \end{bmatrix} = \Phi \quad (22)$$

where

$e_{0_1} = y_{m_1} - y_1$ and $e_{0_2} = y_{m_2} - y_2$ and the vector $\Phi \in \mathfrak{R}^{2 \times 1}$ is the parameter error vector defined as $\Phi = [Q_e(x) - Q(x)]$

MAIN RESULT

The parameter update law

$$\dot{\Phi} = - e_1 \quad (23)$$

$$e_1^T = [e_{11} \quad e_{21}] \quad (24)$$

where

$$e_{11} = \ddot{e}_{0_1} + r_1 \dot{e}_{0_1} + r_0 e_{0_1} \quad \text{and}$$

$$e_{21} = e_{0_2}$$

will yield a bounded tracking error if the transfer functions

$$G_1(s) = \frac{s^2 + r_1 s + r_0}{s^3 + a_2 s^2 + a_1 s + a_0} \quad \text{and}$$

$$G_2(s) = \frac{1}{s + b_0}$$

are strictly positive real (SPR). The definition and the necessary and sufficient conditions for a rational function to be strictly positive real are presented in the Appendix.

Proof:

Using (22) and (24) we get

$$\left. \begin{aligned} e_{11} &= g_1 \Phi_{11} \\ e_{21} &= g_2 \Phi_{21} \end{aligned} \right\} \quad (25)$$

where g_1 and g_2 are the impulse responses of $G_1(s)$ and $G_2(s)$ respectively, $\Phi_{11} = \alpha_e(x) - \alpha(x)$, $\Phi_{21} = \beta_e(x) - \beta(x)$ and $\alpha_e(x)$ and $\beta_e(x)$ are the estimated values of $\alpha(x)$ and $\beta(x)$. By the Kalman-Yacubovitch-Popov lemma [12], the SPR transfer function $G_1(s)$ can be realized in a state-space form, as follows

$$\left. \begin{aligned} \dot{e}_{m1} &= \Gamma_1 e_{11} + b_1 \Phi_{11} \\ e_{11} &= C_1^T e_{m1} \\ \dot{\Phi}_{11} &= -C_1^T e_{m1} \end{aligned} \right\} \quad (26)$$

where $P_1 \Gamma_1 + \Gamma_1^T P_1 = -S_1$, $P_1 b_1 = C_1$ and P_1 and S_1 are symmetric positive definite matrices. A positive definite Lyapunov function $\mu_1(t) = e_{m1}^T P_1 e_{m1} + \Phi_{11}^2$ is selected and its time derivative along the trajectories of (26) is found to be $\dot{\mu}_1(t) = -e_{m1}^T S_1 e_{m1} < 0$. Hence $0 < \mu_1(t) < \mu_1(0)$ for all $t > 0$. Therefore e_{m1} , e_{11} and Φ_{11} are approaching zero as the time is approaching infinity. Also, since the system has no zero dynamics, all the states are asymptotically stable [13]. The proof is quite similar for e_{m2} , e_{21} and Φ_{21} using the state-space realization of $G_2(s)$.

SIMULATION RESULTS

The one step motion control of a PM stepper motor going from phase b to phase a is investigated using the proposed adaptive control law. The motor parameters are [6]:

$R = 10.0 \Omega$, $L = 0.0011H$, $D = 0.001N.m.sec$, $J = 5.7 \times 10^{-6} N.m.sec^2$, $K_m = 0.113$ and $N_r = 50$. If the motor starts from phase b ($i_b = i_0 = 0.75$ and $i_a = 0.0$), the motor position is given by $x_4 = (90/N_r)$ degree. Going to phase a ($i_a = 0.75$ and $i_b = 0.0$), the position is given by $x_4 = 0$ or $x_4 = (180/N_r)$ degree depending on whether the rotation is clockwise or counter-clockwise. The motor is simulated when the adaptive control and the parameter update laws (20), (23) and (24) are incorporated in the dynamical model of the motor. The coefficients of the transfer functions $G_1(s)$ and $G_2(s)$ are selected as: $a_0 = 200$, $a_1 = 3200$, $a_2 = 130$, $r_0 = 1000$, $r_1 = 100$ and $b_0 = 10$.

