

# An Experimental Study of Two-way Ranging Optimization in UWB-based Simultaneous Localization and Wall-Mapping Systems

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**Abstract**—In this paper, we propose a new ultra-wideband (UWB)-based simultaneous localization and wall-mapping (SLAM) system, which adopts two-way ranging optimization on UWB anchor and tag nodes to track the target’s real-time movement in an unknown area. The proposed UWB-based SLAM system captures time difference of arrival (TDoA) of the anchor nodes’ signals over a line-of-sight propagation path and reflected paths. The real-time location of the UWB tag is estimated according to the real-time TDoA measurements. To minimize the estimation error resulting from background noise in the two-way ranging, a Least Squares Method is implemented to minimize the estimation error for the localization of a static target, while Kalman Filter is applied for the localization of a mobile target. An experimental testbed is built based on off-the-shelf UWB hardware. Experiments validate that a reflector, e.g., a wall, and the UWB tag can be located according to the two-way ranging measurement. The localization accuracy of the proposed SLAM system is also evaluated, where the difference between the estimated location and the ground truth trajectory is less than 1 meter.

**Index Terms**—Ultra-wideband, Simultaneous localization and mapping, TDoA, Optimization, Experimental testbed

## I. INTRODUCTION

In recent years, precise indoor localization has been widely studied, which provides real-time tracking services in emergent scenarios, including military security, home surveillance, and medical supervision [1]. Thanks to small size, low cost, and high signal penetration, ultra-wideband (UWB) radio transceivers are used in wireless localization systems to track people or animal movements [2], [3]. In the UWB-based localization system, a UWB tag is attached to the tracking target, as shown in Fig. 1. A number of anchor nodes are deployed to cover an area of interest, such as walking paths in buildings or shops in shopping malls [4]. The anchor nodes are synchronized and continuously broadcast UWB signals. The UWB tag responds the captured UWB signal to the anchor node. Since the service area

layout has considerable effects on signal reflection patterns and received signal strength of the UWB radio, localization accuracy of the UWB-based localization system greatly varies, which results in a unreliable localization service. Moreover, a priori knowledge of the floor plan and the radio pattern in the service area is often unknown. Hence, improving the localization accuracy of the UWB-based localization system in an unknown service area is not trivial.

In this paper, we propose a new UWB-based simultaneous localization and wall-mapping (SLAM) system to track the target’s real-time movement in an unknown area. Specifically, two-way ranging is developed with the proposed UWB-based SLAM to capture the Time-Difference-of-Arrival (TDoA) of channel impulse response (CIR) signals [5], which are transmitted over a line-of-sight propagation path and reflected paths between the anchor nodes and the UWB tag. The real-time location of the UWB tag in the SLAM system is estimated according to the real-time TDoA measurements [6]. To minimize the estimation error resulting from background noise in the two-way ranging, a Least Squares Method [7] is implemented for the localization of a static target, while Kalman Filter [8] is applied for the localization of a mobile target.

Moreover, the proposed two-way ranging utilizes the TDoA of the UWB signal, as well as the localization of the UWB tag in the SLAM system, to determine boundaries of the service area, i.e., the location of walls in a room. Each anchor node is mapped to a virtual transmitter (VT) against the wall of the room. The distance between the VT and the UWB tag is regarded as the length of reflected path according to the wall. By calculating the TDoA of the VTs, the length of the wall can be estimated.

An experimental testbed is built based on off-the-shelf Decawave DWM1001 (a single UWB chip supported by the IEEE 802.15.4 standard) and a portable UWB kit, MDEK1001 [9]. Experiments are conducted in an anechoic

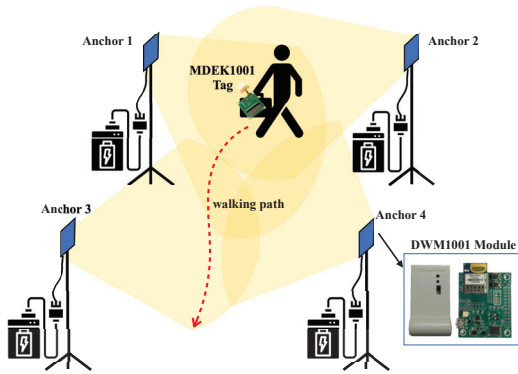


Fig. 1: In the UWB-based localization system, a UWB tag (e.g., MDEK1001) is attached to the tracking target, and a number of anchor nodes (e.g., DWM1001 made by Decawave) are deployed to cover an area of interest.

