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CARBON EMISSIONS INVENTORY GAMES

BY

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ABSTRACT

Carbon emissions reduction has been the center of attention in many organizations during the past few decades. Many international entities developed rules and regulations to monitor and control carbon emissions especially under supply chain context. Furthermore, researchers investigated techniques and methods on how reduce carbon emissions under operational adjustment which can be done by cooperation or coordination. The main contribution of this thesis is to measure to what extend cooperation can contribute to carbon emissions. Many research addresses the advantage of cooperation in reducing cost. However, there isn't a plenty of research addressing the effect of cooperation on carbon emissions when the incentive of the cooperation is to reduce cost only. The aim of this thesis is to show if joint replenishment leads to a reduction in carbon emissions and this to be considered as an advantage to be added to cooperation. Moreover, if a savings occur from cooperation, the aim will be to address the issue of allocating the savings among parties engaged in the coalition.

The thesis methodology adapted and extended cooperative EOQ model and basic inventory model (EOQ) in order to formulate and build an adjusted model to measure carbon emissions. The adjusted model will be used to calculate carbon emission in centralized and decentralized systems with incentives to reduce cost and no incentives to reduce emission. The calculation shall yield the optimum ordering quantity which in turn yields the savings between the two systems. Finally core allocation principles will be leveraged to propose a fair allocation of savings. Furthermore, the model will be extended

to consider some regulation and different environments to which it will cater for carbon-tax regulation and full Truckload system contexts.

Findings indicate that applying inventory game theory leads to a reduction of carbon emissions along with cost. Additionally, the total carbon emissions in centralized system will always be less then decentralized system under all conditions. Moreover, the proposed proportional allocation which was proven to be a core allocation model will be based on the frequency of ordering and the amount of holding emissions.

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CHAPTER 1. INTRODUCTION

The awareness of carbon emissions within supply chain increased among individuals and firms during the past decades. It was estimated that carbon dioxide emissions were around 40 billion tons in 2002, and is expected to increase to 58 billion tons by 2030 (Enkvist et al. 2007). Many entities all over the world embarked on initiatives in an attempt to save the environment by apply regulations and policies over business sectors or individuals. Organizations such as IPCC, Kyoto Protocol, and EU were established to control and manage the amount of carbon emissions.

For firms under governments who signed with the international regulations are in a pressure to reduce their carbon emissions. Their operation management within the supply chain, does not depend on cost only, but carbon emissions became a factor of the decision. These firms are obligated to follow the regulations that are imposed by the government or international entities to reduce carbon emissions within the region. The regulations can be in a form of strict carbon caps, imposing carbon tax for every carbon emission emitted or exceeded the cap, opening the market for carbon trading and offset. Self-initiatives can be another reason for having firm considering carbon emissions within their operation. These firms are aware of the consequences of having carbon being emitted from their daily operation within the supply chain. Other firms starts to build an environmental friendly product in order to satisfy customers who are aware about the environment and to increase their market share. Toyota Company, for example, started to create a new hybrid cars in order to target these type of customers

Many researchers investigated different techniques and methods to reduce carbon emission within sup- ply chain context. Most of the researches addressed investing in a new technology, or adjusting a firm's operation. Investment can be costly where firms has to pay a huge amount to change the currently used technology or building. Since investment can be costly, firms tended to find optimal solutions without having dramatic increase in cost. Solution included changes in operations such as changing the ordering quantity, production quantity, transportation selection mode, storage quantity, facilities location, etc. For example, if the amount of carbon emitted during the holding process is high due the refrigerator, the firm can change the amount of the holding units and store less quantities (Chen et al. 2013). This action can reduce carbon emission pertaining to the holding process. However, the total cost may increase because inventory management is a trade-off between holding and ordering costs. Reducing holding cost will probably lead to an increase to the ordering cost.

Supply chain cooperation and supply chain coordination can be types of operational adjustments were firms can apply to reduce carbon emissions. Supply chain cooperation is coordination between different members within the supply chain. The retailer will adjust its ordering quantity based on the warehouse and or the supplier and opposite. Supply chain cooperation, also known as joint replenishment and shipment consolidation, is firms who are doing the same business will be sharing and joining their resources. Many researchers investigated the reduction of carbon emission under cooperation context. Joint replenishment is a common used method in supply chain where firms coordinate and consolidate their resources, shipment, and/or orders.

Individual firms join their resources to gain the advantages from risk pooling, Negotiation and reduction of cost.

There are several researches addressing reducing carbon emission under supply chain and under consolidated system. However, research addressing the effect of consolidation on carbon emission where the motivation is to reduce cost only is scarce. Similarly, the research addressing the allocation of the saving of carbon emission resulting from the consolidation is also scarce. This thesis will address the effect of using game theory on carbon emission where multiple cases / scenarios will be considered as well as developing a function for saving allocation.

1.1 Scope of the study

The purpose of the thesis is to show the effect and behavior of carbon emission under centralized supply chain as well as to develop an effective model for allocating the savings of carbon emission among the parties within the coalition. The aim is to show that consolidation can lead to a reduction in carbon emission as well as a cost reeducation, and to propose a saving allocation model.

The paper will adapt and alter the game theory model in (Meca et al. 2004 and the basic (EOQ) model in Chen et al. 2013)'s research to address the issue. The newly developed formulas will calculate carbon emission in both decentralized and centralized systems by adapting the optimum quantity that is calculated and used in reducing the total cost. The rational of this approach is to show the behavior of carbon emission in both systems when firms are cooperating in order to reduce cost. Moreover, the thesis endeavor to prove that carbon emission can be reduced.

The study will also contribute to addressing the allocation of the savings/cost within the parties in the game using the method of inventory game. Based on the formulated model, the core allocation of the game will be found and then a proportional allocation will be proposed based on the frequencies of order and holding emissions.

An extended model of carbon emission under carbon-tax regulation will be also presented. Regulations on supply chain can influence cost and ordering quantity and consequently affect carbon emissions. Similar to the above, the carbon-tax regulation model carbon emission formula will be developed and then the core allocation model will be developed. The final extended model (Full Truckload system) will be also presented to show the effect on carbon emission considering that the ordering size is fixed by the truck size.

1.2 Methodology

The methodology of thesis followed the basics of formulated EOQ models under different systems to calculate carbon emission base on the literature surveys. An altered model of EOQ model was formulated to calculate carbon emission under centralized and decentralized. Both models will be using the optimum order quantity obtained from the optimum total cost models. To formulate the model and build it, simple mathematics and the “mathmatica” program were used. First, a model to calculate the total carbon emission under the desired systems with the assist of simple mathematics was formed. Second, the gap of both systems and the saving function were obtained. Third, the saving function, the principle of core allocation, and mathematics were used to find the core allocation of the system. ”Mathimatica” program was used to provide the required graphs. The numerical data were formed in a random in order to prove the results.

1.3 Significance of the Model to Qatar

Participating in reducing the global climate changes is one of Qatar’s 2030 strategic vision goals. As indicated in Qatar National development strategy 2011-2016 [for Development Planning], one of the main stagey goals was to reduce carbon emission and have a green environment. The strategy indicates that Qatar produces more than 7,000 tons of waste daily, and their aim is to reduce this number by 2016. In order to contribute to global warming, Qatar is targeting the adaptation of policies and technologies used by other countries as well as forcing government and private sector to become environmentally friendly.

This study can only be used by companies in Qatar once proper regulation and policies are formed. Nonetheless, the study can serve some sectors within the country by providing an insight about the facts of carbon emissions within the decentralized or centralized system. To that end, firms or decision-makers can have a reference to form the regulation, policies, and adapt best practices.

1.4 Paper Organization

The study is divided to into eight chapters including the current introductory chapter. The remaining chapters will organized as the following:

Chapter 2: Literature review section begins by revising literature pertaining to the background of carbon emission and sustainable operation with examples of some application, next the literature includes joint replenishment main models, usage, and how to use with carbon emission. Finally, the literature touches upon basic introductory of inventory game theory, designs, and outcomes.

Chapter3: Describes the problem as well as limitations and assumptions.

Chapter 4: Explains model formulation. It begins by providing the basic model of EOQ in decentralized and centralized system. Second, altering the basic EOQ formula to cater for carbon emission. Finally, providing the ability to measure the gap between centralized and decentralized system.

Chapter 5: pertains to carbon emission allocation based on the findings of chapter4. The core allocation will be presented and numerical example will be giving.

Chapter6 and 7: Covers the extended models. The first extended model will be an extension of the carbon-tax incentive model. The second extended model will address full truckload model. Finally, the model formulation, findings, allocation and numerical example will be provided for both extended models.

Chapter 8: Highlights main findings, conclusions, and future work.

CHAPTER 2. LITERATURE REVIEW

This chapter presents a wide range summary and comprehensive review of carbon emission in supply chain management, joint replenishment and inventory Game theory. The chapter is divided into three sections, Section (2.1) is an overview of carbon emissions, regulations and sustainable operations. First, it present a brief background about carbon emission in supply chain and Highlights the incentive of firms and organization to move towards sustainable operations and some applications.

Section (2.2) is about Joint replenishments. The section stars with joint replenishments definitions and benefits, then it moves to applications and method used, and models. Next, it addresses applying these methods on carbon emission, and shows all the insights with regards to applying this method.

Section (2.3) addresses game theory model in general by giving a brief about core allocation then shows some applications.

2.1 Carbon Emissions and Sustainable Operations

Between year 1970 and 2004, the GHG emission had increase by 70% (Rice et al.2007). This increase was due to human activities that is associated to GHG gases such as carbon dioxide or (carbon), ”CH4, and nitrous oxide (N2O), perfluorocarbons PFCs, hydrofluorocarbons (HFCs) and Sulphur hexafluoride (SF6) and ozone-depleting substances" (Metz et al. 2007). (Edenhofer et al. 2014) reported that there was an increase of 10 GtCO2eq of GHG between year 2000 and 2010. The increase was directly related to the following human activates: Energy Supply by 47%, industries by 30%, transportation by 11% and building sectors by 3% . If no additional regulations and policies were to be set, the increase in carbon emission can reach 25-90% by year 2030 compared to year 2000 (Rice et al.2007).

There are several international commitments to reduce carbon emissions. Kyoto protocol, that is linked to United Nations Framework Convention on Climate Change, is one of the international agreements in which it binds its parties to reduce emissions [Nations]. Kyoto protocol managed to reduce GHG emission by 12.5% in 1990. It was agreed that in 2010, the emission is to be reduced by 19% under Blair administration's control, and to reach 60% of reduction by 2050 (Martin et al. 2014). As china was one of the parties in Kyoto protocol, the government had announced in 2009 that it will reduce carbon emissions by 40-45% by 2020(Lin et al. 2010). European Union (EU), that manages the activates of carbon emission for almost 50% of European countries, had set up a new regulation to reduce carbon emission (Hoen et al. 2014). EU committed to reduce carbon emission by 20% by 2020 comparing to 1990's level. In 2013, EU managed to reach a total reduction of 19% which keeps them within their target for 2020[European commission].

Greenhouse gases are natural gases that appears in the atmosphere such as carbon dioxide, methane, and some others that are made by industries during supply chain process [NCDC]. As Carbon Dioxide (CO2) is found to be occupying half of the amount of gases that had been emitted and had an effect on the environment, most papers or organizations address greenhouse gases as carbon emission (Floros and Vlachou et al. 2005). The process of calculating the set of greenhouse gases that had been emitted by a firm or by supply chain is defined as carbon Footprinting. Many organizations use Carbon Footprinting to determine the amount of greenhouse gasses that had been emitted throughout the supply chain processes. It can assist in determining the exact process to which the organization has to focus on to reduce carbon emissions from it supply chain (Caro et al. 2013). Reduction of carbon emission can be done by lowering the carbon Footprinting within the direct emissions, indirect emission or both [1] . (Matthews et al. 2008) mentioned that 14% of total carbon emission within the Supply chain comes from direct emissions, while 26% comes from both direct and indirect emissions. Along with emission reduction, some organizations think about minimizing the cost associated with this emission reduction.

Kumar et al.2007 research called for imposing restrictions on carbon emission. The research explained the tradeoff between transportation, storage and carbon emission. However, Kumar did not address the cost of this reduction. In current days, reducing

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[1] [1]GHG protocol had defined the "direct emissions" as the emissions that are controlled and managed by the firm at its premises. "Indirect emissions" are the ones that are associated with the firm's productions and activities but they are accruing at another area that the firm cannot control. [1]Emissions are split into 3 scopes. Scop1 is the direct emission, Scope 2 is the indirect emissions from consumers that are associated with electricity usage and Scope 3 is all type of indirect emissions. Supply chain follows in Scope 3 [5], [55] , [44]

carbon emission is always associated with cost. Organizations try to find the best method to reduce emission with the least cost possible. Investment in supply chin process can be one of the methods. It is a long term strategy for organizations to reduce emission and at the same time be profitable. The widely used type of investment can be changing a currently used technology to a one that is more environmental friendly and reduces emission (Palmer and Burtraw et al.2005). Most of the investments are being used to reduce both direct and indirect emissions. (Dietz et al. 2009) stated that investment can reduce up to 7.4% of US national emission. Investments were categorized into 5 types: type 1, onetime investments for a building or manufacture to reduce carbon emission; type2, investing in new technology. This can be replaced at the end of the useful time; type 3: investment in maintenance; type4: equipment adjustment; type 5: daily operations such as more efficient driving and eliminating standby electricity.

A different type of investments can be a development of a new innovation that targets customers who are concerned about the environments. (Kleindorfer et al. 2005) stated that some companies do invest in new sustainable technologies to surpass their competitors. A case study addressed the invention of Toyota’s hybrid cars. The car is designed to consume half the gasoline and emit half the CO2. This innovation was new to the market and was not proposed by automobile companies that focus on sustainable technologies to which it gave Toyota an advantage. Even though (Kleindorfer et al. 2005) mentioned that investing in sustainable technology can be uncertain with regards to the benefits and development efforts, (Plambeck et al.2012) supported(Kleindorfer et al. 2005) and stated that Walmart was able to increase its revenue just by reducing emission and charging higher for ”green” products. That is because reducing emission can improve the public image to which there are some customers who are willing to pay extra for ”green” products. (Nouira et al. 2013) supported this point by proving that the demand changes depending on the level of ”greenness” of the product, and this greenness comes from the type of transportation mode selected. These changes can lead to an increase in the product cost. Nevertheless, the greener the product is, the higher the demand will be for those customers who care about the environment. A company can have a market share advantage even without reducing the emission of a product. Based on (Kalkanci et al. 2013)'s results, a company can increase its market share by revealing the environmental impact of its products to the public, even if the impact was high. Revealing this information can give a positive image to the company that cares about the environment and thus will increase its market share.

Sustainable operation is a wide field addressed by many researchers and literatures. The aim of the paper is to show the effect on carbon emission under supply chain daily operation and regulations. In order to save the environment and reduce the amount of pollutions and emissions in a whole region or a country, regulations and policies were made by governments and unions to control carbon emissions. Many policies were stablished such as political science. Political science was formed on 1970s, but its engagement to supply chain was not active until mid1990s (Corbett and Klassen et al.2006). That is because at that time, the focus was shifted to the whole supply chain and all its steps rather than a single operation or organization. It was found that the value will be greater if all steps within the chain were to be optimized than just focusing on a single process (Linton et al. 2007).Type of policies associated with carbon emissions within supply chain are: carbon-cap, carbon-tax, cap-and-offset, cap-and-trade and many others, where each of which can have a different impact on the total emission reduction and total supply chain cost. These policies can be used individually or to be combined together.

Carbon-cap policy (carbon capacity) is to set up an upper bound to the amount of carbon emission to be emitted within industries or commercials. Under strict carbon caps, the total carbon emission emitted from industries or commercials' operations should not exceed the cap. The characteristic of the policy are as the following: first, the carbon cap can be applied into a period, or for the whole supply chain (Bai and Mu et al.2014). Second, In case it was imposed on the whole horizon, the emission can be carried from a period to another. Third, the carbon can be emitted from production, ordering process, transportation and inventory holding (Benjaafar et al. 2013).(Song and Leng et al.2012) in his paper made an analysis that in a single period, the policy-maker should set the capacity lower than the company's optimal quantity. That is because in case the optimal quantity was lower than the mandatory capacity quantity, then there won't be any befit to the environment and government to impose this policy. This analysis was a support to (Benjaafar et al. 2013)'s insights with regards of setting the carbon cap in single or multi-firms.

Second type of regulation is Carbon tax. Carbon tax is the tax associated with a unit of carbon emission. Tax can have a variety of forms, but the simplest is to have a cost related to the number of carbon units that had been emitted (Benjaafar et al. 2013). Many European countries started to implement carbon tax on industries and commercials in 1990s. The rate varies from a country to another and it also depends on the sector it is applied on (Almutairi et al.2013). Many researchers addressed the effect of carbon taxes on operational decisions. (Floros and Vlachou et al.2005) stated that the implication of carbon tax reduces carbon emission in Greek manufactures and that it can be used to mitigate carbon emission. Flores and Vlanchou also stated that despite carbon emission reduction, applying this regulation will increase the total cost of the firms. Since the firm will have to pay extra cost associated to emission.

Cap-and-offset policy is allowing firms to exceed the allocated cap associated, but to be financially penalized for doing so by imposing a cost to every unit of carbon that was emitted above their limit. Cap- and-Trade policy is opening the market to firms to buy or sell carbon emissions. For firms that emit less carbon emission than their defined cap, they can sell the extra amount to the public and can buy if it was the other way around. Carbon policies became a main factor in the currently addressed problems and more examples will be stated throughout this literature review.

Investing to reduce carbon emissions or changing in operational decisions can sometimes be costly especially in inventory management operation. There are many researches that addressed the effect of applying environmental policies on supply chain while considering the total cost and carbon emission. Many others provided a model that can assist single firms to select the best optimal ordering quantity that reduces the total cost and minimizes total carbon emissions. (Benjaafar et al. 2013) used a lot-sizing model to present some insights with regards to carbon caps, furthermore, in a single firm with a strict carbon cap, three observations were made. Observation 1: just by changing in operational decision and adjusting the ordering quantity, the emission can be reduced down to 15% while the total average cost will increase by 3%. This reduction in emission depends on the ratio of the ordering to holding cost and the ration of the ordering to holding emission in which it ”continue(s) to play a dominate role in whether or not operational adjustments could have a significant impact on emission.” Observation 2: by adjusting the operational decision only, it could be more cost effective than investing in energy efficient technology. As it is known, investing in environmental friendly technology can reduce the emission, on the other hand, it can cost the company more. Adjusting the operational decision can reduce emission with a slight increase in cost. One of the main insight were that by tightening the Caps, this may result in increasing the carbon emission rather than reducing it. Due to forcing a carbon cap on period base, the reduction of carbon emission will happen in one period, but then it'll be increased in the coming periods. An example, if a firm orders a huge quantity to reduce the emission from ordering, this can lead to an increase in the holding cost of these inventories in the period following the ordering period.

(Chen et al. 2013) used the insights made in the Benjaafar's paper and showed that, in a single firm within Supply chain, the emission can be reduced significantly without increasing cost by only adjusting the operational decisions. Furthermore, it developed economic order quantity (EOQ model) to calculate the total carbon emission emitted from the operation whether it was in ordering or holding cost. While imposing a strict cap policy only, the results showed that it is possible to reduce emissions by modifying the order quantity. Since the cost function is flat at the optimal quantity (Q), another Q at the flat range can be chosen. By only changing Q, the emission can be changed significantly while having a lower relative increase in cost. Additionally, it showed that reducing emission by adjusting order quantity is possible if and only if the ration of the ordering to holding cost does not equal the ration of the ordering to holding emission. The model was extended to show the variant in changes between the optimal Q and the newly calculated Q. the variance model was used to calculate the delta change in both the cost and emission function while using both Qs based on the difference between both rations.

