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Qualitative Dealing with Quantified Assertions

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في هذا البحث، نقدم طريقة جديدة للمعالجة الرمزية للجمل القياسية المحدودة ذات الشكل التالي:
«ك أ ←ب» علماً بأن أ و ب هن مسميات تمثل مجموعات، بينما ك هو قياس لغوي يمثل نسبة مقيمة بطريقة نوعية، نموذجنا هذا، يمكن اعتباره تعميم رمزي من التصورات الاحتمالية الشرطية الإحصائية، وأيضاً تعميم رمزي كلاسيكي للمعامل الاحتمالية، طريقتنا معتمدة بشكل رئيسي على المنطق الرمزي المحدود حيث أن مقياس التدرج في هذا المنطق ممثل بدرجات حقائق منطقية مختلفة، علاوة على ذلك، فإننا نقترح قواعد استدلال رمزية تسمح لنا بالتعامل مع الجمل القياسية المحدودة.
وأخيراً نقدم تشكيل رمزي من مبدأ الاستدلال المباشر يسمح لنا بالتعامل المنطقي الخصوصي.

Key Words: *Knowledge Representation, Linguistic Quantifiers, Syllogistic Reasoning, Direct Inference, Certainty.*

ABSTRACT

In this paper we present a newapproach to a symbolic treatment of quan-tified statements having the following form "*Q A*'s are *B*'s", knowing that *A* and *B* are labels denoting sets,and Q is a linguistic quantifier interpreted as a proportion evaluated in a qualitative way.Our model can be viewed as a symbolic generalization of statistical conditional probability notions as well as a symbolic generalization of the classical probabilistic operators.Our approach is founded on a symbolic finite M-valued logic in which the graduation scale of M symbolic quantifiers is translated in terms of truth degrees. Moreover,we propose symbolic inference rules allowing us to manage quantified statements. Finally, we present a symbolic formalisation of direct inference principle allowing us to reason with particular individuals.

1.Introduction

In the natural language,one often uses statements qualifying statistical information like "Most stu-dents are single", called *quantified assertions*. More formally,these quantified assertions have the fol-lowing form "*Q A*'s are *B*'s", where *A* and *B* are labels denoting sets, and *Q* is a *linguistic quantifier* interpreted as a proportion evaluated in a qualitative way.In this paper,we propose a new approach to such quantified assertions within a *qualitative context*.More precisely,our goal is threefold: (1) to propose a *symbolic representation* of *quantified assertions*, (2) to develop *syllogistic reasoning* allowing us to deduce new quantified assertions from the initial ones,and (3)to deduce new informa-tion concerning a particular individual from a set of quantified assertions.

Zadeh [36]distinguishes between two types of quantifiers:absolute and proportional.An absolute quantifier evaluates the number of individuals of B in A. While a proportional quantifier evaluates the proportion of individuals of B in A.The proportional quantifiers can be precise or vague. A precise quantifier translates an interval of proportions having precise bounds exemplified by "10%", "Between 10%and 20%", etc. While a vague quantifier translates an interval proportions having fuzzy bounds. Thus the vague proportional quantifiers express qualitatively proportions. A proportional quantifier can be viewed as a kind of probabilities assigned to classes of individuals. So,several approaches based on the theory of probabilities have been proposed ([22], [30], [7], [4], [3], [5], [13]) for the modeling of precise proportional quantifiers. Other probabilistic approaches,such as those proposed by ([24], [25], [6]), do not enable an adequate representation of proportional quantifiers,since these approaches are generally introduced to treat uncertainty. These authors interpret the probability degrees assigned to propositions as degrees of certainty or beliefs in the truth of these propositions. They represent statistical assertions of type "Q A's are B's" as uncertain rules of the form: "if A then B" with a belief degree in the truth of the rule (A and B are interpreted as propositions). It has been pointed out by Bacchus [4] that a confusion in the representation is made between the probabilities interpreted as certainty degrees assigned to propositions about particular individuals and those interpreted as proportions assigned to classes of individuals. The probabilities of the first type are called subjective and the second statistical. The statistical probability that corresponds to the proportion is a particular case of probabilities where the distribution is uniform over the finite reference set. For example, the statistical probabil-ity attached to a subset A of the finite reference set Ω, Prop(A), is equal to the absolute proportion of individuals of A, i.e., $\text{Prop}(A) = |A|/|\Omega|$. Similarly, if A and B are two subsets of Ω, the relative proportion of individuals of B in A is expressed by the conditional statistical probability, Prop (B|A), with $\text{Prop}(B|A) = \text{Prop}(A \cap B) / \text{Prop}(A) = |A \cap B|/|A|$.

Some probabilistic approaches ([1], [29], [4], [5]) are interested in a qualitative modeling of the proportional quantifier "Most" or "Almost-all" in the context of default reasoning. The approaches based on the fuzzy set theory ([9], [36], [33], [34], [7], [10]) deal with a vague proportional quantifier as a fuzzy number of the interval [0,1] which can be manipulated by using the fuzzy arithmetic. For example, the membership function of "Most" evaluates the degree to which a given proportion is compatible with the quantifier "Most". The representation of quantified statements involving fuzzy sets is based on the concept of fuzzy subset cardinality. Recently, Dubois et al. [8] have proposed a semi-numerical approach to the vague quantifiers based upon the numerical results obtained in ([7], [3]) for precise quantifiers. It is concerned with a suitable ordered partition of the unit interval [0,1] in several subintervals covering [0,1], each subinterval representing a vague quantifier. The subintervals obtained by applying the inference rules (on the precise quantifiers) to subintervals representing the vague quantifiers are approximately associated to subintervals of vague quantifiers.

Now, being able to represent quantified statements, like "Most birds fly", it is interesting to obtain belief symbolic degrees attached to properties about particular individuals, like "Tweety flies", and this, by using knowledge based upon quantified assertions and certain facts. In other words, it is necesarry to propose a symbolic model based upon a direct inference principle and a choice strategy of the appropriated reference class ([31], [22], [30], [4]). Reasoning about particular individuals is often based upon statistical pieces of information in the sense that the subjective probability associated to a property about a particular individual is derived from the proportion of individuals verifying this property (for example, knowing that "Most smokers may have lung cancer", a doctor thinks that it is very-probable that Martin which is a smoker will have a lung cancer). This direct inference is used to derive symbolic certainty de-grees from quantified assertions and facts (certain information). The proposed model is built upon M-valued predicates logic defined by Pacholczyk [26]. Symbolic quantifiers and symbolic certainty degrees are translated in terms of truth value of two new predicates added to the language and taking into account the two notions of probability. The use of two distinct predicates (called **Prop** and **Cert**) allows to describe the two different notions of probabilities (both in a syntactical and a semantical as-pect). This new predicates are interpreted in different ways and are related to different symbolic degrees. In the first part of this paper (devoted to the first goal of our work), we present a symbolic representation of quantified assertions, and their fundamental properties. In Section 2, we present the M-valued predicate Logic proposed by Pacholczyk in ([26], [27], [28]). Section 3,describes our symbolic representation of statistical probability. The Axioms governing this representation and certain properties generalizing symbolically some classical properties are presented in Section 4. Section 5 is devoted to the second goal of our paper ([15],[16], [17], [18], [19], [20], [21]), that is to say, syllogistic reasoning based on quantified assertions. The third goal of our work concerns the two last sections. In Section 6, we prsent our symbolic representation of subjective probability. In Section 7, we present a symbolic formalisation of direct inference principle leading to areasoning process allowing us to deal with particular individuals ([16]). Finally,in Section 8 we make link with probabilistic works of Bacchus [4] and Bacchus et al. [5].

2. Many-valued Predicate Calculus

We present the many-valued predicate calculus proposed by Pacholczyk in ([26]). Notice that this many-valued logic has been proposed to the processing of imprecise information.

2.1. Algebraic Structures

Let $M \geq 2$ be an odd integer. Let M be the interval [1, M] of integers totally ordered by the relation $\leq$, and n be the mapping defined by $n(\alpha) = \mathfrak{M} + 1 - \alpha$. Then, $\{\mathfrak{M}, \vee, \wedge, n\}$ is a De Morgan lattice with: $\alpha \vee \beta = \max(\alpha, \beta)$ and $\alpha \wedge \beta = \min(\alpha, \beta)$. Let $\mathfrak{L}_M = \{\tau_\alpha, \alpha \in \mathfrak{M}\}$ be a set of M elements totally ordered by the relation $\leq$ such that: $\tau_\alpha \leq \tau\beta \Leftrightarrow \alpha \leq \beta$. Thus $\{\mathfrak{L}_M, \leq\}$ is a chain in which the least element is τ_1 and the greatest element is τ_M. We define in $\mathfrak{L}_M$ two operators and a decreasing involution as follows: $\tau_\alpha \vee \tau_\beta = \tau_{\max(\alpha,\beta)}$, $\tau_\alpha \wedge \tau_\beta = \tau_{\min(\alpha,\beta)}$ and $\sim \tau_\alpha = \tau_{n(\alpha)}$. Linguistically speaking, $\mathfrak{L}_M$ can be viewed as a set of scaling adverbs. For example, by choosing M = 7, we can define $\mathfrak{L}_7$ as follows: $\mathfrak{L}_7$= {not at all, very little, little, moderately, very, almost, totally}. In order to deal with vague predicates within our M-valued logic, L_M will be interpreted as a set of linguistic truth degrees. Then, $\mathfrak{L}_M$ will be denoted as: $L_M = \tau_\alpha$ – truce, $\alpha \in \mathfrak{M}$}. So, previous example gives us: L_7= {not at all-true, very little-true, little-true, moderately-true, very-true, almost-true, totally-true}[1]. On considers that the linguistic expression vα in the statement "x is v_α A" (A becomes a many-valued predicate) is associated with τ_α-true the degree to which x satisfies A, i.e. the truth degree τ_α-true of A(x). So, we have: "John is very tall"is true ⇔ "John is tall" is very-true.