chamber at CSIRO, Australia, to measure the TDoA of the two-way ranging CIR values. The experiments validate that a signal reflector, e.g., the wall, can be detected according to the CIR measurement. In addition, the localization accuracy of the proposed SLAM system is evaluated, where the difference between the estimated location and the ground truth trajectory is less than 1 meter.

The rest of this paper is organized as follows. Section II presents the related work on wireless indoor localization. The system architecture and two-way ranging optimization in SLAM are illustrated in Section III. In Section IV, we implement the proposed SLAM system testbed with the two-way ranging optimization. Experiments are carried out to evaluate the localization performance. Finally, we conclude this work in Section V.

## II. RELATED WORK

In this section, we present the literature review on the existing indoor localization systems based on UWB, WiFi, camera, and motion sensors.

A UWB-based indoor localization system is developed in [10], where TDoA is applied with one way transmission ranging. Given unsynchronized anchors, the one-way transmission ranging is based on the timestamps collected at the anchors. The localization system implements a linear regression to predict next TDoA values of the packets transmitted by the tag. In [11], a leading-edge detection algorithm is studied for processing the UWB tag's requests in the localization system. A localization algorithm is developed to use two moving average windows to improve the accuracy detection of the UWB signals with low signal-to-noise ratios (SNRs) and non-line-of-sight (NLoS) conditions. A UWB-based indoor localization system is presented in [12], where the UWB tag can receive the responses from multiple anchors simultaneously. The tag calculates the TDoA value between the anchors and estimates the location of the tag itself, where the anchors are not required to be synchronized. Roetenberg *et al.* conduct

a data fusion on the UWB and inertial measurements, which can track the human motions [13].

Although WiFi may not always achieve a satisfactory indoor localization performance, the SLAM systems combining motion sensors and WiFi can considerably improve the localization accuracy [14], [15]. One advantage of the WiFi-based localization solutions is the ubiquity of the WiFi devices in indoor environments. A WiFi protocol designing for the indoor localization, i.e., IEEE 802.11mc, has been released aiming for high-performance localization [16]. In [17] and [18], a lightweight people localization system is demonstrated, where a number of anchors are deployed to sniff probe requests periodically polled by targets' smartphones for WiFi connection. The target's presence is detected when the probe request of the smartphone is received by the anchor the target passes by. Therefore, the target's trajectory can be tracked by the probe requests on a series of anchors according to time. An indoor tracking system based on the probe requests broadcast by smartphones is developed in [19]. A probe request interval-based data collection scheme is presented, where the anchor extracts the source MAC address and timestamp from the probe request, and uploads the extracted data according to the probe request interval of the smartphone.

Camera- and laser-based methods [20], [21] can be used for the indoor localization thanks to the high-resolution measurements and high sampling rate. Due to the high energy consumption, powerful computing devices are usually required to process the image or video data for a high-quality localization. In [22], [23], radio signals are utilized for the device-free localization with the deployment of multiple access points. Moreover, the Localization systems based on magnetic induction devices along with motion sensors are tested to accelerate the system deployment process [24], [25]. The lighting change in the indoor environment or Bluetooth connections can also be explored for the SLAM system to improve the localization performance [26], [27].

## III. THE SLAM LOCALIZATION SYSTEM WITH TWO-WAY RANGING

### A. System architecture

Fig. 1 illustrates the proposed UWB-based SLAM system, where 4 Decawave DWM1001 anchor nodes are deployed and battery-powered MDEK1001 UWB tag is attached to the target. The anchors mounting at the same height communicate with the UWB tag in the same channel. Each anchor is connected to and powered by a laptop, where the two-way ranging optimization is conducted to compute the position of the tag. The anchor nodes periodically broadcast beacon messages for access request and information polling. The four anchor nodes initiate the two-way ranging with the UWB tag in time-division multiple access. The two-way ranging beacons contain the source and destination address, and a 16-bit checksum.

