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"STUDY OF A TIME DEPENDENT DOUBLE - DIFFUSIVE SALINITY PROBLEM"

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ABSTRACT

The time-dependent, double diffusive stability problem for a horizontal layer of salty water bounded by two rigid isothermal surfaces is analyzed using a linear perturbation technique. Initially, the layer is subjected to zero temperature and salinity gradients. At time $t = O$, uniform step increases in both temperature and salinity are imposed at the bottom surface while the top surface is impermeable to diffusion of salt. Stability results defined by the critical thermal Grashof number as a function of system parameters namely: solute Grashof number, Prandtl number, Schmidt number, time and wave size are presented graphically.

INTRODUCTION

The convection process in a horizontal fluid layer with steady-state temperature and salinity distributions has been studied extensively for both the linear and non-linear profiles [1−8]. Recently, the effect of vertical motion on the linear stability of a horizontal layer for both finger and diffusive regimes, was investigated theoretically [9]. Very little work has been done to handle the situations in which the fluid layer is heated in a time-dependent manner, however.

The time of the onset of convection in a layer initially stably stratified and then heated from below in a transient manner has been recently considered by Kaviany [10] and Kaviany and Vogel [11]. In these studies, the stability in the diffusive regime, was examined both experimentally and theoretically, for the transient case with initial uniform salinity gradient and zero temperature gradient.

The effect of the presence of a thin mixed layer at either the bottom or the top of the fluid layer on the delay in the onset of convection was examined. In the former case, it was observed that the salinity gradient becomes ineffective if the mixed layer thickness exceeds 25 percent of the fluid layer. In the latter case, it was observed that stability is enhanced provided the heat flux at the bottom is higher than that at the top.

In this paper, the onset of convection in a horizontal layer bounded by two rigid surfaces for the situation in which the time dependent temperature and salinity gradients are initially zero is considered. This situation, finds applications in solar energy systems such as the filling of a salt gradient solar pond and water desalination in a solar still.

FORMULATION

Analysis:

Consider a horizontal fluid layer of Boussinesq fluid of thickness h with the z axis in the opposite direction of gravity confined by two rigid surfaces as shown in Figure (1). The fluid layer is initially at a uniform temperature T_1 and uniform salinity c_1. At time $t = 0$, heating is initiated at the bottom surface by a step increase in the wall temperature to a uniform value T_2. At the same time, the salinity at the bottom is raised to a uniform value c_2, to neutralize the destabilizing effect of heating. The top surface is maintained at temperature T_1 with zero salt flux.

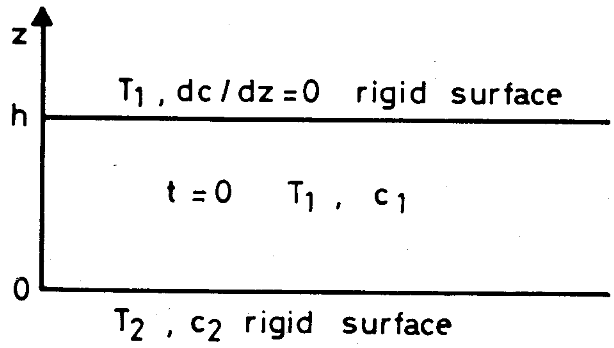

Fig. (1) : Schematic diagram of the fluid layer.

Stability Analysis:

The onset of convection in the stratified layer governed by the following stability problem given in non-dimensional form is given as:

$$[\frac{\partial}{\partial \tau} - \nabla^2] \nabla^2 V = Gt\nabla_1^2 T - Gs\nabla_1^2 C \tag{1}$$

$$[\frac{\partial}{\partial \tau} - \frac{1}{Pr} \nabla^2] T = -(D\bar{T})V \tag{2}$$

$$[\frac{\partial}{\partial \tau} - \frac{1}{Sc} \nabla^2] C = -(D\bar{C})V \tag{3}$$

The perturbed boundary conditions are:

$$\begin{aligned} V &= DV = T = 0 && \text{at} \quad Z = 0, 1 \\ C &= 0 && \text{at} \quad Z = 0, \\ DC &= 0 && \text{at} \quad Z = 1 \end{aligned} \tag{4}$$