This set of coefficients is picked in such a way that the SPR conditions listed in the Appendix are satisfied and satisfactory dynamic response is achieved. The motor position and the direct-axis current along with the reference signals are shown in Fig. 1 and Fig. 2 respectively. These two figures display the tracking behavior between the actual motor outputs x_4 and i_d and the reference signals y_{m_1} and y_{m_2} . The error between the reference signals and the actual outputs of the motor is bounded by zero. The input phase voltages $V_a = u_1 L$ and $V_b = u_2 L$ are shown in Fig.3 and Fig. 4. It is clear that both control input signals are bounded. Note that V_b is switched from the full phase voltage of 7.5 V to zero and that V_a is switched from zero to the full phase voltage indicating that the motor is ready to move from phase a to phase b. The phase currents i_a and i_b are shown in Fig.5 and Fig.6. These figures demonstrate the boundedness and the switching behavior of the motor currents.

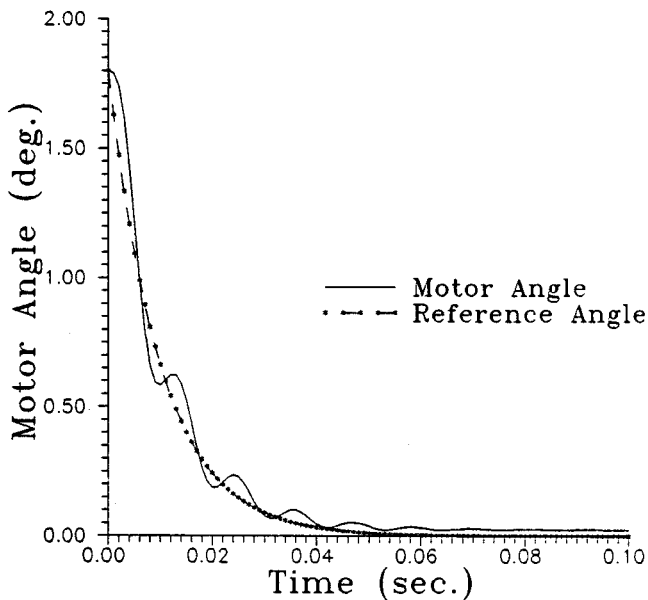


Fig. 1: Motor and reference angles

