

# CREEP BUCKLING OF PLATES UNDER BIAXIAL COMPRESSION

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## ABSTRACT

This work presents an analysis for the creep buckling problem of geometrically imperfect rectangular flat plates under biaxial compression with simple support boundary conditions. The analysis is based on a non-dimensional form of Donnell-type equations for a slightly imperfect flat plate. The elastic constitutive equations for a thin plate are employed. The basic elastic equilibrium equations in the middle surface displacement components are derived through the employment of the principle of virtual displacements. For creep deformations, Odqvist's constitutive equations for steady creep are employed.

Based on the present analysis, a computer program has been developed for the creep buckling of flat plates. The plate ends are assumed to be simply supported. The applied loading is assumed to be biaxial compression.

Numerical results are presented for imperfect isotropic plates under both uniaxial and biaxial compression. For uniaxially compressed plates, the present results are generally in good agreement with previous experimental and analytical results. Numerical results for biaxially loaded plates are finally presented. The present results suggest that each of the level of the axial compressive load, the amplitude of the initial imperfection, and the value of the biaxial load ratio greatly affects the creep buckling times of flat plates.

## INTRODUCTION

In many engineering systems, structural components are required to carry loads while exposed to elevated temperatures. Missile, aircraft, space vehicle, and nuclear reactor structural components are typical examples. In such circumstances, the structural designer must take into consideration the time effects of creep, particularly the possibility of creep buckling of beams, plates, and shells.

Creep buckling of structural components is a failure mode in which a structural element subjected to compressive stress collapses, or a large amount of deformation occurs after the passing of a finite time. In such cases, the designer needs to know how much time it takes for the element deflections to increase to a level which tends to infinity or at which the structural element can no longer perform its structural functions. Such time is called the critical creep buckling time.

In some other cases, however, creep may lead to states of deformation which cause buckling in the classical sense (instaneous collapse or snap through behavior).

Investigations of the creep buckling phenomenon for plates were carried out by a number of investigators, for example (1-12). Excellent discussion of the phenomenon of creep buckling is presented in (8) together with reviews for the early work in this area.

Most of the previous analyses, however, are approximate closed form solutions or computer based numerical solutions of the creep buckling problem of plates under uniaxial compression and constant load and temperature conditions. In most of those analyses, an approximate sandwich plate is used to replace the original plate. Also, the simultaneous redistribution of the elastic stresses during the creep deformation process is not allowed for and the effect of the elastic deformations on the creep buckling times was only accounted for in an approximate manner, see for example (3, 6, 8, and 9). In addition, those analyses do not provide a suitable platform to account for other complicating effects such as time varying applied loads.

Tvergaard (11) proposed an analysis which accounts for elastic, plastic, and secondary creep deformations. He implemented an incremental iterative method based on finite element approximations to obtain solutions. The consideration of plastic deformations considerably complicates the analysis. Tvergaard results, however, showed that accounting only for creep and elasticity was sufficient to obtain good agreement with previous experimental results. This result indicates that the effect of plastic deformations on the creep buckling times may be small enough to be neglected in most practical applications. A similar conclusion was reached by Hoff (8).

In view of the preceding discussions, it seems worthwhile to develop an efficient more general creep buckling analysis which permits consideration of biaxial loading conditions, allows for the simultaneous redistribution of the elastic stresses during the creep process, avoids some of the simplifying assumptions incorporated in previous analyses, and provides a suitable platform for consideration of other complicating factors. Such an analysis is presented herein for the creep buckling behavior of geometrically imperfect rectangular plates under biaxial loading conditions with simple support boundary conditions. The remainder of the paper describes the analysis and presents creep buckling times for a variety of illustrative examples.

ANALYSIS

Elastic Deformations

The analysis presented herein is based on a non-dimensional form of Donnell-type equations, for a slightly imperfect plate. The constitutive equations for a thin isotropic plate with moderate deformations (13), are employed.

The strain energy of a typical plate element (Figure 1) is expressed in terms of the middle surface displacement components  $u$ ,  $v$ , and  $w$  in the  $x$ ,  $y$ , and  $z$  directions, respectively. Applying the principle of virtual displacements, a set of three equilibrium equations in the displacement components can then be determined. The equations are coupled, non-linear, non-homogeneous, partial differential equations in  $u$ ,  $v$ , and  $w$ .

