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Sparse Equalizers for OFDM Signals With Insufficient Cyclic Prefix

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ABSTRACT The cyclic prefix (CP) is appended in orthogonal frequency division multiplexing (OFDM) signals to combat inter-symbol interference (ISI) and inter-carrier interference (ICI) induced by the communication channel, which limits its spectral efficiency. Therefore, inserting an insufficient CP and equalizing the resulting ICI and ISI is a method that has been circulating the literature for a while, aiming at increasing the efficiency of OFDM systems. In this paper, we propose a reduced-complexity sparse linear equalizer and a decision-feedback equalizer for OFDM signals with insufficient CP. A performance-complexity trade-off is highlighted, where we show that it is possible to equalize the received signal with a reduced complexity equalizer while having a limited performance loss. Our proposed equalizer designs are not only less complex to realize, but are shown to provide a higher data rate. The proposed equalizers are further evaluated in terms of the worst-case coherence, a metric determining the effectiveness of our used approach. Numerical results show that we can significantly and reliably reduce the order of the design complexity while performing very close to the conventional complex optimal equalizers.

INDEX TERMS OFDM, sparse approximation, linear equalizers, decision feedback equalizers, insufficient cyclic prefix, worst-case coherence.

I. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM) signaling scheme was adopted in various communication standards due to its robustness against frequency selective channels and the ease of its modulation/demodulation using the inverse fast Fourier transform/fast Fourier transform (IFFT/FFT) algorithm, making it a strong candidate for the upcoming fifth-generation (5G) wireless communication standards [1]–[3]. The cyclic prefix (CP) overhead in OFDM can be significant, especially for long range transmission. For instance, in LTE/LTE Advanced [4], the extended CP length represents 25% of the useful data transmission time. Thus, by reducing the CP length, significant improvement in bandwidth efficiency can be obtained. The reduced complexity modulation/demodulation of OFDM is based on nulling the inter-symbol interference (ISI) and inter-carrier interference (ICI) by the insertion of a CP at the beginning of the OFDM symbol with a duration that is greater than the delay spread imposed by the channel. However, the reduction in the complexity is gained at the expense of the inefficient usage of the time available to communicate data and power consumption. The loss of time and power resources can be quantified by the fraction $\frac{v}{v+N}$, where v is the length of the inserted CP and N is the number of OFDM subcarriers [5]. One can increase the efficiency of OFDM systems by increasing N or reducing v, but this is generally not practical since N is usually constrained in many communication standards by a fixed and relatively small number, and v is chosen such that it is greater than or equal to L, which is the length of the delay spread in samples that is imposed by the channel condition.

Plenty of research articles proposed the insertion of a CP with a length of v, where $v < L$, and design various equalization techniques to rectify the loss of performance caused by the insertion of a short CP. In [6] and [7], the modeling of the ISI and ICI resulting from inserting an insufficient CP has been analyzed, along with an implementation of a zero-forcing (ZF) decision feedback equalizer (DFE) equalizer to compensate for the effects of ISI and ICI. The equalizer assumes a negligible noise at the receiver, which causes a performance degradation if the receiver noise is high, and hence in [8], a minimum mean squared error (MMSE) DFE which inherently considers the receiver noise has been proposed along with forward error correction (FEC) to improve the performance of the equalizer. Furthermore, time-domain equalizers (TEQ) have been proposed with the aim of impulse response shortening (IRS), i.e. shortening the overall impulse response (OIR), which is the convolution of the impulse response of the equalizer and the channel, conditioned on maximizing different received signal evaluation metrics such as the bit-error rate (BER) or the signal-to-interference-plus-noise ratio (SINR) [9], [10], and in [11]–[13], TEQ is applied to shorten, implicitly or explicitly, the effective length of the channel impulse response (CIR). However, even if the introduction of an equalizer compensates for the loss of performance, this compromises the simplicity of the OFDM transceiver design, and hence its utility. Recently, a channel independent interference nulling scheme using precoding for multiple-input multiple-output (MIMO)-OFDM systems with insufficient CP was introduced in [14], and furthermore, iteratively estimating the CIR and using a trellis-based equalization scheme has been proposed for MIMO-OFDM systems in [15] and [16].

Moreover, sparse approximations of finite impulse response (FIR) channel equalizers have been investigated due to their significant complexity reductions while still performing close to non-sparse optimal equalizers. For example, in [17], a framework for designing sparse FIR equalizers is proposed using greedy algorithms, and an interesting application of self far-end crosstalk cancellation was shown for systems adopting discrete multitone modulation, while in [18], a reduced complexity sparsest FIR equalizer was derived for single-carrier linear channel equalizers. Al-Abbasi *et al.* [18] extended their work for the design of sparse FIR MIMO linear and decision-feedback equalizers [19]. However, to the best of the authors' knowledge, no work has previously addressed a sparse equalizer design for OFDM signals suffering from ICI and ISI due to the insertion of an insufficient CP.

In this paper, we propose a reduced-complexity time-domain sparse DFE equalizer for OFDM signals with insufficient CP. We design the equalizer to offer a complexity-performance trade-off by either setting a desired number of active entries or by targeting a specific amount of loss in terms of SINR. We further investigate the worst-case coherence (WCC) metric of our sparsifying dictionary and show by simulations that it is suitable for designing our sparse equalizers with more likelihood of successfully retrieving the non-zero entries. Simulations show that our proposed equalizer preforms closely to the optimal equalizer while significantly reducing the design complexity. We note that the novelty of our proposed equalizer lies in the fact that our design provides flexibility in tuning the OFDM system in terms of: (i) Rate gains over conventional CP designs depending on the delay spread imposed by the wireless channel and (ii) Complexity reductions when compared to the conventional equalization techniques previously presented in the literature. Therefore, by allowing such flexibilities, OFDM will be well adapted to different 5G requirements by allowing the CP length and the design complexity to be adjustable, rather than designed for essentially the worst-case multi-path delay spread or adapting a complex design approach. Interestingly, a similar system was envisioned in [20].

Organization: The rest of the paper is organized as follows. Section II presents the system model. Section III presents the derivations of the reduced-complexity LE and DFE under different constraints while also discussing the complexity and feasability of the proposed schemes. Section IV presents the numerical results, and finally section V concludes the paper.

Notations: Bold upper case letters denote matrices, and bold lower case letters denote vectors. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix, $\mathbf{0}_N$ and $\mathbf{0}_{N\times 1}$ denote an all zeros $N \times N$ matrix and all-zeros $N \times 1$ vector, respectively. For vectors and matrices, $(.)^T$ and $(.)^H$ denote the transpose and Hermitian transpose operations, respectively. The symbols $\mathbb{E}(.)$, Tr(.), vec(.), $\perp$ and $||.||_p$ denote the expectation, trace, vectorization, statistical independence and p^{th} norm operations, respectively.