2.2. Interpretation and Satisfaction of Formulas

Let $\mathfrak{L}$ be the many-valued predicates language and F the set of formulas of $\mathfrak{L}$. We call an interpretation structure $\mathfrak{A}$ of $\mathfrak{L}$, the pair $< \mathfrak{D}, \{R_n | n \in N\} >$, where $\mathfrak{D}$ designates the domain of $\mathfrak{A}$ and R_n designates the multiset[2] associated with the predicate P_n of the language. We call a valuation of variables of L, a sequence denoted as $v = <v_0, \ldots, v_{i-1}, v_i, v_{i+1}, \ldots>$. The valuation s(i/a) is defined by $v(i/a) = <v_0, \ldots, v_{i-1}, a, v_{i+1}, \ldots>$.

Definition 1: For any formula Φ of F, the relation of partial satisfaction "s satisfies Φ to a degree T_α in-$\mathfrak{A}$ "or" s T_α-satisfies Φ in-U", denoted as $\mathfrak{A} =^{v}_{\gamma} \Phi$ is defined recursively as follows:

- $\mathfrak{A} \models^{v}_{\alpha} P_n(z_0, \ldots, z_k) \Leftrightarrow <(v_0, \ldots, v_k)> \in_\alpha R_n$,
- $\mathfrak{A} \models^{v}_{\alpha} \neg \varnothing \Leftrightarrow \{\mathfrak{A} =^{v}_{\beta} \varnothing$ with $\tau_\alpha = \sim \tau_\beta\}$,
- $\mathfrak{A} \models^{v}_{\alpha} \varnothing \cap V \Leftrightarrow \{\mathfrak{A} =^{v}_{\beta} \varnothing$ and $\mathfrak{A} =^{v}_{\gamma} V$ with $\tau_\beta \wedge \tau_\gamma\}$,
- $\mathfrak{A} \models^{v}_{\alpha} \varnothing \cup V \Leftrightarrow \{\mathfrak{A} =^{v}_{\beta} \varnothing$ and $\mathfrak{A} =^{v}_{\gamma} V$ with $\tau_\alpha = \tau_\beta \vee \tau_\gamma\}$,
- $\mathfrak{A} \models^{v}_{\alpha} \varnothing \supset V \Leftrightarrow \{\mathfrak{A} =^{v}_{\beta} \varnothing$ and $\mathfrak{A} =^{v}_{\gamma} V$ with $\tau_\alpha = \tau_\beta \rightarrow \tau_\gamma\}$,
- $\mathfrak{A} \models^{v}_{\alpha} \exists z_n \Psi T_\alpha = \text{Max}\{\tau_\gamma / \mathfrak{A} =^{v(n/a)}_{\gamma} \Psi, a \in D\}$.
- $\mathfrak{A} \models^{v}_{\alpha} \forall z_n \Psi \Leftrightarrow \text{Min}\{\tau_\gamma / \mathfrak{A} =^{v(n/a)}_{\gamma} \Psi, a \in D\}$.

Definition 2: A formula Φ is said to be τ_α-true in U, if and only if, there exists a valuation s such that s τ_α-satisfies Φ in-$\mathfrak{A}$.

3. Symbolic Representation of Statistical Probabilities

The representation of statistical probabilities requires the reference to sets of individuals and also to assign probabilities to these sets. To solve the first problem,we use the concept of placeholder variables in lambda abstraction used by Bacchus [4], where one considers that a Boolean open formula can refer to the

1. Note that "not at all-true"and "totally-true"correspond respectively with the classical truth values "false"and "true".
2. The multiset theory is an axiomatic approach to the fuzzy set theory.In this theory, $x \in \alpha A$, the membership degree to which x belongs to A, corresponds with $\mu_A(x) = \alpha$ in the fuzzy set theory of Zadeh [35].

set of all instances of its free variables, specified as placeholders,satisfying the formula. So given a many-valued predicates language $\mathfrak{L}$, for an interpretation $\mathfrak{A}^*$ with domain of discourse Ω and C_1 the set of open well-formed formulas without bound variables ø of $\mathfrak{F}$ such that,for any valuation v of Ω, ø is totally satisfied in-U* or not at all satisfied in- $\mathfrak{A}^*$. So: $C_1 = \{ø \in F \mid \forall v, u^* =^{v}_{M} ø \text{ or } \mathfrak{A}^* \models^{v}_{1} ø\}$. Since formulas of C_1 contain only free variables,we can consider that free variables of formulas of C_1 stand implicitly for placeholder variables.Thus in interpretation $\mathfrak{A}^*$, each formula of C_1 will be able to make reference to the subset of individuals of Ω that satisfy this formula.

In order to define the symbolic statistical probabilities assigned to subsets referred by formulas of C_1, we add to the language $\mathfrak{L}$, anew many-valued unary predicate, denoted as **Prop**, defined over formulas of C_1 which qualitatively takes into account the notion of proportions of sets referred by formulas of C_1. We are going to extend the structure interpretation of the language $\mathfrak{A}^*$ to $\mathfrak{A}$ with domain $\Omega \cup C_1$, and we suppose that the variable φ designates the argument of **Prop**, and that any valuation v comprises v_0 that is associated to α.

A symbolic proportion Q_α of elements of Ω totally satisfying ø with respect to uniform probability distribution on Ω. So, Q_α can be considered as the symbolic degree of statistical probability of the set A referred by ø in Ω, i.e. the absolute proportion of elements of Ω which are in A. $\mathfrak{L}$in guistically speaking,this can receive the following translation: "A proportion ø of individuals of Ω are in A", which is classically denoted "ø Ω's are A's". Then, the managing of statistical probabilities can be handled by enriching the syntax and the semantic of our $\mathfrak{M}$-valued logic by adding a particular predicate **Prop** with a formation rule based on this predicate,and the axioms governing the use of formulae referring to **Prop**. Of course, in order to obtain a theory of symbolic statistical probabilities, these axioms may be justified at a metalogical level. We can now put the following definition:

Definition 3: The predicate **Prop** is defined as follows[3]:

– For any interpretation $\mathfrak{A}$ $\forall \varphi \in C_1$ **Prop** $(\varphi) \in \mathfrak{F}.\alpha$

– Any interpretation $\mathfrak{A}$ associates to Prop a multiset of C_1, denoted as $\mathfrak{S}$, so that for all valuation v, if ø is an element of C_1, we have: $\mathfrak{A} \models^{v(0/ø)}_{\alpha}$ **Prop** $(\varphi) \Leftrightarrow$ <ø> $\in_\alpha \mathfrak{S} \Leftrightarrow$ **Prop** (ø) is τ_α-true-in-U.

Convention: If no confusion is possible $\mathfrak{A} =_\alpha$**Prop** (ø) stands for $\mathfrak{A} \models^{v(0/ø)}_{\alpha}$ **Prop** (φ)

As noted before, this definition can receive the following translation: "Q_α Ω's are A" (i.e. a proportion Q_α of individuals of Ω are in A). So, it is possible to associate with any $\tau_\alpha \in \mathfrak{L}M$ a vague proportional quantifier denoted Q_α. In the following, we denote as $\mathfrak{Q}M = \{Q_\alpha, \alpha \in \mathfrak{M}\}$ the resulting set of proportional quantifiers.

Choice of quantifiers

By choosing M = 7, we can introduce: $\mathfrak{Q}_7$ = {none, very-few (or almost-none), few, about half, most, almost-all, all} that corresponds to the symbolic degrees of statistical probability.

The previous defintion leads to the following one.

[3] **Prop** has been introduced in a similar way as the predicate **Prob** in [28].

Definition 4: Let $\mathfrak{Q}_M$ be the set of the vague proportional quantifiers $\mathfrak{Q}M = \{Q_\alpha, \alpha \in [1,M]\}$. Then, "$\mathfrak{A} \models {}_\alpha$**Prop** (ø)" will mean that "Q_α, individuals of Ω totally satisfy α in-$\mathfrak{A}$", if and only if, the subset referred by ø belongs to the multiset $\mathfrak{S}$ with a degree τ_α.

