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TIME-OPTIMAL CONTROL OF HIGH-SPEED FLEXIBLE-ROBOT ARM USING PD ALGORITHMS

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ABSTRACT

This paper addresses utilization of proportional-plus derivative (PD) control algorithms for time-optimal control of flexible-robots. Flexibility is modeled using one mode of vibration with negligible structural damping. Two novel methods for time-optimal control were derived and closed form equations for tuning the required PD gains were obtained. The resulting controllers were used to control a high-speed flexible-robot for minimum settling time in response to a step angular motion command. Similarly, for the sake of comparison, three other techniques were used to control the same flexible-robot arm. The first used multi-switch bang-bang control technique. The second used PD approach in which the gains are computed through locating the dominant poles as far left as possible in the left hand side of the complex plane. The third approach used multi-switch bang-bang control followed by PD control. Uncertainties were introduced in the model to evaluate robustness of the methods. Results obtained showed that the novel techniques out performed the other ones.

KEY WORDS: PD Control, High-Gain Control, Time-Optimal Control, Flexible-Robots.

INTRODUCTION

Recently there have been increasing needs for high-speed lightweight robots, especially in chip placement and electronic part assembly. These robots tend to be flexible, leading to vibrations during operation. This motivates researchers to investigate the control of such robots.

PD control, LQR technique, robust servomechanism theory, Lyapunov method and quantitative feedback theory are evaluated in [1] for the control of flexible-robot arm. The optimal torque for motion control and vibrations suppression of robot arm at the end of a prescribed rotation is determined in [2]. The end effector trajectory control of an elastic macro-micro manipulator using inverse and predictive controllers is investigated in [3]. Comparisons between PD controller considering gravity, feedback linearization approach and sliding mode control are made in [4] for trajectory control of flexible joint manipulators.

Integrated structure/control design methodologies of high-speed flexible-robot arm are presented in [5-8]. The minimization of settling time of PD control is performed in [5] by adjusting the real part of the dominant closed-loop pole to be as far left as possible in the complex plane. The traveling time of multi-switch bang-bang control is minimized in [6-8]. The shape of flexible-robot arm is optimized using constant topology and varying arm cross-section size in [5,6]. While in [7,8] varying topology and varying arm cross-section size are used. In [8], air damping was considered and proved to have significant affect. In [6,8], multi switch bang-bang control is followed by PD control to improve the robustness of the system. However, PD control gains and switching time from bang-bang control to PD control are obtained by trial and error method.

In [9], a connection is made between time-optimal control (single switch bangbang control) and PD control for second-order servo systems. It is shown that the performance of PD control is almost identical to bang-bang control provided that the ratio between proportional and derivative gains is computed as a function of initial conditions. Unlike the open-loop bang-bang control, this PD method is a feedback control type. Hence, the time-optimal PD controller could guide the system to the required position in the presence of disturbance, while the open-loop bang-bang can not.

In this paper, an attempt is made to extend the work presented in [9] to include flexible systems. In other words, a new PD algorithm will be derived which is equivalent to multi-switch bang-bang control. Such control to replace the algorithms used in [6,8], and to eliminate the problems associated with gains and switching between the two controls. At first the time optimal PD algorithm presented in [9] is evaluated.

EVALUATION OF TIME-OPTIMAL PD CONTROL (RIGID SYSTEM)

The second-order servo system with input constraints and negligible damping is

described by the following set of state space equations:

$$\dot{x}_1 = x_2
\dot{x}_2 = bu
|u| \le M$$
(1)

Where b and M are positive constants. M represents the upper limit magnitude of the control input. The system of equations (1) can be brought from an initial point $x_1(0)=x_1^0$ and $x_2(0)=0$ to the origin in a minimum time using the following control algorithm [9]:

$$u = -k(\rho x_1 + x_2), \quad k \to \infty \qquad |u| \le M \tag{2}$$

Where (ρ) , the ratio of the proportional gain to the derivative gain depends on initial conditions and is given by:

$$\rho = 2\sqrt{\frac{bM}{\left|x_1^0\right|}}\tag{3}$$

Algorithm (2) is a PD controller with infinite gains. The upper bounds for the PD gains are related to the sampling period, T, [9, 10]:

$$k < \frac{2}{bT} \tag{4}$$

$$\rho < \frac{2}{T}$$
(5)