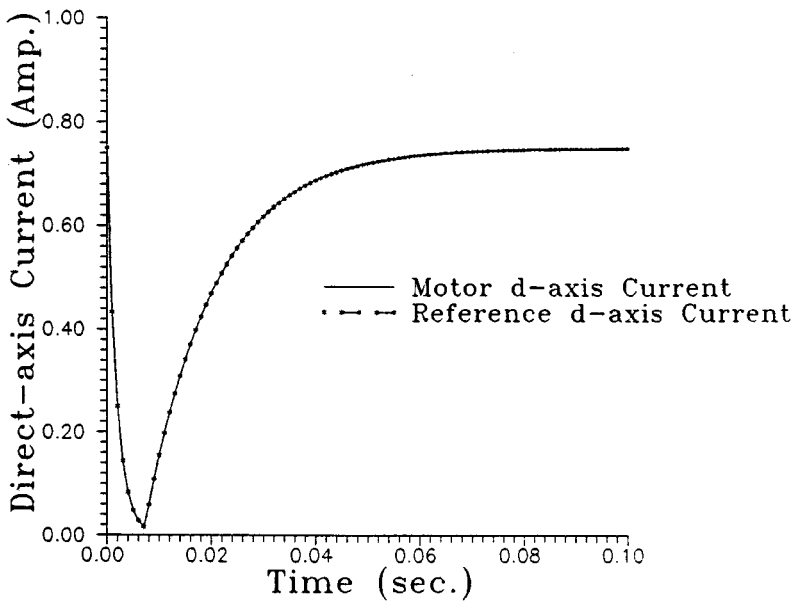


Fig. 2: Motor and reference d-axis currents

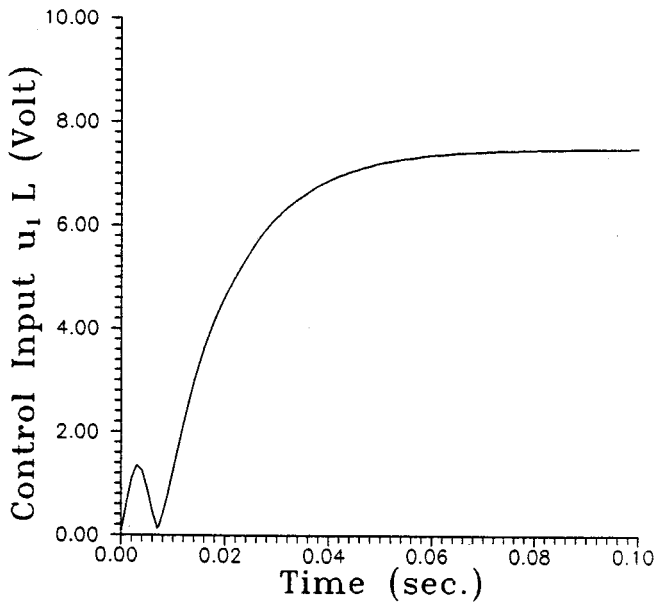


Fig. 3: The control signal V_a

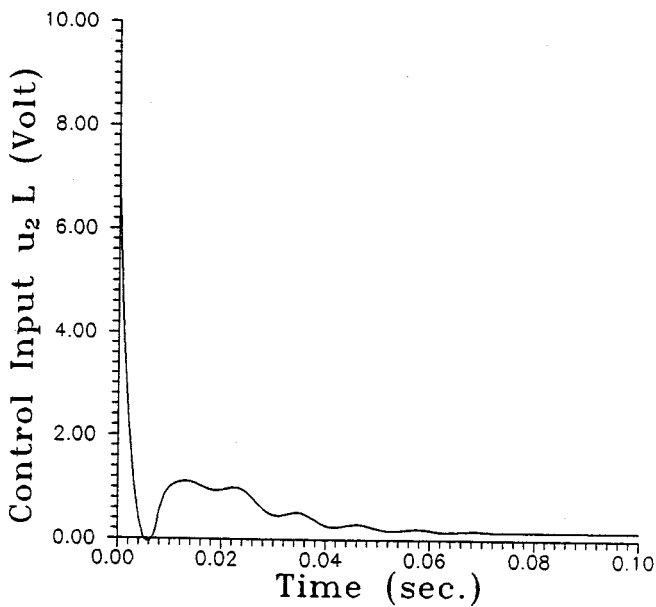


Fig. 4: The control signal V_b

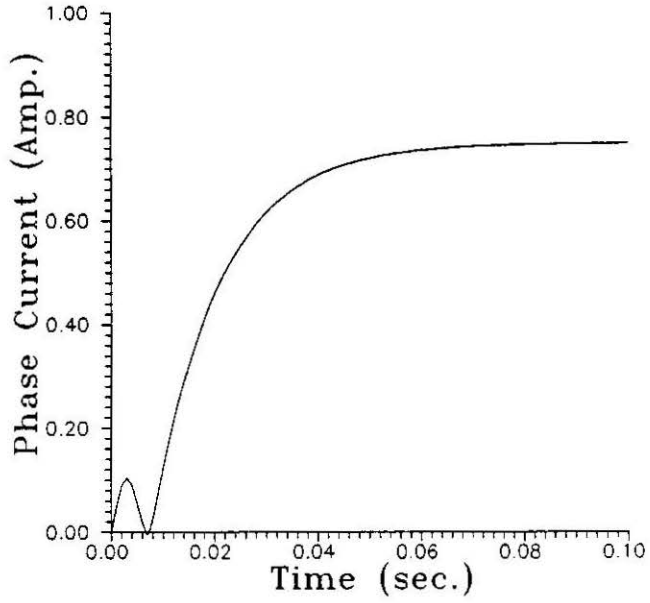


Fig. 5: The phase current i_a

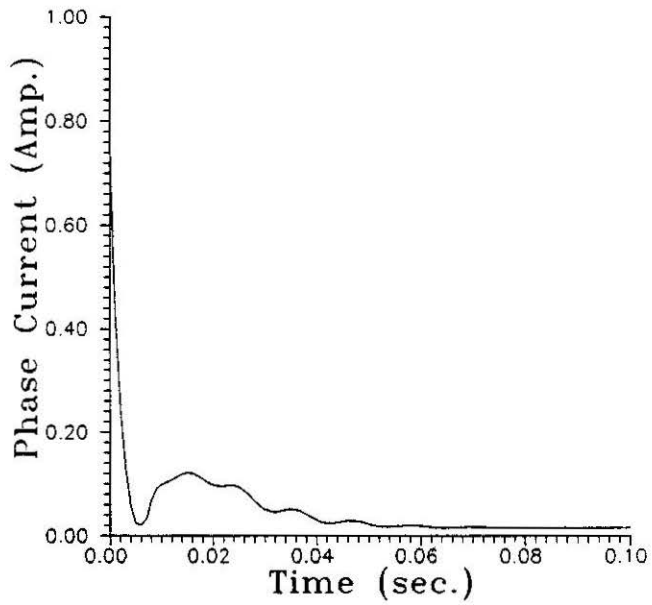


Fig. 6: The phase current i_b

CONCLUSIONS

The paper presents an adaptive controller to achieve output tracking of a PM stepper motor in the presence of parameter uncertainties in the motor model. The controller is based on static feedback in order to linearize and decouple the closed-loop system. An identification scheme is incorporated in the control law to estimate the unknown parameters. The main result shows that the tracking error is guaranteed to be bounded if certain transfer functions are SPR. Simulation results show the effectiveness of the proposed controller in achieving the desired tracking performance.

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APPENDIX