Let us now consider quantified assertions, like "Most birds fly", classically denoted in the following form "Q A's are B's". The idea proposed for the representation of "Q A's are B's" is to interpret it in terms of the symbolic relative (or conditional) proportion of individuals of B in A. Therefore, we can generalize the classical definition of conditional statistical probability in a symbolic context, by using a "symbolic probabilistic division" operator, denoted as **C**, or equivalently a "symbolic probabilistic multiplication" operator, denoted as **I**. These two operators have been defined in ([28],[32]) for the symbolic representation of conditional uncertainty. The operator **I** is an application of $\mathfrak{Q}_M^2$ into $\mathfrak{Q}_M$, that verifies the classical properties of the probabilistic multiplication (commutativity, absorbent element: Q_1, neutral element: QM, monotony, associativity, idem-potence: Q_2). The operator C is an application of $\mathfrak{Q}_2^M$ into $\mathfrak{P}(\mathfrak{Q}_M)$, which is the set of parts of $\mathfrak{Q}_M$. **C** is deduced from **I** by a unique way as follows: $Q_\mu \in C(Q_\alpha, Q_\lambda) \Leftrightarrow Q_\lambda = \mathbf{I}(Q_\alpha, Q_\mu)$. Among the different tables of the operator **C** which verify the axioms chosen in [28], in $\mathfrak{L}_7$ we have chosen Table 1 presented in the annex. The corresponding operator I is defined in the annex by Table 2.

By using the previous definition of absolute statistical probability, we can define the notion of conditional statistical probability of v given ø, denoted as $\mathfrak{A} = {}_\mu$Prop (Ψ|ø), which can be viewed a as symbolic generalisation of conditional probability in classical probability theory.

Definition 5: *Let ø and Ψ be formulas of C_p, the symbolic conditional statistical probability of v given ø, denoted as $\mathfrak{A} \models {}_\mu$**Prop** (Ψ | ø), is defined by the symbolic division of the symbolic degree of **Prop** (Ψ ∩ ø) by that of **Prop** (ø) as follows: {$\mathfrak{A} \models {}_\alpha$**Prop** (ø), $\mathfrak{A} \models {}_\lambda$**Prop** (v ∩ ø), U ⇒ $\mathfrak{A} \models {}_\mu$**Prop** (Ψ|ø) with $Q_\mu \in$ **C** (Q_α, Q_λ).*

In order to obtain an equivalent manipluation of sets and formulas of type "$\mathfrak{A} \models {}_\mu$**Prop** (v | ø)", we can use the usual notation of quantified satatements. If we suppose that ø and v refer respectively to subsets A and B of Ω in the interpretation A, previous equivalence leads to the following definition: "$\mathfrak{A} \models {}_\mu$Prop (Ψ | ø)" ⇔ "$Q_\mu$ A's are B's". Moreover, T being a tautology we have: $\mathfrak{A} \models {}_\alpha$Prop (ø | T) ⇔ $\mathfrak{A} = {}_\alpha$Prop (ø). In other words, absolute probability appears as a particular case of conditional probability: "Q_α Ω's are A's" ⇔ "$\mathfrak{A} \models {}_\alpha$Prop (ø)". Thus, previous definition can be rewritten as follows:

Definition 6: Given an interpretation $\mathfrak{A}$, let us suppose that the formulas ø and v refer respectively to the subsets A and B of Ω. Then, the following assertions are equivalent:

- if {$\mathfrak{A} \models {}_\alpha$Prop (ø) and $\mathfrak{A} \models {}_\lambda$Prop (Ψ ∩ ø)}, then $\mathfrak{A} \models {}_\mu$Prop (Ψ | ø) with $Q_\mu \in C(Q_\alpha, Q_\lambda)$,
- if {Q_α **Ω**'s are A's and Q_λ **Ω**'s are (A ∩ B)'s}, then "Q_μ A's are B's" with $Q_\mu \in S(Q_\alpha, Q_\beta)$.

Remark 1: It appears that: "{Q_α, **Ω**'s are A's and Q_λ **Ω**'s are (A ∩ B)'s}, then "Q_μ A's are B's" with $Q_\mu \in C(Q_\alpha, Q_\beta)$" be viewed as a symbolic generalisation of the classical property: Prop (B |A) = Prop(A ∩ B)/ Prop(A) = |A ∩ B |/|A|.

Example 1: By using Q_7 ,let us suppose that the domain of discourse consists of residents of the city V.

Knowing that: "Most residents of the city V are young" and "Half of residents of the city V are young single". These assertions are respectively translated in our model by: "Q_5 Ω's are Young" and "Q_4 Ω's are Young ∩ Single". Definition 6 gives us: "Q_μ Young are Single" with $Q\mu \in C(Q_5, Q_4) = \{Q_5\}$. Then we obtain: "Most young people are single".

4. Axiomatic and Properties of Symbolic Statistical Probabilities

We can now put the axioms governing the concept of symbolic statistical probabilities. Each of them is justified at a metalogical level.In the following, A and B denote subsets of Ω.

Axiom 1: $A \cap B \neq A$, "Q_α Ω's are A's" and "Q_α Ω's are $(A \cap B)$'s"and $Q \in [Q_3, Q_7] \Rightarrow$ Almost-All A's are B's". (Axiom defining "Almost-all").

Qualitatively the subsets A and $A \cap B$ can have the same symbolic degree of proportions without being equal. This is the case,when $A \cap B$ is equal to the set A without one or some individuals. This can qualitatively be translated by saying that "A and $A \cap B$ are almost equal" or "Almost-all A's are B's". This is not always the case when the proportion of A is very weak (associated with Q_2 = Very-few).

Axiom 2: "Q_α Ω's are A's", $Q_\alpha \in [Q_2, Q_7]$ and "Almost-All A's are B's" ⇒ "Q_α Ω's are $(A \cap B)$'s". (Axiom defining "Almost-all").

When we have "Almost-all A's are B's", we know that $A \neq A \cap B$, but we can say that A and $A \cap B$ are almost equal and therefore A and $A \cap B$ have the same symbolic degree of proportions.

Axiom 3: "Q_α Ω's are A's" ⇔ "$Q_{n(\alpha)}$ Ω's are A's with $n(\alpha) = M + 1 - \alpha$". (Axiom defining the dual quantifier).

Generally the dual quantifier of Q_α corresponds with $Q_{n(\alpha)}$ ("Few" is the dual quantifier of "Most").

Axiom 4: "Q_α Ω's are A's", "Q_β Ω's are B's", $A \cup B \neq \Omega$ and $A \cap B = \emptyset \Rightarrow$ = "Q_r Ω's are $(A \cup B)$'s" with $Q_r \in S(Q_\alpha, Q_\beta)$. (Axiom defining the symbolic proportion of disjoint sets union).

Classically, when A and B are disjoint, the absolute proportion of their union is the sum of their absolute. If the union A and B is different from Ω (otherwise, the symbolic proportion degree of their union is evidently Q_M) and that they are disjoint, then the symbolic proportion degree of their union belongs to the "symbolic sum" of their symbolic proportion degrees. The symbolic sum denoted **S** is introduced in a way that it gives an interval containing one or two values.The lower bound of this interval is greater than or equal to each symbolic value of two arguments of S. Since the set $A \cup B$ is different from Ω, the maximal degree that can take the upper bound of the interval is Q_{M-1}. The use of an interval rather than a single degree is due to the degree Q_2 .It is justified by the fact that the addition of one or some elements (i.e., a very weak quantity) to a set can either preserve its symbolic degree of proportion or increase it at most one degree.

Definition 7: The symbolic addition **S** is a commutative application of $\mathfrak{Q}^M_2$ into $\mathfrak{P}(\mathfrak{Q}_M)$. By supposing that $\alpha + \beta \leq M + 1$, **S** is defined as follows:

$$S(Q_\alpha,Q_\beta) = \begin{cases} \{Q_\alpha\} \text{ if } \beta = 1 \\ [Q_{\alpha+\beta-2}, Q_{\alpha+\beta-1}] \text{ if} \\ \alpha \neq 1, \beta \neq 1, \alpha+\beta \leq M \\ \{Q_{M-1}\} \text{ if } \alpha+\beta = M+1 \end{cases}$$

In agreement with Axiom 3, it is necessary to have $\alpha + \beta \leq M+1$. Indeed, $A \cap B = \emptyset$ implies that $B \subset \bar{A}$. Now Axiom 3 gives $\Omega \subset_{n(\alpha)} \bar{A}$. Intuitively $\beta \leq n(\alpha)$ (for, $B \subset \bar{A}$) therefore, $\alpha + \beta \leq \alpha + n(\alpha) = M + 1$.