The nondimensional initial imperfection is taken in the form

$$w_0(x,y) = e \sin\left(\frac{\alpha\pi x}{a}\right) \cdot \sin\left(\frac{\beta\pi y}{b}\right) \tag{1}$$

where  $x$  and  $y$  are the axial and transverse coordinates of the middle surface, respectively;  $e=W_0/t$  is the nondimensional amplitude of the initial imperfection where  $t$  is the plate thickness;  $a$  and  $b$  are the plate length and width, respectively;  $\alpha$  and  $\beta$  are arbitrary numbers of half waves in the axial and transverse directions, respectively. A solution for the nondimensional displacement components is assumed in the form

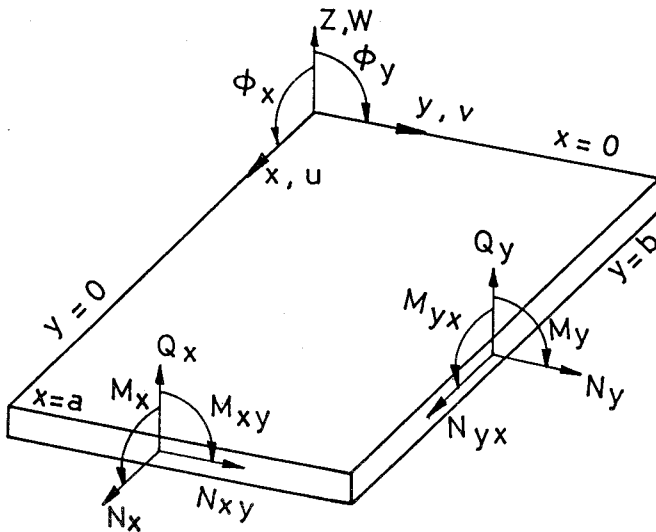


Fig. 1: Notation and sign conventions for a typical plate element.

$$w(x, y) = W \sin\left(\frac{\alpha\pi x}{a}\right) \cdot \sin\left(\frac{\beta\pi y}{b}\right) \quad (2.a)$$

$$u(x, y) = U_0 x + \sum_{i=1}^N \sum_{j=1}^N U_{ij} \cos\left(\frac{i\pi x}{a}\right) \cdot \sin\left(\frac{j\pi y}{b}\right) \quad (2.b)$$

$$v(x, y) = V_0 y + \sum_{i=1}^N \sum_{j=1}^N V_{ij} \sin\left(\frac{i\pi x}{a}\right) \cdot \cos\left(\frac{j\pi y}{b}\right) \quad (2.c)$$

In these equations,  $u$  and  $v$  are the axial and transverse displacement components, respectively;  $w$  is the total lateral displacement component (including the initial imperfection);  $U_0$ ,  $V_0$ ,  $U_{ij}$ ,  $V_{ij}$ , and  $W$  are unknown generalized coordinates (coefficients);  $N$  is the number of series terms which can be taken as large as required. This selection of the displacement components satisfies simple support boundary conditions along the plate ends  $x = 0, a$  and  $y = 0, b$ , namely,

$$\begin{aligned} \text{At } x=0, a: w=M_x=0; \quad & \frac{1}{b} \int_0^b N_x \cdot dy = -\bar{N}_x; \quad \int_0^b N_{xy} \cdot dy = 0 \\ \text{At } y=0, b: w=M_y=0; \quad & \frac{1}{a} \int_0^a N_y \cdot dx = -\bar{N}_y; \quad \int_0^a N_{yx} \cdot dx = 0 \end{aligned} \quad (3)$$

where  $\bar{N}_x = P_x/b$  and  $\bar{N}_y = P_y/a$ ;  $P_x$  and  $P_y$  are the applied axial and transverse compressive loads, respectively,  $N_x$ ,  $N_y$ ,  $M_x$ , and  $M_y$  are stress resultants corresponding to  $u$ ,  $v$ ,  $\phi_x$ , and  $\phi_y$ ;  $\phi_x$  and  $\phi_y$  are the rotations in the axial and transverse directions, respectively. Other selections of the displacement components will yield different boundary conditions.