Another approach was made by (Tracht et al. 2013) were it explained how changing order quantity can reduce carbon emission within supply chain management. The author made an enhancement to EOQ model to calculate the total cost for ordering, holding, Backordering and shortage cost along with the emission cost associated with transportation. Emissions are counted based on the destination and fuel consumed during an order. From this model, it was shown that emissions can be reduced by increasing the order quantity in which it will cause the firm to order less frequently. This reduction in ordering frequency leads to a reduction in transportation. The author also showed how the optimum cost from the model including carbon emission is higher than the one without, and the ordering quantity increases as well.

More elaborations were done with regards to other carbon emissions policies. Hua et al. 2011 studied the impact of carbon cap and carbon price under cap- and-Trade mechanism at the ordering quantity, total cost and total emission in inventory management model(EOQ) . By combining both cost and carbon emission in one model, the numerical results showed that ordering quantity will change between the optimal Ordering Quantity in EOQ model and Optimal ordering quantity that will minimize the carbon emission. Under Cap-and-Trade, based on the carbon price per a unit of carbon and carbon-cap, the total cost varies. With a reasonable price, if the total carbon emission was higher than the Cap, then the firm will buy carbon credits, and will sell if the total carbon emission was less. When the total emissions equal the carbon cap, the company should neither sell nor buy. Furthermore, the author showed that if the carbon price was high, then it may be a chance for firms to decrease their total cost by selling carbon credits. The total cost will decrease if the carbon cap was high and the total carbon emission is below the cap, firms will be able to reduce carbon emissions and sell more carbon credit, and vice versa if the cap was low.

(Benjaafar et al. 2013)'s paper covered carbon and trade as well. The carbon emissions are affected by the price of a unit of carbon instead of the carbon cap's limit. The results showed that the carbon emissions are not affected by the change in carbon caps, but by the change in the price of the trade. Similarly, the higher the carbon price is, the lower the total cost can be. That is because when the cost increases, firms are encouraged to adjust their operational decisions to reduce the carbon emissions and sell these saved carbons to increase their profit. Additionally, the paper also covered carbon off-set where an insight about carbon offset with a tight emission caps was shared and explained that firms tend to offset even when the price of a unit of carbon is high.

Carbon tax was addressed in (Chen et al. 2013)'s paper, where the EOQ model was extended to include carbon prices. It considered having a carbon-tax and cap-and-price. The paper showed that a new Q function under carbon-tax can be generated to find the optimal Q that will minimize the total cost while reducing carbon emissions. The results presented that assigning a tax can reduce the emission cost. The reduction can satisfy the slight increase in the total cost as the order quantity moves away from the optimum value. Carbon-tax will not necessarily decrease carbon emission level. UNDER et al.2013 showed the effect of carbon-tax on both retailer and supplier. It showed that under this regulation, the supplier's annual carbon emission and cost can be reduced, while the retailer can reduce his annual carbon emission, but not the annual cost. (Krass et al. 2013) in his paper showed that if the tax rate was set high, then this can have a negative impact to the system. (Martin et al. 2014) on the other hand, demonstrated the impact of carbon tax on manufactures by having a comparison of firms that pays a fully tax amount and firms that pays a discounted tax amount. It showed that imposing a moderate carbon taxes can help in reducing energy intensity and thus reduce carbon emissions.

(Repetto et al.2013) in his paper "Cap-and-Trade contains global warming better than Carbon tax" stated that carbon-and-trade policy is better to be used in order to reduce carbon emission even though the revenue is fluctuated. Carbon tax is being set and regulated by the government in which the price of carbon tax is imposed to the service that emits emissions such as Fuel. This can have an effect on the product price if the firm used a service that is associated with carbon emission. Carbon-and-Trade, on the other hand, permits the energy user to set the price based on the market. Another point that was raised in the paper is that the services associated to the regulated carbon tax will be affected by the unit price; while in carbon-and-trade the services won't be affected as the amount of carbon emission will be reduced.

Many researches addressed transportation selection mode and facility location such as (Gucwa and Schafer et al.2013 and Hoen et al. 2013). "The transport sector is the second largest carbon emissions contributor in Europe and its emissions continue to increase" (Hoen et al. 2013) . In 2006, the carbon emission from transportation was around 23%, and it is expected to reach 60% in 2030. Hoen et al. 2014 in his latest paper studied the transportation methods and ranked them based on the amount of carbon emission. The author used Order-up-to policy and single-period newsboy problem and extended the model to be able to calculate the emissions from the activities. The paper showed that the emissions for air transportation are the highest, then road, rail and finally water transport to be the lowest. Cost was not mentioned in the previous study, (Lu et al. 2008)'s paper showed that changing the transportation mode can lead to a reduction in carbon emission and sometimes the total cost based on the transportation mode selected and the price of the carbon emission. (Nouira et al. 2013) elaborated more by providing the best transportation mode along with the new facility location. More elaboration were done by (Lu et al. 2008) where it was pointed out that changing the transportation mode in order to reduce carbon emission can have an impact on customer satisfaction level and vice versa. Another idea was proposed by (Cachon et al.2011) where enhancing the supply chain design and transportation model by adding retailers and customers to the model and check if carbon emission and total cost will be affected if the costumer was fully charged for the emission cost.

2.2 Carbon emission under joint replenishment

Unlike previous research where inventory management focuses on reducing total cost and emission from a single firm system, this part of the literature will have a brief about how firms can join resources in order to reduce average cost and carbon emission under such a system. Since the proposed model includes consolidation of two models, a couple of papers with carbon emission and carbon emission regulation under joint replenishment system will be revised.

Before moving to carbon emission under joint replenishment system, a brief about joint replenishment will be presented. Joint replenishment is a term used in supply chain to define the process of having a set of organization/firms, at a different or same level on supply chain, joining their inventory orders into one, or a firm joining multi items in one order instead of ordering it separately. It is also called shipment consolidation or inventory pooling. The main reason of consolidation is reducing total cost for the whole supply chain and individuals. It can also benefit firms in reducing any uncertainty and help in managing demands when demand is stochastic (Elomri et al. 2013). Goyal in 1977 was the first to present the idea of having a coordination between a vendor and a buyer. This concept was the basic of the current models now days Guiffrida et al. 2011.

Joint replenishment is useful when there is a single-retailer and multiple items, or single item and multiple retailers. A single retailer and multiple items, represents a scenario where a firm is sending out multiple items to a retailer, or when multiple items has to be sent to a retail in one order/ transportation. For single item and multiple retailers, a scenario of a firm sending a single item to multiple locations (Silva and Gao et al.2013). It can also help in reducing supplier based. In 1980s, many firms reduced their supplier based. Texas Instruments, for example, reduced it is supplier based by 85% in 2 years. They went from 5000 supplier down to 750 between 1998 and 2000 (Mustafa Tanrikulu et al. 2010).

A single item and multiple locations (retailers) scenario was addressed by many researches. (Cetinkaya and Lee et al.2000) for example, created a model to computed the optimum replenishment quantity for coordinated inventory shipment to multi retailers in VMI system. For a Stochastic demand system with multiple retailers and one item, the author thought of consolidate the shipment of these retailers into one by having a fixed cycle instead for a cycle per retailer. This means that there is a probability of having a backorder to which the author presented it with a waiting penalty. (Cheung and Lee et al.2002) introduced the same concept and included stock rebalancing concept. The author found that even though the cost decreased under coordination, however when the number of retailers increases, the benefit will decrease, the higher the number of retailers, the flatter the graph becomes. This applies to the system with or without stock rebalancing.

(Silva and Gao et al. 2013) solved the same problem under EOQ model. The author considered replenishing inventory level at retails' under a joint shipment in order to reduce the cost of transportation. The author extended the model to include location decisions as well to decide to which retailers the shipments should be consolidated.

Moving to single-retailer and multi-items, many researches addressed reduction of cost through consolidation. (Goyal et al. 1974) w the first to represent the replenishment of multiple items where each item has a strict order frequency. The Author presented an algorithm to find the optimum annual cost by proving the minimum and maximum bounds. (Khouja* et al. et al. 2005) made an enhancement to Goyal's algorithm by considering continues unit cost change while Goyal's algorithm was made with an assumption that the unit cost is fixed and does not change. Khouja also considered the model to be made under Indirect Grouping Strategy environment. Just like Goyal, Khouja found the lower and upper bound and the optimum annual cost. Khouja also found that when the unit cost declines, the life cycle gets shorter; while when it increases, the cycle gets expanded.

According to kouja's in 2012, Wang proposed a method to find the optimum cost under direct grouping. Direct grouping is different from Indirect grouping by having the items grouped to m groups. Each item within each group will have the same cycle time. While in indirect grouping, each items will have a different cycle time. The author solved the model using differential evaluation (DE) and evolutionary algorithm (EA) and found that total cost can be lower for direct grouping under DE method.

More elaborative and complicated system was developed by (Hajji et al. 2009). The author considered finding the optimum annual cost for an uncertainty and an interaction environment on a stochastic supply chain with three stages. The author also considered combining production and ordering costs as one model and equation instead of finding the total cost of each separately. The policy that was developed will help suppliers to plan their inventory level under the uncertainty of not having raw materials on the desired time, or a massive backorder due to unavailability on previous period; and at the same time to meet the retailer's demand. The author obtained a control policy with a combined multi-level base stock policy and state-depend economic order quantity.

(Mustafa Tanrikulu et al. 2010) proposed a new model for multi-items and single supplier. The authors' model was built for a stochastic demand model with multi-items and fixed truck size. Each shipment should not exceed the truck size and backorders were considered. The author created a (s,Q) model where it can be used when backorder cost or service level is high, shipment capacity, and when the lead time is short. The author also proposed that the model can be used with multi supplier and single item.

A more complicated model was presented by (Chen et al.2000). The author proposed a model to find the optimum replenishment quantity in a joint N stages with N items. The model was made in a stochastic demand environment with a passion demand in each stage. The model is to handle a several assembly points with multi units and with multi retailers with various demands. Chen's model was created with a fixed batch size, where various batch sizes were handled later on by (Benjaafar et al. 2006). Benjaafar introduced the same model under stochastic demand with a various batch sizes. The model can help decision makers to decide the number of items to be produced and demands to be satisfied.

All of the above introduced papers were about joint replenishment within the same organization in order to reduce the cost within the firm or to enhance the operation. Next, researches addressing joint replenishment problems among two or three firms were all these firms will join their resources to reduce cost. (Meca et al. 2004) focused on having the initiatives from retailers to place an order, while usually from previous papers, the initiatives comes from the supplier as he knows the demand. The paper extended the inventory management system model by adding multi firms managing a single inventory in the system with a deterministic demand. The model assumes a single supplier where all firms are going to have a joint order from. A reduction in the total cost is possible since all firms will corporate and place one big order. Since orders are joint, the frequency of ordering and the cycle length will be equal for all. The firm with the long cycle will reduce its ordering size and order more frequently than ordering big quantities under the same interval. Reducing ordering size will reduce the inventory in hand and thus reducing the holding total cost of these inventories as well. The paper provided the optimal total cost and the ordering quantity for all firms. Some firms avoid joining due to prevent sharing of private information. However, the paper proposed that it is enough for each firm to reveal their ordering frequency only where their demand and holding cost will remain enclosed. The intermediate will sum all requests and place one big order where it is going to be the only information passed to the supplier. The supplier will only know the total quantity required and will not know the quantity required for each firm.

A condition was introduced by (Nagarajan et al. 2010)'s model. The author proposed how firms can join in order to take an advantage of quantity discount offered by a supplier. Assuming n firms buying from a single supplier where this supplier offers a discount per number of units to be purchased. A firm will try to join his order with other firms in order to get advantage of it if he was not able to by ordering individually. The model was created under deterministic demand system. The paper calculates the saving of the joint under discrete discount schedule and continuous discount schedule.

(Elomri et al. 2013) addressed n retailers joining their replenishment under full truckload system. Each order should not exceed a certain size which is the truck capacity. The model was an enhancement of the topical EOQ model to accommodate n firms and fixed batch size. The paper showed that the ordering cost won't be affected by the number of n firms joining together as the batch size is fixed, but the holding cost will be affected and this is where firms can save cost from.

The concept of joint replenishment can be used to reduce carbon emission as well. Many firms attempt to reduce carbon emissions within their firm solely without paying attention to reduce carbon emission from supply chain as a whole. The absence of collaboration and coordination can result in an increase to the overall carbon emission in supply chain (Benjaafar et al. 2013). One of the advantages of having carbon constraints is allowing firms to coordinate in order to maintain carbon level. Coordination among firms within supply chain can have a great impact in reducing the cost in some cases. With strict cap emissions, firms tends to coordinate to reduce carbon emission along with the total cost.

Guiffrida (2011) developed an analytical development model that measures the performance of supply chain under carbon emission. The model considered two-level supply chain where the total cost is coordinated among parties with regards to lot-size shipment. The demand from the retail was used as the base of the quantity that has to be produced and shipped. The paper considered the contribution to environment as a quality cost function that is added to the total cost of supply chain. This cost function covered the transportation cost from fuel and any other emissions among the cycle. This model was used to help decision maker to select reasonable quality cost values. That is because having a high quality cost can cause a reduction to profit. However, this model did not consider any carbon regulation to reduce down carbon emission.

(Benjaafar et al. 2013) presented an extension to his model from the previous section to address the problem associated with coordination under carbon regulations. The paper showed that the cost under coordination after imposing a strict carbon cap regulation leaded to significant reduction in cost "with the benefit highest when the carbon cap is in the mid-range rather than either very low or very high." When the cap is very high, the policy won't have an effect on the total carbon emission reduction. When the cap is low, the firms will have no room for adjustments which may lead to less or no reduction in cost. However, a reduction can happen due to the coordination among the n firms. This means that the benefit from operation adjustment due to carbon emission is more effective if there is a strict carbon caps.

More regulations were presented by (Zeng et al. 2012). (Zeng et al. 2012) presented 2-eclone system under lot-sizing model. The paper showed that under fixed carbon tax rate, depending on the value of the carbon tax rate, cost and emissions can be affected. With a high carbon tax rate, the emission will start to decrease rapidly and the cost to increase. If the carbon tax rate was set to be too high, the carbon emission will start to decrease gently while the cost will increase linearly. Another approach was to have the regulators defining a low level of carbon (E1) and a high level of carbon (E2). Under progressive carbon tax rate, a normal rate is set to E1, when the carbon emissions are under or equal to E1 level; and a higher rate is set to E2, when the firm's carbon exceeds E2 level. The result showed that, the cost will be affected only when E1's level is low, and the emission will be affected only when E2's level is high. (Jaber et al. 2013) also developed a model based on a 2-echlon Supply chain and studied the effect of carbon tax. It showed that in 2-echlon Supply Chain, by applying taxes only, the total cost of the whole supply chain will not be reduced when having the optimal emission rate. Having the optimal emission rate will have an unnecessarily increase in total cost. When applying penalty policy only, the author provided a formula to calculate the optimal production quantity. The optimal quantity will be the quantity that will reduce carbon emission. This quantity can either be the optimal without a regulation if the cap was high, or the lowest production rate that will keep the emission not exceeded.

Another study was presented by (Bai and Mu et al.2014) in which it used the same concept but in a different system. The paper is about the impact of carbon emissions policies on supply chain based on the theory of System dynamic. The model is under two-echelon supply chain (supplier and retail) in which the demand from the retailer defines the whole productivity at the supplier. The paper showed that, under Cap policy, when placing caps to firms individually, if the caps were reasonable, it'll lead to a reduction in emission and cost. In case the caps were placed at the supply chain as a whole, this will lead to a total reduction in supply chain emissions and cost with a higher profit than having the caps being imposed on firms individually. Results showed that "coordination of supply chain is one of the most effective ways to make full use of carbon caps policies, and maximize the supply chain profit." By implying another policy, the paper showed that under cap-and-trade, reduction of carbon emission and cost depends on the price of a unit of carbon and the carbon cap as well.

Another applications can be found in (Chaabane et al.2013) works. The research developed a model to help decision maker and policy makers to sustain their supply chain. The linear programming model is designed to calculate the optimal carbon-Trade price for the company. It can also be used to find the reasonable carbon-cap that will help in reducing emission without increasing the cost. The paper showed that when fixing the carbon price for each period and varying the carbon cap, the total cost increases linearly while the carbon cap decreases. On the other hand, when the carbon price starts to vary in each period, the total cost increases but not linearly. (Caro et al. 2013) made another contribution by applying the idea to LCA model. The paper used life-cycle assessment and carbon footprint to determine the total emission from a joint production supply chain. The author first determined the footprint from each process within the supply chain with multiple firms, then determined if double counting can be avoided or not. The results showed that, if each firm wants to have their best practice, double-counting cannot be avoided. Firms that cares to reduce the total carbon emission from supply chain, can provide some mechanism to share the cost and the profit of the emission reduction with the firm with the most carbon emission emitted.

Last point presented by Caro supports (Benjaafar et al. 2013) with regards to reducing the cost per firm individually when they are under coordination. Coordination can indeed lead to a reduction in the total overall supply chain cost, but it may not lead to a reduction in the cost and carbons for some firms within the supply chain. Benjaafar stated that Firm 1 can make an adjustment in the ordering or holding quantity in order to coordinate and reduce the emission for Firm 2. This adjustment can lead to an increase in the total cost and emission of Firm1 even though the overall cost is reduced and carbon emissions are within the cap. Coordination can also lead to an investment in physical infrastructure. The investment is done by the firms with the least carbon emission to assist the ones with high carbon emission. For example, if Firm1 has an efficient holding emissions while Firm2 does not, Firm1 can invest in holding techniques and share the inventory holding with Firm2 to reduce the total emission of supply chain. Firms will have to compensate in order to reduce the overall carbon emission. But the question that was raised is how to divide the surplus from the coordination based on each companies' compensations.

2.3 Game Theory

With the deployment of coordination and joint replenishment among several parties with the motivation to reduce cost, the main question is how to allocate cost/ pay-offs among parties. The analysis of cost/ pay-offs allocation among parties in difference inventory management operations is referred to as Game theory.

The analysis of the game theory should assure that the game is stable. Stability means finding all the feasible outcomes to which players will see a benefit from joining resources. In order to make the player to have an incentive to remain in the game, fairness and core allocation is required. The core allocation refers to having one of the feasible outcomes that will prevent a player or a coalition (group of players) to leave the grand coalition and form its/their own (Nagarajan and Sosˇic´ et al. 2008).

Fairness can be hard to maintain specially with a complicated model and number of participants. To find the core allocation, first, the n players are grouped into S group (coalition), where $S \subset N$ and N is the grand coalition. With n players, the number of coalitions that can be formed is $(2n - 1)$.To find the core allocation and to have the best benefit to all players, the following properties should be fulfilled while allocating:

1. All saving to be divided.
2. Each party to be assigned at least as much profit he could obtain when working individually.
3. The profit is allocated so that no sub groups to have a better saving than grouping all together.

Having the above characteristics can guarantee the fairness in allocation cost/ pay-offs among the players (Nagarajan and Sosˇic´ 2008, Gilles et al. 2010).

Cost allocation is an old topic where it started in 1950s by Nash. Through the years, many researches where developed on how to allocate the savings among players based on the model proposed and information given by these parties and the incentive of cooperating. First, this part will introduce models that are closer to this research paper. In the past few years, many papers were written about a cooperative game theory with the basic inventory model under deterministic demand (EOQ). (Meca et al. 2004) ”Inventory Game” paper, which is one of the popular papers, introduced a shipment consolidation model between n parties and showed that the consolidation can lead to a cost saving. The author first showed that players can cooperate without revealing the basic information such as demand, holding cost, and can place an order using the frequency ordering. The author then proposed a way to allocated these saving by showing that proportional allocation of ordering frequencies can be the core allocation. The author then extended his model in his paper “cooperation and competitive in inventory game” by showing the core allocation under economic production quantity (EPQ) and a non-cooperative point of view. Then the author studied the nashi equilibrium of this model.

A simple model with a fixed ordering size was presented by (Elomri et al. 2013) where it was shown an enhancement of EOQ model under a full truckload system. The author showed that the core allocation for the grand coalition and compared it with sharply value allocation. The author presented the limitation of sharply value allocation compared to the core allocation. This thesis was used to express an extension model in this thesis.