The initial temperature and salinity gradients, ($D\bar{T}$ and $D\bar{C}$) appearing in Eqs. (2 & 3) are to be obtained from the solution of the diffusion equations for heat and mass given in the dimensionless form as:

$$\frac{\partial \bar{T}}{\partial \tau} = \frac{1}{Pr} \frac{\partial^2 \bar{T}}{\partial Z^2} \tag{5}$$

and

$$\frac{\partial \bar{C}}{\partial \tau} = \frac{1}{Sc} \frac{\partial^2 \bar{C}}{\partial Z^2} \tag{6}$$

The boundary and initial conditions are:

$$\begin{aligned} \tau = 0 \quad &, \quad \bar{T}(Z) = \bar{C}(Z) = 0 \\ \tau > 0 \quad &, \quad \bar{T}(0)-1 = \bar{C}(0)-1 = \bar{T}(1) = D\bar{C}(1) = 0 \end{aligned} \tag{7}$$

where, Pr = Prandtl number, Sc = Schmidt number.

Applying the separation of variables technique, the solutions to equations (5) and (6) subject to conditions (7) are given by:

$$\bar{T} = (1-Z) - 2 \sum_{m=1}^{\infty} \frac{e^{-\alpha_m^2 \tau / Pr}}{\alpha_m} \sin \alpha_m Z \tag{8}$$

$$\bar{C} = 1 - 2 \sum_{m=1}^{\infty} \frac{e^{-\beta_m^2 \tau / Sc}}{\beta_m} \sin \beta_m Z$$

where

$$\alpha_m = m\pi \qquad \beta_m = (m - 1/2)\pi \tag{9}$$

Typical mean temperature and mean salinity distributions are plotted in Figures (2) and (3) as functions of time. The thermal profile grows very fast such that it covers the whole layer in a time = 0.16 and reaches the steady state at $\tau \cong 1.0$, while the salinity profile grows very slowly reaching the top surface at τ = 7.0 and maintains its steady state at τ = 500.

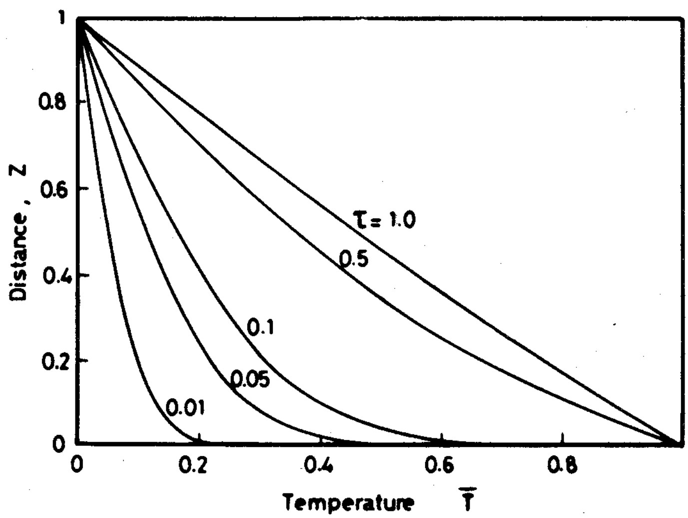

Fig. (2) : T vs. layer thickness Z at different time values,

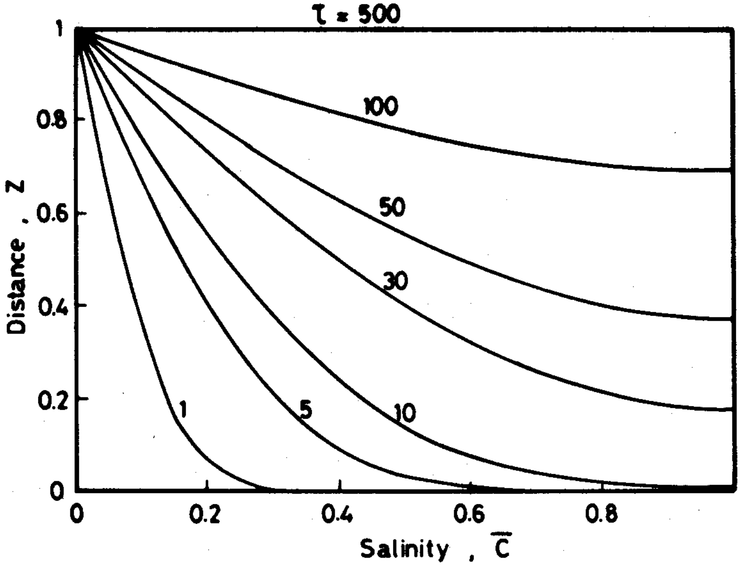

Fig. (3) : $\bar{C}$ vs. layer thickness Z for different values of time, τ .

