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ON THE SOLUTION OF THE JOB-SHOP SCHEDULING PROBLEM UNDER FUZZY ENVIRONMENT

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ABSTRACT

This paper presents a solution algorithm for solving the job-shop scheduling problem involving fuzzy parameters in the constraints. It is considered that the processing times are those fuzzy parameters and the maximum completion time is required to be minimized. The concept of α - level set together with the definition of the fuzzy number and its membership function are introduced. An illustrative numerical example is given to clarify the theory and the solution algorithm.

KEY WORDS: Job-shop scheduling, fuzzy parameters, α - level, membership function.

1. INTRODUCTION

The job-shop scheduling problem refers to most problems found in a typical factory environment where different operations must be carried out to complete a job. A number of jobs are processing at any one time on a number of machines. A schedule must be derived which aims to complete all jobs as quickly as possible on the given production plant.

In classical job-shop scheduling problem (JSSP), n jobs are processed to be completed on m unrelated machines. Each job requires processing on each machine exactly once. For each job, technology constraints specify a complete, distinct routing which is fixed and known in advance.

The processing times are sequence-independent, fixed and known in advance. Each machine is continuously available from time zero, and the operations are processed with preemption. The objective is to minimize the maximum completion time (makespan).

The job-shop scheduling problem is strongly NP-hard i.e., the time required to compute an optimal schedule increases exponentially with the size of the problem and the solution procedures are based on enumeration or on heuristics [2]. For this problem, theoretical results have begun for the two-machines case and in a special three-machines case by S. M. Johnson and R. Bellman in 1954. The analytical results, for example, simple criterion can be obtained by using heuristic methods or dynamic programming type procedures.

A lot of work that deal with the job-shop scheduling problem assume that all the time parameters e.g., processing times are known exactly [2,3,5,6,7,8,9,10]. When the data of the job-shop scheduling problem are imprecise, vague or uncertain, then these data can be only estimated within uncertainty. This uncertainty may be represented by fuzzy numbers and so reduce the errors of imprecision. The main objective of this paper is the implementation of the fuzzy concepts to job-shop scheduling problems rather than the use of the method in obtaining the optimal solution of such problems.

This paper is organized as follows: In Section 2, some basic notations on fuzzy set theory are introduced. In Section 3, we formulate the job-shop scheduling problem involving fuzzy parameters in the constraints. A solution algorithm to solve the problem of concern is suggested and presented in Section 4. In Section 5, a numerical example is given to clarify the theory and the solution algorithm. Finally, the paper is concluded in Section 6

2. FUZZY CONCEPTS

L. A Zadeh advanced the fuzzy theory at the University of California in 1965. The theory proposes a mathematical technique for dealing with imprecise concepts and problems that have many possible solutions. The concept of fuzzy mathematical programming on a general level was first proposed by Tanaka et al (1974) in the framework of the fuzzy decision of Bellman and Zadeh [11]. Now, we introduce some necessary definitions and the reader is referred to [1, 4].

Definition 1.

Let X be a nonempty set. A fuzzy set F in X is characterized by its membership function $\mu_F : X \to [0, 1]$.

Definition 2.

A real fuzzy number $\tilde{a}$ is a fuzzy subset from the real line R with membership function $\mu_{\tilde{a}}$ satisfies the following conditions:

(1) $\mu_{\tilde{a}}$ is a continuous mapping from R to the closed interval [0, 1],

(2) $\mu_{\tilde{a}}(x) = 0 \quad \forall\, x \in (-\infty, a_1\,]$,

(3) $\mu_{\tilde{a}}(x)$ is strictly increasing and continuous on $[\, a_1, a_2]$,

(4) $\mu_{\tilde{a}}(x) = 1 \quad \forall\, x \in [\, a_2, a_3]$,

(5) $\mu_{\tilde{a}}(x)$ is strictly decreasing and continuous on $[\, a_3, a_4]$, ,

(6) $\mu_{\tilde{a}}(x) = 0 \quad \forall\, x \in [\, a_4, +\infty)$.

where a_1, a_2, a_3, a_4 are real numbers and the fuzzy number $\tilde{a}$ is denoted by $\tilde{a} = [\, a_1, a_2, a_3, a_4]$.