Defining **Inf** and **Sup** as two applications of Q^2_M into Q_M, we obtain respectively the lower bound and the upper bound of an interval of Q_M so we can write: $S(Q_\alpha,Q_\beta)$ = [Info **S** (Q_α,Q_β), SupoS (Q_α,Q_β)] or more simply [**InfS** (Q_α,Q_λ), **SupS** (Q_α,Q_β)]. We can prove that the applications **InfS** and **SupS** verify the properties of a T-conorm (commutativity, neutral element,monotony,associativity)

Definition 8: Given **S**, we can define the "symbolic subtraction "denoted D as an application of Ω^2_M into $\mathfrak{P}(\Omega_M)$ such that: if $Q_r \in S(Q_\alpha,Q_\beta)$, then $Q_r \in D(Q_r,Q_\alpha)$, and $Q_r \in D(Q_r,Q_\beta)$. Then **D** can be deduced from **S**:

$$\mathbf{D}(Q_r,Q_\beta) = \begin{cases} \{Q_r\} \text{ if } \beta = 1 \\ \{Q_2\} \text{ if } r = \beta \in [2, M-1] \\ [Q_{r+1-\beta}, Q_{r+2-\beta}] \text{ if} \\ 2 \leq \beta < r \leq M-1 \end{cases}$$

Remark 2: In this paper, for M = 7 we obtain the operators **S** and **D** defined by Tables 3 and 4 (see Annex).

Let A and B be subsets of **Ω**. The following properties can be viewed as symbolic generalization properties of classical statistical probabilities. The proofs of the properties can be found in ([21]).

Proposition 1: If "Q_α **Ω**'s are A's" and $A \subset B$, then "Q_α **Ω**'s are B's" with $Q_\alpha \leq Q_\beta$.

Proposition 2: If "Q_α **Ω**'s are A's", "Q_λ **Ω**'s are $(A \cap B)$'s" and $A \neq \Omega$, then "Q_λ **Ω**'s are $_\gamma A \backslash B$'s" with $Q_\gamma \in D(Q_\alpha, Q_\lambda)$.

Proposition 3: If "Q_α **Ω**'s are A's", "Q_β **Ω**'s are B's", "Q_γ **Ω**'s are $(A \cap B)$'s" and $A \cup B \neq \Omega$, then "Q_r **Ω**'s are $(A \cup B)$'s" with $Q_r \in U(Q_\alpha, Q_\beta, Q_\lambda)$ where $U(Q_\alpha, Q_\beta, Q_\lambda)$ = [InfS (Q_α, InfD (Q_β, Q_λ)), SupS (Q_α, SupD (Q_β, Q_λ))] if $\alpha + \beta - \lambda \leq M - 1$, and $U(Q_\alpha, Q_\beta, Q_\lambda) = \{Q_{M-1}\}$ if $\alpha + \beta - \lambda = M$.

Proposition 4: If "Q_α **Ω**'s are A's", "Q_β **Ω**'s are B's", "Q_r **Ω**'s are $(A \cup B)$'s" and $A \cup B \neq \Omega$, then "Q_λ **Ω**'s are $(A \cup B)$'s" with $Q_\lambda = Q_2$ if $\alpha + \beta - r = 1$ and $Q_\lambda \in$ [InfD (Q_β, SupD (Q_r, Q_α)), Inf {SupD (Q_b, InfD (Q_r, Q_a)), Q_a, Q_b}] otherwise.

5. Syllogistic Reasoning

Reasoning on quantifiers is called by Zadeh [36] *syllogistic reasoning*, where a syllogism is an inference rule that consists in deducing a new quantified statement from one or several quantified state-ments. As an inference scheme,a *syllogism* may generally be expressed in the form:

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ C's are D's

Q_{μ} E's are F's with $Q_{\mu} \in [Q_x, Q_y] \subseteq [Q_1, Q_M]$.

Where E and F are sets resulting from application of operators set on A, B, C or D, and bounds Q_x and Q_y are in accordance with $Q_{\mu 1}$ or $Q_{\mu 2}$.

The quantifier "All" is represented by the implication using the quantifier $\forall$ in classical logic or by the inclusion in set theory. The classical implication and the inclusion propagate inferences by transitivity, contraposition, disjunction or by conjunction. From one or several statements quantified by "All", these inferences enable to generate new statements likely quantified by "All". Nevertheless, most of these inferences are not valid for other quantifiers, i.e., for $Q_{\mu} \in [Q_2, Q_{M-1}]$. For example, from "Most A's are B's" and "Most B's are C's" one can not always have "Most A's are C's". That is due to the fact that the inference by transitivity is not valid for the quantifier "Most". The invalid inference has been considered as a case of total ignorance.

We present some syllogisms. Each of them is illustrated by an example. The proofs of these syl-logisms can be found in ([21]).

Proposition 5. (Relative Duality)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ A's are A \ B and $Q_{\mu 2}$ A's are $\underline{B}$'s

with $Q_{\mu 2} = Q_{n(\mu 1)}$ if $Q_{\mu 1} \neq Q_{n(\mu 1)}$

and $Q_{\mu 2} \in [Q_{n(\mu 1)}, Q_{n(\mu 1)+1}]$ otherwise.

Example 2: Almost all students are unmarried

Very few students are married.

Proposition 6: (Mixed Transitivity):

$Q_{\mu 1}$ A's are B's

All B's are C's

$Q_{\mu 2}$ A's are C's with $Q_{\mu 1} \leq Q_{\mu 2}$

Example 3

Most students are young (less than 25 years)

All young people are non retired

At least most students are non retired.

Proposition 7: (Exception)

Q_μ A's are B's

All C's are A's

All C's are $\overline{B}$

Q_γ A's are C, with $Q \in [Q_\mu, Q_{M-1}]$.

Example 4

Most birds fly

All ostriches are birds

All ostriches do not fly

Most or almost all birds are not ostriches.

Proposition 8: (Union Right)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ A 's are C's

Q_μ A's are $(B \cup C)$'s, with $Q_\mu \in [Q_{Max(\mu 1,\mu 2)}, Q_{M-1}]$.

Example 5

Most students are single

Very few students are taxable

Most or almost all students are single or taxable.

Proposition 9: (Intersection Right)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ A 's are C's

Q_μ A's are $(B \cap C)$'s,with $Q_\mu \leq Q_{Min(\mu 1,\mu 2)}$.

Example 6

Few salaried people are official

Most salaried people are taxable

At least most salaried people are taxable official.

Proposition 10: (Mixed Union Left)

Q_μ A's are C's

All B's are C's

Q_γ $(A \cup C)$'s are B's with $Q_\gamma \in [Q_\mu, Q_{M-1}]$.

Example 7

Most young people are single

All the catholic priests are single

Most or almost all young people or catholic priests are single.

Proposition 11: (Intersection / Product Syllogism)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ $(A \cap B)$'s are C's

$Q_{\mu 1}$ A's are $(B \cap C)$'s, with $Q_{\mu} = I(Q_{\mu 1}, Q_{\mu 2})$.

Example 8

Most students are young

Almost all young students are unmarried

Most students are young and unmarried.

Proposition 12: (Contraction)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ $(A \cap B)$'s are C's

Q_{μ} A's are C's with $Q_{\mu} = [I(Q_{\mu 1}, Q_{\mu 2}), Q_{Max(M-1, \mu 2)}]$.

Example 9

Most students are young

Almost all young students are unmarried

Most or almost all students are young and unmarried

Proposition 13: (Intersection/Quotient syllogism)

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ A 's are C's

$Q_{\mu 3}$ $(A \cap B)$'s are C's

Q_{μ} $(A \cap C)$'s are B's, with $Q_{\mu} \in C(Q_{\mu 2}, I(Q_{\mu 1}, Q_{\mu 3}))$.

Example 10

Most students are young

Most students have no salary

Almost all young students have no salary

Almost all no salaried students are young.

Proposition 14: (Weak Transitivity)

All B's are A's

$Q_{\mu 1}$ A 's are B's

$Q_{\mu 2}$ B's are C's

Q_{μ} A's are C's with $Q_{\mu} \in [I(Q_{\mu 1}, Q_{\mu 2}), Q_{M-1}]$.

Example 11

All salaried people are active

Most active people have no salary

Most salaried people are taxable

Q_{μ} active people are taxable with $Q_{\mu} \in$ [half, almost-all].

5.1 Syllogims with the Quantifier Almost-all

We present three syllogisms with the quantifier "Almost-all". They result from the axioms of quantifier "Almost-all" (Cf.Axiom 1, Axiom 2). These inferences can be viewed as counterparts[4] of inference rules of Adams [1], Pearl [29] and Bacchus et al. [5], where "Almost-all"is interpreted as proportion arbitrarily infinitesimal close to 1.Our symbolic approach leads to the following results which are in accordance with the previous ones, since we obtain a proportion Q_{μ} belonging to a subinterval of Ω_M containing the value "Almost all". It is clear that the meaning associated with the quantifier "Almost all" in our approach, as in the ones of Adams and Pearl, differs from the meaning that it receives in natural language. Indeed, a speaker does not refer to infinitesimal proportion close to 1, since linguistically speaking, "Almost all" is only included in "Most". As noted before, in our approach, "Most"and "Almost-all" define two different values of statistical proportions of Ω_M ,"Most" being less than "Almost all", which is very close to "All", but not included in "Most".

Corollary 15: (Contraction)

Almost-all A's are B's

Almost-all $(A \cap B)$'s are C's

Almost-all A's are C's.

Example 12

Almost all students are young

Almost all young students are single

Almost all students are single.

