Interpretation of Gravity Anomalies Due to Lens-Shaped Structures

by

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ABSTRACT

One of the interesting exploration problems in hydrogeology is finding subsurface geological lens-shaped structures. This paper deals with the interpretation of such structures using their gravity anomalies. A master curve has been constructed, based on the upward continuation method. This curve was used for gravity interpretation. Both the depth and the width and the thickness variation of the burial structure are determined by this curve. The procedure was illustrated by a worked example using theoretical data.

Introduction

The direct interpretation of gravity data has been the subject of mathematical studies for several decades. Numereous papers on this subject have appeared in the geophysical literatures. Complete sets of master curves were prepared by many investigators for interpretation of the gravity anomalies caused by different geometrical bodies.

In this work the method of upward continuation is used for the determination of the parameters of a lens-shaped structure from the gravity profile. A master curve was constructed from derived mathematical expression for evaluating the width, depth and the thickness variation of the caustive body.

Theory

Consider a lens-shaped structure with density contrast $\Delta \rho$ as a geometrical model (Fig. 1). For simplicity the body can be treated as a thin sheet with surface density $\mu(\zeta)$ which increases linearly in the direction to the centre of the body and then decreases linearly again. The gravity effect $\Delta g_{x,0}$ on the plane of observation at a point p is given by the following integral (El-Awady 1979) :

$$\Delta g_{x,0} = 2kt \left[\int_{\zeta=-b}^{0} \frac{\mu_0 + \mu_1(\zeta + b)}{(\zeta - x)^2 + t^2} \, d\zeta + \int_{\zeta=0}^{b} \frac{\mu_0 + \mu_1(\zeta - b)}{(\zeta - x)^2 + t^2} \, d\zeta \right] \quad ..(1)$$

whereas :

k is the universal gravitional constant,

μ_0 is the initial surface density $(= \Delta \rho \, . \, \Delta t_0 = 0)$,

μ_1 is the linearity factor of the surface density variation $(= \Delta \rho \, . \, \Delta t)$,

Δt_0 is the initial thickness $(=0)$,

Δt is the linearity factor of the thickness variation of the structure,

b is the half width and

t is the mean depth.

Evaluating the integral we get:

$$\Delta g_{x,0} = k\sigma_1 \Bigg[2b \left(\tan^{-1}\frac{x+b}{t} - \tan^{-1}\frac{x-b}{t}\right) + 2x \left(\tan^{-1}\frac{x+b}{t} - \tan^{-1}\frac{x-b}{t} - 2\tan^{-1}\frac{x}{t}\right) + t \ln \frac{(x^2+t^2)}{\left[(x+b)^2+t^2\right]\left[(x-b^2)+t^2\right]} \Bigg] \quad ....(2)$$

The gravity at the origin point can be derived by putting $x = 0$, we obtain :

$$\Delta g_{0,0} = 2k\sigma_1 \left[2\,b \tan^{-1}\frac{b}{t} + t \ln \frac{t^2}{b^2+t^2} \right] \quad .... (3)$$

The upward continuation of the gravity effect at a height h above the plane of observation is given by :

$$\Delta g_{0,h} = 2k\sigma_1 \left[2\,b \tan^{-1}\frac{b}{t+h} + (t+h) \ln \frac{(t+h)^2}{b^2+(t+h)^2} \right] \quad .... (4)$$

Dividing equation (4) by equation (3) we get :

$$\frac{\Delta g_{0,h}}{\Delta g_{0,0}} = \frac{2b\left(\tan^{-1}\frac{b}{t+h}\right) + (t+h) \ln \frac{(t+h)^2}{b^2+(t+h)^2}}{2b\left(\tan^{-1}\frac{b}{t}\right) + t \ln \frac{t}{b^2+t^2}} \quad ....(5)$$

Normalizing b and t in terms of the height h, ($B = b/h$, $T = t/h$) we obtain:

$$\frac{\Delta g_{0,h}}{\Delta g_{0,0}} = \frac{2B\left(\tan^{-1}\frac{B}{T+1}\right) + (T+1) \ln \frac{(T+1)^2}{B^2+(T+1)^2}}{2B\left(\tan^{-1}\frac{B}{T}\right) + T \ln \frac{T^2}{B^2+T^2}} \quad .... (6)$$