Figure 1. illustrates the graph of a possible shape of a membership function of a fuzzy number $\tilde{a}$.

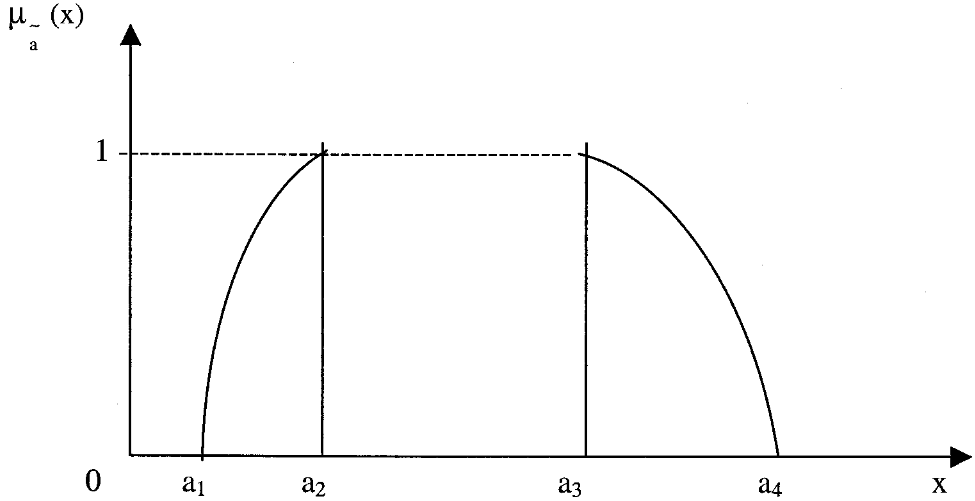

Fig.1. Membership function of a fuzzy number $\tilde{a}$.

Definition 3.

The α- level set of the fuzzy number $\tilde{a}$ is defined as the ordinary set $L_\alpha(\tilde{a})$ for which the degree of their membership function exceeds the level $\alpha \in [0, 1]$:

$$L_\alpha(\tilde{a}) = \left\{ a \in R \,\middle|\, \mu_{\tilde{a}}(a) \geq \alpha \right\}.$$

It is clear that the level sets have the following property:

$$\alpha_1 \leq \alpha_2 \text{ if and only if } L_{\alpha_1}(\tilde{a}) \supset L_{\alpha_2}(\tilde{a}).$$

3. JOB-SHOP SCHEDULING PROBLEM HAVING FUZZY PARAMETERS

In The Constraints $(JSSP)_{\tilde{p}}$

Applegate and Cook in [3] give a formula of the job-shop scheduling problem which has n jobs on m machines. A job has to be processed on each one of the m machines. The route of a job has to follow is predetermined and fixed. The routes of the different jobs are not necessarily the same. The objective is to minimize the makespan. In this formulation P_{ij} denotes the processing times of job j on machine i and the variable Y_{ij} denotes the starting time of this operation. The set N denotes the set of all operations (i, j) corresponding to the nodes in the directed graph. The set A denotes the set of all precedence (routing) constraints (i, j) < (k, j) that require job j to be processed on machine i before it is processed on machine k, i.e. operations (i, j) precedes operation (k, j). The following mathematical program minimizes the makespan.

$$\text{Min} \quad C_{max}, \tag{1.a}$$

subject to

$$Y_{kj} - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) < (k, j) \in A \tag{1.b}$$

$$C_{max} - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) \in N \tag{1.c}$$

$$Y_{ij} - Y_{ir} \geq P_{ir}, \text{ or } Y_{ir} - Y_{ij} \geq P_{ij}, \forall\ (i, r), (i, j); i = 1, 2, \ldots m \tag{1.d}$$

$$Y_{ij} \geq 0 \qquad \forall\ (i, j) \in N \tag{1.e}$$

In the above problem (1.a)-(1.e), the constraints (1.e) represent the fact that same ordering must be exist among the operations of different jobs and processed on the same machines.