Considering, the three-dimensional disturbances to be periodic in X–Y plane, perturbed quantities are written as:

$$F = F^*(Z,\tau) \exp [i(a_1X + a_2Y)] \tag{10}$$

where F = V, T or C, a_1 and a_2 are the wave numbers in X and Y directions respectively, and F^* (Z, τ) is the amplitude of the perturbed quantity.

Introducing Eq. (10) in the stability equations Eqs. (1–3), gives:

$$\left[\frac{\partial}{\partial \tau} - (D^2 - a^2)\right] (D^2 - a^2) V^* = Gta^2T^* - Gsa^2C^* \tag{11}$$

$$\left[\frac{\partial}{\partial \tau} - \frac{1}{Pr} (D^2 - a^2)\right] T^* = - (D\bar{T}) V^* \tag{12}$$

$$\left[\frac{\partial}{\partial \tau} - \frac{1}{Sc} (D^2 - a^2)\right] C^* = - (D\bar{C}) V^* \tag{13}$$

$$
\begin{aligned}
V^* &= DV^* = T^* = C^* = 0 && \text{at} \quad Z = 0 \\
V^* &= DV^* = T^* = DC^* = 0 && \text{at} \quad Z = 1
\end{aligned}
\tag{14}
$$

where $a^2 = a^2_1 + a^2_2$, $D^n = \partial^n / \partial Z^n$, Gt = thermal Grashof number and Gs = solute Grashof number.

Method of Solution:

The system of Eqs. (11−14) is solved numerically using Galerkin's method. In this method, the perturbed quantities. (V^*, T^* & C^*) are constructed as a series of trial functions in such a way to satisfy their boundary conditions. These functions take the form:

$$V^* = \sum_{m=1}^{N} A_m(\tau)\, V_m(Z) \tag{15}$$

$$T^* = \sum_{m=1}^{N} B_m(\tau) \sin(m\pi Z) \tag{16}$$

$$C^* = \sum_{m=1}^{N} C_m(\tau) \sin[(m-1/2)\pi Z] \tag{17}$$

The eigenvectors $V_m(Z)$ together with their eigenvalues are given in Reference [9].

Substituting the above approximate solutions into Eqs. (11−13), utilizing the orthogonality conditions, a set of linear algebraic equations is obtained in the form:

$$\frac{d\vec{X}}{d\tau} = [H]\,\vec{X} \tag{18}$$

where $\vec{X} = (A_m, B_m, C_m)^T$, and [H] is a matrix of order 3Nx3N.

Eq. (18) is solved numerically to determine the critical conditions over the transient period characterized by the thermal Grashof number, Gt and the corresponding frequency, p_i and wave number, a for wide range of Gs, for a NaCl solution having Pr = 3.35 and Sc = 175 (at 60 ° and 10% salinity). The number of terms N needed for an error less than 2% was found to vary from 4 to 12 as Gs was increased from 0 to 10^8. A sample of the results are discussed in the next section.

RESULTS AND DISCUSSION

The effects of a developing salinity gradient on the stability criteria expressed by the critical thermal Grashof number Gt_c and the corresponding wavelength ($2\pi / a$) and frequency (p_i) are presented in Figures (4–6) respectively for an aqueous solution having Pr=3.35 and Sc=175.