Substitution of the assumed displacement field into the equilibrium equations of the plate and minimization of the resulting residuals according to the Galerkin method in which the weighting functions are simply the coordinate functions, leads to  $(2N^2+1)$  algebraic equations in the coefficients  $W$ ,  $U_{ij}$ , and  $V_{ij}$  where  $i, j = 1, N$  (see equations 2). Here, the first  $2N^2$  equations corresponding to the equilibrium equations in the axial and transverse directions are functions of  $W$ ,  $U_{ij}$  and  $V_{ij}$ . The last equation corresponding to the equilibrium equation in the lateral direction, is a function of  $W$ ,  $U_{ij}$ ,  $V_{ij}$  and the external compressive load. For a specific value of the lateral displacement amplitude,  $W$ , the first  $2n^2$  equations are linear functions of  $U_{ij}$  and  $V_{ij}$ . Thus, we can solve  $2N^2$  equations in the coefficients  $U_{ij}$  and  $V_{ij}$ . Characterizing the external compressive loads by a parameter  $K$ , the value of  $K$  can be determined by substitution of  $W$  and the corresponding coefficients  $U_{ij}$  and  $V_{ij}$  into the remaining algebraic equation and solving for  $K$ .

At any stage in the creep buckling analysis, the elastic equilibrium state is determined through an iterative process. Knowing the value of the effective imperfection amplitude, a value for the lateral deflection amplitude  $W$ , is assumed. Employing the solution procedure outlined above, the corresponding applied load parameter,  $K$ , is determined. Depending on whether the determined applied load is larger or smaller than the actual applied load, the assumed value of the lateral deflection amplitude is modified accordingly, and the corresponding elastic solution is determined. This process is repeated until the difference between the values of the actual and the determined applied loads, is within a prespecified limit (for example, difference  $< = 0.1\%$ ).

### Creep Deformations

The analysis of creep deformations presented herein is based on the Odqvist's (generalized Norton's) law for steady creep (11, 14):

$$\dot{e}_{ij}^c = K J_2^m S_{ij} \quad (4)$$

where  $K$  and  $m$  are material constants,  $J_2$  is the second invariant of the stress state. Equation (4) can be put in the form

$$\dot{e}_{ij}^c = \frac{3}{2} B \sigma_e^{n-1} S_{ij} \quad (5)$$

where

$$\dot{e}_{ij}^c = \text{creep strain rate components}$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij}$$

$$\sigma_{ij} = \text{stress components}$$

$$\sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

$$\delta_{ij} = \text{Kronecker delta}$$

$$\sigma_e^2 = 3 J_2 = \frac{3}{2} S_{ij} S_{ij}$$

$$B, n = \text{material constants}$$

For two-dimensional problems, equation (5) reduces to

$$\dot{e}_x^c = B \sigma_e^{n-1} (\sigma_x - \sigma_y/2) \quad (6.a)$$

$$\dot{e}_y^c = B \sigma_e^{n-1} (\sigma_y - \sigma_x/2) \quad (6.b)$$

$$\dot{\gamma}_{xy}^c = 2 \dot{e}_{xy}^c = 3B \sigma_e^{n-1} \tau_{xy} \quad (6.c)$$

$$\sigma_e^2 = \sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2 \quad (6.d)$$

where

$$\begin{aligned} \dot{\epsilon}_x^c, \dot{\epsilon}_y^c, \dot{\epsilon}_{xy}^c &= \text{creep strain rate components} \\ \sigma_x, \sigma_y, \tau_{xy} &= \text{elastic stress components} \\ n &= \text{creep exponent} = \text{material constant} \\ B &= \text{material constant} \end{aligned}$$

rearranging equations (6), stress components  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$  can be expressed in terms of the creep strain rates.