Now, let us go back to problem (1.a)-(1.e). In this model we will consider that processing times are fuzzy parameters and Z denotes the maximum completion time, then this problem can be rewritten as a job-shop scheduling problem involving fuzzy parameters in the constraints $(JSSP)_{\tilde{p}}$ in the following form.

$$(JSSP)_{\tilde{p}}: \quad \text{Min} \quad f(Z) = Z, \tag{2.a}$$

subject to

$$Y_{kj} - Y_{ij} \geq \tilde{P}_{ij}, \qquad \forall\ (i, j) < (k, j) \in A \tag{2.b}$$

$$Z - Y_{ij} \geq \tilde{P}_{ij}, \qquad \forall\ (i, j) \in N \tag{2.c}$$

$$Y_{ij} - Y_{ir} \geq \tilde{P}_{ir}, \text{ or } Y_{ir} - Y_{ij} \geq \tilde{P}_{ij}, \forall\ (i, r), (i, j); i = 1, 2, \ldots m; \tag{2.d}$$

$$Y_{ij} \geq 0 \qquad \forall\ (i, j) \in N \tag{2.e}$$

where $\tilde{P}_{ij}$ and $\tilde{P}_{ir}$ represent fuzzy parameters involved in the constraints where their membership functions are $\mu_{\tilde{p}_{ij}}$ and $\mu_{\tilde{p}_{ir}}$, respectively.

For a certain degree $\alpha \in [0, 1]$ and by introducing the concept of α-level set of the fuzzy numbers $\tilde{P}_{ij}$ and $\tilde{P}_{ir}$, then problem $(JSSP)_{\tilde{p}}$ (2.a)-(2.e) can be understood as the following nonfuzzy problem:

$$(JSSP)_{\tilde{p}}: \quad \text{Min} \quad f(Z) = Z, \tag{3.a}$$

subject to

$$Y_{kj} - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) < (k, j) \in A \tag{3.b}$$

$$Z - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) \in N \tag{3.c}$$

$$Y_{ij} - Y_{ir} \geq P_{ir}, \text{ or } Y_{ir} - Y_{ij} \geq P_{ij}, \forall\ (i, r), (i, j);\ i = 1, 2, \ldots m \tag{3.d}$$

$$P_{ij} \in L_{\alpha}(\tilde{P}_{ij}) \text{ and } P_{ir} \in L_{\alpha}(\tilde{P}_{ir}) \tag{3.e}$$

$$Y_{ij} \geq 0 \qquad \forall\ (i, j) \in N \tag{3.f}$$

where $L_{\alpha}(\tilde{P}_{ij})$ and $L_{\alpha}(\tilde{P}_{ir})$ are the α-level sets of the fuzzy numbers $\tilde{P}_{ij}$ and $\tilde{P}_{ir}$, respectively.

Problem $(JSSP)_p$ (3.a)-(3.f) above can be written as follows:

$$(JSSP)_p \quad : \text{Min} \quad f(Z) = Z, \tag{4.a}$$

subject to

$$Y_{kj} - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) < (k, j) \in A \tag{4.b}$$

$$Z - Y_{ij} \geq P_{ij}, \qquad \forall\ (i, j) \in N \tag{4.c}$$

$$Y_{ij} - Y_{ir} \geq P_{ir}, \text{ or } Y_{ir} - Y_{ij} \geq P_{ij}, \ \forall\ (i, r), (i, j);\ i = 1, 2, \ldots m \tag{4.d}$$

$$\overset{(0)}{h}_{ij} \leq P_{ij} \leq \overset{(0)}{H}_{ij}\ ,\ \overset{(0)}{h}_{ir} \leq P_{ir} \leq \overset{(0)}{H}_{ir}\ \ \forall\ (i, r), (i, j);\ i = 1, 2, \ldots m \tag{4.e}$$

$$Y_{ij} \geq 0 \qquad \forall\ (i, j) \in N \tag{4.f}$$

where $\overset{(0)}{h}_{ij}, \overset{(0)}{H}_{ij}$ and $\overset{(0)}{h}_{ir}$ $\overset{(0)}{H}_{ir}$ are lower and upper bounds on P_{ij} and P_{ir}, respectively.