II. SYSTEM MODEL

We consider a single-input single-output OFDM system with N symbols. The transmitted data symbols are assumed to be independent and the channel is assumed to be known and static at least over one OFDM symbol. Due to the insertion of a short-lengthed (insufficient) CP, the received signal $\mathbf{y}_t \in \mathbb{C}^{N\times 1}$ at time t is impaired by ICI and ISI, and can be expressed by writing

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t - \mathbf{A}\mathbf{x}_t + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{n}_t, \tag{1}$$

where $\mathbf{x}_t \in \mathbb{C}^{N\times 1}$ is the data vector with $\mathbb{E}\{\mathbf{x}_t\mathbf{x}_t^H\} = \sigma_x^2\mathbf{I}_N$, and $\mathbf{x}_t \perp \mathbf{x}_{t-1}$. Further, $\mathbf{H} \in \mathbb{C}^{N\times N}$ is the time-domain circulant channel matrix where its first column is the zero-padded CIR, and L denotes the number of CIR taps. The matrices $\mathbf{A} \in \mathbb{C}^{N\times N}$ and $\mathbf{B} \in \mathbb{C}^{N\times N}$, respectively, represent the ICI and ISI effects [8]. We denote the length of the CP by v, where the matrices $\mathbf{A}$ and $\mathbf{B}$ are non-zero matrices if and only if $v < L$, in which case they can be written as

$$\mathbf{A} = \begin{bmatrix} 0 & \cdots & h_{L-1} & \cdots & \cdots & h_{v+1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & h_{L-1} & \cdots & h_{v+2} & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & 0 & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & h_{L-1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \tag{2}$$

and

$$\mathbf{B} = \begin{bmatrix} 0 & \cdots & 0 & h_{L-1} & \cdots & \cdots & h_{v+1} \\ 0 & \cdots & 0 & 0 & h_{L-1} & \cdots & h_{v+2} \\ \vdots & \ddots & \vdots & \ddots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & 0 & h_{L-1} \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}. \tag{3}$$

We note that the first term of the right hand side in (1) represents the desired signals, while the second and third terms capture the effect of both ICI and ISI, respectively. If $v \geq L$, $\mathbf{A}$ and $\mathbf{B}$ are zero matrices, and hence neither ICI nor ISI exist. However, if the CP length is insufficient, the residual ISI of length equal to $(L-v)$ induces ICI within the current symbol and ISI from the previous symbol and hence, the orthogonality between the different OFDM subcarriers will be lost. Since ISI and ICI may severely downgrade the OFDM performance, an equalization process is needed to counteract these interference effects when the CP is insufficient. Last but not least, $\mathbf{n}_t$ represents the receiver's zero-mean additive-white-Gaussian noise (AWGN) with $\mathbb{E}[\mathbf{n}_t\mathbf{n}_t^H] = \sigma_n^2\mathbf{I}_N$ and $\mathbf{n}_t \perp \mathbf{x}_t \perp \mathbf{x}_{t-1}$.

III. ANALYSIS AND PROBLEM FORMULATION

In this section, we present various techniques of reduced-complexity equalizer designs to mitigate the effects of ISI and ICI at the OFDM receiver. We begin by formulating optimization problems to be used in the design of the proposed reduced-complexity sparse linear MMSE and DFE equalizers. Further, we compare the complexity of our introduced approach with well-known conventional equalizer designs and show that our proposed equalizer can be designed with a lower complexity when compared to them. We finally introduce a method to verify the reliability of our proposed equalizers.

A. LINEAR EQUALIZER DESIGN

To combat the effects of ICI and ISI, the received signal is passed through an $N \times N$ linear MMSE equalizer matrix denoted by $\mathbf{E}_{LE}$. Thus, the compensated received signal can be written as follows

$$\begin{aligned}\hat{\mathbf{x}}_t^{LE} &= \mathbf{E}_{LE}\,\mathbf{y}_t^{LE},\\ &= \mathbf{E}_{LE}\underbrace{(\mathbf{H}-\mathbf{A})}_{\overset{def}{=}\mathbf{G}}\mathbf{x}_t + \mathbf{E}_{LE}\mathbf{B}\mathbf{x}_{t-1} + \mathbf{E}_{LE}\mathbf{n}_t. \end{aligned} \tag{4}$$

Defining the error vector as $\mathbf{v}_{LE} = \hat{\mathbf{x}}_t^{LE} - \mathbf{x}_t$, the mean square error (MSE) is computed by evaluating $\mathbb{E}\{\mathbf{v}_{LE}\mathbf{v}_{LE}^H\}$. By exploiting the linearity of the trace and the expectation operators, the MSE can be expressed as

$$\begin{aligned}\text{MSE}_{LE} &\overset{def}{=} \sigma_{e,LE}^2\\ &= \sigma_x^2\text{Tr}\Big(\mathbf{E}_{LE}\mathbf{G}\mathbf{G}^H\mathbf{E}_{LE}^H - \mathbf{G}^H\mathbf{E}_{LE}^H\\ &\quad - \mathbf{E}_{LE}\mathbf{G} + \mathbf{E}_{LE}\mathbf{B}\mathbf{B}^H\mathbf{E}_{LE}^H + \gamma^{-1}\mathbf{E}_{LE}\mathbf{E}_{LE}^H + \mathbf{I}_N\Big),\end{aligned} \tag{5}$$

where $\gamma \overset{def}{=} \frac{\sigma_x^2}{\sigma_n^2}$. Using the property $\text{Tr}(\mathbf{X}\mathbf{Y}) = \text{vec}(\mathbf{X}^H)^H\text{vec}(\mathbf{Y})$, we can further write (5) as follows

$$\begin{aligned}\sigma_{e,LE}^2 &= \sigma_x^2\text{Tr}\Big(\mathbf{E}_{LE}\underbrace{(\mathbf{G}\mathbf{G}^H + \mathbf{B}\mathbf{B}^H + \gamma^{-1}\mathbf{I}_N)}_{\overset{def}{=}\mathbf{R}_{LE}}\mathbf{E}_{LE}^H\\ &\quad - \mathbf{E}_{LE}\mathbf{G} - \mathbf{G}^H\mathbf{E}_{LE}^H + \mathbf{I}_N\Big),\\ &= \sigma_x^2\left(\mathbf{e}_{LE}^H\mathbb{R}_{LE}\mathbf{e}_{LE} - \mathbf{g}^H\mathbf{e}_{LE} - \mathbf{e}_{LE}^H\mathbf{g} + \mathbf{i}_{N^2}^H\mathbf{i}_{N^2}\right),\end{aligned} \tag{6}$$

where $\mathbf{g} = \text{vec}(\mathbf{G})$, $\mathbf{i} = \text{vec}(\mathbf{I}_N)$, $\mathbf{e}_{LE} = \text{vec}(\mathbf{E}_{LE}^H)$, $\mathbb{R}_{LE} = \mathbf{R}_{LE} \otimes \mathbf{I}_N$, and $\otimes$ denotes the Kroenecker product. Using the well know Cholesky's matrix factorization, we can set $\mathbb{R}_{LE} = \mathbb{L}\mathbb{L}_{LE}^H$, where $\mathbb{L}_{LE}$ is a lower triangular matrix. Hence,

$$\begin{aligned}\sigma_{e,LE}^2 &= \sigma_x^2\Big(\mathbf{e}_{LE}^H\mathbb{L}\mathbb{L}_{LE}^H\mathbf{e}_{LE} - \mathbf{g}^H\mathbb{L}_{LE}^{-H}\mathbb{L}_{LE}^H\mathbf{e}_{LE}\\ &\qquad - \mathbf{e}_{LE}^H\mathbb{L}_{LE}\mathbb{L}_{LE}^{-1}\mathbf{g} + \mathbf{i}_{N^2}^H\mathbf{i}_{N^2}\Big).\end{aligned} \tag{7}$$