Proposition 16: (Cumulativity)

Almost-all A's are B's

Almost-all A's are C's.

$Q\mu$ $(A \cap B)$'s are C's, with $Q_{\mu} \in$ [Most, All].

[4] Pearl's approach is introduced for default reasoning, then his inferences are not exactly syllogisms,but they are rather non-monotonic inferences about particular individuals from defaults.

Example 13

Almost all students are young

Almost all students are single

At least most young students are single.

Proposition 17: (Union Left)

Almost-all A's are C's

Almost-all B's are C's.

Q_μ $(A \cup B)$'s are C's,with $Q_\mu \in$ [Most,Almost-all].

Example 14

Almost all students are single

Almost all priests are single

Most or almost all students or priests are single.

6. Symbolic Representation of Subjective Probabilities

Let us present the main ideas leading to this symbolic approach to subjective probability. Consider statements like "It is probable that the temperature is $\geq 30°$"or "It is very probable that Tweety flies". They have the following form "It is α probable that P" where P is a linguistic proposition, generally of the form "a is A", A a linguistic predicate, and α a scaling adverb. We can rewrite previous statement into the form: "P is α probable"(i.e. "a is A" totally-true) is α probable. Then, it seems convenient to consider that "probable" is representable by a multiset $\mathfrak{P}$ and hence that α stands for the degree to which "P is probable".

More formally,one is led to enrich our M-valued predicate logic by defining a pecular predicate denoted here **Cert**, and by adjoining the following rule of formula formation:

if ø is a formula such that, for any interpretation $\mathfrak{A}$ and any a in the domain Ω, $\mathfrak{A} \models_M ø(a)$ or A =1 ø(a) then **Cert** (ø (a)) is aformula.

In the following, we denote as C_2 the formulae of $\mathfrak{F}$ which are either totally true-in-A or not-at-all true-in-$\mathfrak{A}C_2 = \{ ø \in F \mid \forall a, \mathfrak{A}, \models_M ø(a)$ or $\mathfrak{A} \models_1 ø(a)\}$. This new predicate takes into account the notion of subjective probability of formulas of C_2.

Definition 9: The predicate **Cert** is formally introduced as follows:

- If $ø \in C_2$, then **Cert(ø)** $\in \mathfrak{F}$.
- Any interpretation $\mathfrak{A}$ associates with the predicate Cert the multiset $\mathfrak{P}$.
- Hence, for any ø C_2: $\mathfrak{A} =_\alpha$ Cert (ø (a)) $\Leftrightarrow$ < ø (a) > $\in$ > $\mathfrak{P} \Leftrightarrow$ Cert (ø (a)) is T_α – true – in – $\mathfrak{A}$.

Remark 3: Let U_M be the set of symbolic degrees of subjective probability: $U_M = \{u_\alpha, \alpha \in [1, M]\}$. The basic idea of the definition is to translate the symbolic degree of subjective probability of ø (a) in terms of symbolic degree of truth of the formula **Cert** (ø(a)). Then, we associate with each symbolic degree of truth of the formula **Cert(ø(a))** ,a symbolic degree of subjective probability (or certainty) of the formula ø (a). Therefore, we can write that: ø (a) is u_α-probable-in-A $\Leftrightarrow$ A $=_\alpha$ **Cert** (ø (a)) $\Leftrightarrow$ **Cert** (ø (a)) is T_α-true-in-A.

Remark 4: It is clear that the certainty degree of ø (a) is not its truth value (which is true or false since ø (a) is boolean) but the certainty degree associated with the truth degree of **Cert** (ø (a)).

Choice of subjective probability degrees linked to Ω_7

As we use 7 quantifiers, we introduce 7 symbolic degrees of subjective probability as follows:

U_7 = {Not-at-all-probable (or Impossible), Very-little- probable (or Almost-impossible), Little-probable, Moderately-probable, Very-probable, Almost-certain, Totally-probable (or Certain)}.

This being so,we can give in the next Section a symbolic generalisation of direct inference that allows us to connect our symbolic representation of statistical probabilities with this symbolic represen-tation of subjective probabilities.

7. Symbolic Direct Inference

Another alternative interpretation of statistical probabilities (different from the one related to the proportions of the sets of the domain) is related to the process of "random selection" which consists in selecting individuals from the domain according to their probabilities [4]. In this context, the quantified assertion "α % of individuals of the domain verify a property" can be interpreted as "the probability that a randomly selected domain individual satisfies the property is equal to α". This interpretation of quantified assertion can be seen as a way of justifying the deduction of uncertain conclusions about particular individuals (i.e., subjective probabilities) from statistical knowledge (i.e., statistical probabilities) via the direct inference ([31], [22], [30], [4]). Indeed,the principle of direct inference is based upon the idea that a particular individual in the domain is considered as a member randomly selected in a population, if no particular information distinguishes it from other members of this population.

For example, if all we know about Tweety is that it is bird,then Tweety can be viewed as a randomly selected member of the population of birds since we do not have any other information that distinguishes it from other birds. Then, knowing that Tweety is a bird,the (subjective) probability that Tweety flies is equal to the (statistical) probability that a bird randomly selected from the set of birds flies, i.e. the proportion of flying birds among the birds.

7.1. Symbolic Formalisation of Direct Inference

In the following,we propose a symbolic generalization of direct inference principle allowing us to infer a symbolic subjective probability degree from a symbolic statistical probability degree.Althought the two symbolic probability degrees are different, the generalization takes into account the fact that the two degrees are associated with the same truth degree of predicates **Prop** and **Cert** in a given interpretation. For example, if we know that "Most birds fly" and that all we know about Tweety is that it is a bird,then our direct inference mechanism has to lead to the conclusion that "it is very-probable that Tweety flies". More formally, in our model, considering an interpretation $\mathfrak{A}$, from "$\mathfrak{A} \models_5$ **Prop** (Fly(z) | Bird(z))" and "$\mathfrak{A}$ = Bird (Tweety)",we should deduce that "$\mathfrak{A} \models_5$ **Cert**(Fly(Tweety))".

Our main idea has been to extend previous definition of Bacchus's direct inference [4] using previous symbolic statistical probability theory (Section 3 and 4) and subjective probability theory (Section 5). So, we use M-valued language extended with the two particular predicates **Prop** and **Cert**, denoting the two notions of probability.

For a given M-valued interpretation $\mathfrak{A}$, we will suppose that the current knowledge base KB contains facts about particular individuals and quantified assertions. Then, KB can be formally represented in A as follows.

Definition 10: For a given interpretation A, **the available knowledge** base will be the couple KB = (W, $\mathfrak{R}$) where:

– W is the **conjunction of formulae** representing the **available knowledge about the particular individuals** of the domain,and

– $\mathfrak{R}$ is the set of formulas ($\mathfrak{A} \models_{\mu}$ **Prop** ($\psi|\varnothing$)) associated with the current **quantified assertions** (Q_{μ} A's are B's).

Example 15: Let us consider the classical example: "Tweety is a bird"and "Most birds fly".It can be formally represented by KB = (W, $\mathfrak{R}$) with: W = Bird (Tweety) and $\mathfrak{R}$ = {$\mathfrak{A}$ = 5 Prop (Fly(z) | Bird (z))}.

Definition 11: From the previous set $\mathfrak{R}$ characterizing quantified assertions and by using the **syllogistic inferences** (Section 5) one can deduce a set $\mathfrak{R}$* of formulae characterizing new quantified assertions. In particular, whe have $\mathfrak{R} \subset \mathfrak{R}^*$.

Example 16: Let us suppose that the available knowledge base contains: "Almost all students are young" and "Almost all students are single". Then $\mathfrak{R}$ = {$\mathfrak{A} \models_{6}$Prop (Young(z) | Student(z)), $\mathfrak{A} \models_{6}$Prop (Single(z) | Student(z)}. Then, by using the cumulativity syllogism, we can say that {$\mathfrak{A} \models_{\mu}$Prop (Single(z) | Student(z) $\cap$ Young(z)) with $Q_{\mu} \geq Q_{5}$} $\in \mathfrak{R}^*$. That says "At least most young students are single".

Definition 12: Given the available knowledge base **KB** = (W, $\mathfrak{R}$), **W (a)** will be the conjunction of formulas appearing in **W** mentionning a and ø (a) a formula of C_2. Moreover, we denote as ø (a | z), the formula obtained when textually substituting each occurrence of a in the formula ø (a) by the variable z. Thus, ø (a | z) $\in C_1$.

In particular, W (a | z) is obtained by substituting each occurrence of a in W (a) by z. Intuitively, this substitution denotes a form related to the process of "random selection". The constant a is considered as a "random member"by replacing it in W (a) by the variable z. This leads to suppose that the individual denoted by a is randomly chosen among the individuals sharing all its properties, i.e. the individuals satisfying W (a | z).