A set of master curves is constructed based on equation (6) for different ratios of $\triangle g_{0,h} / \triangle g_{0,0}$ as ordinates, t/h as abscissa and b/h as a parameter. These curves are reproduced in figure 2. Each horizontal line corresponding to a measured $\triangle g_{0,h} / \triangle g_{0,0}$ value will cross various values of lines representing b/h. In applying these curves to field measurements, one assumes that the upward continuation values on the origin axis at three levels say h_1, h_2 and h_3. The $\triangle g_{0,h} / \triangle g_{0,0}$ ratio for each values of h is then computed and drawn as a horizontal line across the appropriate set of the master curves. At each intersection with a curve corresponding to different B, the values of B and T are collected. We then convert these values to b and t simply by multiplying each set of b/h and t/h by the corresponding known value of h. The b values are plotted vs. t on another graph, giving three sets of curves (Fig. 3). The intersection of the three curves gives the actual values of b and t of the structure. Knowing b, t and the density contrast $\triangle \rho$, the thickness variation $\triangle t$ can be calculated using equation (3). The values of the upward continuation at several levels can be obtained as given by Peters 1949, Roy 1966 or Baranov 1953.

Numerical Application

Let us illustrate the application of the master curve by an example using theoretical data (Fig. 1). The gravity profile corresponding to a lens-shaped structure is shown in the figure. Assume that the gravity value at the origin $\triangle g_{0,0}$ equals 1.689 mgal and the gravity value $\triangle g_{0,h}$ varies with the height h as in Table (1).

Table 1

Height h (m)	$\triangle g_{0,h}$ *(mgal)*	$\triangle g_{0,h} / \triangle g_{0,0}$
100	1.416	0.837
200	1.211	0.716
300	1.053	0.623
400	0.929	0.550

Collecting the values of B and T from the intesection of the horizontal lines and

B-curves, we get four sets of B and T. Multiplying B and T by the corresponding height h, we obtain four sets of b and t. The t values are plotted against the values of b as shown in figure 3. The respective curves are seen to intersect near the point t= 315 m and b= 590 m.

Knowing b, t and $\triangle g_{0,0}$, the surface density σ_1 can be calculated using equation (3). The factor of the thickness variation $\triangle t$ of the structure can be obtained by substituting the density contrast $\triangle \rho$ of the structure in the following equations :

$$\sigma_1 = \triangle \rho . \triangle t = 0.1584 \quad g/c.c$$

$$\triangle t = \sigma_1 / \triangle \rho = 0.1584$$

Thickness at the origin point= $\triangle$ t. b= 93.4 m.

The close similarities between the obtained results and the theoretical parameters of the structure are clearly illustrated in the following Table (2).

Table 2

Parameter	*Obtained results*	*Theoretical values*
b	590 m	600 m
t	315	300 m
σ_1	0.1584 g/c.c.	0.1499 G/c.c.
max. thickness	93.4 m	100 m

It should be emphasized that the obtained results are greatly affected by the selected method used for the upward continuation and by the accuracy of the gravity profile.

Conclusion

A master curve for the interpretation of gravity anomalies due to lens-shaped structures has been constructed. The curve can be used to determine the depth, the width and the thickness variation of the disturbing structure. The interpretation is mainly based on the upward continuation of the gravity anomaly at the origin point.