(Dror and Hartmanet al.2007 and Anily and Haviv et al.2007) presented the same model, where an extension of (Meca et al. 2004) was made to cover the major set up cost as well. The model proposed an equation with a holding cost to every item, minor ordering cost to every item, and a fixed major cost for every order. The major fixed cost will be charged at the player on the grand coalition even if he was not part of the players who placed the order in this period. Both papers showed that the function is a core allocation with a non-empty set. Anily and Haviv's model focused on power-of-two policy where it also showed that the saving had a non-empty core system. (Zhang et al.2009) model was closely related to Anily and Haviv's. The author proposed the same model under power-to-policy while allowing items to be stored on a warehouse. The warehouse is responsible for part of the major ordering cost as well. The author used langrangian dual theorem to prove that the saving is non-empty core.

(Van den Heuvel et al. 2007) presented a similar model to meca's where the model involved production cost along to the basic lot sizing model. The model had n retailers and a supplier with a cost function calculating the ordering, production and holding cost. NP production will be done unless the holding quantity was 0. The paper proved that cooperation can lead to saving in production cost along with the ordering cost. The paper also proved that the function is concave and a non-empty core.

Production cost was introduced in (Guardiola et al. 2007)'s paper as well. The author introduced the basic idea of profit allocation under cooperative distribution chain with decreasing production cost depending of order quantity. The author proposed a model with a single supplier, intermediate and non-competitive multiple retailers who receives a single item. The product cost is a decreasing function depends on the quantity of order. The game is an incentive to retailers to join their orders into a big one since the retailer does not know about the each retailer's real order quantity due to having an intermediate. The author showed that by having the retailers joining their orders, the total cost decreases and the saving function is a non-empty core allocation. The author then showed that supplier can be a player in the game as well, which is preferred by him in order to reduce cost. (Nagarajan et al. 2010) also discussed Group Purchase Organization and offered a stable model under several scenarios.

(Meca et al. 2007) had the same primary idea of production discount cost for a large order. The model was designed for n players joining their orders and sharing a warehouse while ordering in order to get the best out of the temporary discount offered by the supplier. The author then produced the p-additive game and showed it is balanced, and had a nonempty core allocation.

Most of the presented researches were about joining items or orders, there are some models addressed sharing facilities and warehouses. (Tijs et al. 2005) had the model of sharing a warehouse. There are n players, to which player 0 has a warehouse, and players 1,2,…,n have items that needs to be stored. The author first showed a cooperative holding game and proved that it is a core allocation; then the author introduced the big boss holding game. The main goal is to find the optimum holding plan and how to allocate the cost.

Another research with regards to facility sharing was (Guardiola et al. 2009)'s paper.

The author designed a cooperative model where the players within the coalition will produce at the cheapest production facility and will store at the cheapest warehouse. The author showed that the model is balanced but not concave. Then the author introduced the own point theory, where every player has to pay the minimum cost of operation. This theory was proved to be a unique fair allocation.

There are not many researches addressing inventory game to reduce carbon emissions. In recent years, (van den Heuvel et al. 2012 and Kellner and Otto et al. 2012) had a similar idea of allocation carbon emission in transportation mode. Both papers showed a predesigned core allocation and showed the best method that can be applied to the proposed situation.

2.4 Summary

This chapter covered the review of carbon emission, sustainable operation, joint replenishment and inventory game. First, the focus was on carbon emission within supply chain and how to sustain the operations. Then the focus moved to joint replenishment in general descripting the method used and models. The usage of joint replenishment to reduce carbon emission was then mentioned and covered. Lastly, the focus moved to inventory game theory within supply chain management. There are not so many research addressing the allocation of carbon emission using inventory management.

Adopting the researches of inventory game to allocate carbon emission under a system where the incentive is reducing carbon emission only and neglecting the effect of cost can be a basic copy of previous models. Many researches addressed measuring and reducing carbon emission along with cost under joint replenishment method; but, not many addressed allocating carbon emissions. This thesis introduced several models in different systems and most of the researches had quite the same result. Many showed that carbon emission can be reduced will having a slight change in total cost.

Next chapter describes the problem statement and what will be covered on this thesis. The basic models that are going to be used will also be introduced.

CHAPTER 3. PROBLEM STATEMENT AND BASIC MODEL

In this chapter the problem statement of the research will be introduced along with the assumptions related to the case used and the formulation of the basic model that was introduced in previous work in the literature review. Section 3.1 will include the problem statement and assumptions, and section 3.2 will include the basic EOQ model formulation.

3.1 Problem Statement

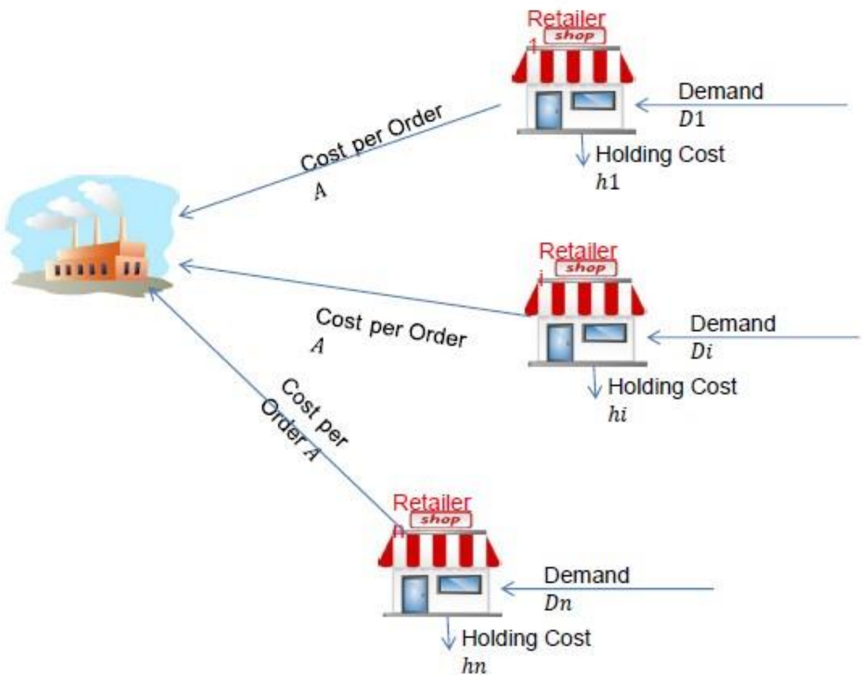
Reducing carbon emissions became one of the cooperate strategies a firm is obligated to have now a days. The reduction of carbon emissions can be done in many ways, either by investment or by operational adjustments. Many firms can coordinate with other members to reduce carbon emissions or cooperate with one another. Cooperation, also known as joint replenishment, inventory pooling or shipment consolidation, in supply chain has three main advantages: risk pooling, negotiation power and reduction of the total cost. Many researches showed that the reduction of the total cost under cooperation for the supply chain can lead to a reduction of the total cost of each parties individually. The same concept can be applied to reduce carbon emissions if the goal of the cooperation was to reduce carbon emissions only without considering the cost. The main contribution of this thesis is to measure up to what extend supply chain cooperation can contribute to carbon emission. The main question that can be addressed in this thesis is finding if cooperation can lead to a reduction in carbon emission. This can be seen as another advantage a firm can gain from cooperating. Another question will be, if cooperation can lead to a reduction, how these achieved savings would be allocated among the parties within the coalition\ game.

The thesis contribution to the field will be that, under deterministic demand system, when n firms cooperate with incentive to reduce cost, they will produce an optimum ordering quantity that will lead to having the optimum total cost. The thesis will study if this optimum ordering quantity can be used in order to reduce carbon emissions as well within the same system. Showing that cooperation can have one more advantage that can be considered, many firms will think about cooperation as an option to save both cost and carbon emissions. The other contribution will be, following the game theory method, the thesis will provide a core allocation to this model in order to divide the obtained savings among the n parties within the game.

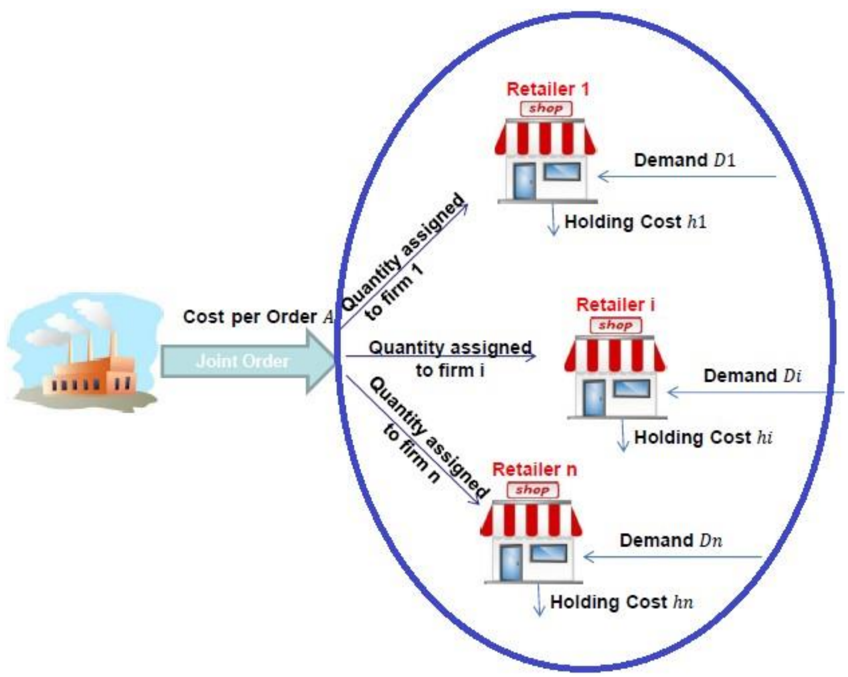
The thesis is going to use the basic EOQ model for both centralized and decentralized supply chain. The used formula will consist of the ordering and holding costs to measure the total cost in both systems; the same will be applied to carbon emissions equation where it will consist of the ordering and holding emissions. No additional costs or emissions parameters to be handled in this model such as the purchase, backorders or fixed costs for simplicity purpose. In order to find the total carbon emission emitted from the system, an adjustment to the EOQ model to both centralized and decentralized supply chain formulas will be applied. The adjustment will be replacing the ordering and holding costs with ordering and holding emissions variables. In addition, it is assumed that the emissions parameters are already calculated and provided. As for the Ordering quantity, the optimum ordering quantity of cost equation will be used into the emission equation, since it is assumed that all firms are going to minimize their total costs.

Before building the model, some assumptions have to be made which are associated with the case environment. It is assumed having a regular EOQ model for both systems as it is shown in figure 1(a) and 1(b). The graph shows that in both decentralized and centralized systems, both systems will have a single supplier replenishing n retailers with a single item. The model is to be built under lot-sizing to which there is no left of inventory at the end of period. It is assumed that the demand is deterministic and known in advanced. Since the demand is known, backorders are not allowed. Moreover, the system will have a zero lead time in which orders will arrive at the time they were ordered. For simplicity, it is assumed that there is unlimited storage for inventory to be held and unlimited capacity for ordering. For all the centralized system, it is assumed that the firms will have one median who is going to arrange the shipment and consolidate them in order to place the orders.

For the first part of the research, it is assumed that there is no regulations assigned with regards to carbon emissions. Carbon emissions will be measured to show the effect of joint replenishment model on carbon emission. Moving forward, regulation will be implemented to the model such as taxes, and the effect of these regulations will be analyzed and compared between the both systems. The same assumptions above will be applied to the modules in both systems while building up the extended tax model. The main objective is to know the effect of carbon tax regulation, on n firms under consolidation, on both cost and amount of carbon emissions where the incentive is to



(a) Decentralized Supply Chain



(b) Centralized Supply Chain

Figure 1. Illustration of Decentralized and centralized supply chain

reduce cost only. It is assumed that the tax rate is fixed per a unit of carbon emission emitted. Finally, to have a more realistic situation, the model will be built assuming that the truck should be fully loaded. This means that each order will have a fix capacity which is the truck size. Firms cannot order more or less that the specified capacity.

3.1 Basic EOQ Model Formulation

The model will be built in both decentralized and centralized systems where they will be compared in order to find the effect of cooperation on carbon emissions. The basic and regular EOQ model under cost context was already created and proved by many researches. This thesis is going to focus on the models that were built in both "the carbon-constrained" article, (Chen et al. et al.2013) and "Inventory Games", (Meca et al. et al.2004).

For the basic inventory model in both systems, there are N firms, where N = {1, 2, ..., n}, joining their resources and consolidating their shipment process. Each firm i within N, it will have a deterministic demand (D_i) in which $D_i \geq 0$. All N firms will be ordering from a single supplier, and it is assumed that there is unlimited capacity. The firms are not allowed to have backorders. And for simplicity, the lead time is equal to zero, which means there is no time between ordering and receiving the order.

The cost equation will be divided into two parts: ordering cost and holding cost. Ordering cost (A) is the cost related to the fees required when placing an order from the supplier and A > 0. The cost will include the delivery cost along with some administration fees. For simplicity, it is assumed that the ordering cost (A) is fixed for all n firm and it does not get affected by the quantity to be ordered or the number of containers required. The second part will be the holding cost which will be the cost of holding or storing one unit of inventory in the warehouse per unit time. For the holding cost, each firm i will have a different cost for holding, donated as h_i where h_i > 0, that is because each firm will be storing and holding the items in their own warehouses and not a shared warehouse.

Since the demand is deterministic and no out of stock or backorder is allowed, each firm i wants to order a quantity, donated as Q_i, to replenish its stock to which it'll satisfy the demand. Q∗ will be the optimal order quantity a firm will use to place an order in order to optimize the total cost. The frequency (f_i) of placing an order per unit of time will be D_i/Q_i. A cycle is the time between replenishments and it can be found from Q_i/D_i. The average quantity per unit period can be donated as Q_i/2.

Both EOQ modules for firms under decentralized and centralized supply chain management in their basic inventory level are adapted from previous work and research mentioned in the literature review. The optimum cost equation is composed from ordering and holding cost. Since the demand is already known, the frequency of ordering (f) can be calculated as Demand (D) divided by the ordering quantity (Q). The ordering cost will be the total of the cost per order (A) multiplied by the frequency (f). The second part of the cost will be the holding cost, which it will be the average of the quantity ordered multiplied by the holding cost (h). The total cost per unit time for one firm i for the basic Inventory model, and based on (Chen et al. et al.2013), is given as

$$C = A\ \frac{D_i}{Q_i} + h\ \frac{Q_i}{2} \tag{1}$$

The total cost of n firms under decentralization will be the summation of the costs for each firm i in N, since each firm is ordering separately and they are not attached.

$$\sum_{i}^{n} C_i \tag{2}$$

Following on the above concept, the inventory model for n firms under consolidation is associated with ordering and holding costs. The demand will be donated by Di, for every i ∈ N. Since all firms will consolidate their orders, the frequency will be $\sqrt{\frac{\sum h_i D_i}{2A}}$ based on "inventory Game" paper, (Meca et al. et al.2004). Since the consolidation will be in the ordering only, each firm will store its goods by its own. The holding cost will be the average holding amount multiplied by the frequency of ordering. Since the holding amount will be less under consolidation, it'll be influenced by the frequency of ordering. The equation will have the following form:

$$C_C^* = A\,f + \frac{1}{f}\sum_{i \subseteq N} \frac{h_i D_i}{2} \tag{3}$$

After finding the basic inventory model in both systems, the model to calculate the emissions within the supply chain will be developed.

CHAPTER 4. MODEL FORMULATION

This chapter is going to follow the previous chapter to build carbon emissions measuring model under centralized and decentralized systems. Section 4.1 presents the basic carbon emissions model in decentralized system and how it is modified to be addressed. Section 4.2 presents the same on centralized system. 4.3 shows the impact of the model and the system on carbon emissions. A comparison to both systems will be presented as well on the same section. Section 4.4 presents a numerical example of the formulated new carbon emissions models and the associated impact.

4.1 Carbon Emission Model for Decentralized Supply Chain

Following the previous chapter of defining the basic model and following the literature reviews, a formula that is similar to EOQ cost formula can be defined to calculate the total carbon emissions associated with placing an order. Since the total carbon emissions model consists of ordering and holding measurements, the yearly average carbon emissions can be calculated by donating $\hat{A}$, $\hat{h}$ to be the amount of emissions to be emitted when ordering and holding a unit respectively. For firm i in a set of N firms, where N = {1, 2,…, n}, the ordering emission will be fixed for all N firms, as $\hat{A}$ where $\hat{A} > 0$. That is because all firms are ordering from the same suppliers and no different charges will be applied to firms based on their location or method used for ordering. For the holding emissions of firm i, the emission will be donated as $\widehat{h_i}$ for each i ∈ N where $\widehat{h_i} > 0$. The holding emissions will differ from a firm to another because each firm will store by its own and in its warehouse. In additions, all emission variables are pre-defined and measured.

After defining the emission's variables, in order to create the total carbon emissions model in the de- centralize system, the cost's variables will be placed with the emission's variables. This model will be used to calculate the total carbon emissions emitted from the systems by all the n parties. The formula will be as the following:

$$E_i = \hat{A}\,\frac{D_i}{Q_i} + \hat{h}\,\frac{Q_i}{2} \tag{4}$$

The total carbon emissions for n firms working individually will simply be the summation of all carbon emissions emitted from the n firms:

$$\sum_{i}^{n} E_i \tag{5}$$

The above formula 5, can be used to measure the total carbon emission in a decentralized system; but it will measure the total optimum carbon emissions based on the emission's variables. That is because the frequency, or optimum ordering quantity, used is the one obtained to minimize the total carbon emissions in the system. In order to find the amount of carbon emissions emitted from the system when firms' incentive is to reduce the total cost, then the frequency obtained from the total cost formula, C_d^*, should be used and placed in the carbon emission function. The total carbon emissions that is measured will be the effect of placing an order on carbon emissions within the decentralized system. The new total carbon emissions formula will be obtained as the following:

$$E_{\Pi}^{d} = \sum \hat{A}\ \mathrm{f}_i + \hat{h}_i\ \frac{Q_i}{2}$$

$$E_{\Pi}^{d} = \sum \hat{A}\,\frac{\mathrm{D}_i}{Q_i} + \hat{h}_i\ \frac{Q_i}{2}$$

$$E_{\Pi}^{d} = \sum \hat{A}\,\frac{\mathrm{D}_i}{\sqrt{\frac{2AD_i}{h_i}}} + \frac{\hat{h}_i}{2}\ \sqrt{\frac{2AD_i}{h_i}}$$

$$E_{\Pi}^{d} = \sum \hat{A}\,\frac{\mathrm{D}_i\sqrt{h_i}}{\sqrt{2AD_i}} + \frac{\hat{h}_i}{2}\ \sqrt{\frac{2AD_i}{h_i}}$$

$$E_{\Pi}^{d} = \sum \hat{A}\,\frac{\sqrt{\mathrm{D}_i h_i}}{\sqrt{2AD_i}} + \frac{\hat{h}_i}{2}\ \sqrt{\frac{2AD_i}{h_i}}$$

$$E_{\Pi}^{d} = \sum \frac{\hat{A}}{A}\frac{\sqrt{\mathrm{D}_i h_i 2A}}{2} + \frac{\hat{h}_i}{h_i}\ \frac{\sqrt{2AD_i h_i}}{2}$$

$$E_{\Pi}^{d} = (\frac{\hat{A}}{A})\sum \frac{C_i^d}{2} + \frac{1}{2}\sum(C_i^d\,\frac{\hat{h}_i}{h_i}) \qquad (6)$$

From the above total carbon emissions formula 6, it is shown that the ordering emission will be influenced by the total optimum cost effected by the ratio of the ordering cost over the ordering emission. Similar goes to the holding emission part; the part is influenced by the total optimum cost effected by the holding cost over the holding emissions. This means that the cost function can be a variable to measure, calculate and influence the amount of carbon emissions emitted from the systems.

4.2 Carbon Emission Model For Centralized Supply Chain

For N firms consolidating their shipment and working jointly under a centralized system, the same concept that was found to obtain the total carbon emissions model in the decentralized system can be followed. The carbon emissions model will have two variables which are the ordering emission $\hat{A}$ and the holding emission $\hat{h}$. As it is known, the ordering emission will be fixed for all firms within the coalition while the holding emission will vary from a firm to another based on the storage techniques since each firm is storing its order individually.