For the case under consideration of thin plates with moderate deformations, it is reasonable to assume that initially plane sections remain plane during creep deformations, and that the effective stress  $\sigma_e$  is constant through the thin plate thickness. In addition, in the expression of  $\sigma_e$  higher order terms are neglected. This yields an approximate constant value for  $\sigma_e$  throughout the plate. In such a case, resultant moments can be expressed in terms of the middle surface lateral creep displacement rates, namely

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \left( \frac{-\zeta \sigma_e^{1-n}}{\Delta T} \right) \cdot \begin{Bmatrix} \Delta w_{,xx}^c + \frac{1}{2} \Delta w_{,yy}^c \\ \frac{1}{2} \Delta w_{,xx}^c + \Delta w_{,yy}^c \\ \frac{1}{2} \Delta w_{,xy}^c \end{Bmatrix} \quad (7)$$

where  $\zeta = t^3/9B$ .  $\Delta T$  is a time increment.  $\Delta w^c$  is assumed in the form

$$\Delta w^c = \Delta W^c \sin\left(\frac{\alpha\pi x}{a}\right) \cdot \sin\left(\frac{\beta\pi y}{b}\right) \quad (8)$$

Also, using the elastic constitutive equations, we have

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D & \nu D & 0 \\ \nu D & D & 0 \\ 0 & 0 & (1-\nu)D \end{bmatrix} \cdot \begin{Bmatrix} -(w_{,xx} - w_{o,xx}) \\ -(w_{,yy} - w_{o,yy}) \\ -(w_{,xy} - w_{o,xy}) \end{Bmatrix} \quad (9)$$

where  $D = Et^3/12(1-\nu^2)$ ,  $t$  = plate thickness,  $E$  = Modulus of elasticity,  $\nu$  = Poisson's ratio.

Next, the plate equilibrium equation in the lateral direction in terms of the stress resultants, is considered. Each of the two expressions for the moment resultants,

eqns (7) and (9), is substituted separately in the equilibrium equation. The two resulting equations are then subtracted from each other to get

$$\zeta \sigma_e^{1-n} \Delta (\nabla^4 w^c) - D \nabla^4 (w - w_0) \cdot \Delta T = 0 \quad (10)$$

Substitution of the assumed displacement components into eqn. (10) and minimization of the resulting residual function according to the Galerkin method in which the weighting function is simply the lateral displacement coordinate function, yields the following relation

$$\Delta W^c = [(W - W_0) \cdot \frac{3BE}{4(1-\nu^2)} \cdot \sigma_e^{n-1}] \cdot \Delta T \quad (11)$$

Equation (11) will be used in an incremental solution to estimate the time increment,  $\Delta T$ , corresponding to an incremental creep deformation amplitude,  $\Delta W^c$ . At the beginning of each increment, the elastic deformation amplitude,  $W - W_0$ , will be calculated and substituted in equation (11) to estimate  $\Delta T$ . This process will be detailed later.

### **Creep Buckling Analysis**

An incremental iterative procedure is implemented for the creep buckling analysis. In this analysis, we postulate that the creep deformations are effectively changing the plate imperfection amplitude. Consequently, creep deformations will cause simultaneous changes in the elastic equilibrium state and corresponding changes in the elastic stresses. Based on this postulation, the following procedure for creep buckling analysis is implemented:

1-Get the elastic solution corresponding to the initial imperfection. This solution is determined by implementing the iterative procedure outlined in section (2.1). The corresponding effective stress function,  $\sigma_e$ , is then estimated.

2-Specifying a reasonably small incremental deflection amplitude due to creep,  $\Delta W^c$ , equation (11) is used to determine the corresponding time increment  $\Delta T$ . In equation (11), the constant B and creep exponent n are specified a priori from previous tests or data available for the particular material under consideration.

3-The incremental creep deflection,  $\Delta w^c$ , is added to the initial imperfection to determine a new effective imperfection for the flat plate. This imperfection is used in conjunction with the iterative procedure of section (2.1) to determine a new elastic equilibrium state (solution) and corresponding new lateral deflection amplitude, W, and effective stress,  $\sigma_e$ .

4-Steps 2 and 3 are repeated, and the time increments are accumulated, until a critical condition, characterized by a limit on the lateral deformation amplitude, W, is reached.

## NUMERICAL RESULTS

### CBAP Computer Program

Based on the preceding analysis, a computer program called CBAP (Creep Buckling Analysis of Plates) has been developed for the creep buckling of flat plates. The plate ends are assumed to be simply supported. The applied loading is assumed to be biaxial compression. All the basic variables in CBAP were put in a nondimensionalized form. Consequently, the input data can be in any system of units. The program was written in Pascal and compiled using the TurboPascal compiler version 6. For a typical case, a single run on a 386-33 PC without a math-coprocessor takes less than 3 minutes to determine the corresponding creep buckling time.