By completing the square, we get

$$\sigma_{e,LE}^2 = \underbrace{\sigma_x^2\Big(N + \mathbf{g}^H\mathbb{L}_{LE}^{-H}\mathbb{L}_{LE}^{-1}\mathbf{g}\Big)}_{\overset{def}{=}\sigma_{e,LE,min}^2} + \underbrace{\sigma_x^2\Big(||\mathbb{L}_{LE}^H\mathbf{e}_{LE} - \mathbb{L}_{LE}^{-1}\mathbf{g}||_2^2\Big)}_{\overset{def}{=}\sigma_{e,LE,exc}^2}. \tag{8}$$

Here, $\sigma_{e,LE,exc}^2$ is the only term in (8) that depends on $\mathbf{e}_{LE}$ and thus can be used to compute the sparse equalizer coefficients. In particular, we use $\sigma_{e,LE,exc}^2$ to control the sparsity level of the designed equalizer. For example, if $\sigma_{e,LE,exc}^2 = 0$, the excess error (the second term in the right-hand side of (8)) will be zero and thus the equalizer is optimal yet not sparse (dense) and its design complexity is computationally demanding. On the other hand, allowing for some tolerable excess error will help reduce the implementation complexity at the cost of a performance loss. To achieve a desirable performance-complexity tradeoff, we formulate the following problem for the design of sparse linear equalizers

$$\hat{\mathbf{e}}_{LE} = \underset{\mathbf{e}_{LE}\in\mathbb{C}_{N^2\times 1}}{\text{argmin}}\ \|\mathbf{e}_{LE}\|_0 \quad \text{s.t. } \sigma_{e,LE,exc}^2 \leq \epsilon_{LE}, \tag{9}$$

where $\|\mathbf{e}_{LE}\|_0$ is the number of nonzero elements in its argument and ϵ_{LE} can be chosen as a function of the noise variance. To solve (9), we propose a general approach presented in the sequel to sparsely design the linear equalizer such that the performance loss does not exceed a pre-specified tolerable limit.

B. DECISION-FEEDBACK EQUALIZER

To better combat the ISI and ICI effects, a weighted sum of past decisions are fed back to help cancel out the interferences they cause in the present signaling interval. Fig. 1 depicts a

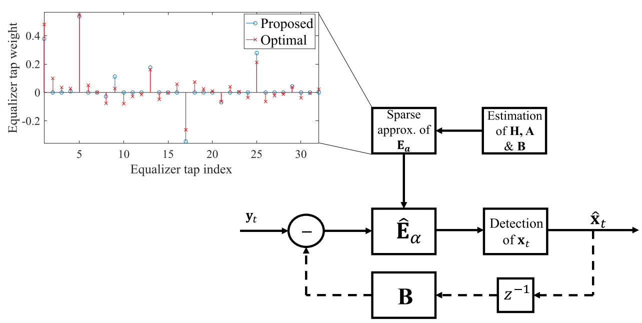

FIGURE 1. A schematic diagram illustrating our proposed system. The dashed line represents the feedback present in the case of using a DFE and absent in the case of using an LE approach.

block diagram of the proposed sparse DFE equalizer. In the DFE model, the received signal for one OFDM symbol at time t can be modeled as [8]

$$\hat{\mathbf{x}}_t^{DFE} = \mathbf{E}_{DFE}\left(\mathbf{y}_t^{DFE} - \mathbf{B}\mathbf{x}_{t-1}\right), = \mathbf{E}_{DFE}\mathbf{G}\mathbf{x}_t + \mathbf{E}_{DFE}\mathbf{n}_t. \tag{10}$$

Note that in the above equation, the error which may be incurred from the previous symbol is subtracted first from the current symbol before the equalization process. The error vector is defined as $\mathbf{v}_{DFE} = \hat{\mathbf{x}}_t^{DFE} - \mathbf{x}_t$, and hence the MSE is computed by applying $\mathbb{E}\{\mathbf{v}_{DFE}\mathbf{v}_{DFE}^H\}$. To avoid cumbersome notations, we drop the time index and further do simplify the MSE to get

$$\begin{aligned}\text{MSE}_{DFE} &\overset{def}{=} \sigma_{e,DFE}^2 \\ &= \mathbb{E}\Big[\text{Tr}\Big(\mathbf{E}_{DFE}\mathbf{G}\mathbf{x}\mathbf{x}^H\mathbf{G}^H\mathbf{E}_{DFE}{}^H \\ &\quad - \mathbf{E}_{DFE}\mathbf{G}\mathbf{x}\mathbf{x}^H + \mathbf{E}_{DFE}\mathbf{n}\mathbf{n}^H\mathbf{E}_{DFE}{}^H \\ &\quad - \mathbf{x}\mathbf{x}^H\mathbf{G}^H\mathbf{E}_{DFE}{}^H + \mathbf{x}\mathbf{x}^H\Big)\Big].\end{aligned} \tag{11}$$

Exploiting the linearity of the expectation operator, we have

$$\begin{aligned}\sigma_{e,DFE}^2 &= \sigma_x^2\text{Tr}\Big(\mathbf{E}_{DFE}\mathbf{G}\mathbf{G}^H\mathbf{E}_{DFE}{}^H - \mathbf{G}^H\mathbf{E}_{DFE}{}^H \\ &\quad - \mathbf{E}_{DFE}\mathbf{G} + \gamma^{-1}\mathbf{E}_{DFE}\mathbf{E}_{DFE}{}^H + \mathbf{I}_N\Big).\end{aligned} \tag{12}$$