In order to create the model of measuring the total carbon emissions emitted in the centralized system, the cost variables, A and h, will be replaced by the emission variables, $\hat{A}$ and $\hat{h}$. The carbon emission formula for centralization will be as the following:

$$E_c^* = \hat{A}\, f + \frac{1}{f}\sum_{i \sqsubseteq N} \frac{\hat{h}_i D_i}{2} \tag{7}$$

Having the above model, 7, won't be enough to calculate the total carbon emissions within the system since it will measure the total carbon emissions when the optimum ordering quantity is used to reduce carbon emissions. Based on the focus of the thesis, the total carbon emissions model needs to be amended in order to measure the amount of carbon emissions obtained based on the order placed in order to reduce the total cost. Similar to the previous section, the frequency, or optimum ordering quantity, used in the total carbon emissions equation will be replaced by the one obtained from the cost equation C_c^*. The new total carbon emissions formula for centralized E_{Π}^{c} can be derived as the following:

$$E_{\Pi}^{c} = \hat{A}\ \mathrm{f} + \frac{1}{f}\sum \frac{\hat{h}_i}{2}\ D_i$$

$$E_{\Pi}^{c} = \hat{A}\sqrt{\frac{\sum h_i D_i}{2A}} + \frac{1}{\sqrt{\frac{\sum h_i D_i}{2A}}}\sum \frac{\hat{h}_i}{2}\ D_i$$

$$E_{\Pi}^{c} = \hat{A}\frac{\sqrt{2A\sum h_i D_i}}{2A} + \frac{\sqrt{2A}}{\sqrt{\sum h_i D_i}}\sum \frac{\hat{h}_i}{2}\ D_i$$

$$E_{\Pi}^{c} = \frac{\hat{A}}{A}\frac{\sqrt{2A\sum h_i D_i}}{2} + \frac{\sum \hat{h}_i D_i}{\sum h_i\ D_i}\frac{\sqrt{2A\sum h_i D_i}}{2}$$

$$E_{\Pi}^{c} = \left(\frac{\widehat{A}}{A}\right)\frac{C_c^*}{2} + \frac{C_c^*}{2}\left[\frac{\sum \hat{h}_i D_i}{\sum h_i\ D_i}\right] \qquad (8)$$

From the above carbon emissions formula in centralized system (8), the same observation can be made that the total carbon emissions model will be influenced and affected by the total cost in the centralized system. The ordering emission part will be influenced by the total cost in the centralized system affected by the ratio of the ordering emission to the ordering cost. The same thing goes for the holding part where the total cost is affected by the holding emission amount to the holding cost amount. As the cost increases, the total carbon emissions will increase as well.

The main argument of this model is to find if joint replenishment can lead to a reduction in carbon emissions if n firms consolidated their shipment with the purpose of reducing cost only. As shown in the literature review, the total cost under cooperation is less than the total cost of firms working individually. The reduction of the cost was achieved after the amendment of the optimal ordering quantity which was calculated in order to find the optimum total cost of the supply chain as a whole and each party in the coalition. Similar results can be found if the N firms were to consider carbon emissions and find an optimum ordering quantity that will minimize the total carbon emissions found along with reducing the total cost. However, the question will be, will it be greener for firms to consolidate their shipment and to benefit from the consolidation to reduce carbon emissions when they are cooperating to reduce cost only?

4.3 The Impact On Carbon Emissions And Findings

Based on the newly formed carbon emissions equations at (6) and (8), the total carbon emissions can be measured and compared to find out the impact of cooperation on carbon emissions. A new advantage can be listed to cooperation and give a motivation to firms to join their resources. After finding a reduction in carbon emissions under a cooperative system, it will assist in developing a theory to distribute this saving among all parties which is the second part of the thesis. Both systems depend on the total optimum cost of the supply chain; and as per the literature review, it was stated that total cost under consolidation will always be less than total cost for firms working individually. In this subsection, the impact of calculating carbon emissions on firms under consolidation will be examined.

Before moving to find the savings of carbon emission obtained from both systems, it will be shown how cooperation can always lead to a reduction in cost (Meca et al. et al.2004). The cost function can be expressed as ($\sum C_i^* = \sum \sqrt{2Ah_iD_i}$) in the decentralized system, and to be expressed as ($C_c^* = \sqrt{2A\sum(h_iD_i)}$) in the centralized system. By comparing both models, it is clear that the centralized system will be less that the decentralized system as the square root of a summation is always lower than the summation of the square root. The comparisons can be equivalent to:

Decentralized formula: Centralized formula:

$$\sum_{n} \sqrt{h_i D_i} \quad \geq \quad \sqrt{\sum_{n} D_i h_i} \tag{9}$$

Theoretically, cooperation can actually assist in reducing carbon emissions as well for several reasons. Based on the proposed scenario, all n firms will consolidate their shipment and order less frequently to reduce the ordering cost. By ordering less frequently, then the amount of carbon emissions that will be emitted from the centralized system should be less than the decentralized system since the ordering part is affected by the number of orders. As for the holding part, since the amount to be held within each firm will be reduced since the quantity ordered will be reduced, then the total holding emission will be reduced as well. In other words, the number of stored items are less, and the holding emissions are measured based on the stored items. Since both parts are proved to be less in the centralized system, then the cooperation can actually lead to a reduction.

This concept can be proved mathematically and can be found in the coming proposition. The proposition States that E_{Π}^{C} will always be lower than E_{Π}^{d} in all conditions. By having N firms consolidating their shipment, this can lead to a reduction in both the total cost and the total carbon emissions.

Proposition 1. *Cooperation can always lead to a reduction in carbon emissions.*

$$E_{\Pi}^{d} = \left(\frac{\widehat{A}}{A}\right) \sum \frac{C_i^d}{2} + \frac{1}{2} \sum \left(C_i^d \, \frac{\hat{h}_i}{h_i} \right) \quad \geq \quad E_{\Pi}^{c} = \left(\frac{\widehat{A}}{A}\right) \frac{C_c^*}{2} + \frac{C_c^*}{2} \left[\frac{\sum \hat{h}_i D_i}{\sum h_i \, D_i}\right] \tag{10}$$

Proof: in order to prove the above proposition, both equation 6 and equation 8 will be compared to find the gap from the total carbon emissions in centralized and decentralized supply chains. Finding the gap can be used in order to find the effect of joint replenishment on carbon emissions.

Both models can be divided into ordering and holding emissions and analyze each part separately. For the ordering emissions part, it is clear mathematically that the ordering emissions in the centralization is lower than the decentralization, where the ordering emissions part can be simplified as the following:

Decentralized formula: Centralized formula:

$$(\frac{\hat{A}}{A})\sum_{n}\frac{\sqrt{2Ah_iD_i}}{2} \geq (\frac{\hat{A}}{A})\frac{\sqrt{2A\sum_n D_ih_i}}{2} \quad (11)$$

$$(\frac{\hat{A}}{A})\sum_{n}\frac{\sqrt{h_iD_i}}{2} \geq (\frac{\hat{A}}{A})\frac{\sqrt{\sum_n D_ih_i}}{2} \quad (12)$$

Equation (12) can be proved as the following: the first reason, ordering emission relays on the total cost of both centralized and decentralized systems. Moreover, as mentioned earlier, it was proven that the total cost in centralized is always smaller than the total cost in decentralized supply chin. Second, since the ratio of ordering emission $(\frac{\hat{A}}{A})$ is constant for both models and both A and $\hat{A}$ ≥0, the ratio can never be negative. Therefore, it is clear that cooperation can lead to a reduction in the ordering emissions.

The challenge will be on total holding emission part. The variables are not equal in both systems and it will be hard to spot the gap as it was on ordering emission part. Since the variables are not equal and hard to be simplified, both equations will be compared as following:

Decentralized formula: Centralized formula:

$$\sum \frac{C_i^d}{2}\frac{\hat{h}_i}{h_i} \quad ? \quad \frac{C_c^*}{2}\frac{\sum \hat{h}_i D_i}{\sum h_i D_i}$$

$$\sum \frac{\sqrt{2Ah_iD_i}}{2}\frac{\hat{h}_i}{h_i} \quad ? \quad \frac{\sqrt{2A\sum h_iD_i}}{2}\frac{\sum \hat{h}_i D_i}{\sum h_i D_i}$$

The ordering cost is constant and can be omitted; and in order to have an equivalent equation, the decentralized function will be multiplied by $\frac{D_i}{D_i}$ .The questions can be equivalent to:

$$\sum \sqrt{h_iD_i}\frac{\hat{h}_iD_i}{h_iD_i} \quad ? \quad \sqrt{\sum h_iD_i}\frac{\sum \hat{h}_i D_i}{\sum h_i D_i}$$

$$\sum \frac{\hat{h}_iD_i}{\sqrt{h_iD_i}} \quad ? \quad \frac{\sum \hat{h}_i D_i}{\sqrt{\sum h_i D_i}}$$

$$\sum \sqrt{\frac{(\hat{h}_iD_i)^2}{h_iD_i}} \quad ? \quad \sqrt{\frac{(\sum \hat{h}_iD_i)^2}{\sum h_iD_i}} \tag{13}$$

Based on the above comparison, it is shown that holding emission under centralized system is less than holding emission in decentralized system. That is because the square root of a summation is smaller than the summation of a square root when both equations have identical variables. Since it was proved that holding emission is also less in centralized system and it was showed before that the ordering emission is less in the centralized system as well, then it is approved that carbon emissions under centralized system is always less and this proves the proposition.

To show the effect of consolidation on carbon emissions, the delta change of carbon emissions will be compared to the delta change in cost. The comparison can show by how much carbon emissions can be affected by the change on the total costs. The delta change for both carbon emissions and cost can be found as the following:

Delta carbon emissions:

$$\Delta E = \frac{EM_d - EM_c}{EM_d}$$

Delta Cost:

$$\Delta C = \frac{C_d^* - C_C^*}{C_d^*}$$

The above delta change in both carbon emissions and total cost will be compared based on several variables that has an effect on the systems. First, both deltas will be compared and calculated based on the change of the ordering cost and ordering emissions. Then, they will be compared based on the holding emissions and holding costs.

First, the effect of ordering cost and ordering emissions will be measures. From graph 2 which represents the effect of changing ordering cost on both the delta change in carbon emissions and delta change in cost, it shows that the variable will only have an effect on the delta change in carbon emissions. The reason will be because in the delta change in cost, the ordering cost is constant and can be omitted from the delta function as it is presented below:

$$\Delta C = \frac{C_d^* - C_C^*}{C_d^*}$$

$$\Delta C = 1 - \frac{\sum \sqrt{2Ah_i D_i}}{\sqrt{2A \sum h_i D_i}}$$

$$\Delta C = 1 - \frac{\sum \sqrt{h_i D_i}}{\sqrt{\sum h_i D_i}}$$

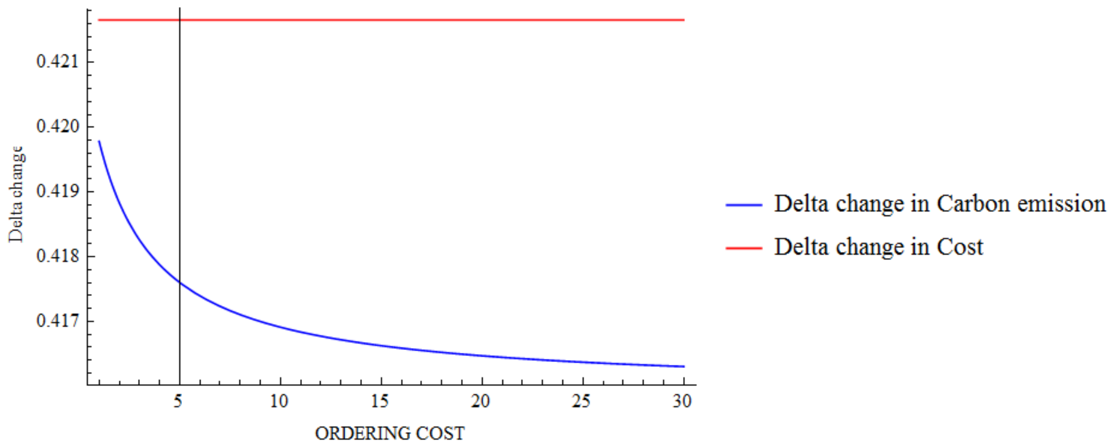


Figure 2. Delta Changes of carbon emissions and costs relative to ordering cost

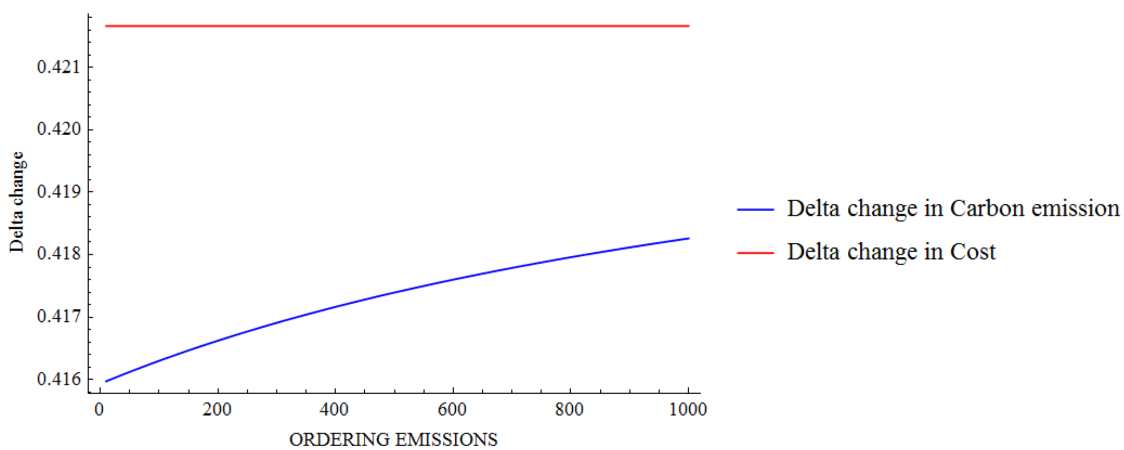


Figure 3. Delta Changes of carbon emissions and costs relative to ordering emissions

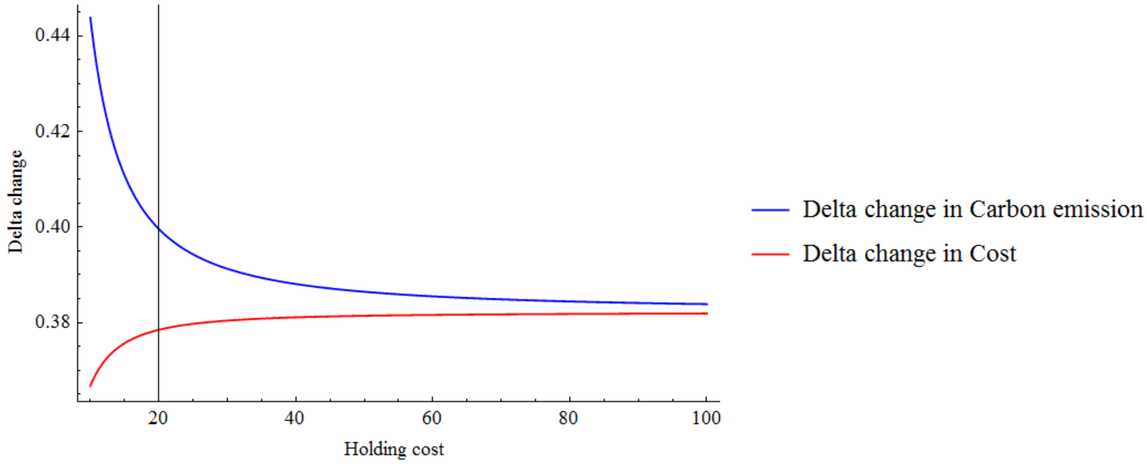


Figure 4. Delta Changes of carbon emissions and costs relative to holding cost

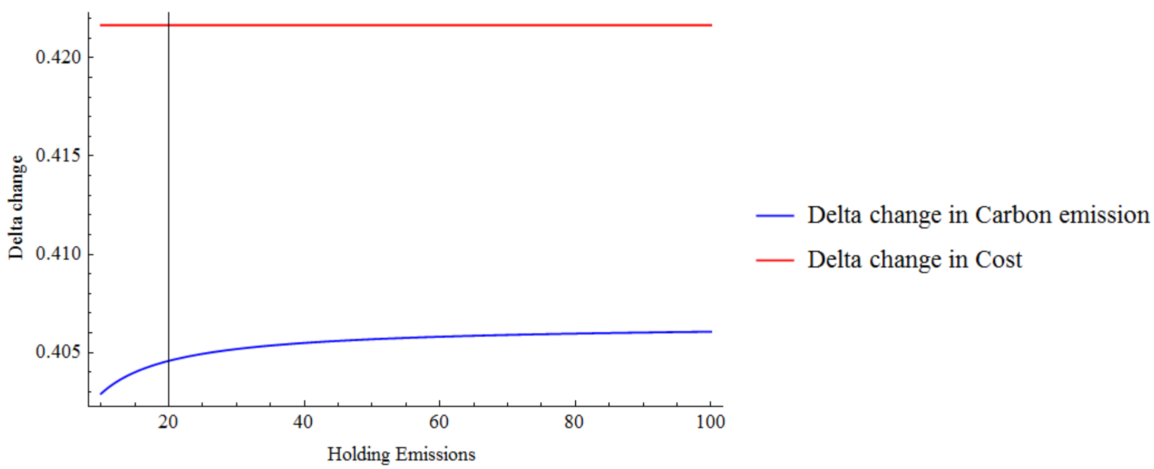


Figure 5. Delta Changes of carbon emissions and costs relative to holding emissions

On the other hands, the larger the value of the ordering cost will be, the lower the delta change on carbon emissions will be until it will reach a flat status. As for the ordering emissions variable shown in graph 3, this variable as well will only have an effect on the delta change in carbon emissions but not the delta change in cost because the variable is not part of the cost model.

For the holding emissions and holding costs, it is assumed that the gap between the firms is fixed and won't change with the increment or decrement of the value of the holding cost or emissions. Having this, when comparing both deltas with regards to holding cost and holding emissions as presented in graph 4 and 5 respectively. It is shown that holding cost will have an effect on both delta change in carbon emissions and costs. The larger the value will be, the greater the delta change in cost will be and the lower the delta change in carbon emission will be. As for the holding emission, it will have an effect on carbon emissions only that is because holding emission is not a variable in the cost model. The larger the holding emissions will get the greater the delta change on carbon emissions will be. The delta change can be almost flat after a certain value.

The above proportion and the delta change suggests that there will be a saving from the total carbon emission if firms decided to joint their resources. If two or more firms consolidated their shipment, then they can do better than having each one of them working individually. To that end, how to allocate the carbon emissions saving among firms; this will be discussed on the next chapter. In the following section, a numerical example of the provided equations found in this section will be presented.

4.4 Numerical Example

A numerical example can be provided to show that the designed model can actually lead to a reduction in carbon emissions. The provided example will be a simple one for clarification purpose only where three retailers will be considered and these retailers are joining their shipment and ordering one type of an item from one supplier. The demand of each retail varies from one to another. The holding costs are not equal as well due to different storage facilities. The ordering cost is the same since it is a fixed cost to be paid to the supplier (different ordering costs based on the location of the retail and trip rout will not be assumed). The results will show the effect on frequencies, total costs and total emissions in both systems. Moreover, the delta change in emission and cost will be presented and compared.