With finite gains it can be shown that the PD algorithm (2) approximate closely time-optimal control. For verification we consider b=1, M=5, and $x_1^0=10$. Consequently ρ is evaluated to be 1.414. Figs. 1 and 2 show the phase plane and displacement response for time-optimal control (bang-bang control) and time-optimal PD algorithm (2). The time-optimal PD algorithm is tested for a high gain k=300 and a low gain k=10. In the case of high gain the time-optimal PD algorithm and time-optimal control show identical results. In the case of low gain the results of time-optimal PD control differ slightly from time-optimal control. It is to be noted that the open-loop bang-bang control required to bring the system of

equation (1) from initial conditions $x_1(0)=x_1^0$ and $x_2(0)=0$ to the origin is: u=-M for $0 \le t < t_f/2$ and u=M for $t_f/2 \le t \le t_f$; where t_f is the total traveling time given by:

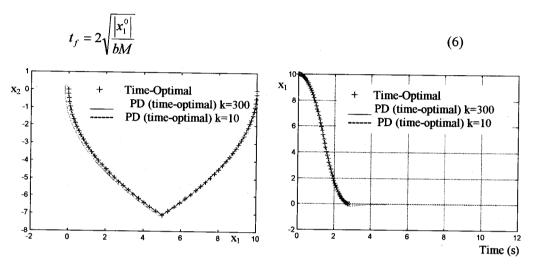


Fig. 1 Phase plane of rigid system

Fig. 2 Displacement response of rigid system

The equivalence between PD algorithm and time-optimal control can be proved using Fig. 3, [9]. In the Figure, curve A is the theoretical optimal trajectory before the switching point s. Curve B is the theoretical optimal trajectory after the switching point s. The Straight line C is passing through the switching point s and the origin. It can be shown that the equation of line C is $\rho x_1 + x_2 = 0$, where ρ is given by equation (3). The control input for both time-optimal control and time-optimal PD algorithm takes the value (-M) on the right hand side of line C and the value (M) on the left hand side. This shows that both controllers are equivalent. It should be noted that for a high value of gain k, the control input for the time-optimal PD control takes the values \pm M except at small regions around the switching point s and the origin.

To help evaluating the time-optimal PD control, it is desired to compare the gains of the time-optimal PD control with those corresponding to minimum settling time. A computer program was written using MATLAB [11] to determine the response and settling time for the system given by (1), for arbitrary values of gains K and ρ . Again. The system parameters and initial conditions are chosen as in the

previous example. Using MATLAB minimization-function, the optimal values of k and ρ were obtained for minimum settling time t_s . This is done for different settling time tolerances (2%, 1%, and 0.5%). Table 1 summarizes the optimization results and shows the corresponding t_s for time-optimal PD control (k=300, ρ =1.414).

	_	-		
	2% t _s	1% t _s	0.5% t _s	
Optimal K	225	238.1044	224.6965	
Optimal p	1.455	1.4337	1.4232	
Optimal t _s	2.457	2.56	2.636	
Time-Optimal PD t _s	2.544	2.623	2.676	
$(K=300, \rho=1.414)$				

Table 1. Optimal gains for different settling times

Fig. 4 shows the response curves for different optimal gains, which are very close to each other. From the result presented in Table 1 and Fig. 4, one can conclude the following:

- 1) The response of the time-optimal PD control is the only one without overshoot.
- 2) The optimal ρ converges to the time-optimal PD control value of 1.414 as the allowable tolerance settling time decreases.
- 3) The optimal t_s converge to the time-optimal PD control t_s as the allowable tolerance settling time decreases.
- 4) The gains of time-optimal PD control can be considered optimal for minimum settling time provided that the allowable tolerance is very small.