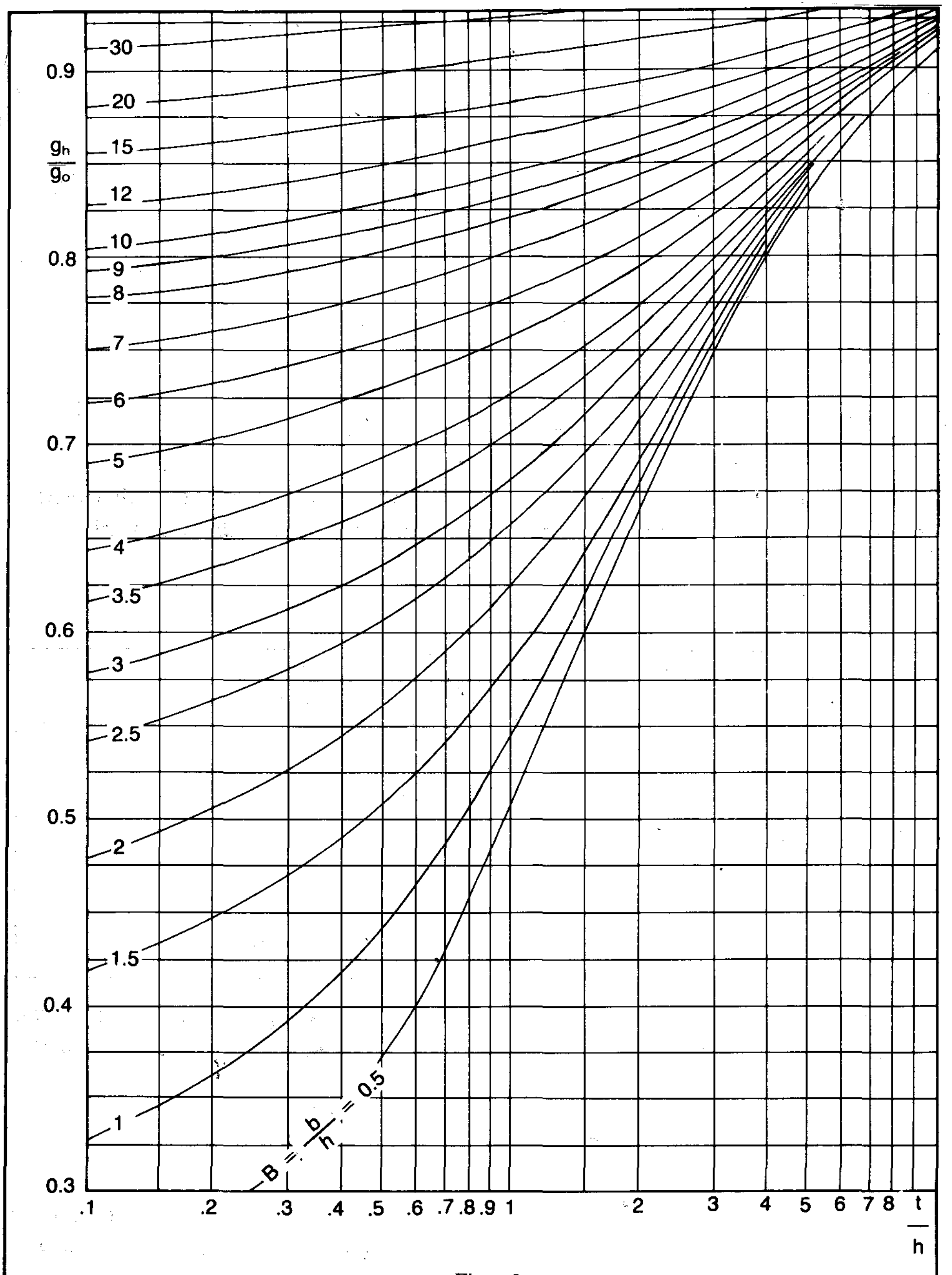
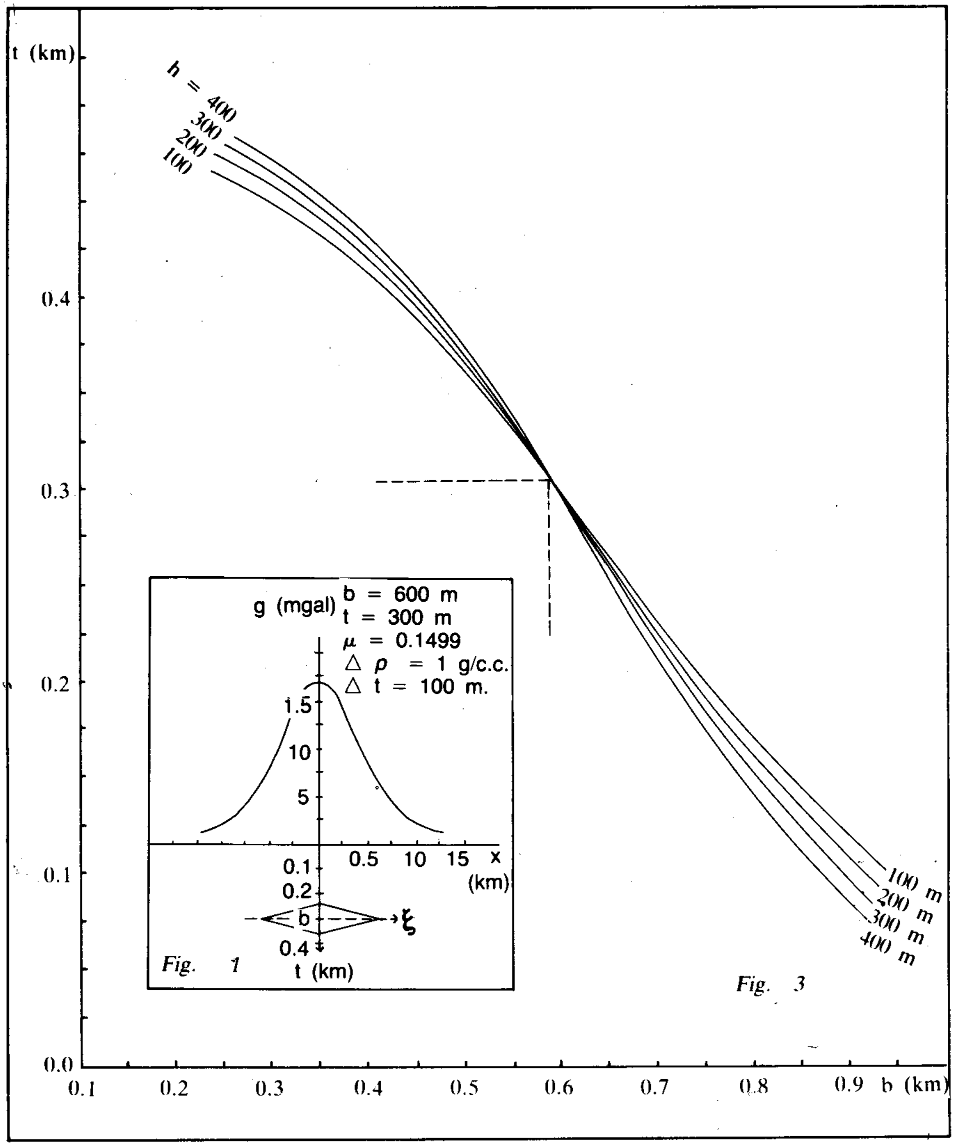

30
20
15
12
10
9
8
7
6
5
4
3.5
3
2.5
2
1.5
1
B = b/h = 0.5
0.9
0.8
0.7
0.6
0.5
0.4
0.3
gh/go
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
2
3
4
5
6
7
8
t/h

Fig. 2



t (km)
0.4
0.3
0.2
0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
b (km)
h = 400
300
200
100
100 m
200 m
300 m
400 m
g (mgal)
b = 600 m
t = 300 m
μ = 0.1499
Δ ρ = 1 g/c.c.
Δ t = 100 m.
1.5
10
5
0.5
10
15
x
(km)
0.1
0.2
b
0.4
ξ
t (km)
Fig. 1

Fig. 3

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تقييم الشـواذ التثاقليـة لبعض الأشكـال الجيـولـوجيـة (العدسات)

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ملخـــــــص

يتناول هذا البحث عرض طريقة جديدة لتقييم الشواذ التثاقلية لبعض الاشكال الجيولوجية مثل العدسات ـ والتي قد تكون ذات أهمية اقتصادية وذلك بالاستفادة من تغير الجاذبية رأسياً الى المستويات الأعلى ووضح ذلك في رسم بيان قياسي يمكن استخدامه في تحديد عمق وسمك واتساع هذه العدسات .