Table 1.Numerical example for centralized and decentralized carbon emissions and cost

Retailer	D_i	h_i	A	$\hat{A}$	$\hat{h}_i$	E_i	E_n	C_i	C_n
1	100	6	20	30	20	374.39	-	154.92	-
2	800	1	20	30	10	1,028.591	-	178.89	-
3	700	1	20	30	30	2635.48	-	167.33	-
		Total				**4,038.46**	**2,356.57**	**501.14**	**289.83**

In table 1, the various values of each variable are presented. The numbers that are provided are random values selected based on previous literature researches. After Computing the total carbon emissions based on the provided variables and the total cost for each firm working individually, it is found that firm 1 emits 374.39 and pays $154.92, firm2 emits 1,028.591 and pays $178.89 and firms 3 emits 2,635.48 and pays $167.33. The total carbon emissions of the decentralized system will be 4,038.46. If firms were to work together, the total carbon emissions of the supply chain will be 2,356.57 to which it is a considerable save. The total cost in the decentralized system is $501.14 and when firms working jointly, the total cost will be reduced to $289.83. The numerical example provided in table 1 showed that cooperation can assist firms to reduce cost along with carbon emissions.

Based on the above calculations and numbers, the delta change in carbon emissions can be calculated and compared based on the provided example.

$$\Delta E = \frac{EM_d - EM_c}{EM_d} = \frac{4{,}038.46 - 2{,}356.57}{4{,}038.46} = 0.416 \quad (14)$$

$$\Delta C = \frac{C_d^* - C_C^*}{C_d^*} = \frac{501.14 \;\; 289.83}{501.14} = 0.422 \quad (15)$$

The delta change in carbon emissions and cost did not differ much. Cooperation had an effect on supply chain by reducing carbon emissions by 41.6% and reducing cost by 42.2%. The delta change for both cost and carbon emissions where almost within the same range of value.

CHAPTER 5. CARBON EMISSION ALLOCATION

This chapter is going to show how to allocate the saving from carbon emission to the participated parties. In section 5.1, a core allocation model will be presented in order to allocate the achieved savings based on the grand allocation properties. Following the model, numerical example will be applied in section 5.2.

5.1 Core Allocation of Carbon Emission

After finding that cooperation can lead to a reduction in carbon emissions, the challenge may lay on allocating this saving among the firms that participated in the shipment consolidation. Allocation should be based on fairness where each firm should find an incentive to join. Fairness can be a relative matter, but it can somehow be calculated so that each firm benefits based on its contribution to the game. Even though it was showed that consolidation can be a cost effective and reduces emission, it does not mean that all firms participating in the consolidation can have a great payoff. Some firms can be doing better by working individually, while others can be better off working with a smaller set rather than the whole group. This theory is called "Game theory." As it was discussed in the literature review, the Game should assure fairness and stability to which there should be no firm or set of firms to break out of the game and start to work individually or to form another set where they can be better off.

A grand coalition in the game should consist N players, where N = 1, 2, ..., n. There should be a coalition S, where $S \subset N$,and S = 1, 2, ..., s. The number of coalitions that can be established from the grand coalition should reach 2n − 1. The Game can have some characteristics in which it can be supper-additive, if for all disjoint sets S,T we have:

$$E(S) + E(T) \geq E(S \cup T) \quad (16)$$

This means that every two subsets can do better if they consolidated their resources. The larger the coalition is, the larger the savings will be. The game is denoted by (N,v) and an allocation is a value that divides the saving X = x_1, x_2, ..., x_i where each player receives X_i value and the core has the following properties:

- **Individual rationality:** the assigned amount of carbon emissions emitted for firm i should be less than or at most equal to the total carbon emissions emitted by the firm working individually.

$$X_i \leq v(i)(\forall i) \quad (17)$$

Where x_1 is the emission assigned to firm i and V(i) is the amount firm i emits in standalone case.

- **Collective Rationality:** the cost should be allocated in a way to which no group would like to leave and that each subset of players should not do better than the grand coalition.

$$\sum_{i \in S} Xi \leq v(S)(\forall S \subset N) \quad (18)$$

- **Efficiency:** To divide all cost among players in the grand coalitions.

$$\sum_{i \in N} Xi = V(N) \quad (19)$$

To facilitate finding a core allocation to the model, the above properties has to be followed to assure fairness to the game. Fairness means that each player is satisfied with the allocation and the amount he should pay based on his contribution to the game. The carbon emission model that is going to be used to which it was donated as (N,E) is presented as the following:

$$EM_c = \frac{\sqrt{2AHD}}{2}(\frac{\hat{A}}{A} + \frac{\sum_i^n \hat{h}_i D_i}{\sum_i^n h_i D_i} \tag{20}$$

First, the thesis is going to prove that the proposed model is supper-additive for all S ⊂ N to which E(S) + E(T) ≥ E(S ∪ T), for all S, T ⊂ N and S ∩ T = φ.

Proposition 2. *The emission function is supper-additive function for all S ⊂ N to which*

E(S) + E(T) ≥ E(S ∪ T), for all S, T ⊂ N and S ∩ T = φ.

Proof: There is a subset S and T where they are non-empty and disjoint:

$$\mathrm{E(S) + E(T)} = \frac{C(s)}{2}\left(\frac{\hat{A}}{A} + \frac{\sum \hat{h}_i D_i}{\sum h_i D_i}\right) + \frac{C(T)}{2}\left(\frac{\hat{A}}{A} + \frac{\sum \hat{h}_i D_i}{\sum h_i D_i}\right)$$

$$= \frac{\sqrt{2A}}{2}\sqrt{\sum_S h_i D_i}\left(\frac{\hat{A}}{A} + \frac{\sum_S \hat{h}_i D_i}{\sum_S h_i D_i}\right) + \frac{\sqrt{2A}}{2}\sqrt{\sum_T h_i D_i}\left(\frac{\hat{A}}{A} + \frac{\sum_T \hat{h}_i D_i}{\sum_T h_i D_i}\right)$$

$$= \frac{\sqrt{2A}}{2}\left(\sqrt{\textstyle\sum_S h_i D_i} + \sqrt{\textstyle\sum_T h_i D_i}\right)((\frac{\hat{A}}{A}) + (\frac{\sum_S \hat{h}_i D_i}{\sum_S h_i D_i} + \frac{\sum_T \hat{h}_i D_i}{\sum_T h_i D_i}))$$

$$\geq \frac{\sqrt{2A}}{2}\sqrt{\textstyle\sum_{S\cup T} h_i D_i}\,(\frac{\hat{A}}{A}) + (\frac{\sum_{S\cup T} \hat{h}_i D_i}{\sum_{S\cup T} h_i D_i}) = E(S \cup T) \tag{21}$$

The proposition implies that the provided model is indeed supper-additive which means that any two or more parties can do better off if they collide with a bigger group than working by themselves. Any player can benefit by joining a larger group than working individually or with a smaller group. The bigger the group is, the better the payoff will be.

By considering that the carbon emission model is the base to have a fair distribution and by following the core allocation rules, a model to distribute the carbon emissions among all parties can be formulated. An interesting solution for this game can be proportional allocation. This proportional allocation to be divided into two parts. The first part will be a proportion allocation of the ordering emission and the second part will be a proportional allocation of the holding emission. Since both parts are joint with an addition sign, then it will be easy to split the model to find a core allocation. The allocation, X, will be the summation of all the distributed cost to all firms resulted from the composition function.

Before trying to find a core allocation function, the carbon emissions function will be simplified. The cost function can also be expressed as 2afn. For simplicity, the carbon emissions function is going be expressed by placing the frequency instead of the cost variable. The above mentioned expression can be replaced for the ordering emission part, while for the holding emission part the cost can be replaced by $\frac{\sum h_i D_i}{f_n}$.

Dividing the model into two parts, for the ordering emissions part $\frac{C_n}{2}\left(\frac{\hat{A}}{A}\right)$, Cn is going to be replaced with 2Afn. By doing this, the ordering emission function can be simplified to be $\hat{A}$fn. It is clear that the ordering emission part is affected by the frequencies to which a proportional allocation for the ordering emission can be, distributing the carbon emissions based on the optimal number of orders (frequency) that had been requested by each party. The allocated function of the ordering emission that should be handled by each party will be as the following:

$$X_i^o = \frac{f_i^2}{f_n^2}\,\hat{A}f_n \qquad (22)$$

Given that $\sqrt{\sum_n f_j^2}$ equals to fn, it can be used as the base of the proportional allocation to prove that the function is a core allocation to the ordering emission. The proof is presented below:

Proposition 3. *The* $X_i^o = (x_1^o, ..., x_n^o) \in \mathbb{R}^n$ *assigned to each retailer i is a core allocation for N players.*

Proof: For All i ∈ N, $X_i = \frac{f_i^2}{f_n^2}\,\hat{A}f_n$ in which it satisfies the following:

1. Individual rationality: for every i$\in N$, $N_N \geq N$. Then $X_o = \frac{f_i^2}{f_n^2}\,\hat{A}f_n = \frac{\sqrt{f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n = \frac{f_i}{f_n}\,\hat{A}f_n = \hat{A}f_i \leq \hat{A}f_i$ = E(i)

2. Efficiency: X(N) = $\sum_{i\in N} X_i^o = \frac{\sum_n f_i^2}{f_n^2}\,\hat{A}f_n = \frac{\sqrt{\sum_n f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n = \hat{A}f_n$ = E(n)

3. Collective rationality: For All $\phi \subset S \subseteq N$, $X_S^o = \sum_{i\in S} X_i^o = \frac{\sum_S f_i^2}{f_n^2}\,\hat{A}f_n = \frac{\sum_S f_i^2}{f_n}\,\hat{A} = \frac{\sqrt{\sum_S f_i^2}}{\sqrt{f_n}}\,\hat{A} \leq \hat{A}f_S = E(S)$

The above is a proof that the first part is indeed a core allocation to the ordering emission of this model. For the second part of the allocation the same concept will be followed by replacing the cost function with $\frac{\sum h_i D_i}{f_n}$. The purpose of the replacement is to reduce the number of variables within the equation in order to find a core allocation. The holding function will now be simplified to $\frac{\sum h_i D_i}{f_n}$ $(\frac{\sum \hat{h}_i D_i}{\sum h_i D_i})$ and to be simplified further to become $(\frac{\sum \hat{h}_i D_i}{f_n})$. After re-expressing the holding emission part, it is clear that the part will be affected by the holding emission amount to which it will be easier to proportionally distribute the emission based on the holding emission value for each firm. A core allocation of the holding emission part can be expressed as:

$$X_i^h = \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i} \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right)$$

It can be proved that this holding emission allocation is a core allocation for this part by showing the following:

Proposition 4. *The* $X_i^h = (x_1^h, ..., x_n^h) \in \mathbb{R}^n$ *assigned to each retailer i is a core allocation for N players.*

Proof: For All i ∈ N, $X_i^h = \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i} (\frac{\sum_n \hat{h}_i D_i}{2f_n})$ in which it satisfies the following:

1. Individual rationality: for every $i \in N$, $N_N \geq N$. Then $x_i = \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i} (\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \frac{\hat{h}_i D_i}{2f_n} \leq \frac{\hat{h}_i D_i}{2f_i} = E(i)$.
2. Efficiency: X(N) = $\sum_{i \in N} X_i^h = \sum_n \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i} (\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = E(n)$.
3. Collective rationality: For All $\phi \subset S \subseteq N$, $X_s^h = \sum_{i \in S} X_i^h = \sum_s \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i} (\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \left(\frac{\sum_s \hat{h}_i D_i}{2f_n}\right) \leq \left(\frac{\sum_s \hat{h}_i D_i}{2f_s}\right) = E(s)$

After finding and proving that both parts are a core allocation to the model, both part will be joint to formulate the total carbon emission amount to be paid for each player i within the game:

$$X_i = \frac{f_i^2}{f_n^2}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n}) \tag{24}$$

Since Xi is the summation on the ordering and holding part, then the proof that it is a core allocation will follow the same principles used above. The above function X will be the amount of carbon emissions each firm will be charged for if they consolidated their shipment. Proposition 5 will show that equation 24 is a core allocation of the system.

Proposition 5. *The X = (X_i, ..., X_n) $\in \mathbb{R}^n$ assigned to each retailer i is a core allocation for N players.*

Proof: For All i ∈ N, $x_i = \frac{f_i^2}{f_n^2}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n})$ in which it satisfies the following:

1. Individual rationality: for every $i \in N, N_N \geq N$. Then $X_i^h = \frac{f_i^2}{f_n^2}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \frac{\sqrt{f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{2f_n} = \frac{f_i}{f_n}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{2f_n} = \hat{A}f_i + \frac{\hat{h}_i D_i}{2f_n} \leq \hat{A}f_i + \frac{\hat{h}_i D_i}{2f_i} = E(i)$.

2. Efficiency: X(N) $= \sum_{i\in N} X_i^h = \frac{\sum_n f_i^2}{f_n^2}\,\hat{A}f_n + \sum_n \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \frac{\sqrt{\sum_n f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n + \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = \hat{A}f_n + \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = E(n)$.

3. Collective rationality: For All $\phi \subset S \subseteq N$, $X_s^h = \sum_{i\in S} X_i^h = \frac{\sum_s f_i^2}{f_n^2}\,\hat{A}f_n + \sum_s \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \frac{\sum_s f_i^2}{f_n}\,\hat{A} + \left(\frac{\sum_s \hat{h}_i D_i}{2f_n}\right) = \frac{\sqrt{\sum_s f_i^2}}{\sqrt{f_n}}\,\hat{A} + \left(\frac{\sum_s \hat{h}_i D_i}{2f_n}\right) \leq \hat{A}f_s + \left(\frac{\sum_s \hat{h}_i D_i}{2f_s}\right) = E(s)$

The above proposition is a proof that the proposed emission allocation model is a core allocation to the system, where its individual rationality, efficiency and collective rationality was proved. No individuals nor subsets would leave the coalition to form another coalition where it will do a better off than the grand coalition. All firms will be paying less if they joint the grand coalition than when working individually or in a sub coalition. All parties in this group will benefit from reducing the total cost of the supply chain and carbon emissions as well.

5.2 Numerical Example

Based on a core allocation model presented in the previous section, a numerical example will be presented to show that the above model is a core allocation and follows all properties. The example and the provided values presented in table (2) are the same as the previously introduced example in section 4.4. The example is going to have three companies that consolidated their shipment to reduce the overall supply chain cost and got the total carbon emissions reduced as well, and they want to divide the achieved savings from carbon emissions fairly.

The ordering cost and ordering emission are fixed values among all parties for every time an order is placed. Each firm will have a different demand, and each firm will have different holding cost and holding emission since the warehouses are not shared. As it was assumed, there will be one item that is going to be ordered and it is going to be ordered from one supplier.

Table 2.Variables of the consolidated system

Retailer	D_i	h_i	**A**	$\hat{A}$	$\hat{h}_i$
1	100	6	20	30	20
2	800	1	20	30	10
3	700	1	20	30	30

Table 3. Carbon emissions amount emitted for the possible coalitions

E(1)	374.3884	**E(2)**	1028.591
E(3)	2635.479	**E(1,2)**	1022.637
E(1,3)	2188.26	**E(2,3)**	2551.552
E(1,2,3)	2356.574		

Table 3 shows the value of carbon emissions for all the possible coalitions that can be formed along with the grand coalition. Based on these numbers, if each firm worked on its own, then the total carbon emissions will be as the following: E1 = 374.3884 , E2 = 1028.591 and E3= 2635.479 with each having a frequency (f= d/Q) of 3.9, 4.5 and 4.2 orders/year respectively. If firms started to work in a group and join their shipment, then the carbon emission will be reduced. From the numerical example, it is noticed that the larger the group will be, the greater the savings will be.

Having the total carbon emissions in the grand coalition, the total carbon emission can be allocated to all firms within the coalition fairly and based on the proposed model. In the ground coalitions, the total frequency of orders will be 7.25 orders/years. The proportional allocation of the carbon emissions of the firms E1,E2, and E3 for each individual and based on the core allocation will be as the following: X1 = 200.119 , X2 = 634.8603, X3 = 1521.595.

From the provided numbers, the difference and the savings from both system is noticeable. The total frequencies was reduced from 12.6 orders/ year in the decentralized system to 7.25 order/ year for the centralized system. Along with that, each firm benefited from the cooperation that the amount of carbon emissions they are entitled to reduce significantly. Moreover, each firm will have to pay a cost of C1=$89.59608, C2= $103.4566, and C3=$96.77482 where the used to pay C1= $154.92, C2= $178.89 and C3=$167.33 in the decentralized system.

The numerical example showed that cooperation can help in reducing the total frequencies of ordering. And thus, the total carbon emissions can be reduced for the whole supply chain and individuals within the cooperation. In addition, the total cost will also be saved for the supply chain as a whole and individuals.

CHAPTER 6. CONSOLIDATION UNDER CARBON TAX REGULATION.

This chapter will demonstrate how the model can be extended to handle some of the carbon emissions regulations, and how the regulations will have an effect on carbon emissions. Section 6.1 presents the formulation of carbon emission model under tax regulation. Section 6.2 shows the findings and the savings from both systems. Core allocation of the savings will be found in section 6.3.

6.1 Model formulation

Regulations on supply chain can play a major role with regards to the cost and ordering quantity and the effect of these changes on carbon emissions. The chapter will measure the effect of carbon emission under consolidation with the presence of tax regulation. However, the incentive of the cooperation will be reducing the total cost of the system. Carbon tax is one type of regularity policy where the government imposes a financial penalty per carbon unit that had been emitted by the firm. Imposing this kind of penalty can help in reducing and controlling carbon emissions throughout the supply chain. There are several types of carbon taxes that can be imposed on firms in which depends on the structure of the supply chain or the business. The most used types of carbon taxes are the fixed tax rate which is imposing penalty for every unit of carbon emissions that had been emitted. Linear tax rate is imposing a rate to all tax players and it can increase based on the quantity. A nonlinear tax rate is where the tax rate changes with regards to carbon quantity.

In this section of the research, carbon tax will be added into the model and study its effect on the fixed tax value. All assumptions that were introduced in section 3.1 are going to be applicable in this model. The tax rate will be a fixed value that is given and known. For simplicity purpose, the tax value will be applied to every carbon that had been emitted from the supply chain.

Since tax rate is a form of a money value, it can be considered as part of the cost equation. The cost function can be enhanced in both centralized and decentralized systems to accommodate the tax cost value associated with the carbon emissions that had been emitted within the supply chain. Let't' donates the tax rate that is going to be paid for every unit emitted and t > 0. Then the new total cost function for firms under decentralization and centralization will be as the following:

$$\sum_{i}^{n} T_i^d = \sum_{i}^{n} (A\frac{D_i}{Q_i} + h_i\frac{Q_i}{2} + tE_i^d) \tag{25}$$

Total cost function for firms under centralization:

$$T_c^* = Af + \frac{1}{f}\sum_{i\subseteq N}\frac{h_i D_i}{2} + tE_{\Pi}^c \tag{26}$$

If t = 0, then no changes will be applied to the cost in both cases and carbon emissions can be neglected by the firms. If t > 0, then the firm will have 2 options. Option 1 will be to neglect the increase in cost especially if the increase was not significant and pay the penalty. Neglecting the cost, means that the firm will have the regular EOQ model and will be placing orders based on the ordering quantity that will reduce the total cost and then add to it the extra fees for the tax. Option 2 will be to reduce carbon emissions from the processes that emits carbon emissions the most and are accountable for the increase in the cost. The reduction of carbon emissions can be in terms of investing on tools that will reduce carbon emissions, or by simply adjusting the operation or the ordering quantity and frequency to adjust the added carbon tax cost. Both cost equations in both systems depend on ordering quantity Q which will affect the total cost. The firms can adjust the optimum ordering quantity Q∗ to which it minimizes the new total cost T c and T d.