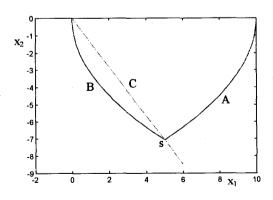


Fig. 3. Switching line for time-optimal control

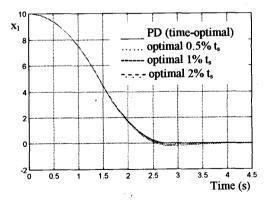


Fig. 4. Displacement response for different optimal gains

PROPOSED TIME-OPTIMAL PD CONTROL (FLEXIBLE SYSTEM)

In the case of dynamical system with rigid mode and one flexible mode with control input constraints, the system can be represented by:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t), \qquad |u| \le M \tag{7}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\boldsymbol{\omega}^2 & 0 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 0 \\ b_0 \\ 0 \\ b_1 \end{bmatrix}$$

 $\mathbf{x}(t)$ is the modal coordinate vector, u(t) is the control function, M is positive constant representing the upper bound control input, ω is the natural frequency of the flexible mode, and b_0 and b_1 are the weighted mode shapes. Both b_0 and b_1 can be obtained using finite element method. The displacement, y_1 , and the velocity, y_2 , of the system can be obtained from:

$$\mathbf{C} = \begin{bmatrix} b_0 & 0 & b_1 & 0 \\ 0 & b_0 & 0 & b_1 \end{bmatrix}$$
 (8)

The system of equations (7,8) can be brought from rest at $y_1(0)=y_1^0$ to rest at $y_1(t_f)=0$ in a minimum time using multi-switch bang-bang control [12] as shown in Fig. 5. The time intervals t_a and t_f can be determined from the following two equations:

$$(t_f/2)^2 - 2t_a^2 = \frac{y_1^0}{b_0^2 M} \tag{9}$$

$$\cos(\omega \ t_f / 2) - 2\cos(\omega \ t_a) + 1 = 0 \tag{10}$$

Fig. 6 shows a flexible system phase plane response for time-optimal control. In the Figure, s_1 , s_2 and s_3 are three switching points, and A and B are two switching

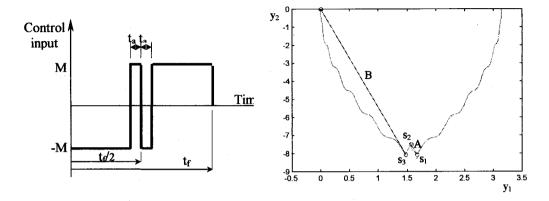


Fig. 5 Bang-bang control of flexible system

Fig. 6 Switching lines of flexible system

lines. Line A passes through s_1 and s_2 and line B passes through s_3 and the origin. It is to be noted that the horizontal coordinate of switching point s_2 is $y_1^0/2$.

Our proposed time-optimal PD algorithm is aimed to employ the results of multi-switch bang-bang controller to achieve minimum time control. To achieve this one can imagine that the phase plane is divided into two halves with a vertical line passing through s_2 separating the two sides. Then, an equivalent time-optimal PD algorithm for rigid system is applied to each side. This step takes into consideration two preconditions. First, intersection between the response curves, and line A and B occurs only at s_1 , s_2 and s_3 and the origin. Second, the two parts of the response curve in each half lie on opposite sides of the corresponding line. For large flexibility these two preconditions may be not satisfied, especially the first one. For such cases, approximation is possible. The PD algorithm for the first half of the phase plane can be written as:

$$u = -k[\rho_1(y_1 - y_1^*) + (y_2 - y_2^*)], \qquad k \to \infty, \qquad |u| \le M$$
 (11)

where ρ_1 is the slope of line A and y_1^* and y_2^* are the coordinates of Point s_2 . The PD algorithm for the second half can be written as:

$$u = -k(\rho_2 y_1 + y_2), \qquad k \to \infty, \qquad |u| \le M \tag{12}$$

where ρ_2 is the slope of line B. The condition required to switch from the first algorithm to the second is $y_1 \le y_1^0/2$. The procedure required to apply the algorithm is:

- 1) Calculate the time intervals t_a and t_f from equations (9,10).
- 2) Calculate the time intervals between switching points:

$$t_{01} = t_{34} = t_f / 2 - t_a, t_{12} = t_{23} = t_a (13)$$

3) Calculate the coordinates of the three switching points y_1^i , y_2^i , i=1,2,3:

$$\overline{M}_{i} = (-1)^{i} \operatorname{sign}(y_{1}^{0}) M
x_{1}^{i} = \frac{\overline{M}_{i} b_{0}}{2} t_{(i-1)i}^{2} + x_{2}^{i-1} t_{(i-1)i} + x_{1}^{i-1}
x_{2}^{i} = \overline{M}_{i} b_{0} t_{(i-1)i} + x_{2}^{i-1}
x_{3}^{i} = \frac{b_{1} \overline{M}_{i}}{\omega^{2}} (1 - \cos \omega t_{(i-1)i}) + \frac{x_{4}^{i-1}}{\omega} \sin(\omega t_{(i-1)i}) + x_{3}^{i-1} \cos(\omega t_{(i-1)i})
x_{4}^{i} = \frac{b_{1} \overline{M}_{i}}{\omega} \sin(\omega t_{(i-1)i}) + x_{4}^{i-1} \cos(\omega t_{(i-1)i}) - x_{3}^{i-1} \omega \sin(\omega t_{(i-1)i})
y_{1}^{i} = b_{0} x_{1}^{i} + b_{1} x_{3}^{i}
y_{2}^{i} = b_{0} x_{2}^{i} + b_{1} x_{4}^{i}$$
(14)

Note that: $x_1^0 = y_1^0/b_0$, $x_2^0 = x_3^0 = x_4^0 = 0$.

4) Calculate ρ_1 and ρ_2 :

$$\rho_{1} = \left| \frac{y_{2}^{2} - y_{2}^{1}}{y_{1}^{2} - y_{1}^{1}} \right|$$

$$\rho_{2} = \left| \frac{y_{2}^{3}}{y_{1}^{3}} \right|$$
(15)

5) Assume finite value for gain k beyond its limit and check the limits of ρ_1 and ρ_2 .

APPLICATION AND EVALUATION OF TIME-OPTIMAL PD CONTROL (FLEXIBLE SYSTEM)

The new algorithm was applied to a flexible single robot arm found in [6]. The parameters of the robot arm are: total inertia $J=0.151 \text{ kg-m}^2$, first pole P=211.5 Hz, first zero Z=92.9 Hz, maximum input torque M=15 N-m, and initial position $y_1^0=\pi/10$ rad. The natural frequency and weighted mode shapes were computed from the following equations [8]:

$$b_0 = \frac{1}{\sqrt{J}}, \qquad \omega = P, \qquad b_1 = b_0 \sqrt{\frac{P^2}{Z^2} - 1}$$
 (16)

Time intervals t_f and t_a were calculated from (9,10) and found to be 0.1125 and 3.5E-04 sec. The coordinates of s_1 , s_2 and s_3 were calculated from (14) and their horizontal (rad) and vertical (rad/sec) coordinates were found to be (0.15888, -5.2694), (0.15708, -5.0445), and (0.15528, -5.2694), respectively. The gain ratios ρ_1 and ρ_2 were calculated from (15) and found to be 124.9 and 33.9, respectively. k is assumed to be 125. The phase plane and displacement response for the proposed algorithm and for multi-switch bang-bang control were determined using MATLAB program. The results are shown in Figs. 7 and 8. The curves for both approaches are almost identical. The results indicate the equivalent of the two methods. The 2% settling time of the proposed algorithm is 0.1006 sec.

