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PERFORMANCE EVALUATION OF BIPED ROBOT OPTIMAL GAIT BASED ON GENETIC ALGORITHM

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ABSTRACT

A Genetic Algorithm (GA) gait synthesis method for walking biped robots is considered in this paper. The walking occupy most of the time during the task performance, therefore its gait is analyzed based on the minimum consumed energy (CE) and minimum torque change (TC). The biped robot optimal gait is considered starting from static standing state and continuing with normal walking. The proposed method can be applied for wide ranges of step lengths and step times and for other tasks that might to be performed by humanoid robot. By using GA as an optimization tool it is easy to include constraints and add new variables to be optimized. The biped robot gait is generated without neglecting the stability, which is verified by the zero moment point ZMP concept. Simulations are realized based on the parameters of "Bonten-Maru I" humanoid robot. The evaluation by simulations shows that the proposed method has a good performance and energy is significantly reduced.

KEY WORDS: Biped Robots, Genetic Algorithms, Consumed Energy, Torque Change, Gait Synthesis.

NOMENCLATURE

x_w, z_w	coordinates of the robot waist
$\ddot{x}_w, \ddot{z}_w$	accelerations of the robot waist
m_i	mass of link "*i*"
$\bar{x}_i$, $\bar{z}_i$	coordinates of the center of mass of link "*i*"
$\ddot{x}_i, \ddot{z}_i$	accelerations of the center of mass of link "*i*"
g_z	gravitational acceleration

θ	position vector
$\dot{\theta}$	angular velocity vector
$\ddot{\theta}$	angular acceleration vector
X_{zmp}	ZMP position
ZMP_{jump}	ZMP position at the beginning of the step
ZMP_{f},	ZMP position at the end of the step
t_f	step time
τ	torque vector
$\Delta\tau_{jump}$	torque necessary to jump the ZMP
C	constraint function
c	penalty function vector
En	minimum energy cost function
$J_{torque\ change}$	14 minimum torque change cost function
$J(\theta)$	mass matrix
$X(\theta)$	matrix of centrifugal coefficients
$Y(\theta)$	matrix of Coriolis coefficients
$Z(\theta)$	gravity terms

1. INTRODUCTION

Autonomous humanoid robots, which can substitute humans in many hazardous-working environments, are inevitably restricted to a limited amount of energy supply. For this reason, it will be advantageous to consider the energy consumption, when cyclic movements like walking are involved.

One of the factors that influence the energy consumption is the gait synthesis. In most of the previous papers related to biped robots [1-2], the prescribed angle trajectories of the leg part are based on data taken from humans. The motion of upper body is calculated in order to have the ZMP inside the sole region. Roussel et al. [3] treated the minimum CE gait synthesis during walking, considering a four link biped robot. The angle trajectories are considered unconstrained, generated by piecewise constant inputs. Channon et al. [4] analyzed the CE related to the walking velocity and step length. The biped robot optimal gait is generated based on the calculus of variation method. Because the number of degrees of freedom is high, the calculation of partial differential is complex. Also, when the model or the constraints are changed, the formulation of the optimum control is needed again. In this method, the calculation time increases exponentially with the number of variables to be optimized.

In this paper, we present a GA gait synthesis method for biped robots during walking. The cost functions to be minimized are Consumed Energy and Torque

Change. Our goal is to create modules for many tasks considered to be performed by "Bonten-Marru I" humanoid robot. In this paper optimal gait for walking is presented. The optimal gait for going up-stairs is already considered. Based on the information received by the eye system, the appropriate module will be simulated to generate the optimal motion.

When solving for optimal gaits, some constrains must be considered. In our work, the most important constraint is the stability. GA makes easy handling the constraints by using the penalty function vector, which transforms a constrained problem to an unconstrained one. GA has also been known to be robust for search and optimization problems (Goldberg 1989). It has been used to solve difficult problems with objective functions that do not possess properties such as continuity, differentiability, etc. These algorithms manipulate a family of possible solutions that allows for the exploration of several promising areas of the solution space at the same time.

The paper is organized as follows. We start with a brief overview of GA in Section 2. In section 3, the gait and upper body motion are treated. In section 4, the problem formulation and proposed method are discussed. Boundary conditions and GA variables are treated in section 5. Simulation results are given in section 6. The implementation issues are considered in section 7. Finally, conclusions and future work given in section 8.