### Illustrative Numerical Examples

Numerical results are presented next to provide check cases, illustrate the scope of the present analysis, and to arrive to some important conclusions. The following notation is employed in the subsequent examples:

- ☆ Geometric properties:  $t$  = thickness;  $a$  = length in axial direction;  $b$  = plate width;  $e = W_0/t$  = dimensionless initial imperfection amplitude.
- ☆ Material properties:  $E$  = Young's Modulus;  $\nu$  = Poisson's Ratio.
- ☆ Load parameters:  $\bar{\sigma}_x$  = applied axial compressive stress =  $P_x/tb$ ;  $\bar{\sigma}_y$  = applied transverse stress =  $P_y/ta$ ;  $\sigma_E$  = Classical Buckling stress for a square simply supported plate =  $4\pi^2 D/tb^2$ ;  $D = Et^3/12(1-\nu^2)$ ;  $\lambda = \bar{\sigma}_x/\sigma_E$ ;  $\omega = \bar{\sigma}_y/\bar{\sigma}_x$ .
- ☆ Other parameters:  $T_r$  = Reference Time =  $(\sigma_E/E)/B \bar{\sigma}_x^n$ ;  $B$  = material constant;  $n$  = material constant (creep exponent).

In the following examples, an initial imperfection with a single half wave in the transverse direction and a half wave length in the axial direction equals the plate width, is considered. This selection leads to reasonably conservative estimates of the creep buckling times. In addition, the number of series terms in the elastic displacement field,  $N$ , is taken to be 3. This choice was found to be sufficient to accurately represent the elastic equilibrium state. Finally, in employing the previously outlined creep buckling analysis, a critical limit of the total lateral displacement amplitude was generally taken to be equal to double the plate thickness (unless otherwise stated).

Axially compressed plates (Ex. 1). Experimental results for 33 test specimens manufactured of 2024-T3 aluminium alloy, were reported in (9). In those tests, sheets having a thickness of 0.020 in. and a length varying from 6 in. to 11 in., were bent into square boxes with side width of 1.5 in. The boxes were tested under axial



compression at 600° F. At this temperature, the modulus of elasticity of the material was found to be  $7.2 \times 10^6$  psi. The use of thin walled square tubes insured the existence of ideal simple support conditions along the longitudinal boundaries of the various plate elements ( $y = 0$  and  $y = b$ ). Hoff (9) used the results of those experiments in conjunction with previously derived theoretical formulas to estimate the material constants B and n. B was found to equal  $3.4 \times 10^{-7} \text{ h}^{-1} (\text{ksi})^{-6}$  and n was found to equal 6. Figure 2 shows good agreement between creep buckling times predicted by the present analysis and those predicted by various theoretical formulas derived by Hoff and his collaborators (3, 8, 9), assuming the same initial imperfection amplitude,  $e = 0.01$ . To obtain the present analysis predictions, the values for the material constants E,  $\nu$ , B, and n estimated in (9), were used. Figure 2 also indicates that the predictions of the present analysis provide a reasonably conservative envelope for the various experimental results.

Effect of the creep exponent n (Ex. 2). In this example, a comparison is presented of the creep buckling times predicted by the present analysis for a thin plate element under uniaxial compression, with those of Hoff (9) and Tvergaard (11). Two cases of the creep exponent  $n = 3$  and  $n = 6$  were considered. Tvergaard