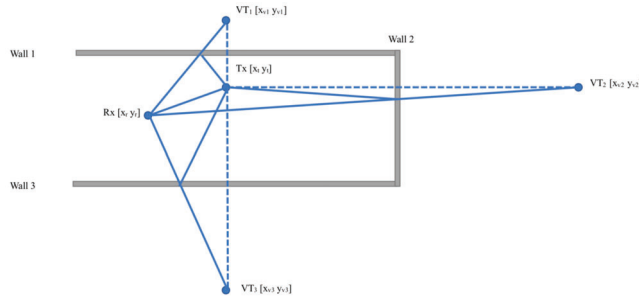


Fig. 2: Multipath reflection of the anchor node (the virtual transmitter) and the UWB tag.

To determine the boundary of the monitoring area, i.e., the walls, the proposed SLAM system investigates virtual transmitters (VT) that are geometrically symmetric to the tracked UWB tag corresponding to the wall. In particular, the distance between the UWB tag and a VT indicates the length of the reflected path, since the reflected path of the anchor node's signal can be regarded as a direct path between the VT and the tag. Fig. 2 studies an example for multipath reflection of the anchor node (the VT) and the UWB tag. Tx is the location of the anchor node and Rx is the location of the UWB tag. The VT's locations according to the three walls are denoted as  $VT_1$ ,  $VT_2$  and  $VT_3$ , respectively.

The coordinates of  $VT_1$ ,  $VT_2$  and  $VT_3$  can be determined by

$$[x_v \ y_v] = [x_t \ y_t] - \frac{2 \left( [a \ b] \times \begin{bmatrix} x_t \\ y_t \end{bmatrix} + c \right)}{(a^2 + b^2)} \times [a \ b] \quad (1)$$

where  $[x_v \ y_v]$  is the location of the VT, and  $[x_t \ y_t]$  is the location of the actual anchor node.  $a$ ,  $b$  and  $c$  are the parameters of the straight-line equation of the wall, i.e.,  $ax + by + c = 0$ .

The signal reflection is regarded as a line-of-sight (LoS) path and a reflected path. The LoS distance between the UWB tag and the anchor node can be obtained by

$$d_{TxRx} = \sqrt{(x_t - x_r)^2 + (y_t - y_r)^2} \quad (2)$$

The length of the reflected path is the distance between the locations of the actual anchor node and the VT. Given  $VT_1$ ,  $VT_2$ , and  $VT_3$ , the length of the three reflected paths is

$$d_{RxVT_m} = \sqrt{(x_{vm} - x_r)^2 + (y_{vm} - y_r)^2}, \quad (3)$$

where  $m = 1, 2$ , or  $3$ . Furthermore, the difference of the LoS and the reflected paths is

$$dT_m = \sqrt{(x_t - x_r)^2 + (y_t - y_r)^2} - \sqrt{(x_{vm} - x_r)^2 + (y_{vm} - y_r)^2} \quad (4)$$

where  $m = 1, 2$ , or  $3$ . Based on (4), the TDoA can be obtained by  $dT_m/c_l$ , where  $c_l$  is the speed of the light.

## B. The SLAM system optimization

To minimize the estimation error of the walls' locations, we optimize the VT's position based on the Least Squares Method. With TDoA of the three reflection paths, we have

$$\min \sum_{i \in N, j \in M} \left( \left\| \|VT_i - Rx_j\| - \|Tx_v - Rx_j\| \right\| - dT_m \right)^2 \quad (5)$$

where  $N$  and  $M$  are the numbers of anchor nodes and UWB tags, respectively. Based on the optimization in (5), the estimated distance between the anchor node and the wall is closest to the actual distance, which provides an accurate wall's location.