Following the same steps for the case of the LE, and setting $\mathbf{R}_{DFE} \overset{def}{=} \mathbf{G}\mathbf{G}^H + \gamma^{-1}\mathbf{I}_N = \mathbf{L}_{DFE}\mathbf{L}_{DFE}^H$, the MSE for the case of the DFE can be written as

$$\begin{aligned}\sigma_{e,DFE}^2 &= \underbrace{\sigma_x^2\left(N + \mathbf{g}^H\mathbb{L}_{DFE}^{-H}\mathbb{L}_{DFE}^{-1}\mathbf{g}\right)}_{\overset{def}{=}\sigma_{e,DFE,min}^2} \\ &\quad + \underbrace{\sigma_x^2\left(||\mathbb{L}_{DFE}^H\mathbf{e}_{DFE} - \mathbb{L}_{DFE}^{-1}\mathbf{g}||_2^2\right)}_{\overset{def}{=}\sigma_{e,DFE,exc}^2}.\end{aligned} \tag{13}$$

where $\mathbb{R}_{DFE} = \mathbf{R}_{DFE} \otimes \mathbf{I}_N$ and $\mathbb{L}_{DFE} = \mathbf{L}_{DFE} \otimes \mathbf{I}_N$. $\sigma_{e,DFE,exc}^2$ is the only term in (13) that depends on $\mathbf{e}_{DFE}$ and thus can be used to compute the sparse equalizer coefficients. Specifically, we suggest to use $\sigma_{e,DFE,exc}^2$ to control how sparse is the designed DFE equalizer. Thus, for example, if $\sigma_{e,DFE,exc}^2 = 0$, the excess error will be zero and thus the equalizer is optimal. However, the DFE is not sparse and its design complexity is computationally expensive. On the other hand, when allowing for some negligible excess error, the implementation complexity can be reduced at the cost of a tolerable loss in the system performance. To handle such performance-complexity tradeoff, we formulate an optimization problem for the design of sparse DFE equalizers as follows

$$\hat{\mathbf{e}}_{DFE} = \underset{\mathbf{e}_{DFE}\in\mathbb{C}_{N^2\times 1}}{\text{argmin}} \ \|\mathbf{e}_{DFE}\|_0 \quad \text{s.t. } \sigma_{e,DFE,exc}^2 \le \epsilon_{DFE}, \tag{14}$$

where $\|\mathbf{e}_{DFE}\|_0$ is the number of nonzero elements in its argument and ϵ_{LE} can be chosen as a function of the noise variance. Next, a generalized approach to design a sparse DFE equalizer is presented.

C. PROPOSED DESIGN APPROACH

We now present our proposed sparse equalizer design approach for both LEs and DFEs. Our proposed design allows the flexibility of controlling the sparsity level of these equalizers, which can be controlled based on different criteria, namely the number of active entries of the equalizer and tolerable losses in the SINR or the data rate. The choice of the number of active entries in the equalizer matrix represents a performance-complexity tradeoff and further, under our formulation, the designer has a direct control on the desired number of the nonzero entries. The second choice is to design the equalizer such that the loss in the performance is upper bounded by a certain value. The bound on the performance loss can be on either the data rate or the SINR. Thus, we formulate the following optimization problem

$$\hat{\mathbf{e}}_\alpha = \underset{\mathbf{e}_\alpha\in\mathbb{C}_{N^2\times 1}}{\text{argmin}} \ \|\mathbf{e}_\alpha\|_0 \quad \text{s.t. } \sigma_{e,\alpha,exc}^2 \le \epsilon, \tag{15}$$

where $\alpha \in \{LE, DFE\}$ and ϵ is a design parameter used to control the sparsity level of the equalizer. Clearly, this problem is not convex, and a relaxation can be performed to make it convex by substituting the ℓ_0-norm in (15) with the ℓ_1-norm. Under this relaxation, the above problem can be solved using any convex optimization solvers. However, besides its complexity, the resulting solution does not exactly yield zero entries in $\hat{\mathbf{e}}_\alpha$. Another approach is to use some of the available greedy algorithms, as for example the orthogonal-matching pursuit (OMP) algorithm [21], which we denote by the function OMP(dictionary matrix, data vector, c), where c is a stopping criterion that can be set as the number of active entries in the equalizer matrix or a bound on the performance loss. This allows us to have a flexible stopping criterion depending on the constraints faced by the designer. Next, we define the decision-point average SINR for both linear and decision-feedback equalizers. The decision-point SINR ζ_α can be written as

$$\begin{aligned}\zeta_\alpha(\mathbf{e}_\alpha) &\overset{def}{=} \frac{\mathbb{E}[\text{Tr}(\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H])}{\mathbb{E}[\mathbf{v}_\alpha^H\mathbf{v}_\alpha]}, \\ &= \frac{\mathbb{E}[\text{Tr}(\mathbf{x}\mathbf{x}^H])\mathbb{E}[\text{Tr}(\mathbf{H}^H\mathbf{H})]}{\mathbb{E}[\mathbf{v}_\alpha^H\mathbf{v}_\alpha]}, \\ &= \frac{||\mathbf{h}||_2^2\sigma_x^2}{\sigma_{e,\alpha,min}^2 + \sigma_{e,\alpha exc}^2}, \\ &\overset{(a)}{\ge} \frac{||\mathbf{h}||_2^2\sigma_x^2}{\sigma_{e,\alpha,min}^2 + \epsilon_{\text{SINR},\alpha}}, \\ &= \frac{\zeta_\alpha^{\max}(\mathbf{e}_\alpha)}{1 + \frac{\epsilon_{\text{SINR},\alpha}}{\sigma_{e,\alpha,min}^2}},\end{aligned} \tag{16}$$

where $\epsilon_{\text{SINR},\alpha}$ is a design parameter, and $\sigma_{e,\alpha,exc}^2 \le \epsilon_{\text{SINR},\alpha}$, which is the reason behind the inequality in (a). In other words, $\epsilon_{\text{SINR},\alpha}$ is a performance penalty a designer is willing to afford for designing a reduced-complexity equalizer. The parameter $\epsilon_{\text{SINR},\alpha}$ will be used as an input to the OMP algorithm to determine the desired sparsity level of the equalizer.

Further, $\psi_{\mathrm{SINR},\alpha} \overset{def}{=} \frac{\zeta^{\max}_{max,\alpha}(\mathbf{e}_\alpha)}{\zeta_\alpha}$, where $\zeta_{max,\alpha}(\mathbf{e}_\alpha) \overset{def}{=} \frac{||\mathbf{h}||_2^2\sigma_x^2}{\sigma^2_{e,min,\alpha}}$. Then, the performance loss is quantified by $\psi_{\mathrm{SINR},\alpha}$ (in dB), i.e.,

$$\begin{aligned}\psi_{\mathrm{SINR},\alpha} &= 10\log_{10}\left(\frac{\zeta_\alpha^{\max}(\mathbf{e}_{opt})}{\zeta_\alpha(\mathbf{e}_\alpha)}\right),\\ &\leq 10\log_{10}\left(1+\frac{\epsilon_{\mathrm{SINR},\alpha}}{\sigma^2_{e,\alpha,min}}\right) \overset{def}{=} \psi^{\max}_{\mathrm{SINR},\alpha}.\end{aligned} \tag{17}$$

Then, we compute $\epsilon_{\mathrm{SINR},\alpha}$ based on an acceptable value for $\psi^{\max}_{\mathrm{SINR},\alpha}$ and compute the sparse solution $\mathbf{e}_\alpha$ accordingly from equation (15).

Now, we quantify the losses in the data rate resulting from the sparse approximation of the optimal equalizer solution. As previously discussed, the insertion of a CP results in a loss of the bandwidth efficiency. To overcome this problem, one can use a CP of insufficient length and rectify the effects of the ISI and ICI by employing an equalizer at the receiver side, which could be an LE or a DFE. Therefore, we next analyze the rate gains resulting from reducing the CP length after the sparse approximation of the optimal equalizer.