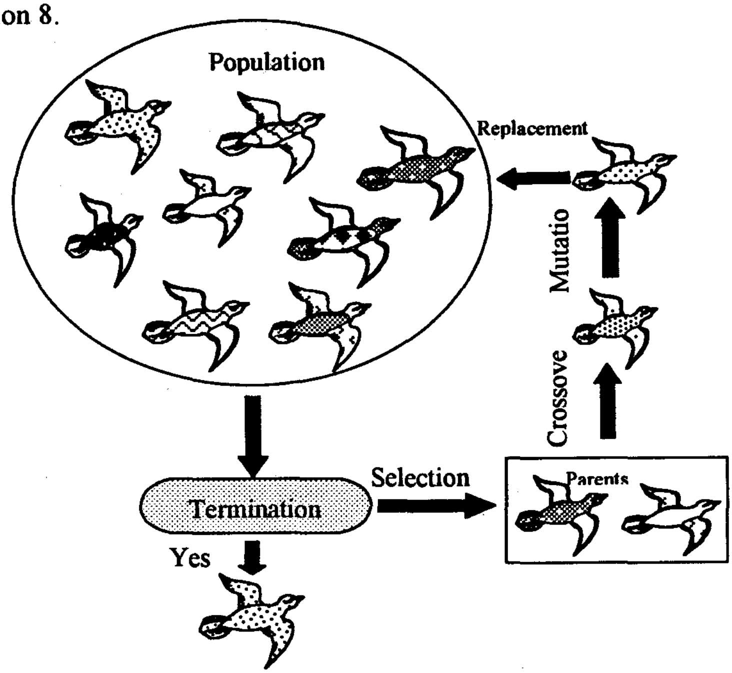


Fig. 1. Symbolic presentation of GA.

2. GA

GA is a search algorithm based on the mechanics of natural selection and population genetics. The search mechanism is based on the interaction between individuals and the natural environment. GA comprises a set of individuals (the population) and a set of biologically inspired operators (the genetic operators). The individuals have genes, which are the potential solutions for the problem. The genetic operators are crossover and mutation. GA generates a sequence of populations by using genetic operators among individuals. Only the most suited individuals in a population can survive and generate offspring, thus transmitting their biological heredity to the new generations. The symbolic representation of GA is shown in Fig. 1 and the main steps are shown in the following:

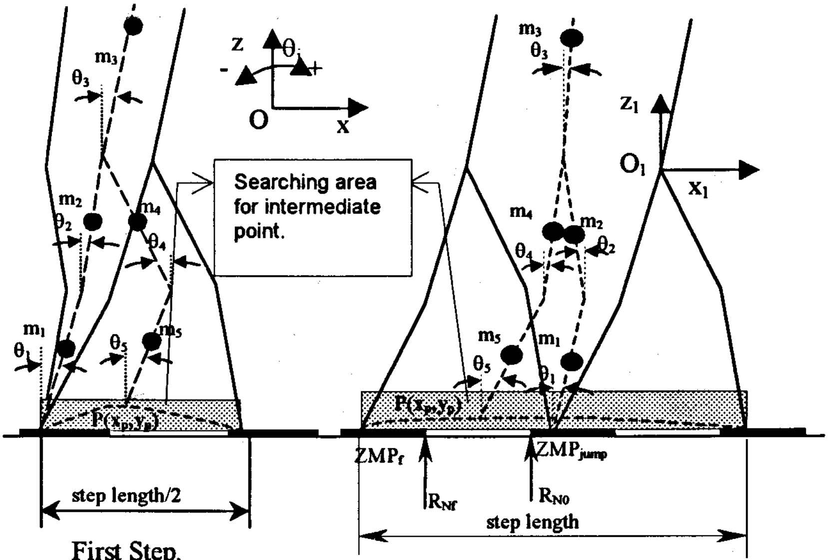
1. Supply a population P_0 of N individuals and respective function values;
2. $i \leftarrow 1$;
3. $P_i' \leftarrow$ selection_function ($P_i - 1$);
4. $P_i \leftarrow$ reproduction_function (P_i');
5. Evaluate (P_i);
6. $i \leftarrow i+1$;
7. Repeat step 3 until termination;
8. Print out the best solution found.

3. GAIT AND UPPER BODY MOTION

During walking, the arms of humanoid robot will be fixed in the chest. Therefore, it can be considered as a five-link biped robot walking in two-dimensional sagittal plane, as shown in Fig. 2. This model assembles most of the characteristics of the human walking. The motion of biped robot is considered composed from a single support phase and an instantaneous double support phase. During the single support phase, the ZMP must be inside the foot, so the contact between the foot and the ground will not be violated. To have a stable periodic walking, when the swing foot touches the ground, the ZMP must jump in its sole. To realize the jumping of the ZMP, we consider the upper body link acceleration. To have an easier relative motion of the upper body, the coordinate system from the ankle joint of supporting leg (OXZ) is moved transitionally to the waist of the robot ($O_1X_1Z_1$). With respect to the new coordinate system, the ZMP position is written as follows:

$$\overline{X}_{ZMP} = \frac{\sum_{i=1}^{5} m_i(\ddot{\overline{z}}_i + \ddot{z}_w + g_z)\overline{x}_i - \sum_{i=1}^{5} m_i(\ddot{\overline{x}}_i + \ddot{x}_w)(\overline{z}_i + z_w)}{\sum_{i=1}^{5} m_i(\ddot{\overline{z}}_i + \ddot{z}_w + g_z)}, \tag{1}$$

where m_i is mass of the particle "*i*", x_w and z_w are the coordinates of the waist with respect to OXZ coordinate system, $\overline{x}_i$ and $\overline{z}_i$ are the coordinates of the mass particle "*i*" with respect to $O_1X_1Z_1$ coordinate system, $\ddot{\overline{x}}_i$ and $\ddot{\overline{z}}_i$ are the acceleration of the mass particle "*i*" with respect to $O_1X_1Z_1$ coordinate system. Based on the formula (1), if the position, $\overline{x}_i, \overline{z}_i$, and acceleration, $\ddot{\overline{x}}_i, \ddot{\overline{z}}_i$, of the leg part (i=1,2,4,5), the body angle, θ_3, and body angular velocity, $\dot{\theta}_3$, are known, then because $\ddot{\overline{x}}_3, \ddot{\overline{z}}_3$ are functions of $l_3, \theta_3, \dot{\theta}_3, \ddot{\theta}_3$, it is easy to calculate the body angular acceleration based on the ZMP position.