It should be noted that the constraints (3.e) have been replaced by the equivalent constraints (4.e). The presence of the either-or-constraints posses a difficulty since the model is no longer in the linear programming format. This difficulty is treated by introducing the binary variables $x_{ij,ir}$ defined by

$$x_{ij,ir} = \begin{cases} 0 & \text{if operation } (i, r) \text{ precedes operation } (i, j), \\ 1 & \text{if operation } (i, j) \text{ precedes operation } (i, r), \end{cases} \tag{5}$$

For M sufficiently large, the either-or- constraints become equivalent to the two simultaneous constraints:

$$M\, x_{ij,ir} + (Y_{ij} - Y_{ir}) \geq P_{ir}, \tag{6.a}$$

and

$$M\, (1 - x_{ij,ir}) + (Y_{ir} - Y_{ij}) \geq P_{ij},\ \forall\ i, j, r. \tag{6.b}$$

4. SOLUTION ALGORITHM

In what follows, we describe a solution algorithm for solving problem $(JSSP)_{\tilde{p}}$ (2.a)-(2.e) in finite steps. The proposed algorithm consists of two main phases. In Phase A, problem $(JSSp)_{\tilde{p}}$ can be converted into its equivalent nonfuzzy version.

In Phase B, where problem $(JSSp)_{\tilde{p}}$ becomes in its nonfuzzy form, a heuristic rule is suggested to prefer between the different operations to be scheduled. The first phase of the solution algorithm can be summarized as follows:

Phase A

Step 0. Set $\alpha = \alpha^* = 0$.

Step 1. Determine the points (a_1, a_2, a_3, a_4) for each fuzzy parameter in problem $(JSSp)_{\tilde{p}}$ with the corresponding membership function of these fuzzy parameters.

Step 2. Convert the problem $(JSSp)_{\tilde{p}}$ (2.a)-(2.e) in the nonfuzzy version $(JSSp)_{\tilde{p}}$ (3.a)-(3.f).

Step 3. Read the values of n and m.

Before we go further, we arrange the different operations according to its precedence relations. The steps of Phase B of the proposed algorithm can be described as follows:

Phase B

Step 4. Put the operations that have no predecessors in column I, operations that have directly by those in column I are replaced in column II, operations that are immediately preceded by those in column II are replaced in column III, etc.

Step 5. On the Gantt chart, shift the operations to the left as far as possible from the first column without violating precedence constraints.

Step 6. Solve the resulting problem using any available integer linear programming software package.

Step 7. Set $\alpha = (\alpha^* + \text{step}) \in [0, 1]$ and go to step 1.

Step 8. Repeat the above procedure until the interval [0, 1] is fully exhausted. Then, stop.

5. AN ILLUSTRATIVE EXAMPLE

Consider the 3x2 job-shop scheduling problem (3 machines and 2 jobs). The precedence structure is given as follows:

Job 1: $m_1m_3m_2$ and Job 2: $m_2m_1m_3$.

Table (1) below contains the fuzzy parameters which are characterized by the following fuzzy numbers.

$\tilde{P}_{11} = (2, 4, 5, 7)$	$\tilde{P}_{12} = (0, 3, 4, 7)$
$\tilde{P}_{21} = (3, 4, 5, 7)$	$\tilde{P}_{22} = (1, 5, 7, 10)$
$\tilde{P}_{31} = (0, 3, 5, 6)$	$\tilde{P}_{32} = (1, 6, 7, 9)$

Table (1).