We define the average data rate of one OFDM symbol R_α, after equalization, as follows

$$\begin{aligned}R_\alpha &= \delta\mathbb{E}\left\{\log_2\left(1+\frac{\mathrm{Tr}(\mathbf{H}\mathbf{x}\mathbf{x}^H\mathbf{H}^H])}{\mathbf{v}_\alpha^H\mathbf{v}_\alpha}\right)\right\},\\ &\overset{(b)}{\leq} \delta\log_2(1+\zeta_\alpha),\\ &\overset{(c)}{\approx} \underbrace{\delta\log_2\left(\frac{||\mathbf{h}||_2^2\sigma_x^2}{\sigma^2_{e,\alpha,min}}\right)}_{R_{max,\alpha}} - \delta\log_2\left(1+\frac{\sigma^2_{e,\alpha,exc}}{\sigma^2_{e,\alpha,min}}\right),\end{aligned} \tag{18}$$

where the inequality in (b) follows from Jensen's inequality and the approximation in (c) is valid under the assumption that we operate in the high SNR regime, $\delta = \Delta f\frac{N}{N+v}$, where Δf denotes the subcarrier spacing. In (17), we have defined a design control parameter which is a function of $\psi_{\mathrm{SINR},\alpha}$ that controls the amount of SINR loss. We now define another design parameter denoted by $\epsilon_{\mathrm{Rate},\alpha}$. This control parameter is a function of the rate loss, i.e., $\psi_{\mathrm{Rate},\alpha}$. Next, after some straight-forward manipulations, $\epsilon_{\mathrm{Rate},\alpha}$ can be written as

$$\epsilon_{\mathrm{Rate},\alpha} \geq \left(2^{\frac{\psi_{\mathrm{Rate},\alpha}}{\delta}}-1\right)\sigma^2_{e,\alpha,min}, \tag{19}$$

where $\psi_{\mathrm{Rate},\alpha} \overset{def}{=} R_{max,\alpha} - R_\alpha$.

To this end, we have shown that the problem of designing the sparse LE and DFE's matrix entries can be cast into a sparse approximation of a vector by a fixed matrix. The general form of this problem is given by (15). In order to solve this problem, we can use any greedy algorithms to determine the locations and weights of the equalizer matrix entries. Here, for simplicity, we use the well-known OMP greedy algorithm [21] that estimates $\mathbf{e}_\alpha$ by iteratively selecting a set, e.g., S, of the dictionary matrix columns that are most correlated with the data vector and then solving a restricted least-squares problem using the selected columns. Then, after properly selecting the equalizer design constraint, we can realize $\mathbf{e}_\alpha$ by applying any one of the following functions

$$\hat{\mathbf{e}}_\alpha = \mathrm{OMP}\left(\mathbb{L}_\alpha^H, \mathbb{L}_\alpha^{-1}\mathbf{g}, \lceil p\times N^2\rceil\right), \tag{20}$$

$$\hat{\mathbf{e}}_\alpha = \mathrm{OMP}\left(\mathbb{L}_\alpha^H, \mathbb{L}_\alpha^{-1}\mathbf{g}, \epsilon_{\mathrm{SINR},\alpha}\right), \tag{21}$$

or

$$\hat{\mathbf{e}}_\alpha = \mathrm{OMP}\left(\mathbb{L}_\alpha^H, \mathbb{L}_\alpha^{-1}\mathbf{g}, \epsilon_{\mathrm{Rate},\alpha}\right), \tag{22}$$

where p is a parameter that defines the desired percentage of active entries in $\mathbf{e}_\alpha$. Recall that $\mathbb{L}_\alpha^H$ is the dictionary matrix, $\mathbb{L}_\alpha^{-1}\mathbf{g}$ is the data vector we use to estimate $\mathbf{e}_\alpha$, and the final argument c can be a condition on the number of active entries in the equalizer matrix ($\lceil p\times N^2\rceil$), an upper-bound of the SINR losses ($\epsilon_{\mathrm{SINR},\alpha}$), or a predefined limit on the data rate ($\epsilon_{\mathrm{Rate},\alpha}$). Algorithm 1 presents the steps of the OMP algorithm required to provide the sparse equalizer $\mathbf{e}_\alpha$ when the design is restricted by the number of active entries. This algorithm was described in details in [21]. In line number 3, Δ represents the disjunctive union symbol. Extending Algorithm 1 to encompass the performance loss restrictions presented above is straight-forward.

Algorithm 1 OMP Algorithm

Input: Matrix $\mathbb{L}_\alpha^H$, vector $\mathbb{L}_\alpha^{-1}\mathbf{g}$, scalar $\beta = \lceil p\times N^2\rceil$
Output: Vector $\mathbf{e}_\alpha$
1 **Initialize:** $S = [\,]$; $j = 1$; $I \in \{1,\ldots,N^2\}$;
$\hat{\mathbf{e}}_\alpha^H = \mathbf{0}_{N^2\times 1}$; $\mathbf{t} = [\,]$;
2 **while** $(j \leq \beta)$
3 $\bar{S} = S\Delta I$;
4 $k = \mathrm{argmax}(|\mathbb{L}_\alpha^H(:,\bar{S})\mathbb{L}_\alpha^{-1}\mathbf{g}|)$;
5 $S = S\cup k$;
6 $\mathbf{t} = \mathbb{L}_\alpha^{-H}(:,S)\mathbb{L}_\alpha^{-1}\mathbf{g}$;
7 $j = j+1$;
8 **end**
9 $\mathbf{e}_\alpha(S) = \mathbf{t}$;

We would finally like to note that the analysis of the MSE described above that will be used in the realization of our proposed sparse equalizer can be done in a different way. It has been shown in [8] that, while satisfying the orthogonality principle that results in the realization of the optimal equalizer, we can write

$$\mathbf{G} - \mathbf{R}_\alpha\mathbf{E}_\alpha^H = \mathbf{0}_{N\times N}, \tag{23}$$

or

$$\mathbf{g} - \mathbb{R}_\alpha \otimes \mathbf{e}_\alpha^H = \mathbf{0}_{N^2\times 1}. \tag{24}$$

Therefore, the sparsifying dictionary matrix is no more a decomposed lower-triangular matrix constructed by the Cholesky factorization technique. Furthermore, it has been shown in [18] that the Cholesky factorization written in e.g. (7) outperforms the approach in (24). Therefore, we focus our study on the Cholesky factorization based technique.

We conclude this section by noting that several equalization structures follow as special cases of our general design approach presented here including:

- The one-tap frequency-domain equalizer where a sufficient CP is considered. The equalizer is realized by taking the FFT of the time domain channel response while setting the sparsity level to zero, i.e., $p = 0$, then scaling each OFDM subcarrier by a single tap.
- For any ν, the optimal equalizer in the MMSE sense can be considered as a special case of our design by fully equalizing the effects of the ISI and ICI.
- In the DFE setup, the ZF equalizer can also be generated as a special case of our approach when the SNR tends to infinity.