First Step.

Normal Step.

Fig. 2. Five link biped robot.

Let $_{(0)}$ and $_{(f)}$ be the indexes at the beginning and the end of the step, respectively. At the beginning of the normal step, $\ddot{\theta}_{30}$ makes the ZMP to jump in the position ZMP_{jump}. At the end of the normal step, the angular acceleration, $\ddot{\theta}_{3f}$, is calculated in order to have the ZMP at the position ZMP_f, such that the difference between $\ddot{\theta}_{3f}$ and $\ddot{\theta}_{30}$ to be minimal. Consequently, the torque necessary to change the acceleration of the upper body link will be minimal.

4. PROBLEM FORMULATION AND PROPOSED METHOD

Problem Formulation

The problem, for the first and normal step, consists on finding the joint angle trajectories that connect the first and last posture of biped robot for which the CE or TC is minimal. Related with the CE, because the joint torque is proportional to the current it can be assumed that the energy to control the position of the robot is proportional to the integration of the square of the torque with respect to time. Therefore, the minimum CE problem converts on minimizing the joint torque. The cost function En, which is a quantity proportional with the energy required for the motion, is defined as follows:

$$\mathrm{E}n = \frac{1}{2}\left(\int_0^{t_f} \tau^{\mathrm{T}}\tau \mathrm{dt} + \Delta\tau_{jump}^2 \Delta t + \int_0^{t_f} \mathrm{Cdt}\right) \tag{2}$$

where: t_f is the step time, τ is the torque vector, $\Delta\tau_{\mathrm{jump}}$ and Δt are the torque applied to the body link to cause the ZMP to jump and its duration time respectively, and C is the constraint function given as follows:

$$\mathrm{C} = \begin{cases} 0 \text{ - if the constraints are satisfied,} \\ c_i \text{ - if the constraints are not satisfied,} \end{cases}$$

C denotes the penalty function vector. The following constrains are considered:

1. The ZMP to be in the sole length;
2. The distance between the hip and ankle joint of the swing leg must not be longer then the extended leg;
3. The swing foot must not touch the ground prematurely.

To clarify the effect of the proposed method, the minimum CE angle trajectories are compared with the minimum TC angle trajectories [6]. The cost function is given by:

$$\mathrm{J}_{\text{torque change}} = \frac{1}{2}\left(\int_0^{t_f} \left(\frac{\mathrm{d}\tau}{\mathrm{dt}}\right)^{\mathrm{T}} \left(\frac{\mathrm{d}\tau}{\mathrm{dt}}\right)\mathrm{dt} + \left(\frac{\Delta\tau}{\Delta \mathrm{t}}\right)^2 + \int_0^{t_f} \mathrm{Cdt}\right). \tag{3}$$

Proposed Method

The block diagram of the proposed method is presented in Fig. 3. Based on the initial conditions and the range of the searching variables, an initial population is generated. Every angle trajectory is presented as polynomial of time. Its degree is determined based on the number of angle trajectory constraints and the coefficients are calculated to satisfy them. The torque vector is calculated from the inverse dynamics of five-link biped robot [7] as follows:

$$J(\theta)\ddot{\theta} + X(\theta)\dot{\theta}^2 + Y\dot{\theta} + Z(\theta) = \tau \,. \tag{4}$$

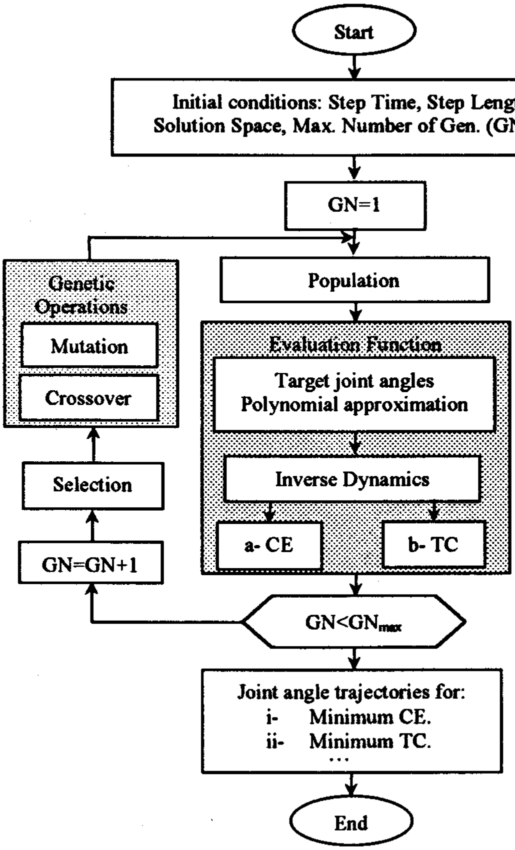


Fig. 3. Block diagram of the proposed method.