As for Q∗ under decentralization, the formula was calculated and according to previous work stated on chapter 2, $Q = \sqrt{\frac{2(A+t\hat{A})D_i}{h_i+t\hat{h}_i}}$

As for centralization, the new Q_t^* will be $Q_t^* = D_i \sqrt{\frac{2(A+t\hat{A})}{\sum h_i D_i + t \sum \hat{h}_i D_i}}$ and was driven according to the following:

$$T_c^* = Af + \frac{1}{f}\sum_{i\subseteq N}\frac{h_i D_i}{2} + tE_\Pi^c$$

$$T_c^* = A\frac{D_i}{Q_i} + \frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{h_i D_i}{2} + t(\hat{A}\frac{D_i}{Q_i} + \frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{\hat{h}_i D_i}{2})$$

$$T_c^* = A\frac{D_i}{Q_i} + \frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{h_i D_i}{2} + t\,\hat{A}\frac{D_i}{Q_i} + t\frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{\hat{h}_i D_i}{2}$$

In order to find Q_t^*, the equation to be derived with regards to Q.

$$\frac{d(T)}{dQ} = A\frac{D_i}{Q_i} + \frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{h_i D_i}{2} + t\,\hat{A}\frac{D_i}{Q_i} + t\frac{Q_i}{D_i}\sum_{i\subseteq N}\frac{\hat{h}_i D_i}{2}$$

$$\frac{d(T)}{dQ} = -A\frac{D_i}{{Q_i}^2} + \frac{1}{D_i}\sum_{i\subseteq N}\frac{h_i D_i}{2} - t\,\hat{A}\frac{D_i}{{Q_i}^2} + t\frac{1}{D_i}\sum_{i\subseteq N}\frac{\hat{h}_i D_i}{2}$$

$$-A\frac{D_i}{{Q_i}^2} - t\hat{A}\frac{D_i}{{Q_i}^2} = -\frac{1}{D_i}\sum_{i\subseteq N}\frac{h_i D_i}{2} - t\frac{1}{D_i}\sum_{i\subseteq N}\frac{\hat{h}_i D_i}{2}$$

$$-\frac{D_i}{{Q_i}^2}(A + t\hat{A}) = -\frac{1}{2D_i}(\sum_{i\subseteq N} h_i D_i - t\sum_{i\subseteq N}\hat{h}_i D_i)$$

$$-D_i(A + t\hat{A})2D_i = -{Q_i}^2(\sum_{i\subseteq N} h_i D_i - t\sum_{i\subseteq N}\hat{h}_i D_i)$$

$$\frac{2{D_i}^2(A + t\hat{A})}{(\sum_{i\subseteq N} h_i D_i - t\sum_{i\subseteq N}\hat{h}_i D_i)} = {Q_i}^2$$

$$Q_t^* = D_i\sqrt{\frac{2(A + t\hat{A})}{\sum h_i D_i + t\sum \hat{h}_i D_i}}$$

The total optimal Q_t^*will be $\sum Q_i^t$.

From the newly funded optimum quantity, the value will be affected by the ordering and holding emissions along with the ordering and holding costs. The change in the emissions variables will have an effect on the ordering quantity and the total cost. Moreover, the carbon tax rate will also play a role in deciding the number of quantities to be replenished within each order.

Using the above Q value, the cost equation can be simplified. For the decentralized system, the equation will be $T_i^{\mathrm{d}} = \sqrt{2(A + t\hat{A})D_i(h_i + t\,\hat{h}_i)}$. For more simplification, the equation will have $\widetilde{\mathrm{A}}$ variable to present the formula of $2(A + t\hat{A})$ and $\widetilde{H_i}$ variable to present the formula of $(h_i + t\,\hat{h}_i)$. The new cost for the decentralized system will be:

$$\sum_{n} T_i^d = \sum_{n} \sqrt{2\tilde{A}D_i\widetilde{H}_i} \tag{27}$$

Same concept will be followed for the centralized system. The cost equation can be presented as $T_C^* = \sqrt{2(A + t\hat{A})(\sum_{i\subseteq N} h_i D_i - t\sum_{i\subseteq N} \hat{h}_i D_i)}$. The equation can be simplified by the presentation of $\widetilde{A}$. In the centralized system, $\overline{H_\iota}$ will be used to express the formula$(\sum_{i\subseteq N} h_i D_i - t\sum_{i\subseteq N} \hat{h}_i D_i)$. The cost equation will have the following form

$$T_C^* = \sqrt{2\tilde{A}\overline{H_\iota}} \tag{28}$$

Carbon emissions model for both systems can be simply found after using the above cost and ordering quantity, Q. Since a new Q that is associated with tax rate value will be used to calculate the cost, carbon emissions equation won't be that same as the previous section; it has to change depending on the new ordering quantity value. The new carbon emissions equation associated with tax can be as the following:

For the decentralized system:

$$\sum E_t^d = \left(\frac{\hat{A}}{\tilde{A}}\right)\sum \frac{T_i^d}{2} + \sum \frac{\hat{h}_i}{\widetilde{\overline{H_\iota}}}\,\frac{T_i^d}{2} \tag{29}$$

The total emission for the centralized system:

$$E_t^c = \left(\frac{\hat{A}}{\tilde{A}}\right)\frac{T_c^*}{2} + \frac{T_c^*}{2}\,\frac{\sum \hat{h}_i D_i}{\overline{H_\iota}} \tag{30}$$

Equation (29) was derived as the following:

$$E_d = \sum_{n} (\hat{A}\,\frac{D_i}{Q_i} + \hat{h}_i\,\frac{Q_i}{2})$$

$$E_d = \sum_n (\hat{A}\frac{D_i}{\sqrt{\frac{2(A+t\hat{A})D_i}{h_i+t\hat{h}_i}}} + \frac{\hat{h}_i}{2}\sqrt{\frac{2(A+t\hat{A})D_i}{h_i+t\hat{h}_i}}\ )$$

$$E_d = \sum_n (\hat{A}\frac{D_i\sqrt{h_i+t\hat{h}_i}}{\sqrt{2(A+t\hat{A})D_i}} + \frac{\hat{h}_i}{2}\sqrt{\frac{2(A+t\hat{A})D_i}{h_i+t\hat{h}_i}}\ )$$

$$E_d = \sum_n (\hat{A}\frac{D_i\sqrt{2(A+t\hat{A})D_i(h_i+t\hat{h}_i)}}{2(A+t\hat{A})D_i}$$

$$+\frac{\hat{h}_i}{2}\frac{\sqrt{2(A+t\hat{A})D_i\ (h_i+t\hat{h}_i)}}{h_i+t\hat{h}_i}\ )$$

$$E_d = \sum_n (\frac{\hat{A}}{2(A+t\hat{A})}\ T_i^d + \frac{\hat{h}_i}{2(h_i\ +t\hat{h}_i)}\ T_i^d)$$

As for the centralized system in equation (30), the drive will be the same:

$$E_c = \hat{A}\ \frac{D_i}{Q_i} + \frac{1}{f}\ \frac{\sum_n \hat{h}_i D_i}{2}$$

$$E_c = \hat{A}\ \frac{D_i}{D_i\sqrt{\frac{2(A+t\hat{A})}{\sum h_i D_i + t\sum \hat{h}_i D_i}}} + \frac{D_i\sqrt{\frac{2(A+t\hat{A})}{\sum h_i D_i + t\sum \hat{h}_i D_i}}}{D_i}\ \frac{\sum_n \hat{h}_i D_i}{2}$$

$$E_c = \hat{A}\ \frac{\sqrt{\sum h_i D_i + t\sum \hat{h}_i D_i}}{\sqrt{2(A+t\hat{A})}} + \frac{\sqrt{2(A+t\hat{A})}}{\sqrt{\sum h_i D_i + t\sum \hat{h}_i D_i}}\ \frac{\sum_n \hat{h}_i D_i}{2}$$

$$E_c = \hat{A}\ \frac{\sqrt{2(A+t\hat{A})(\sum h_i D_i + t\sum \hat{h}_i D_i)}}{2(A+t\hat{A})} + \frac{\sqrt{2(A+t\hat{A})(\sum h_i D_i + t\sum \hat{h}_i D_i)}}{\sum h_i D_i + t\sum \hat{h}_i D_i}\ \frac{\sum_n \hat{h}_i D_i}{2}$$

$$E_c = \frac{\hat{A}}{2(A + t\hat{A})} T_c^* + \frac{\sum_n \hat{h}_i D_i}{2(\sum h_i D_i + t \sum \hat{h}_i D_i)} T_c^*$$

Having the formulated carbon emission equation within both systems in 29 and 30, and after simplifying both models, it can be noticed that the structure of both models will have the same structure as the regular model provided in chapter 4. Noticing that, it can be predicted that cooperation can also lead to a reduction in carbon emission under tax regulation as well. This can be shown and proved in the coming section.

6.2 Comparing cost and carbon emissions in both systems

Previously, the total carbon emission model was formulated with regards to carbon tax. Since it was proved in previous chapters that consolidation can reduce both cost and carbon emissions, same result can be found if carbon emission tax rate was also added to the system. This section is going to show if cooperation can help in reducing cost and carbon emissions under carbon tax regulation as well when the incentive of the cooperation is to reduce cost only.

First, the effect on costs in both systems by comparing the total cost equations with taxes will be found. From equations (27) and (28), it can be shown that the total cost in centralized supply chain with tax will always be less than the total cost with tax in decentralized supply chain and it can be proved mathematically. The comparison of both systems will be as the following:

Decentralized formula: Centralized formula:

$$\sum_{n} \sqrt{2(A + t\hat{A})D_i(h_i + t\,\hat{h}_i)} \quad ? \quad \sqrt{2(A + t\hat{A})(\sum_{i \subseteq N} h_i D_i - t \sum_{i \subseteq N} \hat{h}_i D_i)} \tag{31}$$

$$\sum_{n} \sqrt{D_i(h_i + t\,\hat{h}_i)} \quad \geq \quad \sqrt{(\sum_{i \subseteq N} h_i D_i - t \sum_{i \subseteq N} \hat{h}_i D_i)} \tag{32}$$

Based on the above comparison, it is shown that decentralized system will be greater than the centralized system. Since the ordering cost part will be emitted because it is constant, then the comparison will only be on the holding amount. The holding part under centralized system is smaller than holding part in decentralized system. That is because the square root of a summation is smaller than the summation of a square root when both equations have identical variables. Mathematically, this proves that there will be a reducing in cost in a cooperative environment.

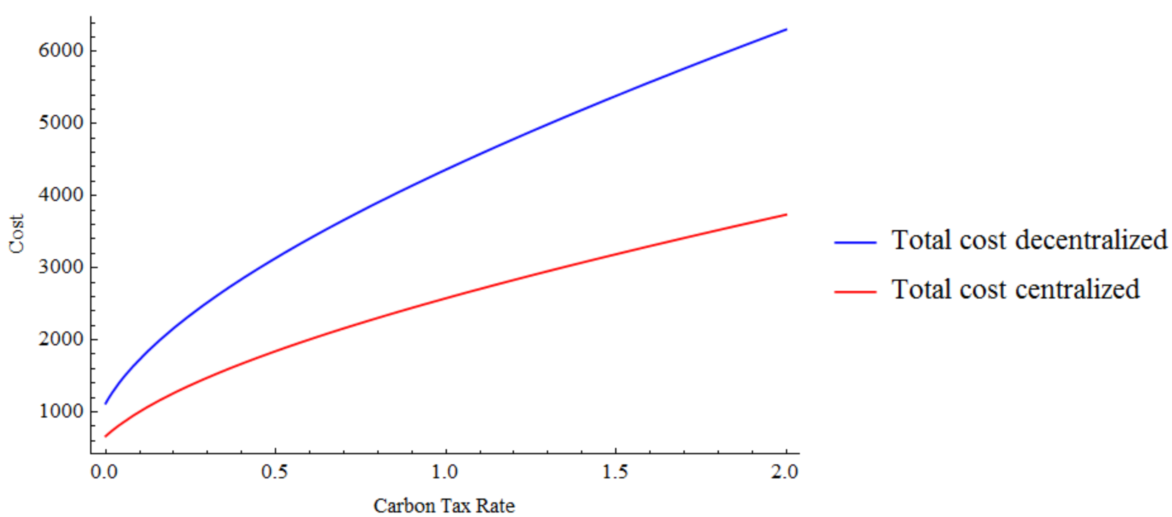


Figure 6. Affect of tax rate on cost function in both systems

Aside from the formula above, the results from figure (6) shows that operational adjustment in the form of joint replenishment can be an advantage to firms if a tax penalty was imposed. By having the cost function to be graphed with relation to the tax rate, it shows that the total cost will keep on increasing linearly with the increment of the tax rate. On the other hand, in centralized system, the total cost will always re- main smaller than the decentralized system even with the incremental of the tax rate and there is a sufficient differences between both modules as well.

Moving to the second part related to saving in carbon emission obtained from the carbon emissions models in both systems, from previous chapters, it was shown that consolidation was able to reduce the amount of carbon emission within the system. Since there is a regulation imposed on supply chain for this part, it will make sense that the carbon emission will be less than previous section. This was proved by several researches in the literature review section. Most papers agreed that tax rate should be set at a reasonable rate in order to get the best result out of it. If the tax rate was set too little, then the emission won't be reduced as much. The same thing can be applied if the tax rate was set too high, then the carbon emission reduction will barely be noticeable and can be avoided.

Cooperation can be used to assist in reducing carbon emission from the supply chain along with costs as well and under carbon tax regulation, and this can be proved theoretically and mathematically. Since the emission under tax regulation has a similar structure to the regular emission function formulated in chapter 4, then it is clear that a reduction can be applied in the ordering emission part. The amount of carbon emissions that are going to be measured for the ordering part depends on the number of frequencies, and the number of frequencies will be reduced in the centralized system. The holding emissions part will be reduced as well since the company are going to hold less than what they did in the decentralized system since the number of items that had been ordered are less.

Following the above theoretical concept, mathematically, it can be shown that carbon emission under tax regulation can be reduced with consolidation even if the firm's main aim is to reduce cost only.

Proposition 6. *Cooperation can always lead to a reduction in carbon emissions under tax regulation.*

$$E_d = \sum_n \left(\frac{\hat{A}}{2\left(A + t\hat{A}\right)} T_i^d + \frac{\hat{h}_i}{2(h_i + t\hat{h}_i)} T_i^d \right) \geq E_c = \frac{\hat{A}}{2(A + t\hat{A})} T_c^* + \frac{\sum_n \hat{h}_i D_i}{2(\sum h_i D_i + t \sum \hat{h}_i D_i)} T_c^* \quad (33)$$

Proof: For the current environment where firms will care about cost rather than carbon emissions, carbon emissions equations in both systems that was introduced earlier, (equations 29 and 30) will be com- pared to find the gap between them. For simplification purpose, both equations will be split into ordering and holding to which each of which will be compared separately. This is possible since both parts are joint by additional sign.

The ordering part of the equation is influenced by the total cost in both models. Based on the previous prove that the total cost will be reduced under cooperation, this finding can be used to prove that the ordering emission will be less in centralized system than the decentralized as the following:

Decentralized formula: Centralized formula:

$$\sum_n \frac{\hat{A}}{2\left(A + t\hat{A}\right)} T_i^d \qquad ? \qquad \frac{\hat{A}}{2(A + t\hat{A})} T_c^*$$

$$\sum_{n} \frac{\hat{A}}{2(A+t\hat{A})} \sqrt{2(A+t\hat{A})D_i(h_i + t\,\hat{h}_i)} \quad ? \quad \frac{\hat{A}}{2(A+t\hat{A})} \sqrt{2(A+t\hat{A})(\sum_{i \subseteq N} h_i D_i - \ t \sum_{i \subseteq N} \hat{h}_i D_i)}$$

The ordering costs can be omitted since it is constant, and the formula will be equivalent to:

$$\sum_{n} \sqrt{D_i(h_i + t\,\hat{h}_i)} \quad \geq \quad \sqrt{(\sum_{i \subseteq N} h_i D_i - \ t \sum_{i \subseteq N} \hat{h}_i D_i)}$$

From the above comparison, mathematically it is proved that the ordering part will be greater in the decentralized system since the ordering emission part is equivalent to the cost comparison which was proved earlier.

After showing that the ordering will always be greater in decentralized system, the holding emission can be proved as well by having the following:

Decentralized formula: Centralized formula:

$$\sum_{n} \frac{\widehat{h}_i}{2(h_i + t\widehat{h}_i)} T_i^d \quad ? \quad \frac{\sum_n \widehat{h}_i D_i}{2(\sum h_i D_i + t \sum \widehat{h}_i D_i)} T_c^*$$

$$\sum_{n} \frac{\widehat{h}_i \sqrt{2(A+t\hat{A})D_i(h_i + t\,\hat{h}_i)}}{2(h_i + t\widehat{h}_i)} \quad ? \quad \frac{\sum_n \widehat{h}_i D_i \sqrt{2(A+t\hat{A})(\sum_{i \subseteq N} h_i D_i - \ t \sum_{i \subseteq N} \hat{h}_i D_i)}}{2(\sum h_i D_i + t \sum \widehat{h}_i D_i)}$$

To which it is equivalent to the below equation after the omitting of the ordering cost as it is constant and adding $\frac{D_i}{D_i}$ ratio to the decentralized system for equivalent purpose:

$$\sum_{n} \frac{\widehat{h}_i \sqrt{2D_i(h_i + t\,\hat{h}_i)}}{2(h_i + t\widehat{h}_i)} \quad ? \quad \frac{\sum_n \widehat{h}_i D_i \sqrt{2(\sum_{i \subseteq N} h_i D_i - \ t \sum_{i \subseteq N} \hat{h}_i D_i)}}{2(\sum h_i D_i + t \sum \widehat{h}_i D_i)}$$

$$\sum_{n} \frac{\widehat{h}_i D_i \sqrt{2D_i(h_i + t\,\hat{h}_i)}}{2D_i(h_i + t\widehat{h}_i)} \quad ? \quad \frac{\sum_n \widehat{h}_i D_i \sqrt{2(\sum_{i \subseteq N} h_i D_i - \ t \sum_{i \subseteq N} \hat{h}_i D_i)}}{2(\sum h_i D_i + t \sum \widehat{h}_i D_i)}$$

$$\sum_{n} \frac{\hat{h}_i D_i}{\sqrt{2D_i(h_i + t\hat{h}_i)}} \quad ? \quad \frac{\sum_n \hat{h}_i D_i}{\sqrt{2(\sum h_i D_i + t \sum \hat{h}_i D_i)}}$$

$$\sum_{n} \sqrt{\frac{(\hat{h}_i D_i)^2}{2D_i(h_i + t\hat{h}_i)}} \quad ? \quad \sqrt{\frac{(\sum_n \hat{h}_i D_i)^2}{2(\sum h_i D_i + t \sum \hat{h}_i D_i)}} \qquad (34)$$

The above equation proved that the holding emission part under centralized system is less than the holding emission part under decentralized system with regards to tax rate. By proving that both ordering and holding parts can lead to a reduction of carbon emissions under cooperation, then the total carbon emissions will be reduced when firms consolidate their shipment. From this finding, the delta change in both total cost and total emissions can be presented to find the effect of the costs and emissions' variable on the systems.

The delta change of the cost function and the carbon emissions functions can be presented below: Delta change in cost:

$$\Delta C = \frac{C_t^d - C_t^c}{C_t^d}$$

$$\Delta C = 1 - \frac{C_t^c}{C_t^d}$$

$$\Delta C = 1 - \frac{\sqrt{2(A + t\hat{A})(\sum_{i \subseteq N} h_i D_i - t \sum_{i \subseteq N} \hat{h}_i D_i)}}{\sum \sqrt{2(A + t\hat{A}) D_i (h_i + t\,\hat{h}_i)}}$$

$$\Delta C = 1 - \frac{\sqrt{(\sum_{i \subseteq N} h_i D_i - t \sum_{i \subseteq N} \hat{h}_i D_i)}}{\sum \sqrt{D_i (h_i + t\,\hat{h}_i)}} \qquad (35)$$

Delta change in carbon emissions:

$$\Delta E = \frac{E_t^d - E_t^c}{E_t^d} \tag{36}$$

The delta change in cost between the two systems depends on the ratio of $\sqrt{(\sum_{i \subseteq N} h_i D_i - t \sum_{i \subseteq N} \hat{h}_i D_i)}$ over $\sum \sqrt{D_i(h_i + t\,\hat{h}_i)}$ since the ordering cost is constant in both systems, it won't have an effect on the change. In other words, neither the ordering cost nor ordering emission will have an effect on the saving of cost that will occur from consolidation. This can be clearly observed from graph 7.