D. COMPUTATIONAL COMPLEXITY

Here, we compare the orders of the complexities of some conventional equalizers versus our proposed equalizer design approach. For both cases of linear and decision-feedback equalizers, it has been shown in [8] that a ZF equalizer can be constructed by applying

$$\mathbf{E}_{ZF} = \mathbf{G}^{-1} = (\mathbf{H} - \mathbf{A})^{-1}. \tag{25}$$

To invert the matrix in (25), the computational complexity is of the order of $\mathcal{O}(N^3)$ [22]. The channel matrix $\mathbf{H}$ can be inverted by applying $\mathbf{H}^{-1} = \mathbf{F}^H\bar{\mathbf{H}}^{-1}\mathbf{F}$, where $\mathbf{F}$ is the unitary FFT matrix and $\bar{\mathbf{H}}$ is a diagonal matrix where its diagonal entries represent the frequency response of the wireless channel. However, $\mathbf{A}$ is singular and inverting it is not possible. Moreover, inverting $\mathbf{G}$ without inverting $\mathbf{A}$ is possible using a modified version of the Woodbury formula reported in [23], where it shows that it is possible to write

$$(\mathbf{H} - \mathbf{A})^{-1} = \mathbf{H}^{-1} + \underbrace{(\mathbf{I}_N - \mathbf{H}^{-1}\mathbf{A})^{-1}}_{\Lambda}\mathbf{H}^{-1}\mathbf{A}\mathbf{H}^{-1}, \tag{26}$$

and to compute Λ, we use Newman series approximation which states that [24]

$$\Lambda = \sum_{n=0}^{\infty}(\mathbf{H}^{-1}\mathbf{A})^n, \quad \text{iff } \rho(\mathbf{H}^{-1}\mathbf{A}) < 1, \tag{27}$$

where $\rho(.)$ denotes the spectral norm function, i.e. the maximum eigenvalue of the matrix $\mathbf{H}^{-1}\mathbf{A}$. Hence, we can now invert $\mathbf{G}$ while just having to invert $\mathbf{H}$, which involves the eigen-decomposition of $\mathbf{H}$ requiring the computation complexity of $\mathcal{O}(Nlog(N))$. However, we cannot guarantee the condition in (27) since we are dealing with random matrices that result in random values of the spectral norm. Further, even if the condition $\rho(\mathbf{H}^{-1}\mathbf{A}) < 1$ is satisfied, using Woodbury's formula will actually increase the complexity of realizing $\mathbf{E}_{ZF}$ since n in (27) might be large to give satisfactory results.

With respect to the optimal MMSE equalizer design, it requires the inversion of $\mathbf{R}_\alpha$, which also costs an order of $\mathcal{O}(N^3)$ operations. However, our proposed sparse equalizer can be realized through the OMP algorithm with a complexity of $\mathcal{O}\left(N^2W\right)$, where $1 \leq W \leq N$, depending on how sparse is our proposed equalizer. For the sake of comparison, we compare our proposed sparse equalizer with another sparse equalizer which we refer to as the "*significant entries*" equalizer. The significant entries equalizer is constructed by thresholding the optimal equalizer (in the MMSE sense) in terms of the maximum absolute values of the equalizer entries. The number of entries depends on how many of the active entries are needed (which determines the sparsity level of the equalizer accordingly). However, the optimal MMSE equalizer needs to be computed first to estimate the significant entries of the equalizer. Hence, to design an equalizer based on the significant entries approach, $\mathcal{O}(N^3)$ computations are needed. The complexity comparisons for our proposed method and some other approaches are summarized in Table I.

TABLE 1. Comparison between the order of the complexities of our proposed equalizer designs to some other selected equalizers.

Equalizer Type	Design Complexity
Conventional MMSE Equalizer	$\mathcal{O}\left(N^3\right)$
Significant-entries Equalizer	$\mathcal{O}\left(N^3\right)$
ZF Equalizer	$\mathcal{O}\left(N^3\right)$
Proposed Equalizer	$\mathcal{O}\left(N^2W\right)$

E. WORST-CASE COHERENCE

A useful metric to evaluate the performance of our proposed equalizer is the WCC metric. The WCC operator, which we denote by $\mu(.)$, can be written as [18], [19]

$$\mu(\mathbf{R}_\alpha) = \max_{i\neq j} \frac{|\langle \mathbf{R}_\alpha(i), \mathbf{R}_\alpha(j)\rangle|}{||\mathbf{R}_\alpha(i)||_2||\mathbf{R}_\alpha(j)||_2}, \tag{28}$$

where $\langle ., .\rangle$ denotes the inner product operator. We note that the normalization in (28) sets $\mu(\mathbf{R}_\alpha)$ to be in the range [0,1]. A lower $\mu(\mathbf{R}_\alpha)$ implies that the OMP algorithm will likely recover the equalizer entries. More discussion about the WCC metric can be found in e.g. [18], [19]. Further, it is worth noting that other metrics, which are similar to the WCC metric, have been studied in the literature as in e.g. [25]. The metric that has been studied there was the maximum expected coherence (MEC) metric since it is easier to analyze. However, the WCC presents the worst-case-scenario performance of the dictionary matrix's coherence, and thus was selected to be included in our simulation studies instead of the MEC. In the sequel, the results of the WCC simulations are discussed for the cases of LE and DFE equalizers, and will show that a Cholesky factorization approach to design the dictionary matrix will yield better results when compared with selecting $\mathbf{R}_\alpha$ as the dictionary matrix. Next, we will present the results of our numerical experiments to evaluate the performance of our proposed designs considering both LE and DFE equalizers.

IV. NUMERICAL RESULTS

In our simulations, we use the OFDM-based IEEE 802.16 Mobile WiMax standard described in [26] where the key IEEE 802.16 standard parameters are shown in Table 2.

TABLE 2. Values of the parameters used in our simulations.

Parameter	Value
Number of Subcarriers	128
Subcarrier Spacing	10.94 KHz
Cyclic Prefix Length	16
Subcarrier Modulation	16-QAM
OFDM Symbol Time	102.83 μs
OFDM Symbol Time (Useful)	91.41 μs

Furthermore, we use an equal tap channel (ETC) model [27] where the channel is composed of 4 taps with normalized delays of (0, 4, 8, 12) samples and an average gain of 0.25 for each tap, where each tap is generated as a complex independent Gaussian random variable. Further, we use coded OFDM using a convolutional encoder [133, 171] with a rate of 1/2 and a constraint length of 7. At the receiver, we use a Viterbi decoder with hard decision decoding. In our simulation settings, the vector $\mathbb{L}_{\alpha}^{-1}\mathbf{g}$ is segmented into N vectors, and then the OMP algorithm is applied in a parallel fashion. This was performed to further reduce the computational time since N in our simulations is set to 128, which will make the data vector's size to be 16384. This increase in dimension (from N to N^2) consumes larger memory and increases the simulation time. Thus, parallelization helps reduce the simulation time.