Based on the equations (2) and (3), the cost function is calculated for minimum CE and minimum TC, respectively. This value is attached to every individual in the population. The GA moves from generation to generation, selecting parents and reproducing offspring until the termination criterion (maximum number of generations GN_{max}) is met. Based on GA results, the gait synthesis is generated for minimum CE and minimum TC, respectively.

5. BOUNDARY CONDITIONS AND GA VARIABLES

We consider the boundary conditions and GA variables for the normal step and first step. We will start with normal step, because it occupies most of the walking time. The first step is considered as transition phase between the static standing position and the normal step.

Normal Step

To have a continuos periodic walking, we consider that the posture is the same at the beginning and at the end of the step. Therefore, the following relations must be satisfied:

$$\theta_{10}=\theta_{5f},\ \theta_{20}=\theta_{4f},\ \theta_{1f}=\theta_{50},\ \theta_{2f}=\theta_{40},\ \theta_{30}=\theta_{3f}. \tag{5}$$

In order to find the best posture during the normal step, the best value of θ_{10}, θ_{20} and θ_{30} must be determined by GA. For a given step length, it is easy to calculate θ_{40} and θ_{50}. Related with angular velocity, based on the link rotation direction we can write:

$$\dot{\theta}_{10}=\dot{\theta}_{20}=\dot{\theta}_{40}=\dot{\theta}_{2f}=\dot{\theta}_{4f}=\dot{\theta}_{5f}=0. \tag{6}$$

Link 1 continues to rotate in the same direction at the end of the step. This can be written in the form $\dot{\theta}_{1f}=\dot{\theta}_{50}\geq 0$. To find its best value, we consider it as one variable of GA. The angular velocity of the upper body link must be the same at the beginning and at the end of the step, so $\dot{\theta}_{30}=\dot{\theta}_{3f}$. GA will determine its best value. The following relations are considered for the angular acceleration:

$$\ddot{\theta}_{10}=\ddot{\theta}_{5f}, \ddot{\theta}_{20}=\ddot{\theta}_{4f}, \ddot{\theta}_{1f}=\ddot{\theta}_{50}, \ddot{\theta}_{2f}=\ddot{\theta}_{40}. \tag{7}$$

In this way, during the instantaneous double support phase is not needed to apply an extra torque to change the angular acceleration of the links. To determine the angle trajectories of the leg part, the coordinates of an intermediate point $P(x_p,z_p)$ and their passing time, t_p, are also considered as GA variables. The searching area for the intermediate point is shown in Fig. 2. At the end of the normal step, the upper body goes in its first position. To find the upper body angle trajectory, an intermediate angle θ_{3p} and its passing time t_3 will be considered as GA variables.

Based on the number of constrains, the degree of the polynomials for angle trajectories of θ_1 and θ_2 is 3, θ_3 is 7, and θ_4 and θ_5 is 6.

First Step

The first step is a transition between the static standing position and normal step. For the standing position, we can write the following relations:

$$\begin{aligned} &\theta_{10}=\theta_{50},\ \theta_{20}=\theta_{40}, \\ &\dot{\theta}_{10}=\dot{\theta}_{20}=\dot{\theta}_{30}=\dot{\theta}_{40}=\dot{\theta}_{50}=0, \\ &\ddot{\theta}_{10}=\ddot{\theta}_{20}=\ddot{\theta}_{30}=\ddot{\theta}_{40}=\ddot{\theta}_{50}=0. \end{aligned} \tag{8}$$

To find the best posture at the standing position, the GA variables will be: θ_{10}, θ_{20} and θ_{30}. For a continuous walking at the end of the first step every link state, angle, angular velocity and angular acceleration becomes equal with those at the end of the normal step, which can be written as:

$${}_{\text{first}}\theta_{if} = {}_{\text{normal}}\theta_{if},\quad {}_{\text{first}}\dot{\theta}_{if} = {}_{\text{normal}}\dot{\theta}_{if},\quad {}_{\text{first}}\ddot{\theta}_{if} = {}_{\text{normal}}\ddot{\theta}_{if}, \tag{9}$$

where i=1, …,5.

The coordinates of an intermediate point $P(x_p,z_p)$ and their passing time t_p are used to determine the best angle trajectories of the swing leg, as shown in Fig. 2. An angle θ_{3p} and its passing time t_3 will be considered to decide the upper body motion. Based on the link state constrains, each angle trajectory is presented as polynomial of time. The degree of the polynomials for θ_1, θ_2, θ_3, θ_4, θ_5 are 5, 5, 6, 6, and 6, respectively.

6. SIMULATION RESULTS

The simulations are realized based on the parameters of “Bonten-Maru I” humanoid robot. The parameter values are given in Table 1 and the robot is shown in Fig. 4. The “Bonten-Maru I” is 1.2 m high and 32 kg. Each leg has 6 degrees of freedom and is composed by three segments: upper leg, lower leg and the foot. A DC servomotor actuates each joint. Potentiometers measure the angel and angular velocity of every link, interfaced to the computer via RIF-01 (ADC). The control platform is based on Common Object Request Broker Architecture (CORBA), which makes easy updating and addition of new modules. A Pentium base microcomputer (PC/AT compatible) is used to control the system.