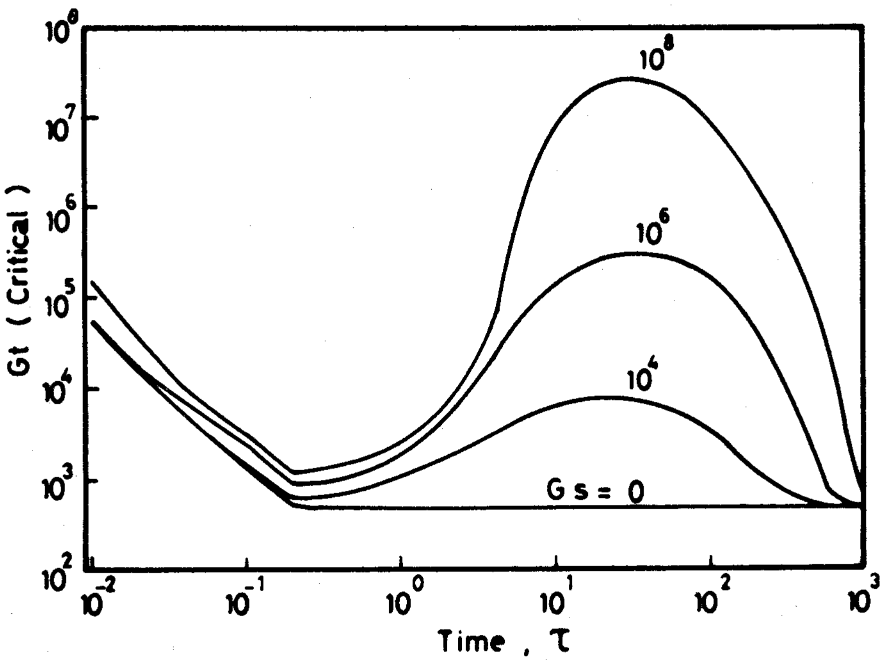

Fig. (4) : Thermal Grashof number, Gtc vs. time, τ , for different Gs values.

CONCLUSION

The stability of the horizontal fluid layer initially subjected to a step increase in both temperature and salinity at its bottom surface is summarized as follows:

(1) When the heating begins, the salinity profile is very weak in comparison with the thermal profile, thus the stability of the layer is thermally dominated. For values of time greater than 0.16, the growth of salinity profile acts to neutrailize the destabilizing thermal effect reaching its maximum at $\tau = 30$. For $\tau > 30$ the effect delays and the problem becomes thermally dominated at $\tau = 500$.

(2) The wave spectrum as defined by wave number and frequency at the onset of convection is greatly influenced by time changes. For small values of time, $\tau < 1.5$ instability is initiated as stationary cells while for $\tau \geq 1.5$ the instability sets in as travelling waves.

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NOMENCLATURE

a	:	wave number
a_1, a_2	:	wave numbers in x, y directions respectively
c, c'	:	mean and perturbed salinities, % weight
c_1	:	initial salinity
c_2	:	salinity at bottom surface
C	:	mean dimensionlesss salinity, $(c - c_1) / (c_2 - c_1)$
D	:	$\partial / \partial Z$
D_s	:	solute diffusivity

$D\bar{C}$	:	mean salinity gradient
$D\bar{T}$	:	mean temperature gradient
g	:	gravitational acceleration
h	:	thickness of horizontal layer
Gs	:	solute Grashof number, $g\beta(c_2 - c_1)h^3/\nu^2$
Gt	:	thermal Grashof number, $g\gamma(T_2 - T_1)h^3/\nu^2$
p_i	:	frequency
Pr	:	Prandtl number, ν/α
Sc	:	Schmidt number, ν/D_s
t	:	time
T_1	:	initial temperature, temperature at top surface
T_2	:	temperature at bottom surface
v	:	perturbed vertical velocity
V	:	non-dimensional perturbed velocity, $v.h/\nu$
x, y, z,	:	cartesian coordinates
X,Y,Z	:	$(x, y, z)/h$

Superscript

* perturbed quantities as a function in Z and τ

Greek Letters

α	:	thermal diffusivity
β	:	coefficient of solute expansion
γ	:	coefficient of thermal expansion
ν	:	kinematic viscosity
τ	:	Fourier number, $\nu t/h^2$
∇^2_1	:	$\partial^2/\partial X^2 + \partial^2/\partial Y^2$
∇^2	:	$\partial^2/\partial X^2 + \partial^2/\partial Y^2 + \partial^2/\partial Z^2$

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