Throughout the simulations, our proposed OMP-based equalizer designs are compared with the significant-entries (Sig. entries) equalizer, Sufficient CP based design and ZF equalizer. We further compare our equalizer design approach with the optimal MMSE equalizer defined as $\mathbf{E}_{MMSE} = \mathbf{G}^H\mathbf{R}^{-H}$ [8]. Note that the optimal equalizer follows as a special case of our approach by setting the excess losses to zero, i.e., full equalization with 100% active taps. The Sufficient CP equalizer (when $L \leq \nu$) and the optimal MMSE equalizer can be considered as benchmarks for our comparative study. The OMP algorithm is applied to the MMSE-DFE case since, as it will be shown in the results, the MMSE-DFE equalizer performs the closest to the Sufficient CP based design, and our aim is to perform as close as possible to the Sufficient CP based design while reducing the complexity of realizing the equalizer.

First, we study the effect of the sparse equalizer designs on the performance where we plot the percentage of the active entries (i.e, entries with nonzero weights) of the total equalizer length N^2 versus the maximum SINR loss. In Fig. 2, we can see that our proposed approach requires far less active entries when compared with the Sig. entries based design for different received SNR values, and far less active entries are required when more performance loss is tolerated. As expected, the higher the received SNR, a higher number of active entries is needed in order to cancel the effects of the ISI and ICI.

In Fig. 3, we plot the coded BER for a varying received SNR and a CP of length 2, i.e., $\nu = 2$. We see that the best

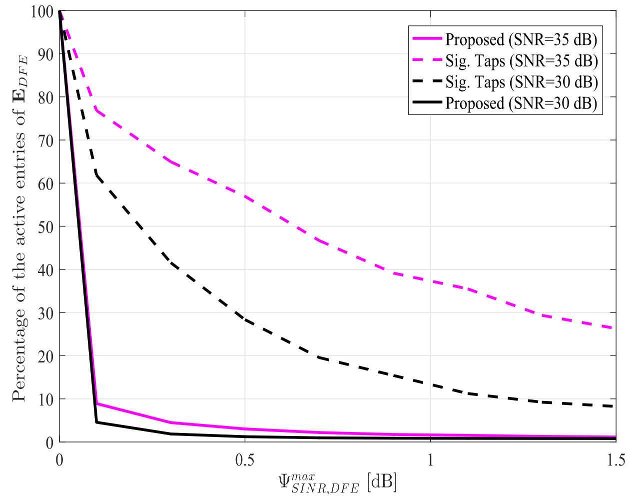

FIGURE 2. Percentage of the active entries of the equalizer matrix (p) vs. the loss in SNR in dB for different received SNR. In this case, $\nu = 8$.

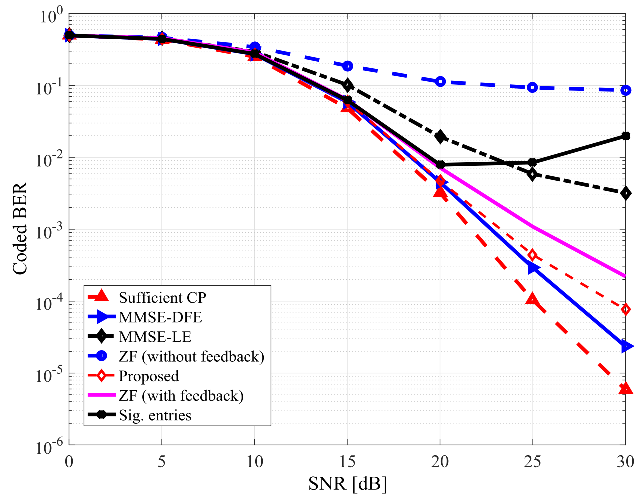

FIGURE 3. Coded BER vs. the received SNR for $\nu = 2$ when different types of equalizers are implemented.

performance of all tested methods in terms of coded BER is the OFDM system deploying sufficient CP, followed by the optimal MMSE equalizer (full equalization with 100%). Thresholding based techniques such as the Sig. entries approach does not perform well as it fails to choose the best entries that cancel-out the effects of ISI and ICI. This means that relying only on the largest magnitudes to select the equalizer entries and ignoring its phase components deteriorates the performance of the equalizer when the receiver AWGN is negligible. Also, both ZF and MMSE LEs perform much worse than the ZF and MMSE DFEs. In Fig. 4, the coded BER is plotted against the percentage of the active number of entries of the Sig. entries and our proposed equalizers. As it can be seen from the figure, our proposed equalizer converges fast to the optimal equalizer's BER as p increases, where it almost performs as well as the optimal equalizer with only 40% of its taps being active.

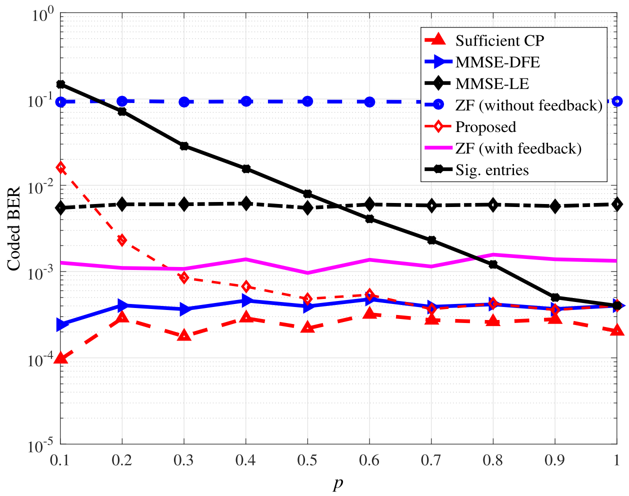

FIGURE 4. Coded BER vs. p with $v = 2$ and a received SNR of 25 dB for different types of equalizers.

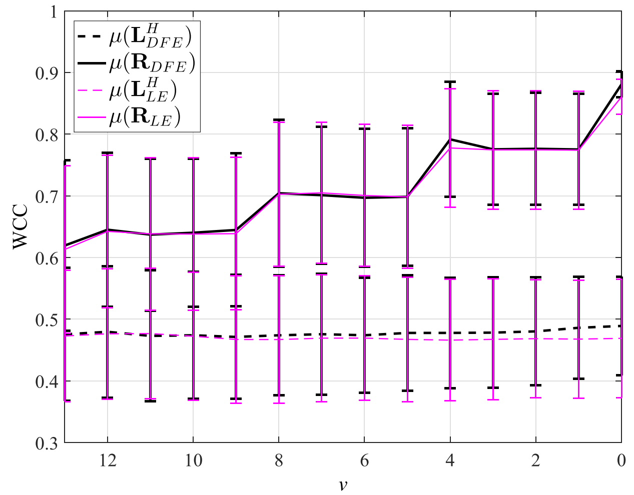

FIGURE 6. WCC vs. v wwhen the received SNR is set to 15 dB.