Assume that the membership function corresponding to the fuzzy numbers is in the form:

$$\mu_{\tilde{a}}(a) = \begin{cases} 0, & a \le a_1, \\ 1-(\dfrac{a-a_1}{a_1-a_2})^2, & a_1 \le a \le a_2, \\ 1, & a_2 \le a \le a_3, \\ 1-(\dfrac{a-a_3}{a_4-a_3})^2, & a_3 \le a \le a_4, \\ 0, & a \ge a_4, \end{cases}$$

where $\tilde{a}$ corresponds to each $\tilde{P}_{i1}$ and $\tilde{P}_{ir}$ in the $(JSSP)_{\tilde{p}}$. Let $\alpha = 0.36$, then we get:

$$2.4 \le P_{11} \le 6.6,\ 0.6 \le P_{12} \le 6.4, \quad 3.2 \le P_{21} \le 6.6,$$
$$2.8 \le P_{22} \le 9.4,\ 0.6 \le P_{31} \le 5.8, \quad 2 \le P_{32} \le 8.6.$$

The nonfuzzy $(JSSP)_p$ can be written as follows:

$(JSSP)_p$: Min $f(Z) = Z$,
subject to

$$Y_{21} - Y_{11} \geq P_{11} \qquad Y_{12} - Y_{22} \geq P_{22} \qquad Y_{32} - Y_{31} \geq P_{31}$$

$$Z - Y_{11} \geq P_{11} \qquad Z - Y_{12} \geq P_{22} \qquad Z - Y_{21} \geq P_{21}$$

$$Z - Y_{22} \geq P_{22} \qquad Z - Y_{31} \geq P_{31} \qquad Z - Y_{32} \geq P_{32}$$

$$M\, X_{11,12} + Y_{11} - Y_{12} \geq P_{12}, \qquad M\,(1 - X_{11,12}) + Y_{12} - Y_{11} \geq P_{11}$$

$$M\, X_{21,22} + Y_{21} - Y_{22} \geq P_{22}, \qquad M\,(1 - X_{21,22}) + Y_{22} - Y_{21} \geq P_{21}$$

$$M\, X_{31,32} + Y_{31} - Y_{32} \geq P_{32}, \qquad M\,(1 - X_{31,32}) + Y_{32} - Y_{31} \geq P_{31}$$

$$2.4 \leq P_{11} \leq 6.6, \quad 0.6 \leq P_{12} \leq 6.4, \quad 3.2 \leq P_{21} \leq 6.6,$$
$$2.8 \leq P_{22} \leq 9.4, \quad 0.6 \leq P_{31} \leq 5.8, \quad 2 \leq P_{32} \leq 8.6.$$

$$Y_{11}, Y_{12}, Y_{21}, Y_{22}, Y_{31}, Y_{32} \geq 0,$$
$$X_{11,12} \geq 0, X_{21,22} \geq 0, X_{31,32} \geq 0,$$
$$1 - X_{11,12} \geq 0, 1 - X_{21,22} \geq 0, 1 - X_{31,32} \geq 0.$$

For M = 100, we have the following results:

$(Z, Y^*_{11}, Y^*_{12}, Y^*_{21}, Y^*_{22}, Y^*_{31}, Y^*_{32}, X^*_{11,12}, X^*_{21,22}, X^*_{31,32}) =$
10, 0, 5.8, 6.4, 2.4, 5.7, 6.6, 1, 0, 1) with the α-optimal parameters:

$$(P^*_{11}, P^*_{12}, P^*_{21}, P^*_{22}, P^*_{31}, P^*_{32}) = (2.5, 0.8, 3.6, 4, 0.9, 3.4).$$

6. CONCLUSION

In this paper we suggested a solution algorithm, described in finite steps, for solving the job-shop scheduling problem with fuzzy parameters in the constraints. We have considered that the processing times are those fuzzy parameters and it is required that the maximum completion time has to be minimized. However, there are many open points for future research in the area of the job shop scheduling problems. Some of these points are follows:

(i) A procedure is needed to solve the cyclic job-shop scheduling problem with different jobs,

(ii) A procedure is needed to solve the cyclic job-shop scheduling problem with fuzzy parameters,

(iii) A procedure is needed to solve the cyclic job-shop scheduling problem with random parameters,

(iv) A procedure is needed to solve the early and tardy job-shop scheduling problem with fuzzy and with random parameters.

(v) An algorithm is needed to solve multiple-objectives job-shop scheduling problems involving fuzzy parameters and others involving random parameters.

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