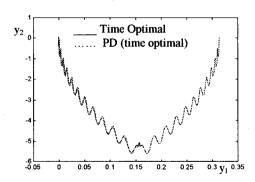


Fig. 7 Phase plane of flexible system

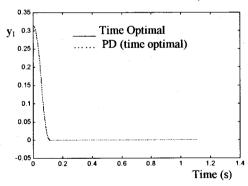


Fig. 8 Displacement response of flexible system

Since the three switching points s_1 , s_2 and s_3 are close to each other, it is thought that it might be feasible to use only one PD algorithm instead of two provided that

the gain ratio ρ is determined as the slope of a line passing through an imaginary switching point and the origin. The coordinates of the arbitrary assigned switching point are determined from the average coordinates of the three switching points. The phase plane and the displacement response for this new single-line PD control are shown in Figs. 9 and 10. The 2% settling time is 0.1021 sec. The results differ slightly from the previous double lines time-optimal PD control.

For comparison, the time-optimal PD control for rigid system (described in section 2) was applied to the same flexible single robot example. Hence, the gain ratio ρ was determined neglecting the flexibility of the system. Fig. 11 shows the displacement response of the algorithm with slight overshot. The 2% settling time equals to 0.1187 sec. Although, the algorithm for rigid system is simpler than the proposed algorithms, the performances of the proposed algorithms are better in the presence of flexibility.

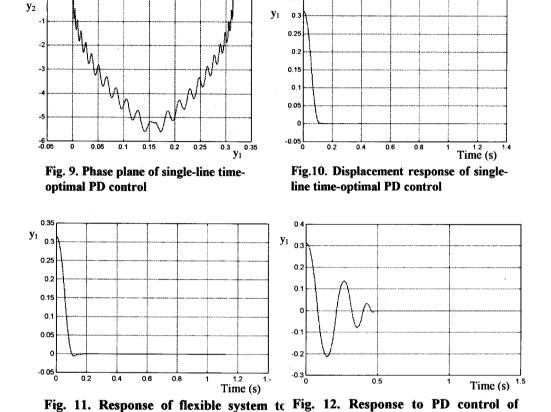
In [5], minimizing the settling time of flexible-robot arm was obtained through adjusting the real part of the dominant closed-loop pole to be as far left as possible in the complex plane. The optimal gains $(k_p = k \ \rho)$ and $(k_v = k)$ are determined by the following equations in the case of $b_1^2/b_0^2 \le 4$.

$$k_{p} = \frac{\omega^{2} b_{0}^{2}}{\left(b_{0}^{2} + b_{1}^{2}\right)^{2}} \tag{17}$$

$$k_{v} = \frac{2\omega b_{1}}{\left(b_{0}^{2} + b_{1}^{2}\right)^{3/2}} \tag{18}$$

In the case of $b_1^2/b_0^2 \ge 4$, the optimal gains can be computed numerically so that the three poles have identical real parts and the other pole has the least real part. The ratio b_1^2/b_0^2 in the example carried out here is 4.183. Hence, the optimal gains were calculated numerically. The corresponding k and ρ were found to be 69 and 145, respectively. The corresponding poles are -834, -519!0.4i, and -519. Fig. 12 shows the displacement response, which indicates large overshoots. The 2% settling time equals to 0.4722 sec. This settling time is bout 270% higher than the value obtained in the proposed time-optimal PD control. Consequently, in the case of bounded control input these gains cannot be considered as optimal. They are optimal only if there is no limitation on the control input, which is not practical. Thus similar to the case of rigid system with bounded control input, the gains of the proposed algorithm can be considered optimal for flexible system.

0.35



EVALUATION OF THE PROPOSED ALGORITHMS IN THE PRESENCE OF MODELING UNCERTAINTIES

time-optimal PD control of rigid system

optimal gains according to [5]

Many researchers have attempted different approaches [5,6,8] to obtain minimum settling time of high-speed flexible-robot arm. In [6,8], multi-switch bang-bang control was used. This open loop control could guide the system to the required position only if there were no disturbances or modeling uncertainties, which is not practical. Therefore, the bang-bang control was followed by PD control. In [8], the PD control was applied at the end of the bang-bang control. In

[6], the PD control was applied after 90% of the theoretical traveling time of bangbang control. The PD control gains were determined by trial and error method.