To track the UWB tag, the anchor node sends beacons to the tag every 0.3s, in other words, the location is sampled every 0.3s. The samples are taken as the input to a Kalman Filter. In particular, a location sample indicates the previous system state in the filter, which can be used to predict the current state. The following samples can be seen as observation values that correct the predicted current system state. The state of the UWB tag consists of its location and velocity, which can be represented as a matrix with two elements,  $x_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix}$ , where  $p_t$  is the position of the tag and  $v_t$  is the velocity [28]. Hence, the previous state of  $x_t$  is given by

$$p_t = p_{t-1} + v_{t-1} \times \Delta t + u_t \times \frac{\Delta t^2}{2}, \quad (6)$$

$$v_t = v_{t-1} + u_t \times \Delta t, \quad (7)$$

where  $u_t$  is the velocity of the tag, and  $\Delta t$  is the time when the states are sampled. Furthermore, the system state can be given as a matrix:

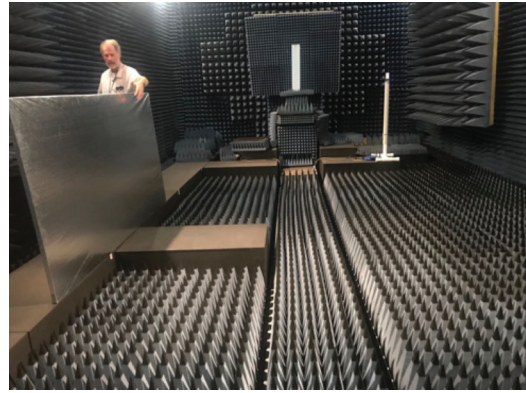
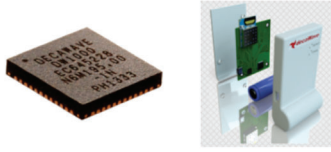
$$\begin{bmatrix} p_t \\ v_t \end{bmatrix} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ v_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix} u_t$$

We can observe the transformation matrix of Kalman Filter, which gives  $F_t = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$ . The control matrix that illustrates how the controlled variable  $u_t$  acts on the current state is  $B_t = \begin{bmatrix} \frac{\Delta t^2}{2} \\ \Delta t \end{bmatrix}$ .

Let “ $\hat{\cdot}$ ” denote the estimated value, and “ $-$ ” denote the value that is predicted optimally in accordance with the previous state. Predicting the current state in Kalman Filter can be formulated as

$$\hat{x}_t^- = F_t \hat{x}_{t-1} + B_t u_t. \quad (8)$$

Moreover, sampling noise results in uncertainties on the state's prediction. A covariance matrix that calculates the correlation between the states can quantify the uncertainty of the prediction. The covariance matrix comes from the process noise and propagation of the uncertain predicted state. Based on the property of the covariance matrix, i.e.,



(a) Hardware of the off-the-shelf Decawave DW1000 chip. (b) CIR measurements in the indoor anechoic chamber.

Fig. 3: Experiments based on our SLAM testbed are conducted to characterize the CIR measurements in an anechoic chamber at CSIRO, Australia.

$cov(Ax, By) = Acov(x, x)B^T$ , the covariance matrix prediction is formulated as

$$P_t^- = FP_{t-1}F^T + Q, \quad (9)$$

where  $Q$  is the covariance matrix of the sampling noise.

The signal measurements are the observations  $z_t$ , which yields

$$z_t = Hx_t + v, \quad (10)$$

where  $H$  is the observation model matrix, and  $v$  is the measurement noise of the observation. Let  $K_t$  denote the Kalman filter, which is

$$K_t = P_t^- H^T (HP_t^- H^T + R)^{-1} \quad (11)$$

where  $R$  is the noise.

We define a measurement residual as the difference of the observation and the estimated value, namely,  $z_t - H\hat{x}_t^-$ . With the obtained prediction state  $\hat{x}_t^-$ , the measurement residual and Kalman factor can be used to correct the prediction state, which gives

$$\hat{x}_t = \hat{x}_t^- + K_t(z_t - H\hat{x}_t^-). \quad (12)$$

The noise distribution of the optimal estimation is updated for next iteration by using

$$P_t = (I - K_t H) P_t^-. \quad (13)$$

#### IV. THE SLAM TESTBED AND EXPERIMENTS

In this section, we first demonstrate the experimental testbed which is built based on the off-the-shelf Decawave DWM1001 UWB kit. Next, experiments are carried out to measure CIR values in the indoor anechoic chamber, and track the UWB tag's movement.