Positive Real and Strictly Positive Real Functions

A rational function $G(s)$ of the complex variable $s = \sigma + j\omega$ is positive real (PR) if :

1. $G(\sigma) \in \Re$ for all $\sigma \in \Re$
2. $\text{Re} [G(\sigma + j \omega)] \geq 0$ for all $\sigma > 0, \omega \geq 0$.

The function $G(s)$ is strictly positive real (SPR) if, for some $\varepsilon > 0$, the function $G(s - \varepsilon)$ is PR. It can be shown [14], [15] that a strictly proper transfer function $G(s)$ is SPR if and only if it satisfies all of the following conditions :

1. $G(s)$ is stable, i.e. the denominator polynomial is Hurwitz
2. $G(s)$ is minimum phase, i.e. the numerator polynomial is Hurwitz.
3. $\text{Re}[G(j\omega)] > 0$, for all $\omega \geq 0$.
4. $\lim_{\omega \rightarrow \infty} \omega^2 \text{Re}[G(j\omega)] > 0$.

In order that $G_1(s)$ and $G_2(s)$ are SPR, the above conditions are applied resulting in the following coefficient constraints:

1. $a_0 > 0, a_2 > 0, a_1 a_2 > 0$
2. $r_0 > 0, r_1 > 0$
3. $a_2 > r_1, a_0 r_0 > 0$

$$4. \begin{vmatrix} a_1 & r_0 \\ a_2 & r_1 \end{vmatrix} > a_0$$