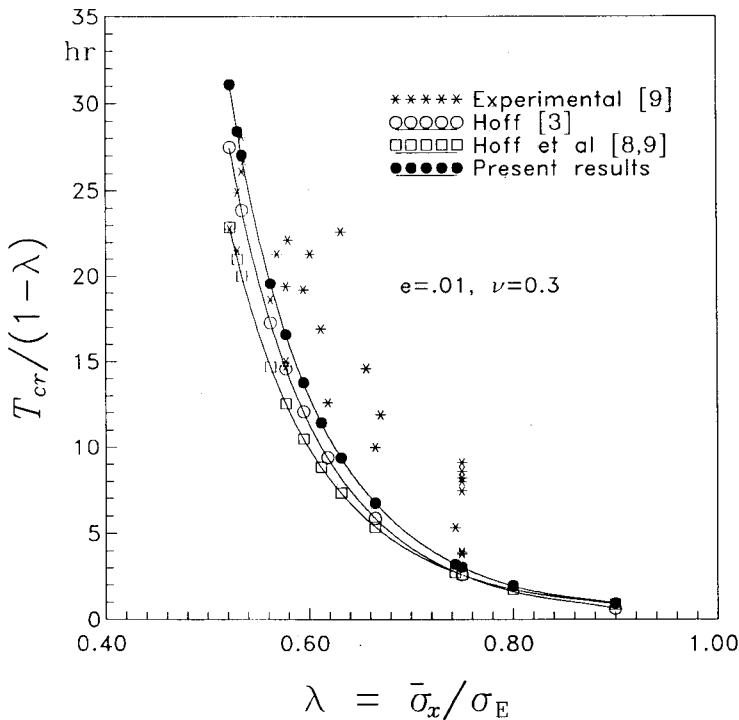


Fig. 2: Creep buckling times of axially compressed plates, example 1.

results are based on a finite element analysis of the creep buckling problem for two cases of transverse boundary conditions. The first case considered constant normal stress along the transverse boundaries of the plate,  $\sigma_x = \text{constant}$  (along  $x = 0$  and  $x = a$ ). The second case considered constant axial displacement along the transverse boundaries,  $\delta_x = \text{constant}$ . Figure 3 shows that the predictions of the present analysis are in good agreement with those of Hoff and are generally more conservative than those of Tvergaard.

For the case of a creep exponent  $n = 6$ , finite critical times exist at which nearly vertical asymptotes appear for the lateral displacement curves. For the case of a creep exponent  $n = 3$ , no finite critical time exists as the lateral deflections do not increase suddenly beyond all bounds. However, in the vicinity of the critical condition ( $W > 2t$ ), the lateral deflection rate increases considerably and shortly after, large deformations develop. Consequently, the corresponding time is a reasonable estimation for the creep buckling time. It is to be noted that the lateral deformation curve predicted by the present analysis for the case of  $n = 3$ , is similar in shape and trend to numerical results presented by Hoff in (8) for similar plates.

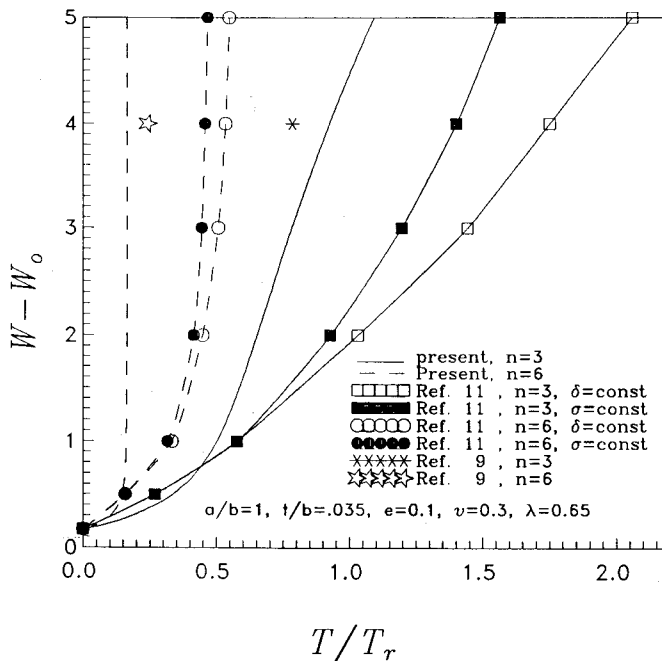


Fig. 3: Lateral Deflection versus time curves for square plates under Axial compression, example 2.

Effects of the initial imperfection amplitude (Ex. 3). Figure 4 shows creep buckling times for plates with various initial imperfection amplitudes under uniaxial loading conditions. Figure 4 indicates that each of the amplitude of the initial imperfection and the level of the applied compressive load has an apparent effect on the creep buckling times of plates.

Biaxially Loaded Plates (Ex. 4). Here we consider the case of simply supported imperfect square plates under various levels of compressive loading and various biaxial load ratios. Figure 5 indicates that both the level of the applied load and the level of the biaxial load ratio greatly affect the creep buckling times of plates.