##### A. UWB measurements in the indoor anechoic chamber

In our SLAM testbed, a number of anchor nodes are deployed to cover an indoor area. The anchor nodes are synchronized and continuously broadcast UWB signals. The UWB tag that is attached to a person responds the anchor nodes. The anchor nodes and the tag are developed with the off-the-shelf Decawave DW1000 chip and MDEK1001 kit, as shown in Fig. 3(a).

Based on the SLAM testbed, we carry out experiments to characterize the CIR measurement in an anechoic chamber at CSIRO, Australia, which is shown in Fig. 3(b). The experiments aim to capture features of the CIR strength variation according to the location of the obstacle. The localization with the Least Squares Method and Kalman Filter is implemented in Python, which carries out the two-way ranging to determine the UWB tag's location. A metal panel is placed as the signal reflector in the field, which leads to a multi-path signal reflection on the transmitted UWB signal of the anchor node. Moreover, Fig. 4 demonstrates the setup of the anchor node, UWB tag and the reflector in the chamber. Specifically, the distance between the anchor node and the tag is about 7.010 meters. The distances from the metal panel to the anchor and the tag are about 4.699 and 4.547 meters, respectively. Thanks to the signal absorption, the CIR in the chamber can experience two paths, one is the LoS link between the anchor and the tag, and the other is the reflected signal by the panel.

Fig. 5 presents the collected data samples regarding the CIR strength value in the chamber. Due to the LoS signal and the reflected one from the metal panel, we can observe the two highest peaks of the CIR measurement. The Time-of-Arrival (ToA) of the LoS signal is 747ns and that of the reflected one is 754ns. Hence, the experimental TDoA between the LoS signal and the reflected one is about 7ns. Given the speed of signal transmission about  $2.99 \times 10^8$  m/s, the distance difference between the anchor and the tag, and the anchor and the metal panel can be estimated

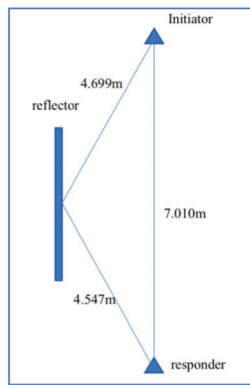


Fig. 4: The setup of the anchor node, UWB tag and the reflector.

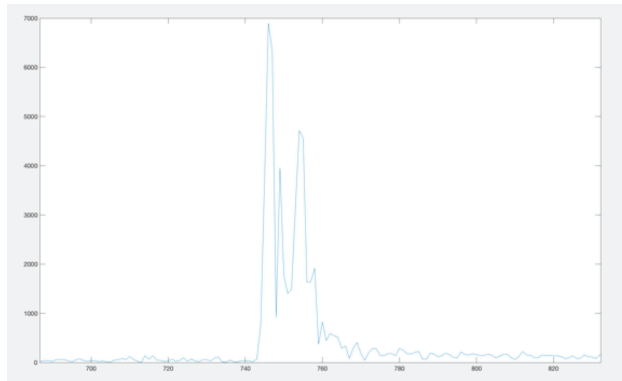


Fig. 5: CIR measurements based on the SLAM testbed.

by  $2.99 \times 10^8 \text{m/s} \times 7 \text{ns} = 2.09 \text{m}$ . Given the measured distances 4.699m and 4.547m, the distance between the anchor node and the tag can be estimated as  $(4.699 \text{m} + 4.547 \text{m}) - 2.09 \text{m} = 7.156 \text{m}$ . Compared to the measured distance 7.010m, the estimation error is only 14.6cm, which validates the reliability of the proposed localization optimization.

### B. Localization performance

Fig. 6 compares the localization performance of the proposed optimization with the Least Squares Method and Kalman Filter and the one without the Kalman Filter. In particular, Fig. 6(a) shows that the localization without the Kalman Filter is at most 50cm away from the ground truth which is marked red. Given a total of 19 waypoints, only 4 of them are tracked. Fig. 6(b) shows that Kalman Filter highly reduces the localization error below 15cm, while 14 of the 19 waypoints are successfully tracked.