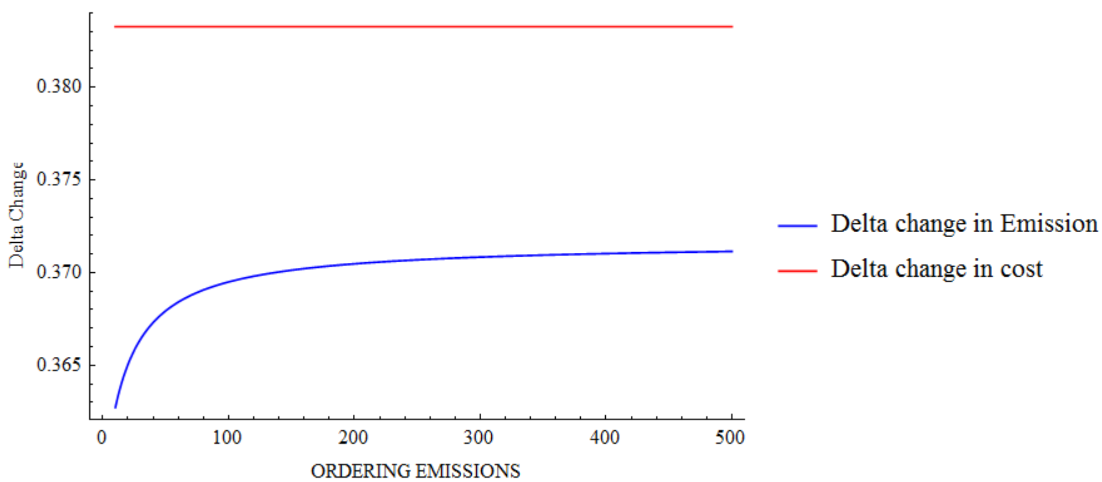


Figure 7. Affect of ordering emissions on the delta change of the cost and carbon emission functions

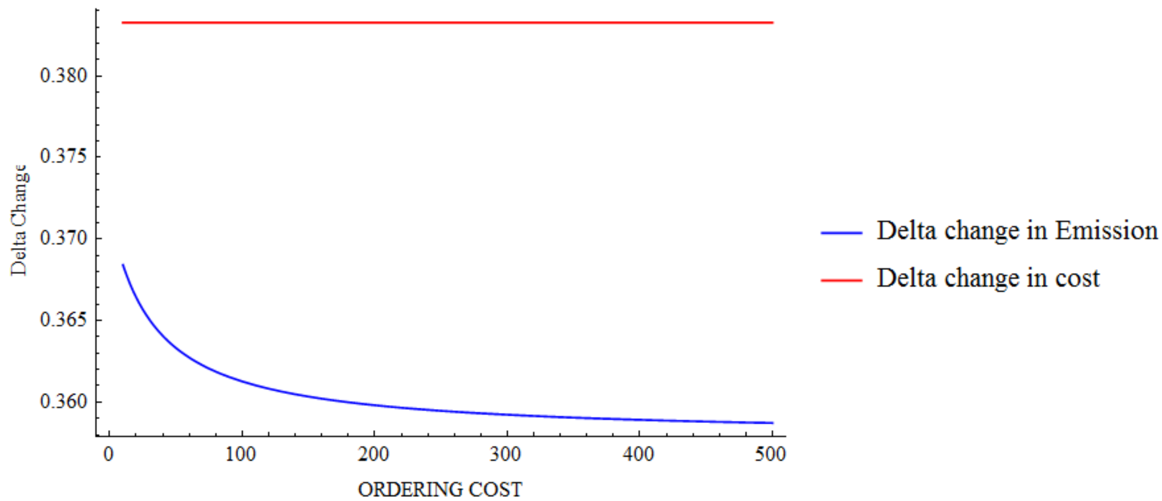


Figure 8. Affect of ordering cost on the delta change of the cost and carbon emission functions

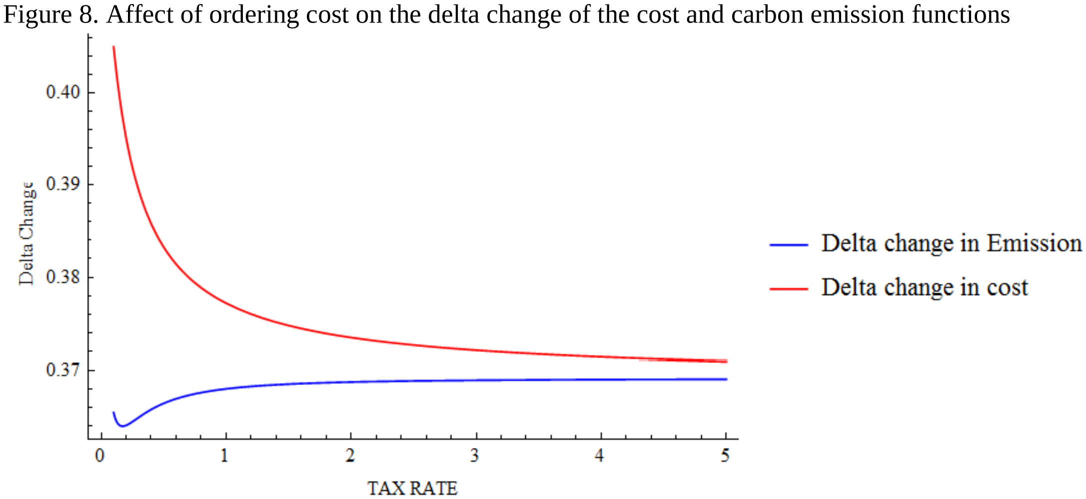


Figure 9. Affect of carbon tax rate on the delta change of the cost and carbon emission functions.

The delta change in carbon emissions and total cost will be compared bases on various variables. Both deltas will be plotted based on the changed in ordering emissions, ordering cost, and tax rate. Graph 7 shows that ordering emission variable can have an effect on the change of carbon emissions but not the change in cost due to having the variable to be omitted from the delta change. The greater the emission value will be, the greater the delta change in emission will be until it reaches a flat status. The same goes for the ordering cost variable in graph 8. The change will be applied to the carbon emissions function only and the change will get on decreasing as the value increases. As for the carbon tax rate, the variable will have an effect on both carbon emissions and cost models. The affect will be the opposite in both models due to the position of the variable in the function. The greater the value is, the delta change in cost will be reduced while the delta change in carbon emissions will be increased. This can be shown in graph 9.

In additions to the graphs, a numerical example can show the amount of carbon emissions that had been saved by consolidating the shipment. The numerical example for this part is a continuation of the previous scenario. In table 2, all the values for three firms were listed of the following variables: Ordering cost and ordering emission, holding cost and holding emission, and the demand. In order to calculate the tax regulation, it was assumed that the government set the carbon tax rate, t, to be $0.3 for every unit of carbon that had been emitted. Having all these values, the total carbon emission for each firm working individually and together will be calculated. The values are randomly generated and considered based on previous researches.

Table 4. Numerical example for centralized and decentralized Emissions under tax regulation

Retailer	E_i	E_n
1	356.3061	-
2	761.3509	-
3	1285.348	-
Total	2403.005	1526.179

A summary of all carbon emission that had been calculated after placing all the values into the emission equation are presented in table 4. Additionally, it can be observed that regulations can play a great roles in reducing the carbon emission unit emitted by each firm within the system. Compared to the previous examples, just by having tax rate imposed into the system, this can have a great impact on carbon emissions even if the incentive is to reduce the total cost. From the results found in table 2 and table 4, it can be seen that carbon emission was reduced in the decentralized system from 4,035.46 to 2403.005; and in the centralized system, it was reduced from 2.356.57 to 1526.179. This means that with imposing a small tax to the system, carbon emissions can be reduced significantly. It can also be noticed through graph 10.

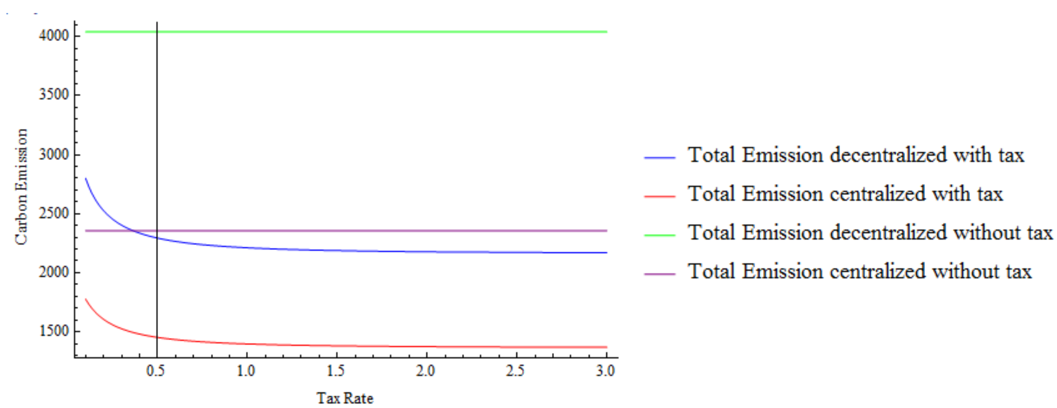


Figure 10. The effect on the total Carbon Emission with and without tax regulation

6.3 Carbon Emission Allocation

In order to find a core allocation formula to distribute the carbon emissions among the parties in the coalitions, the carbon emissions model under tax regulation will be checked if it has the characteristics of a supper-additive coalition. This means that the greater the coalition is, the better the saving will be. The carbon emissions function in the centralized system donated as (N,E) can be presented as the following and to be tested:

$$E_c = \frac{\hat{A}}{2(A + t\hat{A})} T_c^* + \frac{\sum_n \hat{h}_i D_i}{2(\sum h_i D_i + t \sum \hat{h}_i D_i)} T_c^* \quad (37)$$

The emission equation has one of the characteristics that is it is a super-additive function. It means that firms will do better off if all consolidated their shipment instead of having two big groups.

Proposition 7. *The carbon emission function under tax regulation is supper-additive function for all S ⊂ N to which E(S) + E(T) ≥ E(S ∪ T), for all S, T ⊂ N and S ∩ T = φ.*

Proof: There is a subset S and T where they are non-empty and disjoint:

$$E(s) + E(T) = \frac{C(s)}{2}\left(\frac{\hat{A}}{\tilde{A}} + \frac{\sum_s \hat{h}_i D_i}{2(\sum_s h_i D_i + t \sum_s \hat{h}_i D_i)}\right) + \frac{C(T)}{2}\left(\frac{\hat{A}}{\tilde{A}} + \frac{\sum_T \hat{h}_i D_i}{2(\sum_T h_i D_i + t \sum_T \hat{h}_i D_i)}\right)$$

$$= \frac{\sqrt{2\tilde{A}(\sum_S h_i D_i - t \sum_S \hat{h}_i D_i)}}{2}\left(\frac{\hat{A}}{\tilde{A}} + \frac{\sum_s \hat{h}_i D_i}{2(\sum_s h_i D_i + t \sum_s \hat{h}_i D_i)}\right) + \frac{\sqrt{2\tilde{A}(\sum_T h_i D_i - t \sum_T \hat{h}_i D_i)}}{2}\left(\frac{\hat{A}}{\tilde{A}} + \frac{\sum_T \hat{h}_i D_i}{2(\sum_T h_i D_i + t \sum_T \hat{h}_i D_i)}\right)$$

$$= \frac{\sqrt{2\tilde{A}}}{2}\left(\sqrt{\sum_S h_i D_i - t \sum_S \hat{h}_i D_i} + \sqrt{\sum_T h_i D_i - t \sum_T \hat{h}_i D_i}\right)\left(\left(\frac{\hat{A}}{\tilde{A}}\right) + \frac{\sum_s \hat{h}_i D_i}{2(\sum_s h_i D_i + t \sum_s \hat{h}_i D_i)} + \frac{\sum_T \hat{h}_i D_i}{2(\sum_T h_i D_i + t \sum_T \hat{h}_i D_i)}\right)$$

$$\geq \frac{\sqrt{2\tilde{A}}}{2}\left(\sqrt{\sum_{S \sqcup T} h_i D_i - t \sum_{S \sqcup T} \hat{h}_i D_i}\right)\left(\left(\frac{\hat{A}}{\tilde{A}}\right) + \frac{\sum_{S \sqcup T} \hat{h}_i D_i}{2(\sum_{S \sqcup T} h_i D_i + t \sum_{S \sqcup T} \hat{h}_i D_i)}\right) = E(S \sqcup T) \quad (38)$$

In order to find a core allocation function, the same concept as the previous chapter will be followed. Instead of using the usual cost function, it will be changed to the cost with regards to frequency. The new cost function with relevance to frequency that is associated with the ordering emission will be $2\tilde{A}f_n$, while the one associated with the holding emission will be $\frac{\sum_n h_i D_i + t \sum_n \hat{h}_i D_i}{f_n}$. The new Emission function will be as the following:

$$E_c = \frac{\hat{A}}{\tilde{A}} \frac{T_c^*}{2} + \frac{T_c^*}{2}\left[\frac{\sum_n \hat{h}_i D_i}{\sum h_i D_i + t \sum \hat{h}_i D_i}\right]$$

$$E_c = \frac{\hat{A}}{2\tilde{A}}\,(2\widetilde{\mathrm{A}}f_n) + \frac{\sum_n h_i D_i + t\sum_n \hat{h}_i D_i}{f_n}\left[\frac{\sum_n \hat{h}_i D_i}{2\sum h_i D_i + t\sum \hat{h}_i D_i}\right]$$

$$E_c = \hat{A}f_n + \frac{\sum_n \hat{h}_i D_i}{2f_n} \qquad (39)$$

In equation (39) the allocation can be split into two parts, ordering and holding. For the ordering part $\widehat{\mathrm{A}}\mathrm{f}_{\mathrm{n}}$, the emission will be allocated to be proportionally distributed based on the frequency of ordering of each firm. The proportional allocation will be the frequency of each firm f_{i} over the overall frequency f_{n}. As for the holding emission, since it depends mainly on the amount of the holding emissions, the proportional allocation will be distributed based on it. The allocation equation will be as the following:

$$X_i = \frac{f_i^2}{f_n^2}\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\;\frac{\sum_n \hat{h}_i D_i}{2f_n} \qquad (40)$$

The below proposition proves that the above Xi equation is the core allocation for distributing the emitted emissions among all the participated firms in the inventory game:

Proposition 8. *The X = (X_i, ..., X_n) $\in \mathbb{R}^n$ assigned to each retailer i is a core allocation for N players.*

Proof: For All i ∈ N, $x_i = \frac{f_i^2}{f_n^2}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n})$ in which it satisfies the following:

1. Individual rationality: for every i $\in N, N_N \geq N$. Then $x_i = \frac{f_i^2}{f_n^2}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\,(\frac{\sum_n \hat{h}_i D_i}{2f_n}) = \frac{\sqrt{f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{2f_n} = \frac{f_i}{f_n}\,\hat{A}f_n + \frac{\hat{h}_i D_i}{2f_n} = \hat{A}f_i + \frac{\hat{h}_i D_i}{2f_n} \leq \hat{A}f_i + \frac{\hat{h}_i D_i}{2f_i} = E(i).$

2. Efficiency: X(N) = $\sum_{i\in N} x_i = \frac{\sum_n f_i^2}{f_n^2}\,\hat{A}f_n + \sum_n \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = \frac{\sqrt{\sum_n f_i^2}}{\sqrt{f_n^2}}\,\hat{A}f_n + \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = \hat{A}f_n + \left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = E(n).$

3. Collective rationality: For All $\phi \subset S \subseteq N,\quad x_s = \sum_{i\in S} x_i = \frac{\sum_s f_i^2}{f_n^2}\,\hat{A}f_n + \sum_s \frac{\hat{h}_i D_i}{\sum_n \hat{h}_i D_i}\left(\frac{\sum_n \hat{h}_i D_i}{2f_n}\right) = \frac{\sum_s f_i^2}{f_n}\,\hat{A} + \left(\frac{\sum_s \hat{h}_i D_i}{2f_n}\right) = \frac{\sqrt{\sum_s f_i^2}}{\sqrt{f_n}}\,\hat{A} + \left(\frac{\sum_s \hat{h}_i D_i}{2f_n}\right) \le \hat{A}f_s + \left(\frac{\sum_s \hat{h}_i D_i}{2f_s}\right) = E(s)$

The above proposition proves that Xi is a core allocation. All firms within the game will like to remain in the game and consolidate its resources because it will have a fair distribution of carbon emissions. None of the participated firms will have to emit more than what it emits when working individually. Moreover, no subgroup would leave the game to form a new one by themselves because they will have to pay more than when they are part of the grand coalition.

Table 5. Carbon Emission under tax regulation for the possible consolidation grouping

E(1)	356.3061	**E(2)**	761.3509
E(3)	1285.348	**E(1,2)**	835.357
E(1,3)	1323.884	**E(2,3)**	1491.246
E(1,2,3)	1526.179		

The above proposition can be prove by providing a numerical example. Following the previous example of carbon emissions in table 2, the total carbon emission will be calculated for all subsets within the games. Since there are three retails, the subset will be 5 subsets. The total carbon emissions of each subset is summarized in table 5. It shows that working all together as a group will reduce the total number of carbon emission. This is because the frequency of ordering jointly will be 14 orders/ year, while if each firm ordered individually the frequency will be 4.5, 7.5 and 10.9 respectively with a total of 22.9 orders/year totally.

The allocation of carbon emission among all firms will be based on the proved carbon emissions allocation formula above. The firms will have to allocate the total of 1526.179 among themselves. As mentioned above, the frequency of ordering will be 14 orders/ year and the number of carbon emissions each firm is assigned to will be as the following: X1 = 115.601, X2 = 403.3736 and X3 = 1007.204; where they used to emit when working individually were as the following: X1 = 356.3061, X2 = 1285.348 and X3 = 132.884.

CHAPTER 7. CARBON EMISSION UNDER FULL TRUCKLOAD SYSTEM.

In this chapter, the model will be extended to handle a more realistic scenario of having a fixed order size when placing an order. The research will study the system's effect on the carbon emissions. Section (7.1) presents the formulation of the carbon emission formula under full Truckload system and show the finding based on decentralized and centralized system. Section (7.2) will present a core allocation of savings.

7.1 Model Formulation and Findings

The regular EOQ model can be modified to handle a more realistic scenarios such as full truckload scenario under consolidation. All shipments needs to be fully loaded in order to reduce the cost of ordering. Full Truckload means having a fix size when ordering which is the size of the truck. (Elomri et al. 2013)'s model will be used and adjusted for this thesis. Two models were introduced, one is having a full truckload for a single firm, and the second is a consolidated shipments between N firms. The research showed, and as it was explained in the literature review, that consolidation can actually reduce the cost among the N firms for this system. The question will be for this thesis is whether this consolidation will have an effect of carbon emission as well under full truckload system. Will carbon emission be reduced or will it be increased.

The notations for this model are presented as the following: it will be assumed having

N firms in both systems, centralized and decentralized. Each firm will have a demand Di. The system will deal with one item that is provided to all firms for simplicity purpose. The volume of the item will be measured as V for all firms and items. Trucks will be of the same size and will carry the same amount of items in it. The capacity of the truck is donated as CAP. The ordering cost and operation handling for each truck will be fixed and donated as A. The holding cost per item unit will vary from a firm to another and will be donated as hi since each firm will hold the quantity in its own warehouse. For simplicity purpose, the holding cost will donated as Hi, where Hi = $h_i \frac{Q_i}{2}$.

In decentralized system where each firm orders separately, the replenishment, Q, of each firm will depends on the number of items that will fit into the truck, to which Q will be $\frac{CAP}{V}$. The frequency of ordering will be $\frac{D_i}{Q}$ or $\frac{D_i V}{CAP}$. This means that the total ordering quantity won't change from a firm to another. The difference will be in the frequencies which represents the number of trucks required.

The cost for decentralized will be as the following:

$$C_i = Af_i + \frac{H_i}{f_i} \tag{41}$$

The cost for the N firms in the decentralized will be basically the summation of Ci for every firm i $\sum_n C_i = A \sum_n f_i + \sum_n \frac{H_i}{f_i}$.