For the optimization of the cost function, a real-value GA was employed in conjunction with the selection, mutation and crossover operators. Many experiments comparing real value and binary GA have proven that the real value GA generates better results in terms of the solution quality and CPU time [8]. To ensure a good result of the optimization problem, the best GA parameters are determined by extensive simulation that we have performed, as shown in Table 2. The maximum number of generations is used as the termination function. The GA converges within 40 generations (see Fig. 5). The average of cost function En against the number of generations is shown in Fig. 6. The 33-th generation has the lowest value of the average of cost function En.

Table 1. "Bonten-Maru I" parameters.

	Upper body	Lower leg	Upper leg	Lower leg + foot
Mass of the link [kg]	12	2.93	3.89	4.09
Moment of inertia [kg m2]	0.19	0.014	0.002	0.017
Length [m]	0.3	0.2	0.204	0.284
CoM from lower joint [m]	0.3	0.09	0.1	0.136
CoM from upper joint [m]	0.0	0.11	0.104	0.136

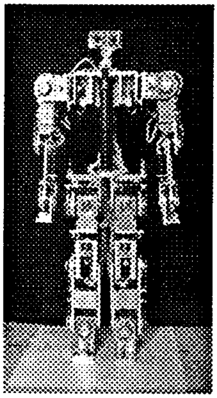


Fig. 4. "Bonten-Maru I" humanoid robot.

Table 2. Functions and parameters of GA.

Function Name	Parameters
Arithmetic Crossover	2
Heuristic Crossover	[2 3]
Simple Crossover	2
Uniform Mutation	4
Non-Uniform Mutation	[4 GNmax 3]
Multi-Non-Uniform Mutation	[6 GNmax 3]
Boundary Mutation	4
Normalized Geometric Selection	0.08

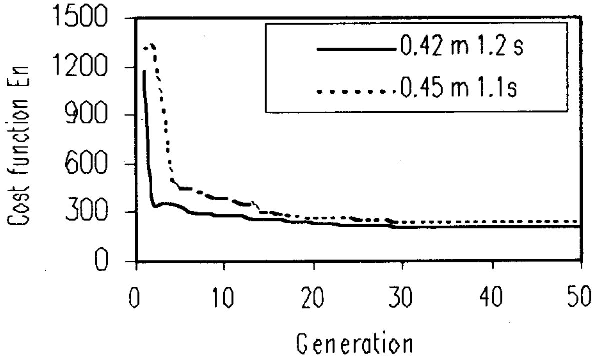


Fig. 5. Cost function En vs. generations.

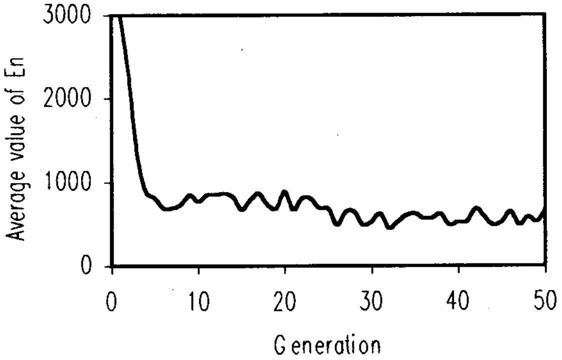


Fig. 6. Average of the cost function En vs. generations.

Normal Step

In simulations, the normal step length varies from 0.3 m to 0.5 m. The value of En cost function versus the step time for different step lengths is presented in Fig. 7. From this graph it is clear that every step length is optimal at a particular walking velocity. The searching space of the variables to be optimized and GA results are presented in Table 3 for step length 0.4 m and step time 1 s. Based on these data, the joint angles trajectories (θ_i), the torque vector (τ_i) and the walking pattern are generated for minimum CE and TC cost functions, as shown in Fig. 8 and Fig. 9, respectively. As can be seen from Fig. 8(a) and Fig. 9(a) the boundary conditions for the normal step are satisfied. The angular velocity of the lower leg of supporting foot at the end of the step is greater when minimum CE is used as cost function. Comparing Fig. 8(b) and Fig. 9(b), the torques change more smoothly when TC is used as a cost function. Based on the simulation results, the posture is straighter when minimum CE is used as a cost function (see Fig. 8(c) and Fig. 9(c)). The ZMP position is presented in Fig. 10 for minimum CE and TC. The ZMP is all the time in the sole length, which guaranty a stable motion. At the end of the step, the ZMP is at position ZMP_f, as shown in Fig. 2. At the beginning of the step, the ZMP is not exactly at the position ZMP_{jump} because the mass of the foot is not neglected. It should be noted that the mass of the lower leg is different when it is in supporting leg or swing leg. The values of En cost function, for minimum CE and TC gait synthesis, are presented in Fig. 11. The minimum CE gait synthesis reduces the energy by about 25% compared with minimum TC.