Let us now present the basic notions leading to our basic definition of **symbolic direct inference**. Given a knowledge base KB,we suppose that a denotes an **individual constant** of the domain Ω, and we suppose that a appears in W. The main object of the direct inference can be presented as follows: given an individual constant a, and ø (a) a formula of C_2, we research its certainty degree u_{α}-probable (or the true degree T_{α} of **Cert** (ø (a))) resulting from the available knowledge base KB = (W, R). This certainty degree (or subjective probability degree) is defined in the following way:

Definition 13: (Direct Inference Principle)

By using previous notations,a denotes an individual constant of the domain, z a variable, and KB = (W, R) the available knowledge base. Given a formula ø (a) of C_2, the degree u_α-probable of the subjective probability of ø (a) results from the following **direct inference principle**. If there exists a syllogism such that ($\mathfrak{A} \models_\alpha$ **Prop** (ø (a | z)| W (a | z))) ∈ $\mathfrak{R}$*, then ø (a) will be said u_α-probable, i.e. $\mathfrak{A}$ = $_\alpha$**Cert** (ø (a)).

Formally,it means that the truth degree of **Cert** (ø (a)), associated to the symbolic degree u_α of the subjective probability of ø (a), is the same truth degree as Prop (ø (a |z) | W (a | z)), associated to the symbolic degree Q_α of the proportion of individuals satisfying ø (a | z) among the ones satisfying W(a | z). So, the symbolic meaning does not concern the equality (as, in numerical approaches of the two degrees (since they are symbolicly different) but on their association to the same truth degree of **Prop** (ø (a | z) | W (a | z)) and Cert (ø (a)).

Example 17: Let us consider the previous example: "Tweety is a bird" and "Most birds fly".We have: KB = (W, $\mathfrak{R}$) with W = Bird (Tweety) and R = {$\mathfrak{A} \models_5$Prop (Fly (z) | Bird (z))}. The symbolic degree of certainty of "Fly(Tweety)" is deduced as follows: W (a) = W (Tweety) = Bird (Tweety) (which is totally-true in-$\mathfrak{A}$), W (a | z) = Bird (z), ø (a) = Fly (Tweety), ø (a | z) = Fly (z). Since Prop (Fly (Tweety | z) | Bird (Tweety | z)) is equal to Prop (Fly (z) | Bird (z)), we have ($\mathfrak{A} \models_5$ Prop (Fly (Tweety |z) | Bird (Tweety | z))) ∈ $\mathfrak{R}$*. In other words, the Direct inference principle gives us: $\mathfrak{A} \models_\alpha$**Cert** (Fly (Tweety)). This means that "it is very probable that Tweety flies".

Example 18: Let us add to the base of example, "Tweety is a penguin", "All penguins are birds" and "Few penguins fly". Then, we have: W (Tweety) = Bird (Tweety) ∩ Penguin (Tweety) and $\mathfrak{R}$ = {$\mathfrak{A} \models_5$ Prop (Bird (z) | Penguin (z)), $\mathfrak{A} \models_5$Prop (Fly(z) | Bird(z)), $\mathfrak{A} \models_3$ Prop (Fly(z) |Penguin(z))}. By using the syllogism of mixed cumulativity[5] from:{$\mathfrak{A} \models$ Prop (Bird(z) |Penguin(z)), $\mathfrak{A} \models_3$Prop (Fly(z) | Penguin(z))}, we deduce that: $\mathfrak{A} \models_3$ Prop (Fly(z) | Penguin(z) ∩ Bird(z)). Applying direct inference leads to deduce the symbolic degree of certainty of "Fly (Tweety)" as follows: Since, we have: ($\mathfrak{A} \models_3$Prop (Fly (Tweety | z) | Penguin (Tweety | z) ∩ Bird (Tweety | z))) ∈ $\mathfrak{R}$*, then we obtain: $\mathfrak{A} \models$3 Cert (Fly (Tweety)), that is to say "it is little proba-ble that Tweety flies".

Example 19: Let us consider the following example proposed in [11]: "Most native speakers of German are not born in America", "All native speakers of Pennsylvanian Dutch are native speakers of German", "Most native speakers of Pennsylvanian Dutch are born in Pennsylvania", "All people which are born in Pennsylvania are born in America" and "Hermann is a native speaker of Pennsylvanian Dutch". The knowledge base is made of W (Hermann) =P Dutch-speak (Hermann) and $\mathfrak{R}$ = {$\mathfrak{A} \models_5$ Prop (¬ America(z) | German-speak (z)), $\mathfrak{A} \models$Prop (German-speak(z) | P Dutch-speak (z)), $\mathfrak{A} \models$5 Prop (Pennsylv(z) | P Dutch-speak (z)), $\mathfrak{A} \models$Prop (America(z) | Pennsylv(z))}. Applying the syllogism of mixed transitivity, we have {$\mathfrak{A} \models_5$ Prop (Pennsylv(z) | P Dutch-speak(z)), $\mathfrak{A} \models$Prop (America(z) | Pennsylv(z))}, R* containing ($\mathfrak{A} \models_\mu$Prop (America (Hermann | z) | P Dutch-speak (Hermann | z)) with $Q_\mu \geq Q_5$) = ($\mathfrak{A} \models_\mu$Prop (America(z) | P

[5] that is {"All A's are B's", "Q_α A's are C's"} ⇒ "Q_α (A ∩ B)'s are C's".

Dutch-speak(z))). Then, the direct inference leads to deduce $\mathfrak{A} \models_{\mu}$America (Hermann), with $Q_{\mu} \geq Q_5$. In other words, it is at least very probable that Hermann is born in America.We can note that, there is no syllogism allowing to deduce a syllogism from the quantified assertions $\mathfrak{A} \models_5$ Prop (¬ merica(z) | German-speak (z)) and $\mathfrak{A} \models$Prop (German-speak(z) |P Dutch-speak (z)).

Definition 14: Given the available knowledge base KB = (W, R) and a an individual constant, a **reference class of a given KB** for a formula ø (a) (in which one wants to generate a certainty degree) is a subset of the domain Ω to which belongs the individual a. More particularly, **W (a | z) characterizes a reference class of individual a given KB.**

Remark 5: It is important to point out that this direct inference principle is not always applicable: It is the case when we do not have any meaningful information for the reference class W (a | z), i.e. we have $\mathfrak{A} \models_{[1,M]}$**Prop** (ø (a | z) | W (a | z)) (that is a case of total ignorance). Moreover, in some cases one can be confronted to the existence of conflictual reference classes. The next section is devoted to this conflictual situation.

7.2 Choice of a Reference Class

One is often confronted to the existence of conflictual reference classes.One can distinguish three conflict types:

- Conflict between less and more specific classes,
- Conflict between classes associated with less and more precise information,
- Conflict between incomparable classes.

To solve the first and the second conflict type, we are going to modify the basic definition of the direct inference by a symbolic formalization of the specificity rule of Reichenbach [31] and the strength rule of Kyburg [22]. For the third type, we are going to propose a combination function of symbolic degrees associated to incomparable reference classes.

7.2.1. Formalization of the Specificity Rule

The specificity rule of Reichenbach consists of choosing among reference classes, the smallest (specific)class for which we have meaningful information. So, given a knowledge base translated in KB = (W, R). We propose a symbolic formalization of the specificity rule allowing us to infer the certainty symbolic degree in ø (a) from KB, by choosing information associated to the smallest reference class designed by W'(a | z), when we ignore the degree as-sociated to W(a | z).

Definition 15: (Specificity Rule) Let us suppose that: KB = (W, $\mathfrak{R}$). The **specificity rule** allows us to infer "$\mathfrak{A} \models_{\alpha}$Cert (ø (a))" if the three following conditions are satisfied:

1) We have ($\mathfrak{A} \models_{[1,M]}$Prop (ø (a | z)| W(a | z))),

2) ∃ W'(a |z) such that: $\mathfrak{R}$* containing ($\mathfrak{A} \models_{\alpha}$Prop (W' (a | z) | W(a | z))), ($\mathfrak{A} \models_{\alpha}$Prop (ø (a | z)| W'(a | z))),

3) ∄ W" (a | z) such that: $\mathfrak{R}$* containing (A $\models$Prop (W'(a | z) |W" (a | z)))), $\mathfrak{A} =_{\beta}$Prop (ø (a | z)| W" (a | z))).

Intuitively, the three conditions above express respectively:

1) We do not have any meaningful information for the reference class W (a|z). Otherwise,the basic definition of the direct inference is applicable.

2) The existence of a reference class W' (a|z) for which we possess a meaningful information.

3) There is not a smaller reference class that W' (a|z) for which we possess a meaningful infor-mation.

Remark 6: Let us suppose that $W(a) = W'(a) \cap W''(a)$. The specificity rule allows us to consider that the information W" (a) is no relevant for the derivation of the certainty degree in ø (a).

Example 20: Let us suppose that we have the fol-lowing knowledge base:

- Most elephants are gray: $\mathfrak{A} \models_5$ Prop (Gray(z) | Elephant(z)),
- Few royal elephants are gray: $\mathfrak{A} \models_3$Prop(Gray(z) | Elephant(z) $\cap$ Royal(z)),
- Clyde is an African royal elephant: $\mathfrak{A} \models$ Elephant (Clyde) $\cap$ Royal (Clyde) $\cap$ African (Clyde).