In order to compare the strategy in [6,8] with the proposed method, parametric uncertainty is introduced. Based on the same flexible robot arm used in section 3, a 5% uncertainty is assigned to the inertia parameter b_0 . In our simulation, uncertainty has resulted in large overshot at the end of the bang-bang control. Therefore, the PD control is switched on after 90% of the theoretical traveling time of the bang-bang control. The PD control gains were determined through optimization techniques in which the settling time of the system is minimized. It is believed that this technique can imitate the trial and error method used in [6.8] and insure best performance of the strategy. The gain k and p are found to be 38,4025 and 134.5193, respectively. Fig. 13 shows the displacement responses for [6,8] and the method of optimal gains [5]. For comparison, the responses for time-optimal PD-control algorithms (double lines and single line) are shown in the same Figure. The 2% settling times for these algorithms are presented in Table 2. The best settling time is that of the proposed method (double lines). The worst is that of the optimal gains [5]. The proposed method (single line) comes in second place, while the strategy of [6,8] comes in third place. It is to be noted that trying different switching time between the two controls used in [6,8] have shown similar results. And, only our proposed algorithms responses have shown no overshoot. Fig. 14 shows the displacement response for time-optimal PD control (double lines) for different values of parametric uncertainty of b₀. For 10% to 30% uncertainty in b₀, the increase in settling time is only 5% to 18%. This indicates the robustness of the proposed method.

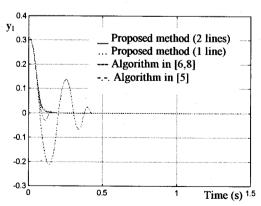


Fig. 13. Comparison between different Approaches

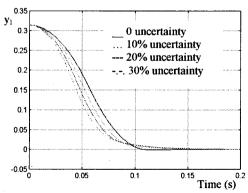


Fig. 14. Time-optimal PD control response in presence of uncertainties

Table 2 Settling times of different approaches

	Proposed method (double lines)	Proposed method (single line)	Approach in [6,8]	Approach in [5]
2% t _s (sec.)	0.1188	0.1309	0.1349	0.4146

The out performance of the proposed method, time-optimal PD control, over the strategy of [6,8] is based not only on settling time comparison but also on the robustness of the algorithm. In [6,8], the gains are determined using trial and error method for every joint rotational angle. This is of course impractical. On the other hand, the proposed algorithm computes the best gains for each angle directly using closed form equations.

CONCLUSION

To develop time-optimal PD algorithm for flexible robot we started by evaluating the algorithm developed in [9] to compute time-optimal PD control gains for controlling a second order servo system with input constrains (rigid system). Gains for minimizing settling time for specified settling tolerances (2%, 1%, 0.5%) were obtained. Results showed that the gains for time-optimal PD control can be considered as optimal for minimum settling time provided that the allowable tolerance is very small. Successful optimization results motivated deriving time-optimal PD control algorithms for flexible robot. The first algorithm uses two PD tuning formulas. The first formula is applied from the initial to midpoint position. The second formula is applied from the midpoint to final position. The PD gain ratios for both formulas depend on the initial conditions of motion. An approximation is made to replace the two tuning formulas with one. Both, the new time-optimal PD control and its approximation was applied to a flexible single robot arm. The responses of the new algorithms were found almost identical to that of time-optimal control (multi-switch bang-bang control). The response of the approximation is slightly less efficient. The new algorithms were compared to the strategies given by [5] and [6,8]. The optimal gains method presented in [5] yielded large overshoot and settling time. This is due to the fact that the method is optimal only in the case of unbounded control input, which is not practical. The method of [6,8] vielded larger settling time than the proposed method in the presence of modeling uncertainties of 5%. Furthermore, the proposed methods show relative low settling time even in the presence of modeling uncertainties up to 30%, which indicates its robustness. Results obtained showed that the new algorithm out performed the other ones.

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