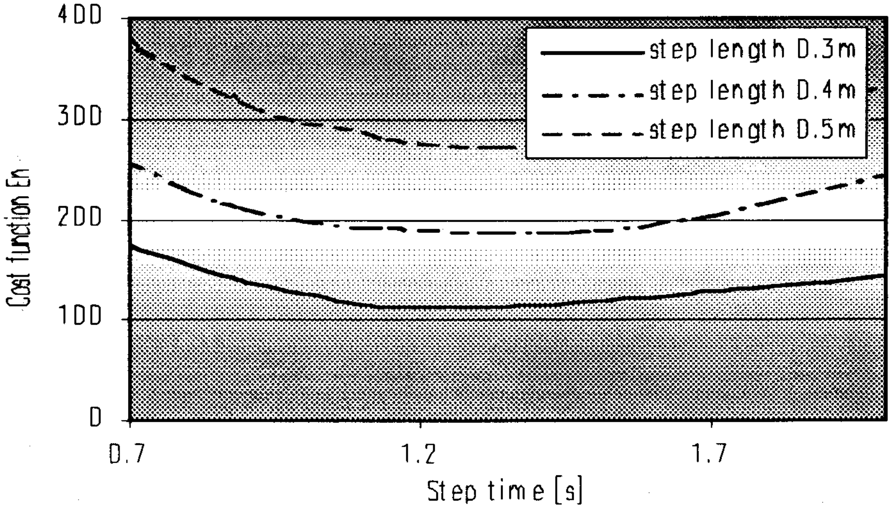
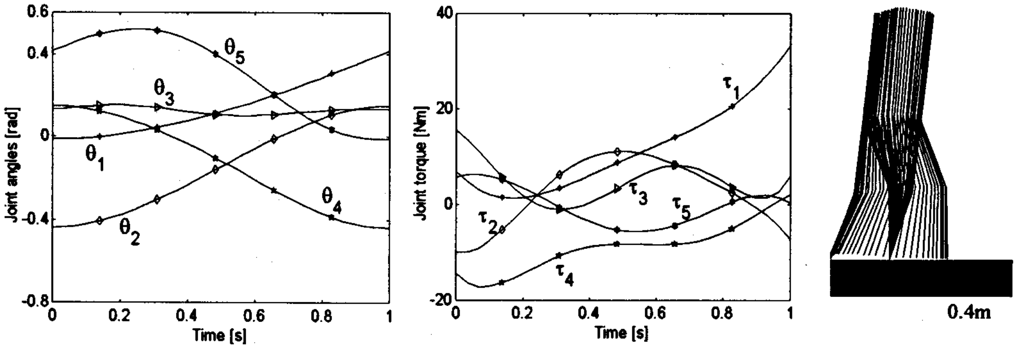


Fig. 7. Optimum step time for different step lengths.

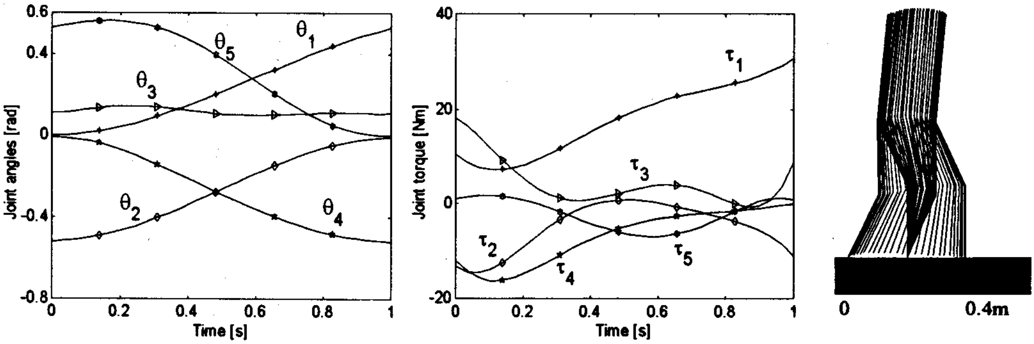
Table 3. GA results for normal step.

GA Variables	Limits	GA Results	
		CE	TC
θ_{10}	-0.3~0	-0.137015	-0.000691
θ_{20}	-.7~-.3	-0.437428	-0.519955
θ_{30}	0 ~ 0.3	0.130960	0.110055
θ_{3p}	-0.1~0.2	0.099799	0.100112
t_3	0.3~0.7	0.567068	0.590031
x_p	-0.2~0.2	-0.086162	-0.058547
y_p	.01~.04	0.014289	0.011892
t_p	0.1~0.9	0.451618	0.447172
$\dot{\theta}_{1f}$	0 ~ 2	0.661357	0.393585
$\dot{\theta}_{30}$	-1 ~ 1	-0.029474	0.171488



(a) Angle trajectories. (b) Joint torque. (c) Walking pattern.

Fig. 8. Minimum CE cost function results for the normal step.



(a) Angle trajectories. (b) Joint torque. (c) Walking pattern.

Fig. 9. Minimum TC cost function results for the normal step.

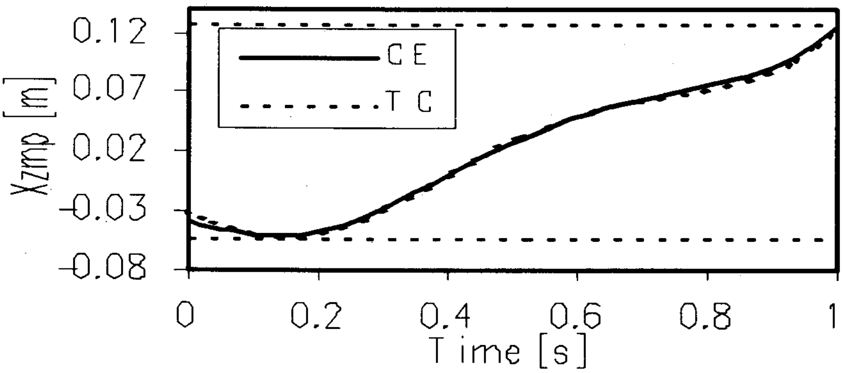


Fig. 10. ZMP position for normal step.