For the centralized system where all N firms has to work jointly, the number of frequencies which is the number of orders won't be affected due to having to fully load the trucks. Since the truck has to be fully loaded, then the number of trucks required and trips won't differ even if they were ordered in a different time period. The frequency formula will simply be A $\sum_n f_i = A \sum_n \frac{D_i V}{CAP}$. The holding cost will be the total number of holding for all n firms over the total frequency. That is because the quantity that will be held per each firm will be less in consolidation systems than items used to be held in a decentralized system. The joint cost will be as following:

$$C_n = A\sum_n f_i + \frac{\sum_n H_i}{\sum_n f_i} = Af_n + \frac{H_n}{f_n} \tag{42}$$

Following the same concept in previous chapters to formulate carbon emissions models, the cost's variables will be replaced by the emission's variables. As it was introduced, the ordering emission will be donated as $\hat{A}$ and the holding emissions will be donated as $\widehat{H}_i$, where $\widehat{H}_i = \hat{h}_i \frac{Q_i}{2}$. The equation will simply be:

$$E_i = \hat{A}f_i + \frac{\widehat{H}_i}{f_i} \tag{43}$$

And

$$E_n = \hat{A}f_n + \frac{\widehat{H}_n}{f_n} \tag{44}$$

In order to measure the amount of carbon emissions emitted from both decentralized and centralized systems where firm consolidated their shipment for a better performance and to reduce cost, the optimum frequencies that optimize the total cost will be used in the emission function to show the effect of cooperation on carbon emissions. The cost model uses a fixed ordering quantity to which it is calculated as the capacity of the truck over the volume of one item. It is the quantity needed to fill out one truck to which any order cannot exceed the size of the truck which is the ordering quantity, and at the same time cannot be less than the truck size as well. Since the order quantity is fixed, the frequency will remain the same in carbon emission model. For that, the emission equation will have the same structure presented in 43 and 44, with the introduced frequencies $\frac{D_i V}{CAP}$. It means that the emission equation won't be affected by the ordering nor the holding costs.

Since the amount of carbon emissions won't be affected by the costs values, and the order quantity and frequencies will be the same in both the cost and carbon emissions model, then the carbon emissions under centralized system will always be less than the total carbon emissions in decentralized system. This can be shown in the below proposition.

Proposition 9. *Cooperation can always lead to a reduction to carbon emissions.*

$$E_i = \hat{A}f_i + \frac{\hat{H}_i}{f_i} \quad \geq \quad E_n = \hat{A}f_n + \frac{\hat{H}_n}{f_n} \tag{45}$$

Proof: From the above equation, both carbon emissions model can be split into two parts: ordering and holding emission. Each part will be proved separately that it can lead to a reduction. From the ordering emissions part, the number of frequencies are the same in both systems since the order should be based on the truck size. Additionally, since the ordering emission variable is constant, then both system will have an exact equal account of carbon emissions emitted from ordering process.

Decentralized formula: Centralized formula:

$$\sum_n \hat{A}f_i \qquad ? \qquad \hat{A}f_n$$

Moving to the holding emission part, mathematically the model can be proved that decentralization will emit more carbon emissions from the system than centralization and this can be seen below. Theoretically, that is because the average number of the holding items will be less in centralized system since firms are going to order jointly and the quantity will be spilt among the firms in the coalition.

$$\sum_{n} \left(\frac{\widehat{H}_i}{f_i} \right) \quad \geq \quad \frac{\widehat{H}_n}{f_n}$$

The summation of a division is mathematically larger than the sum of variable A over sum of variable b.

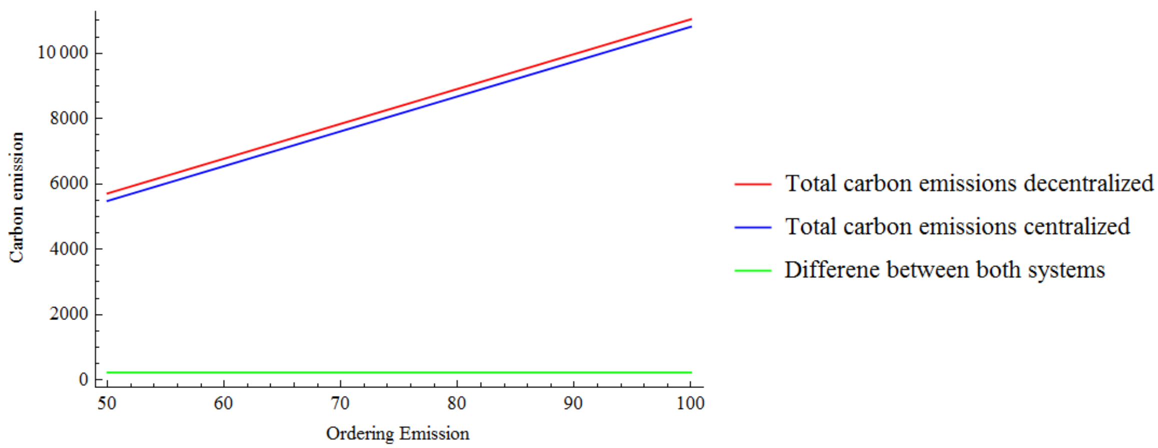


Figure 11. Affect of Ordering emissions on two systems under FTL system

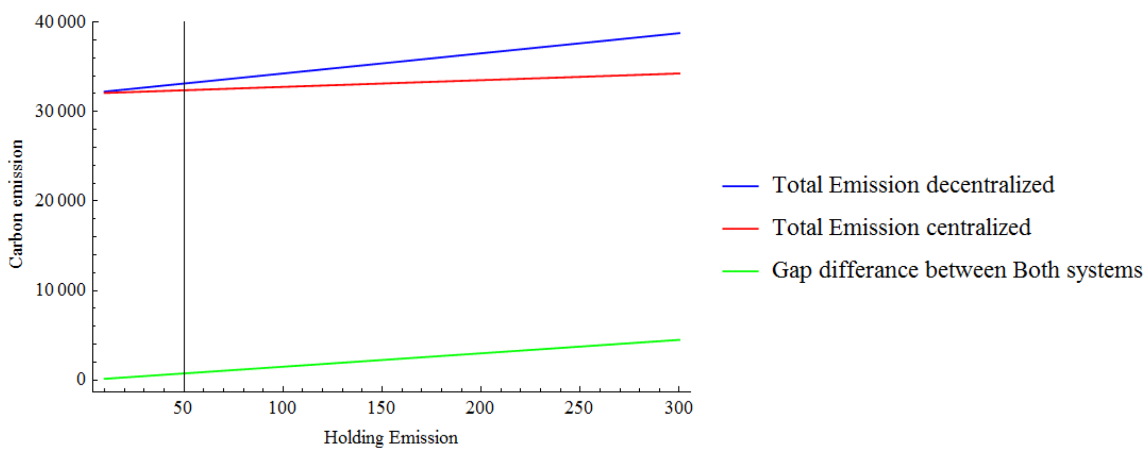


Figure 12. Affect of Holding emissions on two systems under FTL system

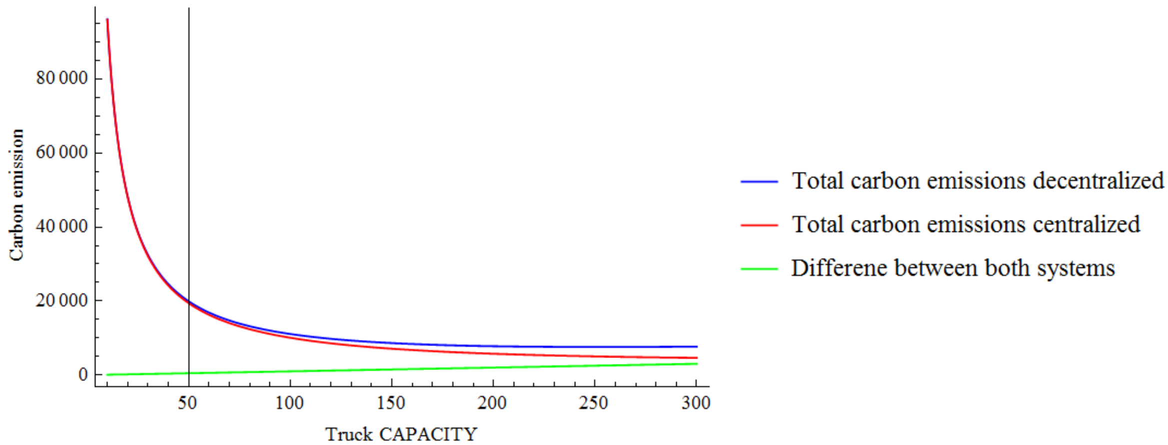


Figure 13. Affect of Holding emissions on two systems under FTL system.

The above proposition cab be presented in graphs to which these graphs will show the effect of each emission variable on carbon emissions in both systems and the difference obtained. Graph 11 shows the effect of ordering emissions on both models. The greater the ordering emission will be, the greater the carbon emissions emitted from both systems will be. The centralized system will emit less than the decentralized system even with the increment of the variable. However, the difference between the both systems or the savings won't be affected by the increment of ordering emissions. Secondly, the effect of holding emissions is presented in graph 12. The centralized system will do a better off than the decentralized system. The greater the value will be, the greater the carbon emissions will be. Moreover, the difference between both systems will differ and starts to increase with the increment of holding emissions' value. Lastly, chancing the truck capacity size; the change in this value can have an effect on carbon emissions in both systems and the difference between them. The greater the value will be, carbon emissions will be reduced in both systems, but the amount emitted from the centralized system will always be lower.

Table 6. Numerical example for centralized and decentralized carbon emissions in FTL system

Retailer	D_i	**CAP**	**V**	$\hat{A}$	$\hat{h}_i$	E_i	E_n
1	100	30	2	300	20	350	-
2	800	30	2	300	10	1675	-
3	700	30	2	300	30	1625	-
		Total				**3650**	**3345.31**

A numerical example based on the above proposal can be found in table 6. The table shows that carbon emission can be reduced when firms starts to work jointly. Based on the provided values which were generated in random, the decentralized system will emit around 3650 carbon emissions throughout its process; while in the centralized system, the total carbon emissions emitted will be 3345.31. The saving obtained from cooperation for this scenario will be around 304.69 carbon emissions.

7.2 Carbon Emission Allocation

Since it was proved that shipment consolidation, cooperation, can lead to a saving in the total carbon emissions, this savings can be allocation and shared among the n parties in the grand coalition. Before moving to allocate the savings, it will be proved that the total carbon emissions is a supper-additive model. This means that no matter how large the coalition is, the players will do better off if they joint a larger group (grand coalition). The greater the group will be, the greater the savings will be. The following proposition will show the mathematically proof:

Proposition 10. *The emission function is supper-additive function for all $S \subset N$ to which $E(S)+E(T) \geq E(S \cup T)$, for all $S, T \subset N$ and $S \cap T = \varphi$.*

Proof: there is a subset S and T where they are non-empty and disjoint:

$$E(S) + E(T) = \hat{A}\sum_{S} f_i + \frac{\sum_S \widehat{H_i}}{\sum_S f_i} + \hat{A}\sum_T f_i + \frac{\sum_T \widehat{H_i}}{\sum_T f_i} \quad = \hat{A}(\sum_S f_i + \sum_T f_i) + \left(\frac{\sum_S \widehat{H_i}}{\sum_S f_i} + \frac{\sum_T \widehat{H_i}}{\sum_T f_i}\right) \geq \hat{A}\sum_{S\sqcup T} f_i + \frac{\sum_{S\sqcup T} \widehat{H_i}}{\sum_{S\sqcup T} f_i} = \mathrm{E}(S \sqcup T)$$

After proving that the model is supper-additive and since there is a saving, the next step will be finding a formula to allocate the carbon emissions among the n players in the grand coalition. Having the total carbon emissions model, formula 44, the model can be divided into two parts, ordering and holding emissions. For the ordering emission part, and referring to previous section, it was proved that the amount of ordering emission is equal in both centralized and decentralized system since the amount of items to be ordered is fixed due to having the truck size so the frequency will be the same. For that, all firms will have to be charged the same amount when they are working separately or jointly. The ordering emission for each firm will be $\hat{A}f_i$ where $f_i = \frac{D_i V}{CAP}$ and $f_n = \sum f_i$.

As for the holding emission part, the amount of items to be held will depend of the total number of items ordered in each replenishment of all firms. Which means that the number of items to be held will be less when ordering jointly. Each firm will be charged for $\frac{\widehat{H}_i}{\sum_n f_i}$ and where $\widehat{H}_n = \sum \widehat{H}_i$.

The cost allocation function for firm i will be as the following:

$$X_i = \hat{A}f_i + \frac{\widehat{H}_i}{\sum_n f_i} \tag{46}$$

The above equation can be proved that it is a core allocation in the following proposition:

Proposition 11. *The* $X = (X_i, ..., X_n) \in \mathbb{R}^n$ *assigned to each retailer i is a core allocation for N players.*

Proof: For All i ∈ N, $X_i = \hat{A} f_i + \frac{\hat{H}_i}{\sum_n f_i}$ in which it satisfies the following:

1. Individual rationality: for every $i \in N, N_N \geq N$. Then $X_i = \hat{A} f_i + \frac{\hat{H}_i}{\sum_n f_i} \leq \hat{A} f_i + \frac{\hat{H}_i}{f_i} = E(i)$

2. Efficiency: X(N) = $\sum_{i \in N} X_i = \sum_n \hat{A} f_i + \frac{\hat{H}_i}{\sum_n f_i} = \hat{A} \sum_n f_i + \frac{\sum_n \widehat{H_i}}{\sum_n f_i} = \hat{A} f_n + \frac{\hat{H}_n}{f_n} = E(N)$

3. Collective rationality: For All $\phi \subset S \subseteq N$, $X_s = \sum_s \hat{A} f_i + \frac{\hat{H}_i}{\sum_n f_i} = \hat{A} f_s + \frac{\hat{H}_s}{f_n} \leq \hat{A} f_s + \frac{\hat{H}_s}{f_s} = E(S)$.

The above allocation function was proved to be a core allocation function. The carbon emission function of each firm will depend of the frequency and the holding emissions values. The proposition showed that the provided formula follows the properties of a core allocation. It was proved that cooperation can actually have a firm to do a better off than when working individually. It was also proved that the formula assures that the all the costs will be distributed among the players and that the players will do better in the grand coalition than when working in a sub coalition.

The difference between this model and previous models is that the ordering quantity is fixed, for that the emission function structure will be identical to cost function. Since the ordering quantity is fixed, the emission function won't be affected by the ordering and holding cost at all, and will be only affected by the ordering, holding emission and truck size (CAP) variables. Moreover, the savings will only be obtained from the holding emission amount.

Table 7. Emission under Full Truckload of the possible consolidation grouping

E(1)	350	**E(2)**	1675
E(3)	1625	**E(1,2)**	188.33
E(1,3)	1815.63	**E(2,3)**	3145
E(1,2,3)	3345.31		

A numerical example is provided to show that the X is a core allocation. Following the previous example of carbon emissions in table 6, the total carbon emission can be calculated for all subsets within the games which are 5 subsets. The total carbon emissions of each subset is summarized in table 7. As it was shown previously, the ordering frequency will remain unchanged for both systems. The Total number of orders will be 106.67 orders where retailer 1 will place 6.67 orders, retailer 2 will place 53.33 orders and retailer 3 will place 46.67 orders.

The allocation of carbon emission among all firms will be based on the core allocation of the carbon emission formula above. The firms will have to allocate the total of 3345.313 among themselves. The number of carbon emissions that has to be emitted by each firm will be: X1 = 209.375, X2 = 1637.5 and X3 = 1498.438 which is less than when these firms used to pay individually. Firms used to pay in the decentralized system as he following: E1: 350, E2: 1675 and E3: 1625.

CHAPTER 8. CONCLUSION AND FUTURE WORK

Carbon emissions reduction efforts gained a considerable attention in the past few decades. The reduction effort of carbon emissions can be costly when applied to supply chain systems. To that end, having a balance between cost and environmental can be hard to achieve in many cases. Many firms tend to find a balance between both parameters by complying with carbon emissions requirements and maintaining the allocated budget. The aim of this thesis follows a different bath where the main objective is to show the effect on carbon emission under joint replenishment where n players cooperate and share their resources in order to reduce the total cost from the supply chain. In view of this, the thesis seeks to develop a model to measure the amount of carbon emissions that is being emitted by both centralized and decentralized systems and understand the influence of cooperation on carbon emissions. Regular EOQ model, and carbon emissions equation were all used to build the model. by showing that cooperation can lead to a reduction in carbon emissions along with cost, the thesis aims to develop a core allocation model that helps in distributing and allocating the payoffs of carbon emissions among the n parties within the coalition by using inventory game techniques,. Furthermore, the allocation model can address the three cases covered by this research: regular cooperative EOQ model, extended model with fixed tax rate regulation, and extended model with full truckload model.

The first question in this thesis was pertaining to finding the effect on carbon emissions under cooperation context. Results of comparing centralized and decentralized system for the three cases showed that centralized system always lead to a reduction in carbon emissions. Additionally, indicate that companies can benefit from reducing the total cost and the total carbon emissions when joining resources with other companies.

The Second question in this thesis was pertaining to savings allocation. To have an effective cooperation, the coalition game should be fair for all parties so that no entity will have an incentive to leave the coalition. The thesis proved that the developed allocation model is a core allocation model in which ad- dresses all required characteristics. Moreover, the thesis demonstrated that the model used in centralized system to measure the amount of carbon emissions is actually a supper additive formula. Therefore, the developed allocation model can be seen as a fair and stable model which will encourage companies to join and stay in the game.

Other findings include the relation between ordering and holding costs variables in the three models. Ordering and holding costs variables seem to have a direct influence on both the regular model and extended model with fix tax rate regulation in terms of carbon emissions. Conversely, the full truckload carbon emissions calculation model was not affected by the cost variables as the order quantity is fixed.

The findings and conclusion can encourage firms to form and assess coalitions and outcomes by using the models and evaluating the reduction in cost and carbon emissions as well as by understanding saving allocation which in turn creates more attractive conditions for cooperation. It can also assist firms to show the amount of carbon emission omitted during the process to increase customer satisfaction levels. Furthermore, the findings can be used by decision makers, governments, entities, and related firms in order to measure carbon emissions omitted from the supply chain. The measurement can help in defining the carbon caps, tax rates or any other regulations within the system. Even though two extended models were presented, the results may not remain constant based on the models used or the type of extension.

8.1 Future Work and Extended Models:

The model can be extended to handle different scenarios and more costs and regulations. It can be sophisticated by adding the backorders at each period of time and can be more realistic by including the setup and purchase costs. Adding fixed costs to the model may or may not have a change in the overall structure of the model and it may follow the same findings and results obtained in this thesis.

Another type of extension can be considering the model under a variable ordering emissions or a fixed holding emissions. The model can handle a fixed ordering cost to be applied to all firms and a variable unit cost that varies from a firm to another. The structure of the model will differ and it may not have the same structure. This is an interesting extension to which it considers the destination difference for each firms and the delivery method used. This extended model can lead to another question that can be consider which is how to measure the increase of ordered items within a replenishment period. If more trucks are needed or bigger ones, then this means that the ordering emissions will change and cannot be treated as the regular one. To have a fixed holding emissions is when firms joints their warehouses and stores in the cheapest warehouse.

The regular model can be extended to handle multiple items or multiple suppliers or to add suppliers to the model. The model can be enhanced to handle the three scenarios: multi items ordered by a single retailer, or single item ordered from multi suppliers, or multi items and multi suppliers. Having a multi items system with a single retailer, or multi suppliers and a single item can have the same structure. The more sophisticated model can be having a multi suppliers and retailers and items. Another type will be adding the supplier to the model, the whole structure and concept of supply chain will change. In this model, the supplier's influence and decisions can play a major role to the orders and the total costs.

Lastly, an interesting point can be considered if the system is not fully an enclosed system. Currently, the holding cost is an enclosed information which is not shared by firms. A future work can be done to have an allocation emission function where is requires firms to share all of their information truthfully such as holding costs, or maybe ordering variable and this depends on the model.

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