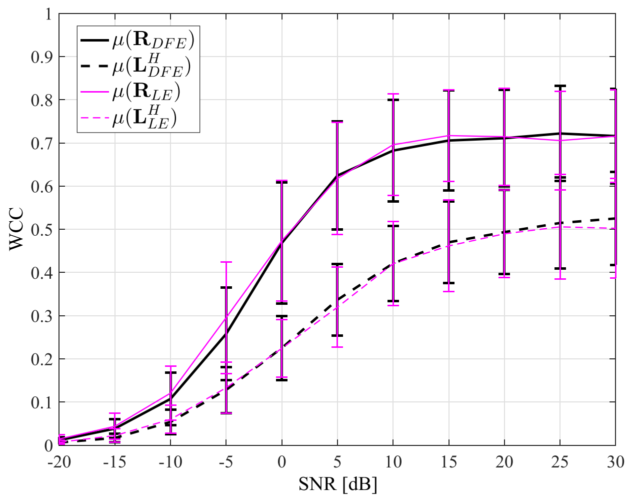

FIGURE 5. WCC vs. the received SNR with $v = 8$.

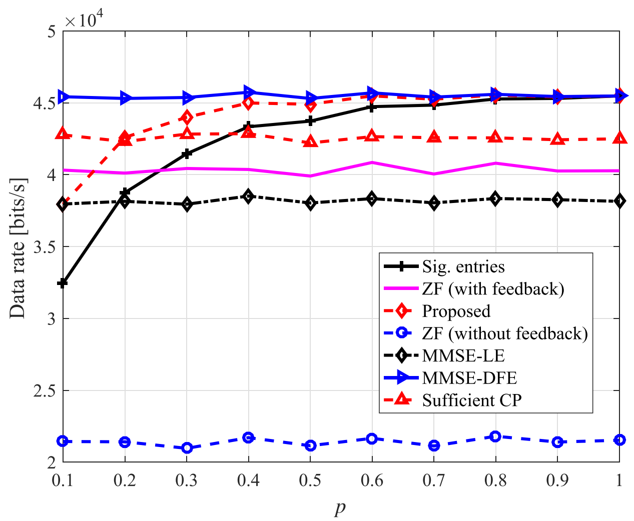

FIGURE 7. Data rate vs. p for different equalizer designs when the received SNR is equal to 20 dB and $v = 2$.

In Fig. 5, the average WCC is plotted for different values of the received SNR when $v = 8$. The WCC of $\mathbb{L}_{LE}$, $\mathbb{L}_{DFE}$, $\mathbb{R}_{LE}$ and $\mathbb{R}_{DFE}$ are investigated. We note that $\mu(\mathbf{L}_{DFE}) = \mu(\mathbf{I}_N \otimes \mathbf{L}_{DFE})$. Furthermore, we can see that $\mu(\mathbb{R}_\alpha)$ and $\mu(\mathbb{L}_\alpha)$ are below 1 which reflects the high likelihood of our approach to estimate the non-zero entries perfectly. We notice that the WCC of all selected dictionary matrices saturate at high SNR as the coherence becomes independent of the SNR. On the other hand, at low SNR, the noise dominates over the channel taps and therefore the dictionary matrices become close to identity matrices which, hence, have almost zero WCC. Furthermore, interestingly, the WCC for the LE and the DFE are almost identical.

Moreover, to shed some light on the behavior of the WCC when v is varied, Fig. 6 depicts the effect of decreasing v on $\mu(\mathbb{R}_{DFE})$, $\mu(\mathbb{L}_{DFE})$, $\mu(\mathbb{R}_{LE})$ and $\mu(\mathbb{L}_{LE})$. We notice that $\mu(\mathbb{R}_{DFE})$ significantly depends on v, although inserting a CP with length 1 is similar to inserting a CP with length 4, and inserting a CP with length 5 will have a similar effect as inserting a CP with length 8, etc. This is due to the structure of the sparse ETC channel model that we have used in our simulations. However, for the cases of $\mu(\mathbb{L}_{DFE})$ and $\mu(\mathbb{L}_{LE})$, we see that the WCC does not change with decreasing the CP length. That makes our design approach more attracting and adds a value for considering it in practical applications to improve the bandwidth efficiency. Since the coherence of $\mu(\mathbb{L}_{DFE})$ and $\mu(\mathbb{L}_{LE})$ is still low for even small values of v, we can still obtain a good performance with a high likelihood of well estimating the locations and weights of the non-zero entries. Due to this independence of v, our approach is considered as a robust method.

Figs. 7 and 8 depict the data rate of an OFDM symbol when the percentage of the active entries and the CP length in samples are varied, respectively. Clearly, reducing the length of the CP length will increase the data rate as more useful data

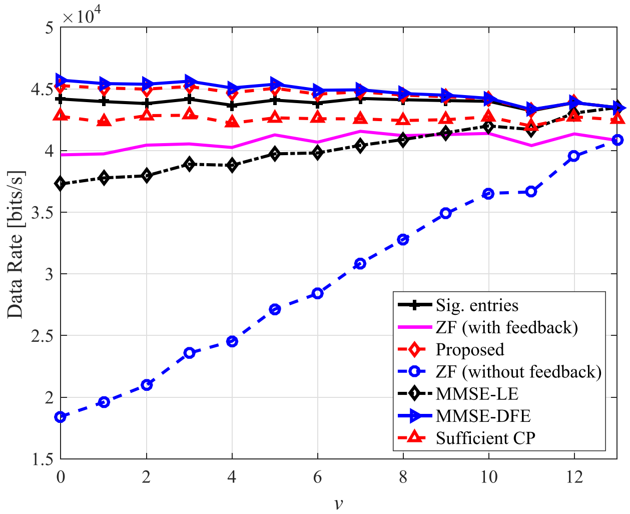

FIGURE 8. Data rate vs. *v* for different equalizer designs when the received SNR is equal to 20 dB and $p = 50\%$.

is transmitted. For both figures, we set the CP length for the Sufficient CP case to be equal to 16, totally eliminating the ISI effect. In Fig. 7, our proposed equalizer performs better than the significant entries equalizer for all p's and performs very close to the optimal equalizer when $p = 40\%$. Most importantly, our proposed equalizer results in a higher rate than the Sufficient CP case after using only 20% of the equalizer's entries, and the Sig. entries equalizer starts yielding better rate results after 40% of its entries are activated. Again, we emphasize on the point that our proposed equalizer not only outperforms the significant entries equalizer in terms of performance, but is less complex to design as the Sig. entries equalizer requires first to get the dense and complex optimal equalizer result before thresholding it. In Fig. 8, since the CP length for the Sufficient CP case is set to 16 samples, our proposed equalizer always results in a higher data rate even when only 50% of its entries are being active.

V. CONCLUSION

In this paper, we studied the equalization of OFDM signals suffering from ICI and ISI due to the insertion of an insufficient CP. We formulated the equalizer design problem as a convex optimization problem, and then, using the OMP algorithm, we controlled the level of sparsity of our equalizer by either a predefined number of the non-zero entries or by bounding the amount of losses that can be tolerated. Using only 40% of the equalizer entries, our proposed reduced-complexity sparse equalizer performs as good as that of the optimal MMSE equalizer (full dense equalization) in terms of coded BER and data rate. We have further tested the compatibility of our proposed equalizer through investigating the WCC metric and have shown that our dictionary matrices have low coherences which makes our approach a robust method.

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