We have two reference classes having meaningful information, for Gray (Clyde): Elephant(z) and Elephant(z) $\cap$ Royal(z). This last is the smallest class, because we have: ($\mathfrak{A} \models$ Prop (Elephant (Clyde|z) | Elephant (Clyde|z) $\cap$ Royal (Clyde|z))) $\cap \mathfrak{R}^*$. Applying the specificity rule, we obtain: $\mathfrak{A} \models_3$Cert (Gray (Clyde)), i.e., "it is little probable that Clyde is gray".

7.2.2. Formalization of the Strength Rule

The strength rule of Kyburg [22] is used in the cases where information associated with reference classes are intervals.It considers that a class of reference is better than another one, if associated information is more precise than one associated with the other. In our symbolic context, we put the following definition:

Definition 16: (Strength Rule) Given a knowledge base KB = (W, R). Then, the **strength rule**, allows us to derive that: $\mathfrak{A} \models_\alpha$Cert (ø (a)) with $u \in [u_c, u_d]$, if the following conditions are satisfied:

- $\mathfrak{R}^*$ containing ($\mathfrak{A} \models_{\alpha 1}$Prop (ø (a | z) | W (a | z))) with $Q_{\alpha 1} \in [Q_a, Q_b]$),
- W'(a |z)is a reference class,
- $\mathfrak{R}^*$ containing ($\mathfrak{A} \models_{\alpha 2}$Prop (ø (a | z) | W ' (a | z))) with $Q_{\alpha 2} \in [Q_c, Q_d]$), and $[Q_c, Q_d]$ $[Q_a, Q_b]$).

Remark 7: The priority between the two rules is given by the strength rule. Therefore, the specificity rule can be applied when the the strength rule condition is not verified.

7.2.3 Incomparable Reference Classes

The reference classes can be incomparable, i.e., neither the specificity rule nor the strength rule can be used. A solution can be proposed in this case like in [5]: the certainty degree results from a combination of certainty degrees associated with the incomparable reference classes.

Definition 17: A combination function **Comb** is an application of U_M^2 into U_M possessing the following properties:

[Cb1] $\forall\alpha, \beta \in [2.. M]$, Comb (u_α, u_β) = Comb (u_β, u_α),

[Cb2] $\forall\alpha, \beta \in [2.. M]$, Comb $(u_\alpha, u_\beta) = \in (u_{max(\alpha, \beta)}, u_{min(\alpha, \beta)})$,

[Cb3] $\forall\alpha, \in [2.. M]$, Comb $(u_\alpha, u_M) = u_M\ (u_{min(\alpha, \beta)}]$, u_M is an absorbent element for any $\alpha \in [2.. m]$.

[Cb4] $\forall\alpha, \in [1.. M-1]$, Comb $(u_\alpha, u_1) = u_1$ is an absorbent element for any $\alpha \in [1.. m]$.

[Cb5] $\forall\alpha, \in [2.. M-1]$, Comb $(u_\alpha, u_{n(\alpha)}) = u_4$ Conflict related to the ambiguity

[Cb6] $\forall\alpha, \beta, \delta \in [1.. M]$, Comb (Comb (u_α, u_β), $u_\delta =$ Comb $(u_\alpha,$ Comb $(u_\beta, u_\delta))$, Associativity

[Cb7] $\forall\alpha, \beta, \delta \in [2.. M-1]$, Comb $(u_\alpha, u_\beta), = u_\delta \Rightarrow$ Comb $(u_\alpha, u_{\beta+1}) \in (u_\delta, u_{\delta+1})$, Monotonicity.

Choice of Comb: We can choose the function **Comb** as follows: $\forall\alpha, \beta \in [2.. M-1]$:

$$\textbf{Comb}(u_\alpha, u_\beta) = \begin{cases} u_{\lfloor(\alpha+\beta)/2\rfloor} & \text{if } \alpha+\beta \leq M \\ u_{\lceil(\alpha+\beta)/2\rceil} & \text{if } \alpha+\beta > M \end{cases}$$

where $\lfloor r \rfloor$ (resp. $\lceil r \rceil$) denotes the greatest integer lower than (resp. lowest integer greater than) or equal to r.

It is clear that the following table corresponds with the function **Comb**.

Comb	u_1	u_2	u_3	u_4	u_5	u_6	u_7
u_1	u_1	u_1	u_1	u_1	u_1	u_1	
u_2	u_1	u_2	u_2	u_3	u_3	u_4	u_7
u_3	u_1	u_2	u_3	u_3	u_4	u_5	u_7
u_4	u_1	u_3	u_3	u_4	u_5	u_5	u_7
u_5	u_1	u_3	u_4	u_5	u_5	u_6	u_7
u_6	u_1	u_4	u_5	u_5	u_6	u_6	u_7
u_7		u_7	u_7	u_7	u_7	u_7	u_7

Definition 18: Given a knowledge base KB = KB = (W, $\mathfrak{R}$) for which we have:

W(a) $= A_1(a) \cap \ldots \cap A_n(a)$, with $n \geq 2$; R* containing $((\mathfrak{A} \models_{\alpha 1}$Prop (B (z) $|A_1(z)))$, ..., $(\mathfrak{A} \models_{\alpha n}$Prop (B (z) | An (z))) where $\exists\, \alpha_i \in [1..n]$, $\exists\, a_j \in [1..n]$ such that $\alpha_i = 1$ and $\alpha_j = M$).

The classes referred by $A_i(z)$ being incomparable, the certainty degree u_α such that $\mathfrak{A} \models_\alpha$Cert (B(a)) results from a combination, using the function **Comb**, of the certainty degrees $u_{\alpha i}$ associated with these classes.

Example 21: Given the following knowledge base (the Nixon Diamond):

Almost all Quakers are pacifist

Almost all republicans are not pacifist

Nixon is a republican Quaker.

Then, W = Q (Nixon) $\cap$ R (Nixon),

$\mathfrak{R} = \{\mathfrak{A} \models_6$Prop (P(z)|Q(z)), A $\models_6$Prop $(\neg$P(z)|R(z))$\}$.

From $\mathfrak{A} \models_6$Prop $(\neg$P(z)|R(z)) $\in$ R, the relative duality syllogism implies $\mathfrak{A} \models_2$Prop (P(z)|R(z)) $\in \mathfrak{R}^*$. The two reference classes having an information to determine the certainty symbolic degree of P(Nixon) are classes Q(z) and R(z). This conflict between the two classes relates to the ambiguity (Property [Cb5]). By using the function Comb, we get Comb $(u_6, u_2) = u_4$. Therefore, we obtain: "$\mathfrak{A} \models_4$Cert (P(Nixon))", i.e., "it is moderately probable that Nixon is pacifist". Thus, the derived certainty symbolic degree of P(Nixon) and one of its negation are the same, i.e. none of these two conclusions is more probable than the other.

That is a satisfactory result,since in terms of specificity, none should be preferred to the other.

Example 22: Given the following knowledge base:

S1: Very few students are salaried

S2: All students are adult

S3: Most adults are salaried

S4: Most people of the active population are salaried

S5: Between half and almost all taxable people are salaried

W(Paul) = Student (Paul) $\cap$ Adult (Paul) $\cap$ Active (Paul) $\cap$ Taxable (Paul)

We look for the certainty symbolic degree of Salar (Paul).We have:

Student $\subset$ Adult $\Rightarrow$ "Student" is more specific that "Adult".

{Most} $\subset$ [Half, Almost-all] $\Rightarrow$ "Active" is more stronger that "Taxable".

The classes "Student" and "Actives" are incomparable, therefore, we use certainty degrees combination:

Comb (Very-little-probable, Very-probable) = Little-probable, i.e., one obtains "It is little probable that Paul is salaried".

Remark 8: The probabilistic non-monotonic reasoning does not suppress conclusions, as it is the case in non-monotonic approaches. It preserves the conclusions, but it decreases or increases its probability value. The certainty symbolic degree inferred by this principle can be revised,when new information is added. Indeed, the added information can imply a new reference class or a new combination. Therefore, the certainty symbolic degree can increase or decrease.

8. Comparison with other Approaches

We can justify the correctness of our approach to linguistic quantification and the soundness of our results. The basic notions defining (1) the representation of linguistic modifiers, and (2) the deductive process dealing with quantified assertions results from the papers of Bacchus [4] and Bacchus et al.[5]. There are three levels where we can com-pare our approach to the one proposed by Bacchus. The first level concerns the representation of statistical quantified assertions,the second level deals with syllogistic reasoning and the last level is the process of direct inference.

8.1 Representation

At the representational level, Bacchus extends first-order classical logic by introducing a new operator (denoted by "[]")in order to define numerical proportions. In our approach, unless to add a new operator, we have introduced in our M-valued symbolic logic a new predicate (that can be viewed as an element of a metalogic) and we have given the axioms governing it.This can be interpreted (see paragraphs 3,4 and 5) as a symbolic generalization of classical absolute and conditional statistical probabilities (proportions).