As shown in Fig. 7, the proposed SLAM system estimates the location of the three walls based on the two-way ranging CIR measurement. The three walls are in a rectangle shape, where 6 anchor nodes are placed in the field. Thanks to the Least Squares Method, our SLAM system achieves the localization error below 2 meters on the wall detection.

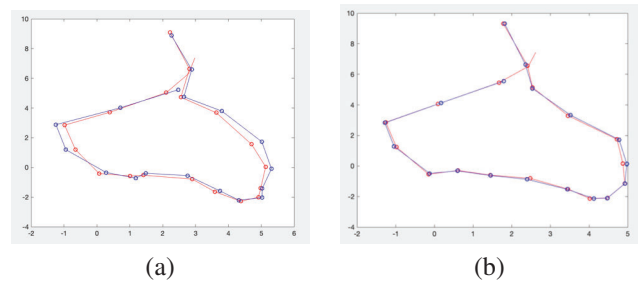


Fig. 6: Performance comparison, where the ground truth trajectory is marked red line. The blue line in (a) stands for the localization based on the CIR measurement without the Least Squares Method and Kalman Filter, and the one in (b) is the proposed localization optimization.

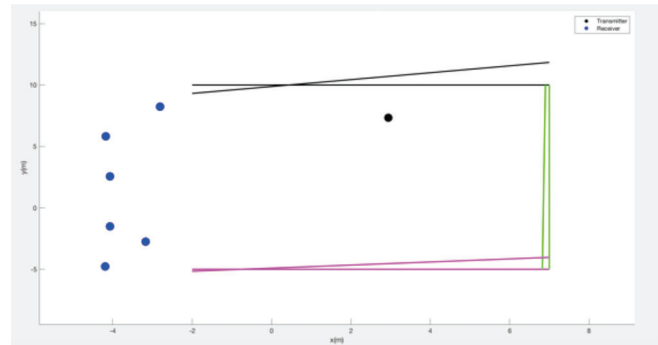


Fig. 7: The two-way ranging CIR measurement for the walls' localization, where 6 anchor nodes are placed in the field.

In Fig. 8, the proposed SLAM system adapts the Least Squares Method and Kalman Filter to estimate the location of the three walls (marked by a red dash line) while tracking the trajectory of the UWB tag (which is the green line). Since the location is sampled every 0.3s, we also plot several sample values which are the blue dots in the figure. Based on the samples, the proposed localization estimates the trajectory of the UWB tag. As observed, the trajectory is effectively tracked along the ground truth (i.e., the pink line).

## V. CONCLUSION

In this paper, we demonstrated a new UWB-based SLAM system to track the target's real-time movement in an unknown area. The proposed UWB-based SLAM system captures the TDoA of the anchor nodes' signals over an LoS propagation path and reflected paths. The real-time location of the UWB tag is estimated according to the real-time TDoA measurements. The Least Squares Method and Kalman Filter are developed in the proposed SLAM system to minimize the estimation error resulting from the background noise in the two-way ranging. An experimental testbed is built based on off-the-shelf UWB hardware. Experiments validate that the wall and the UWB tag can

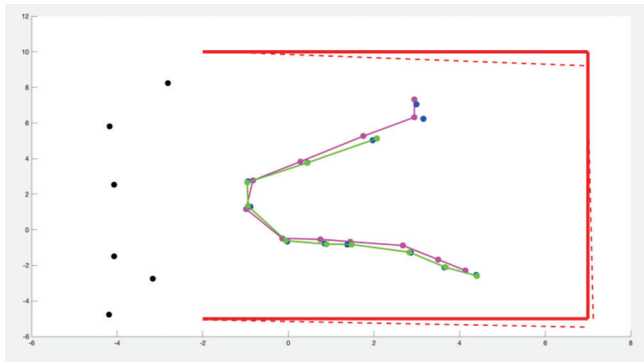


Fig. 8: The proposed SLAM system achieve the localization of the walls and the UWB tag.

be located according to the two-way ranging CIR measurement. The localization accuracy of the SLAM system is evaluated, where the difference between the estimated location and the ground truth trajectory is less than 15cm.

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