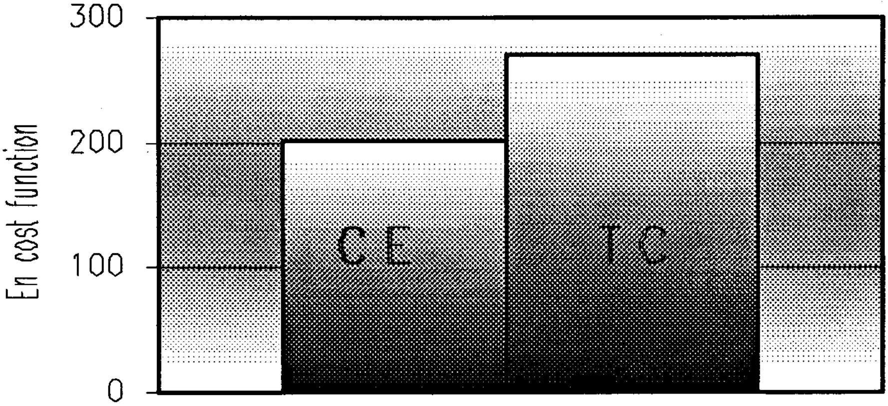


Fig. 11. Comparison of En cost function.

First Step

The time for the first step is determined after many simulations. The value of En cost function versus the step time is presented in Fig. 12. The step time for which the CE is minimal, is 0.8s. The searching space of the variables and GA results, for minimum CE and minimum TC, are presented in Table 4. The angle trajectory, torque vector, and walking pattern are presented in Fig. 13 and Fig. 14 for minimum CE and minimum TC, respectively. Based on Fig. 13(a) and 14(a) the links states at the end of the first step are equal with those of normal step. The posture is straighter for minimum CE cost function. Fig. 15 shows that the ZMP is ,all the time, in the sole length and; at the end of the first step, it is at the position ZMP_f.

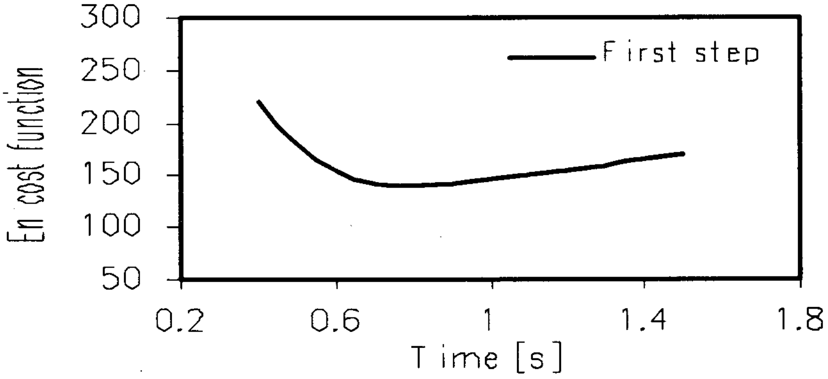
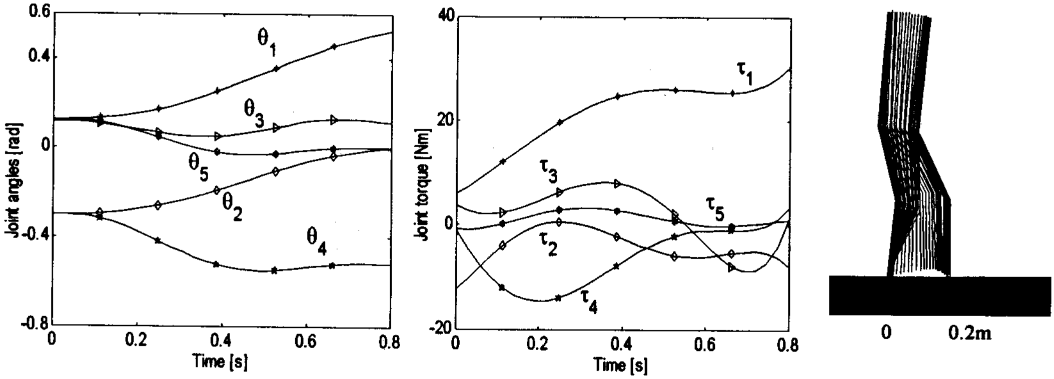


Fig. 12. Relation between En cost function and step time.