About the quantifiers, Bacchus' framework is defined upon statistical assertions using numerical values but it is only used for the symbolic values "majority" (which is interpreted as a proportion > 0.5)and "minority". Bacchus call his quantifier expressing the majority "most". So, he only focuses on one linguistic quantifier (and its dual one) defining typicality. In our work, the aim is different since we do not want to represent the notion of majority but to capture the whole set of symbolic proportions. That's why we use several different quantifiers (seven in this paper) which describe a scale of quantifiers. Clearly, this scale defines a set of symbolic quantifiers which can be seen as a refinement of the symbolic quantifiers used by Bacchus.

8.2 Syllogistic Reasoning

If we focus now on the syllogistic reasoning, we can verify that our approach leads to find similar syllogisms as the ones that can be found with Bacchus' proposal.

If Q is associated with a numerical value (or a numerical interval [a, b]) then in Bacchus's approach,we can give the following syllogisms:

- 1 - Mixed Transitivity

Q A's are B's

1 B's are C's (1 is equivalent to 100 % or "All")

[Q, 1] A's are C's (i.e. Q'A's are C's with $Q' \geq Q$).

Our approach gives the same result (Cf. Section 5):

$Q_{\mu 1}$ A's are B's

All B's are C's

$Q_{\mu 2}$ A's are C's with $Q_{\mu 2} \geq Q_{\mu 1}$.

- 2 - Intersection/Product syllogism

Q1 A's are B's

Q2 $(A \cap B)$'s are C's

Q1*Q2 A's are $(B \cap C)$'s (where *stands for the multiplication operator)

Our approach gives a similar result (Cf.Section 5), since the operator **I** stands for an operator having in $\mathfrak{L}_M$ the properties of a multiplication operator [28].

$Q_{\mu 1}$ A's are B's

$Q_{\mu 2}$ $(A \cap B)$'s are C's

Q_{μ} A's are $(B \cap C)$'s,with $Q_{\mu} = I(Q_{\mu 1}, Q_{\mu 2})$.

It is easy to verify that propositions 16 to 19, and 21 to 23, of Section 6 lead to similar results.

It is clear that they correspond to the same syllogisms since, for each syllogism, the resulting assertion is the same and the quantifier is obtained is the same way in the numerical and in the symbolic setting (that is using the same combination of the operators).

Moreover, the operators **C** (division), **I** (product), **S** (addition), **D** (difference) are the symbolic counterparts of the four classical operators (see Annex A and paragraph 4). The operators defined for the symbolic setting respect the properties of classical operator such as, depending on the considered operator, associativity, existence of a neutral ele-ment, commutativity ,monotony,...

The behavior of our syllogistic reasoning when dealing with precise values is then depending on the symbolic operators used for the syllogism. The question is to verify that they are in accordance with the classical operators used in a numerical setting.

The problem is that it is not possible to prove that the symbolic operators are in total accordance with numerical operators. Indeed, it does not exist an isomorphism between the numerical and the symbolic settings (as shown, for example, by Kaufmann [14]). So, it is not possible to give an interface between numerical and symbolic quantifiers allowing to compare the behaviors of the two different systems.

Finally, let us say that, on one hand, Bacchus' proposal is adapted when the given data are expressed with precise values but it would not be suitable when the informations are symbolic. On the other hand, our work is useful when there do not exist precise values but when the information are only expressed in terms of symbolic values (which is suitable with the initial aim of our work).

8.3 Comparison with Bacchus direct inference

As noted before, Bacchus is interested in representing the quantifier "most" (denoting the majority). In our approach ,this quantifier can be represented either by the quantifier "most", either "almost-all", either "all" that corresponds to "at least most". Then, we obtain results like the ones found in Bacchus. In the set of examples used by Bacchus, we do not give here the simplest ones but we focus on the most important ones.

First, let us take the well-known Nixon Diamond: "Most Quakers are pacifist", "Most republicans are not pacifist", "Nixon is a republican Quaker". Bacchus do not decide if Nixon is pacifist or not. In our framework,the same result is obtained (see Section 8.2.3, example 20 by using quantifiers Q5 instead of Q6 in order to use "Most" instead of "Almost-all") since we deduce both that it is moderately probable that Nixon is pacifist and it is moderately probable that Nixon is not pacifist.

Another example is the following one: "Most native speakers of German are not born in America", "All native speakers of Pennsylvanian Dutch are native speakers of German", "Most native speakers of Pennsylvanian Dutch are born in Pennsylvania", "All people which are born in Pennsylvania are born in America" and "Hermann is a native speaker of Pennsylvanian Dutch". In his paper, Bacchus deduces that the probability that Hermann is born in America is > 0.5, that is to say it is probable that Hermann is American. In our framework, we find (see Example 18) that it is very probable that Hermann is born in America. The two results are in accordance.

So ,the results found in our framework are in accordance with the ones found with Bacchus' direct inference.

The notions of reference classes used in our paper are based on the same strategies used by Bacchus. Hence, we propose a definition of specificity rule and strength rule that is in accordance with the ones proposed by Bacchus. Moreover, for dealing with incomparable classes, we have introduced a combination function (Section 6.2.2).

8.4 Comparison with Bacchus et al. s direct inference

The comparison with the work proposed by Bac-chus et al. needs a preliminary clarification.We have to notice that the notion of direct inference proposed in their paper differs from the one we use (and from the one proposed by Bacchus in his previous work).

Their aim is not the syllogistic reasoning but the default reasoning with defining an inference rule verifying postulates of rational inference relation. Their definition of direct inference is defined on the semantic of random worlds.

In this work, they introduce a new operator to deal with quantifiers of the form "approximately x%". This allows to represent the quantifier "Almost-all",expressing a default rule,by "approximately 1". Their approach implies choice strategies of reference classes and a combination function of information associated with incomparable refer-ence classes.

Then, it is possible to compare their "numerical" results with the ones obtained with our approach. As for the previous section, we only give here the comparison for one example (the one they use to explain the combination function). For "The Nixon Diamond "problem, our approach leads to "it is moderately probable that Nixon is pacifist" (see Example 20). By putting, the quantifier associated with "Most Quakers are pacifists" and approximatively equal to 1 and the quantifier associated to "Most republicans are pacifists" approximatively equal to 0, they obtain the value 0.5 (when considering that the rate of exceptions is the same for each default). The deductions are in accordance for the two frameworks.

So, as far as the comparison between the different frameworks is possible, we can say that the results we find are in accordance with the results found by Bacchus and Bacchus et al.

9. Conclusion

In this paper we have firstly presented a symbolic approach to quantifiers used in the natural language to express a qualitative evaluation of proportions. This approach allows us to reason qualitatively on quantified assertions,since we provide inference rules based upon statements involving linguistic quantifiers. Moreover, in order to obtain belief symbolic degrees attached to properties about particular individuals, and this, by using knowledge based upon quantified assertions and certain facts, we have also proposed a symbolic model based upon a direct inference principle and a choice of the appropriated reference class. It appears that the main contribution of our approach to the management of incomplete information, expressed through quantified assertions, results from the fact that we have clearly distinguished, (1) reasoning with a quantifier, which concerns the particular use of a statis-tical quantifier Q_α applied to two subsets A and B of discourse universe Ω satisfying "Q_α A's are B's", from (2)reasoning with particular indivduals. Since, in our approach, reasoning on particular individuals constitutes a non monotonic rea-soning process, it will be interesting to verify that the properties associated with our process fullfil the basical postulates of a non monotonic relation, like the ones defining System P [23]. This point is actually on study.

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Annex: Tables of operators C, I,S and D

Remark 9: In the following tables,$Q_{a,b}$ stands for interval $[Q_a,Q_b]$

Table 1: Operator C

C	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Q_1	$Q_{1,7}$	0	0	0	0	0	0
Q_2	$\{Q_1\}$	$Q_{2,7}$	0	0	0	0	0
Q_3	$\{Q_1\}$	$Q_{2,5}$	$Q_{6,7}$	0	0	0	0
Q_4	$\{Q_1\}$	$Q_{2,4}$	$\{Q_5\}$	$Q_{6,7}$	0	0	0
Q_5	$\{Q_1\}$	$Q_{2,3}$	$\{Q_4\}$	$\{Q_5\}$	$Q_{6,7}$	0	0
Q_6	$\{Q_1\}$	$\{Q_2\}$	$\{Q_3\}$	$\{Q_4\}$	$\{Q_5\}$	$Q_{6,7}$	0
Q_7	$\{Q_1\}$	$\{Q_2\}$	$\{Q_3\}$	$\{Q_4\}$	$\{Q_5\}$	$\{Q_6\}$	$\{Q_7\}$

Table 2: Operator I

I	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1	Q_1
Q_2	Q_1	Q_2	Q_2	Q_2	Q_2	Q_2	Q_2
Q_3	Q_1	Q_2	Q_2	Q_2	Q_2	Q_3	Q_3
Q_4	Q_1	Q_2	Q_2	Q_2	Q_3	Q_4	Q_4
Q_5	Q_1	Q_2	Q_2	Q_3	Q_4	Q_5	Q_5
Q_6	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
Q_7	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7