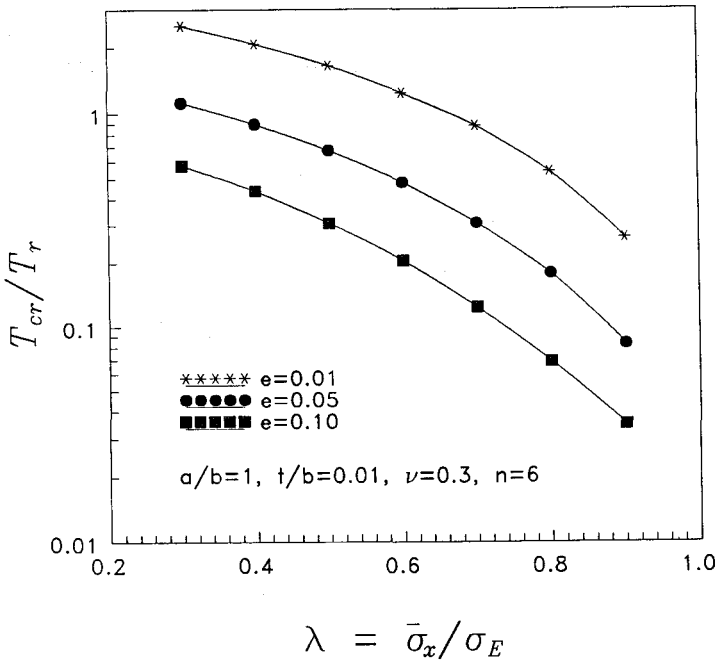


Fig. 4: Effect of the initial imperfection amplitude on the creep buckling times of axially compressed plates, example 3.

### CONCLUSION

An efficient incremental analysis for the creep buckling problem of geometrically imperfect isotropic flat plates under biaxial compression with simple support boundary conditions is presented. Based on the present analysis, a computer program (CBAP) has been developed for the creep buckling of flat plates.

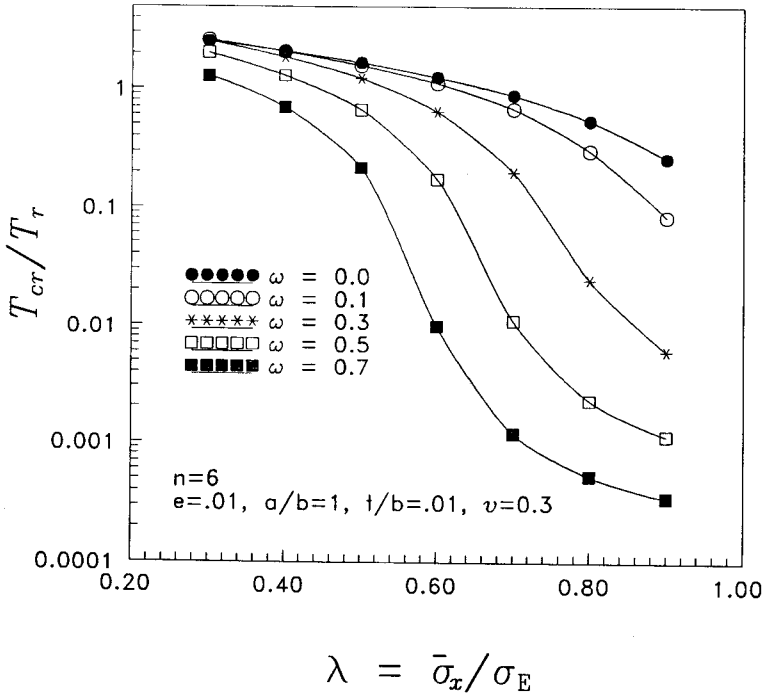


Fig. 5: Creep buckling times of biaxially loaded plates, example 4.

Numerical results are presented for imperfect isotropic plates under both uniaxial and biaxial compression. For uniaxially compressed plates, the present results are generally in reasonably good agreement with previous experimental and analytical results. The present results suggest that each of the level of the axial compressive load, the amplitude of the initial imperfection, and the value of the biaxial load ratio greatly affects the creep buckling times of flat plates.

The results indicate that the present incremental analysis provides reasonably conservative predictions for the creep buckling times of simply supported flat plates. In addition, the present incremental approach presents a suitable platform for creep buckling analyses under such complicating factors as time varying loads. This will be addressed in future work.

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