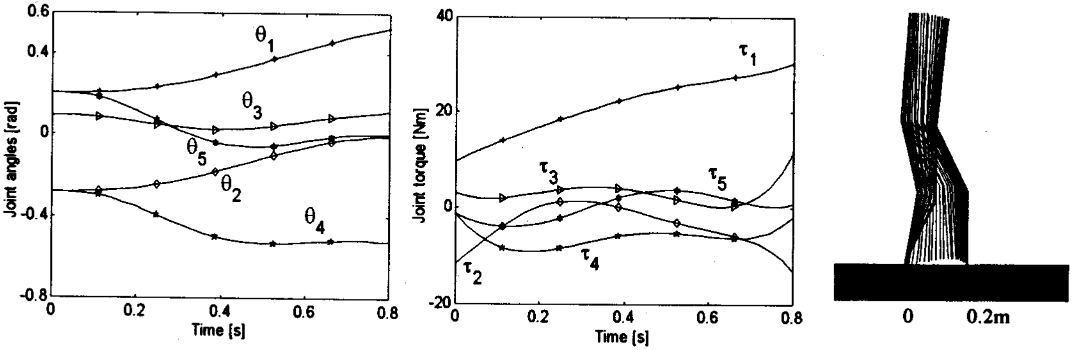
Table 4. GA results for the first step.

GA Variables	Limits	GA results	
		CE	TC
θ_{10}	0 ~ 0.3	0.125321	0.201593
θ_{20}	-.4 ~ 0	-0.290167	-0.28218
θ_{30}	-0.1~0.3	0.116132	0.090900
θ_{3p}	0 ~ 0.3	0.049391	0.022801
t_3	0.2~0.6	0.355701	0.395216
x_p	0 ~ 0.19	0.120298	0.151571
y_p	.01~.05	0.017801	0.015091
t_p	0.1~0.7	0.394859	0.460654



(a) Joint angle trajectories. (b) Joint torque. (c) Walking pattern.

Fig. 13. Minimum CE cost function results for the first step.



(a) Joint angle trajectories. (b) Joint torque. (c) Walking pattern.

Fig. 14. Minimum TC cost function results for the first step.

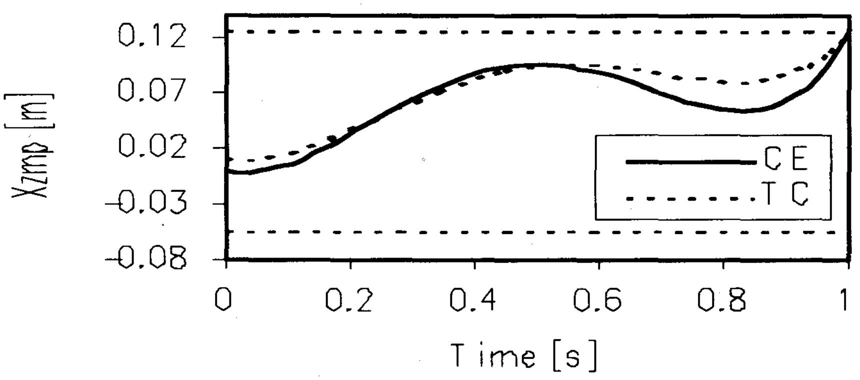


Fig. 15. ZMP position for first step.

7. IMPLEMENTATION ISSUES

In contrast to other optimization methods, GA needs more time to get the optimal solution. In our simulations, it needs about 10 minutes. But often during walking, the humanoid robot must change the step length and walking velocity. Based on the new step length and step time, the angle trajectories must be generated. Also, when humanoid robot must overcome an obstacle or move up the stair, based on the information received by eye system, the angle trajectories must be generated in a short time. For these reasons, we are considering the following methods for a real time implementation of our method.

- **Parallel Processing Architecture.**

Cantu-Paz [9] and Wang et al. [10] have shown that by using parallel processing architecture, the GA can generate the solution in real time.

- **Teaching a Neural Network (NN) based on the GA results.**

With the data received from GA, for different step lengths and step times, a NN can be taught. The input variables of NN will be the step length and step time and the output will be the same with the searching variables of GA. The NN can generate the solution in very short time.

- **Sequential Quadratic Programming (SQP).**

SQP provides the GA with a local improvement operator, which can greatly enhance its performance. GA is capable of quickly finding promising regions of the searching space, thus the combination of GA and SQP can reduce the running time.

8. CONCLUSIONS AND FUTURE WORKS

In this paper, a GA based approach for optimal gait generation of biped robots, is proposed. By using GA as optimization tool the system makes easy to include constrains and add new variables to be optimized. The proposed method can be applied to generate the angle trajectories for other that might be performed by humanoid robot, like moving up and down the stairs, overcoming obstacle etc. In this paper, we considered the gait synthesis during walking, starting from a static standing position. Our method can be applied for a wide range of step lengths and step times. Because the double support phase is very short, to ensure a stable walking the jumping of the ZMP is realized by accelerating the upper body link. The performance evaluation is carried out by simulation. Based on the simulation results, we conclude:

- for each step length there is a particular walking velocity for which the CE is minimal;
- the angular velocity of the lower leg at the end of the normal step is greater when minimum CE is used as a cost function;
- the stability is important to be considered during optimal gait generation;
- the biped robot posture is straighter when the minimum CE is used as cost afunction;
- the minimum CE optimal gait reduces the energy nearly 25 % compared with minimum TC gait.

In the future the following problems will be considered:

- optimal gait of biped robot during obstacle overcoming, moving up and down the stairs, etc;
- real time implementation of the proposed method.

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